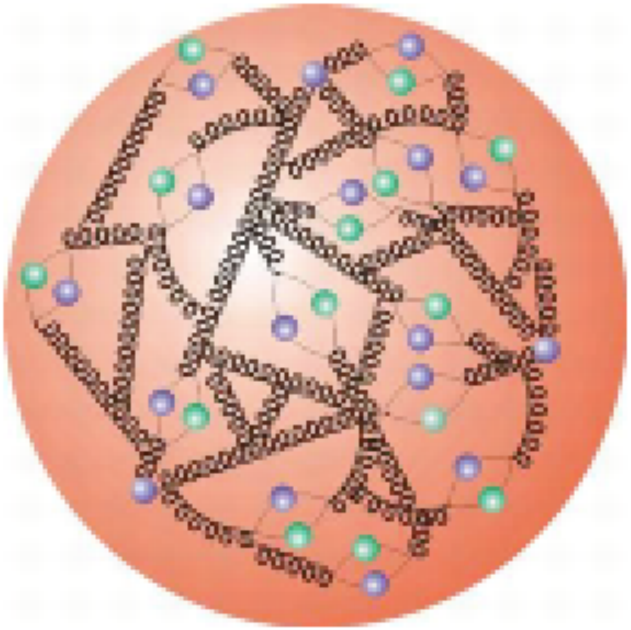
One-Loop Hybrid Renormalization Matching Kernels for Quasi-Parton Distributions

Jiunn-Wei Chen

National Taiwan U.

w/ Chien-Yu Chou 2204.08343

The complicated world inside a proton



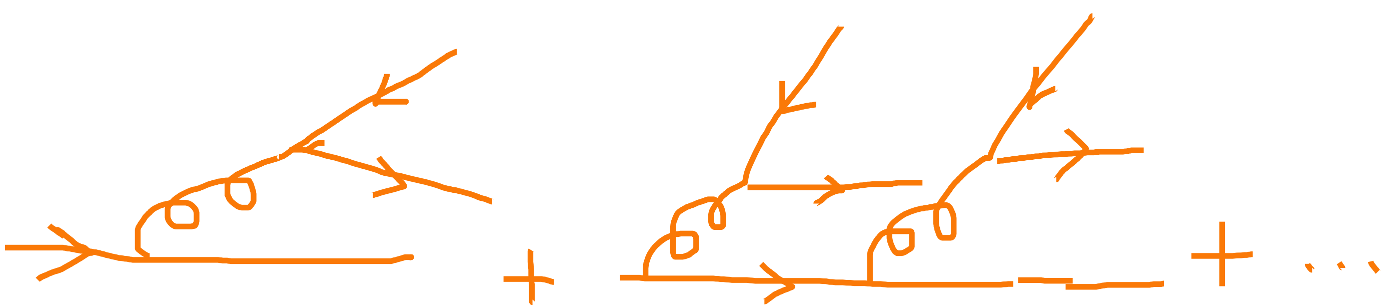
Parton structures: 1d mom+spin PDF to 3d GPD & TMD to Wigner (and beyond?) [BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …] to applications (Higgs, new physics…)

Can we determine these distributions theoretically?

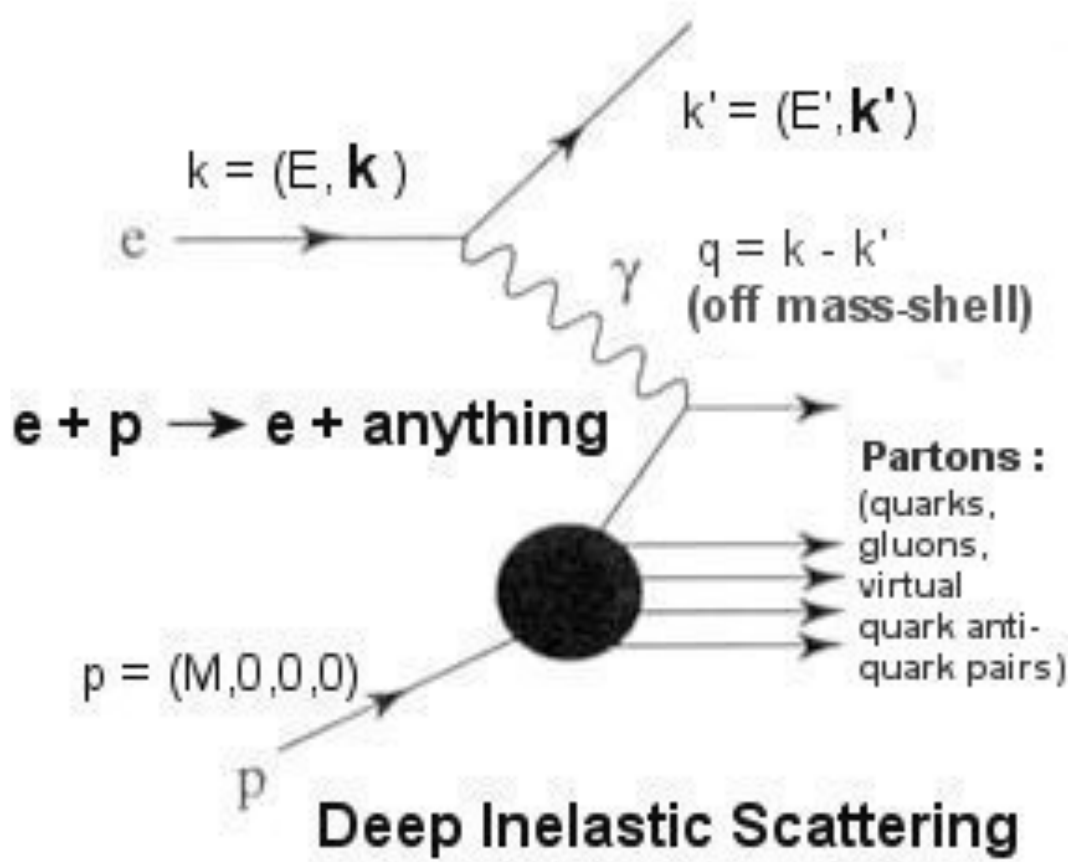
PDFs from QCD---a light cone problem!

- The number of quark anti-quark pairs diverges (manifestation of non-perturbative nature of the problem): an infinite body problem!
- Lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators

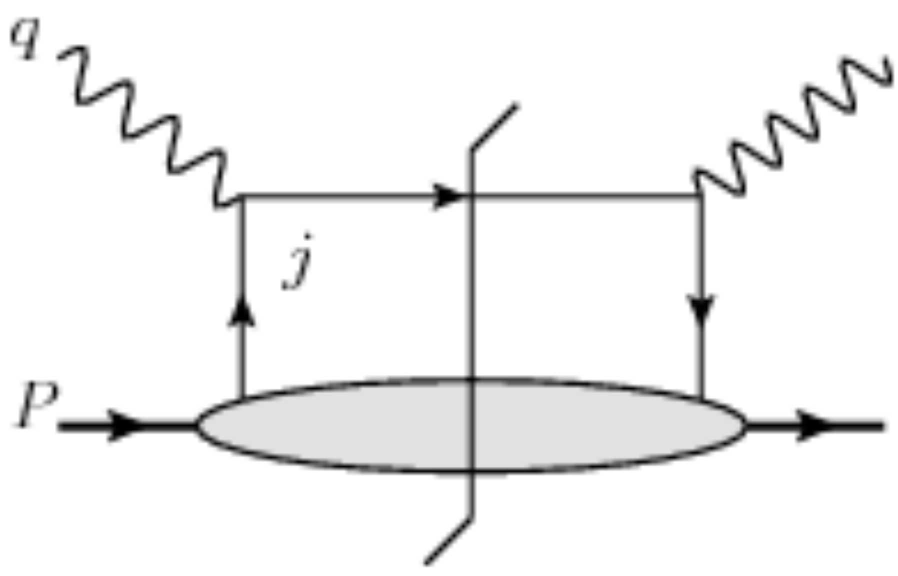
$$t^2 - \mathbf{r}^2 = 0$$
$$-t_E^2 - \mathbf{r}^2 = 0$$



Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \right\rangle$$

PDFs from QCD---a light cone problem!

- Euclidean lattice: light cone operators cannot be distinguished from local operators
- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$\langle x^n \rangle$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF; Qiu & Ma
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x -dependence directly. (variation: pseudo-PDF, Radyushkin; w/ Karpie, Orginos, Zafeiropoulos)

Ji's idea

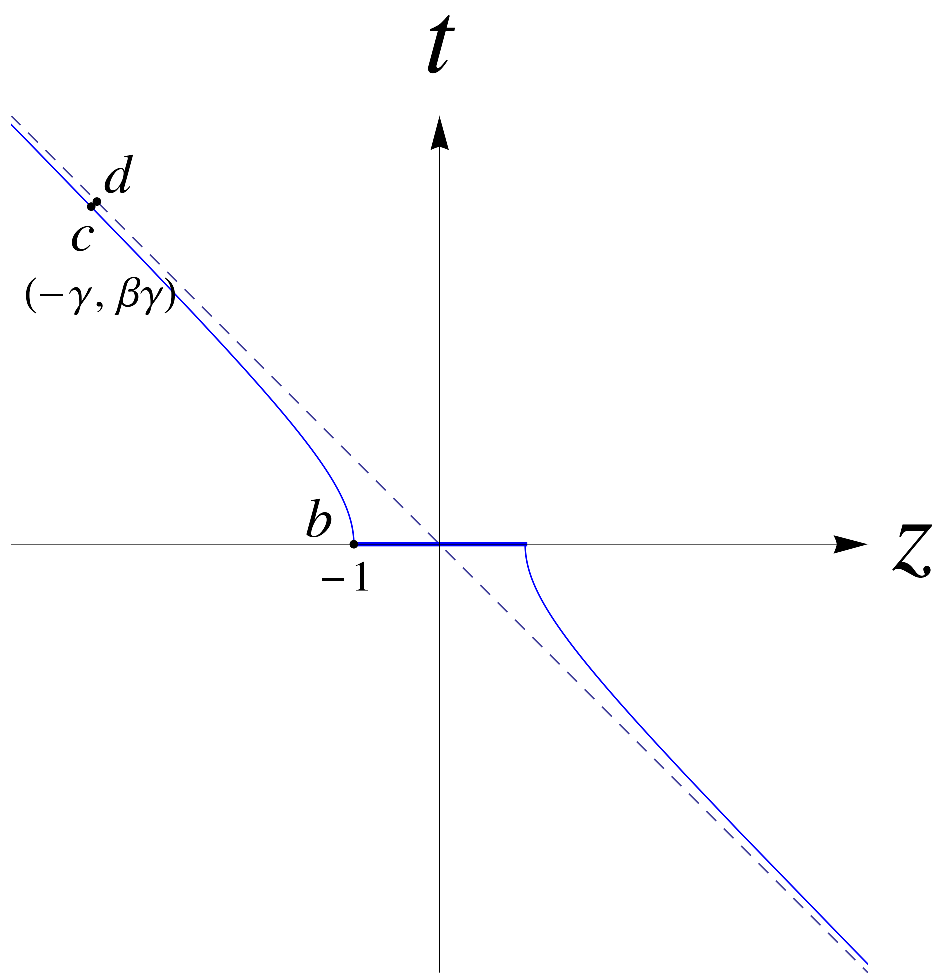
- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.



- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

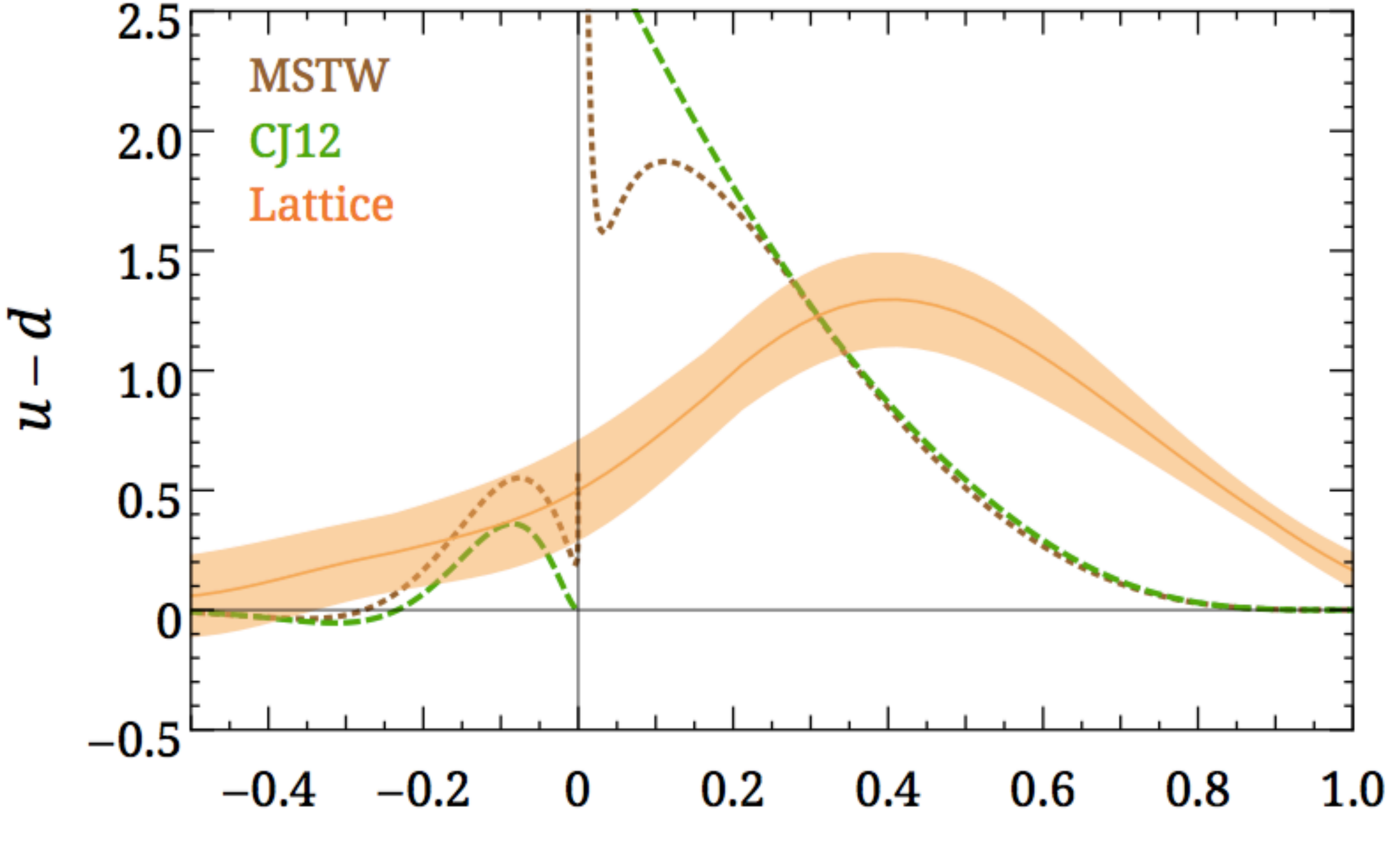
$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dz}{4\pi} e^{-izk} \times \left\langle \vec{P} \right| \bar{\psi}(z) \gamma_z e^{ig \int_0^z A_z(z') dz'} \psi(0) \left| \vec{P} \right\rangle$$

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

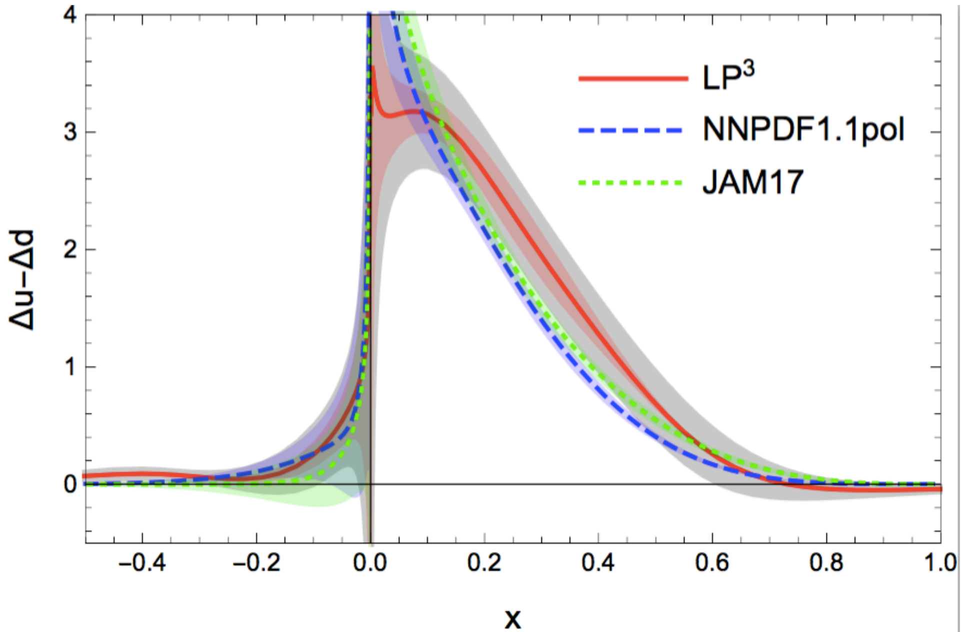
$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)q(y,\mu)+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2},\frac{M^2}{P_z^2}\right)+\ldots.$$

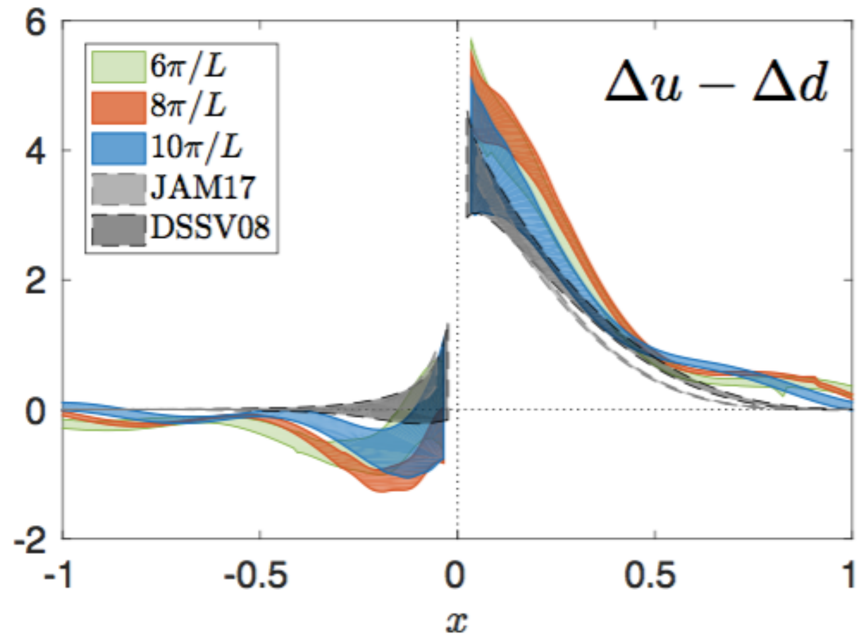
LP3 (1402.1462)



$q(-x) = -\bar{q}(x)$

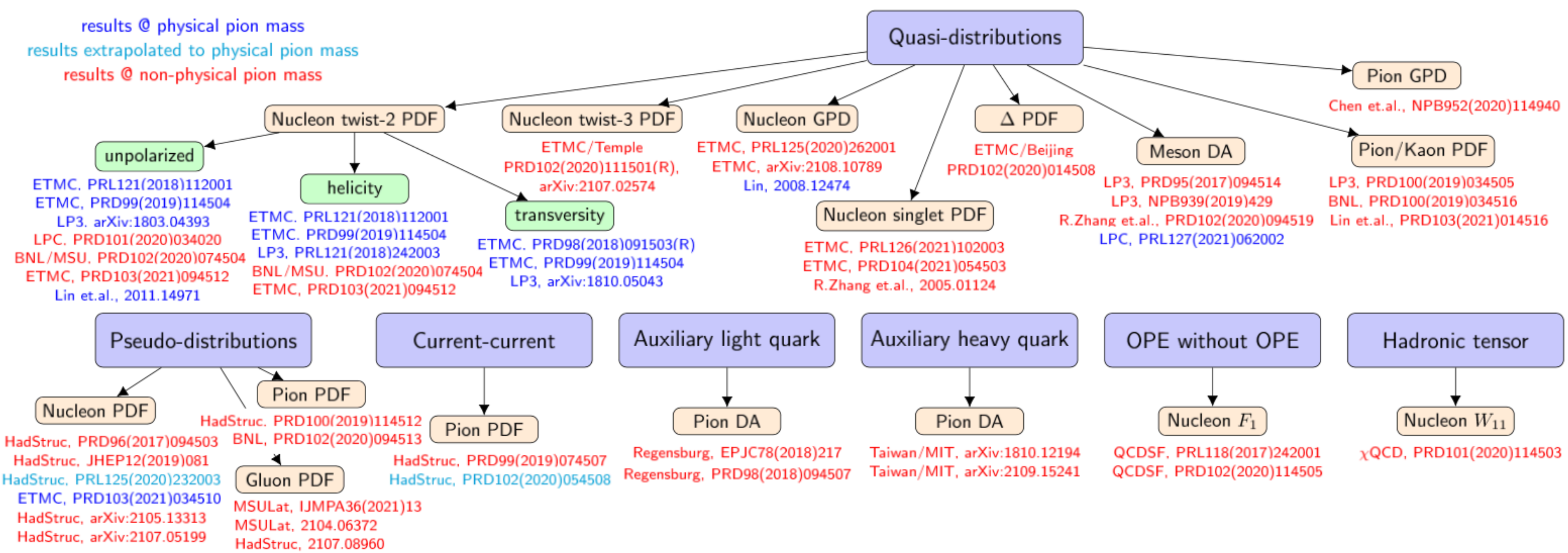
Helicity





LP3(1807.07431,PRL) ETMC(1803.02685,PRL)

A lot more recent lattice computations. See Krzysztof Cichy's review on Lattice2021 (2110.07440).



Back to Factorization

$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)q(y,\mu)+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2},\frac{M^2}{P_z^2}\right)+\ldots.$$

$$q(x,\mu^2)=\int\frac{d\xi^-}{4\pi}e^{ix\xi^-P^+}\left\langle P\left|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda)\right|P\right\rangle$$

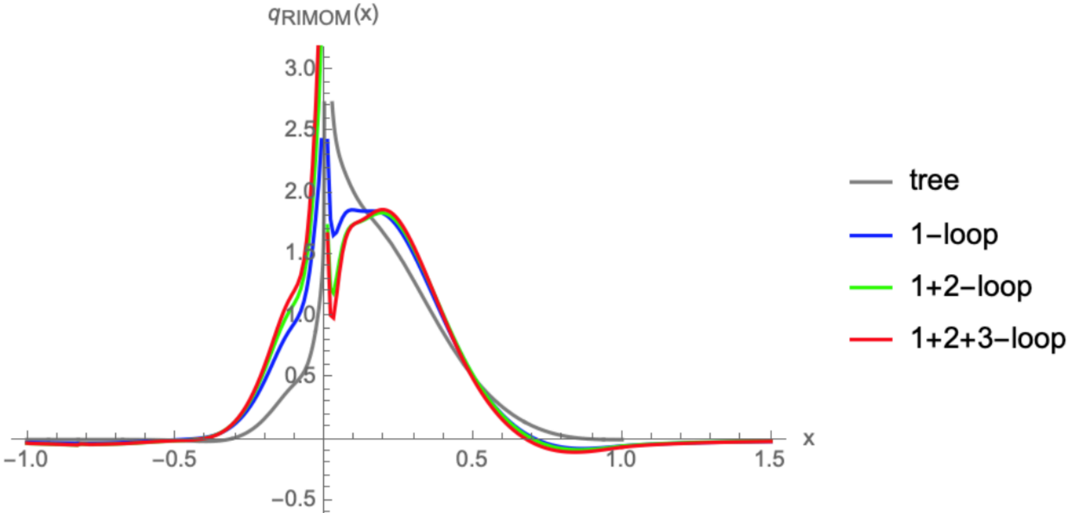
$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dz}{4\pi}e^{-izk}\times$$

$$\left\langle\vec{P}\right|\bar{\psi}(z)\gamma_z e^{ig\int_0^z A_z(z')dz'}\psi(0)\left|\vec{P}\right\rangle$$

Power Corrections

- $\mathcal{O}(M^2/(P^z)^2)$ corrections computed to all orders (JWC et al. 1603.06664)
- Renormalon effect: Braun, Vladimirov, Zhang (1810.00048) $\mathcal{O}(\Lambda^2_{\rm QCD}/x^2 P_z^2)$; But the slow convergence is not seen in bubble diagrams at 3-loops

(w/ Wei-Yang Liu 2010.06623)



Matching Kernel

$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)q(y,\mu)+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right)+\ldots.$$

- compensating the UV difference of quasi-PDF and PDF; renormalization scheme and scale dependent.
- PDF in MS-bar, quasi-PDF in lattice spacing---lattice action dependent, slow convergence (linear divergence, Wilson line mass subtraction scheme, Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang;)

NPR

- Non-perturbative renormalization (NPR): quark bilinear operators multiplicatively renormalized, ratio scheme (same operator, different states, Radyushkin), RI/MOM (loop corrections removed at off shell momentum, Yong & Stewart; Constantinou et al), to continuum limit, no lattice discretization dependence (ChQCD (2012.05448): might not for RI/MOM)

Hybrid Renormalization

- Ratio & RI/MOM schemes remove UV divergence, but kernel has non-perturbative IR effect. Long (> Zs~0.3 fm) Wilson line op. using Wilson line mass subtraction scheme---Hybid renormalization (X. Ji, Y. Liu, A. Schäfer, W. Wang, Y.B. Yang, J.H. Zhang, Y. Zhao)

Our Contribution

- Hybrid-RI/MOM (q-PDF) to MS-bar (PDF) one loop matching kernel for any hadron (isovector, unpolarized, helicity, and transversity PDFs and skewless GPDs).
- Hybrid-Ratio as a special example (mu_R = 0, pz_R=0)
- Self-renormalization (LPC) also a special case (Zs = 0), some modification at short distance needed

Scheme Conversions

- Multiplicative renormalization

$$\tilde{Q}^{B}_{\gamma^\mu}(z, P^z, \epsilon) = \frac{1}{2P^\mu}\langle P|\bar{\psi}(z)\gamma^\mu W(z,0)\psi(0)|P\rangle.$$

$$\tilde{Q}^{B}(z, P^z, \epsilon) = \tilde{Z}^{X}(z, P^z, \epsilon, \tilde{\mu})\, \tilde{Q}^{X}(z, P^z, \tilde{\mu}),$$

$$Z^{X}_{\overline{\rm MS}}(z, \tilde{\mu}, \tilde{\mu}') \equiv \frac{\tilde{Q}^{X}(z, P^z, \tilde{\mu})}{\tilde{Q}^{\overline{\rm MS}}(z, P^z, \tilde{\mu}')} = \frac{\tilde{Z}^{\overline{\rm MS}}(z, P^z, \epsilon, \tilde{\mu}')}{\tilde{Z}^{X}(z, P^z, \epsilon, \tilde{\mu})}$$

- Factorization proved in MS-bar; can be converted to other schemes

MS-bar to MS-bar matching

- Loose ends in Izubuchi, Ji, Jin, Stewart, Zhao:

Epsilon expansion and Fourier transform commute? Fermion number conservation and delta function at infinite x/y

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu)$$

$$-\int \frac{dzp^z}{2\pi} e^{ixzp^z} \ln(z^2\mu^2 e^{\gamma_E}) = -\left[\frac{d}{d\eta}\int \frac{dzp^z}{2\pi} e^{ixzp^z}\left(z^2\mu^2 e^{\gamma_E}\right)^\eta\right]\Bigg|_{\eta=0}$$

$$-\int \frac{dzp^z}{2\pi} e^{ixzp^z} \ln(z^2\mu^2 e^{\gamma_E}) = -\int \frac{dzp^z}{2\pi} e^{ixzp^z} \ln\left(\frac{z^2\mu^2 e^{\gamma_E}}{K^2}\right) - \int \frac{dzp^z}{2\pi} e^{ixzp^z} \ln K^2$$

$$= \tilde{f}^C(x) - \ln K^2\left[\frac{1}{2}\left(\frac{1}{x^2}\delta^+\left(\frac{1}{x}\right) + \frac{1}{(-x)^2}\delta^+\left(-\frac{1}{x}\right)\right)\right]$$

Ratio to MS-bar matching

- Kernel has non-perturbative IR contribution

$$\tilde{Z}^{\overline{\mathrm{MS}}}(\tilde{\mu}, \epsilon) = 1 + \frac{\alpha_s C_F}{2\pi} \frac{3}{2} \frac{1}{\epsilon_{UV}} + \mathcal{O}(\alpha_s^2).$$

$$Z_{\overline{\mathrm{MS}}}^{\mathrm{ratio}}(z, \tilde{\mu}) = 1 - \frac{\alpha_s C_F}{2\pi} \left(\frac{3}{2} \ln \frac{\tilde{\mu}^2 z^2}{4e^{-2\gamma_E}} + \frac{5}{2} \right) + \mathcal{O}(\alpha_s^2).$$

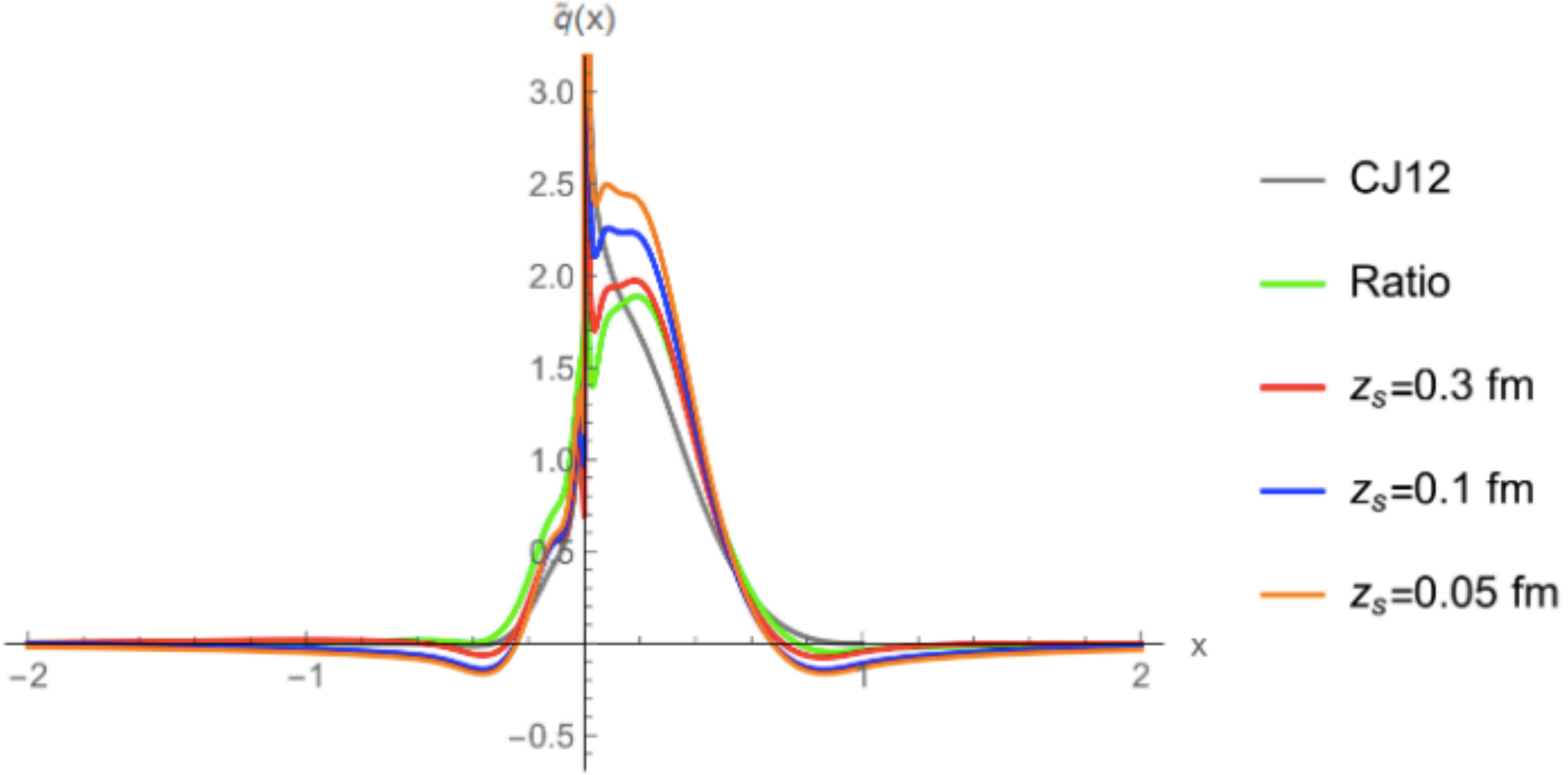
- Use Wilson line mass subtraction scheme for z > Zs, conversion factor is constant in z in dim reg

$$C^2 \exp(-\delta m |z|)$$

$$Z_{\overline{\mathrm{MS}}}^{\mathrm{hybrid\text{-}X}}(z, z_s, \tilde{\mu}, \tilde{\mu}')$$

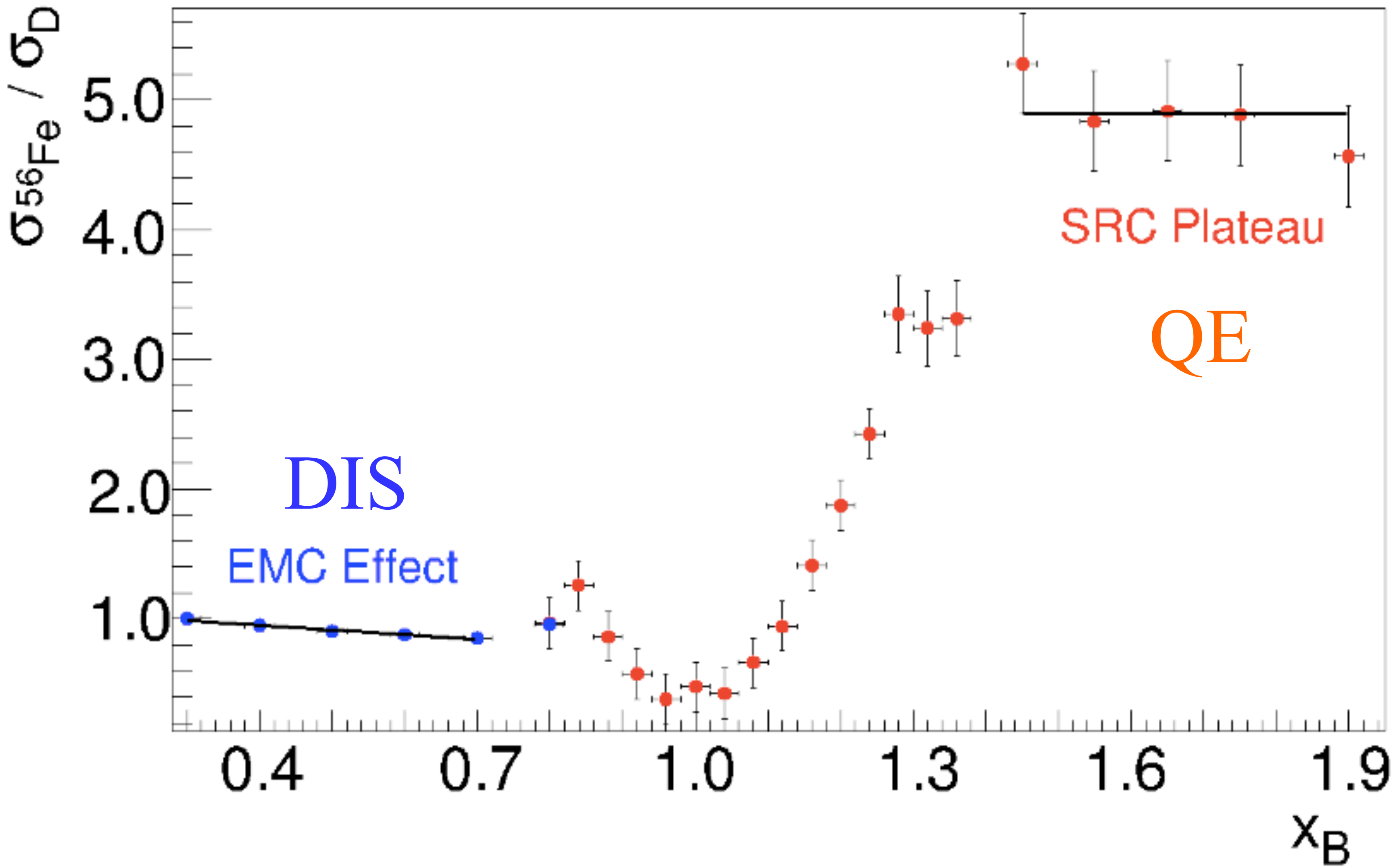
- $$= Z_{\overline{\mathrm{MS}}}^{X}(z, \tilde{\mu}, \tilde{\mu}')\, \theta(z_s - |z|) + Z_{\overline{\mathrm{MS}}}^{X}(z_s, \tilde{\mu}, \tilde{\mu}')\, \theta(|z| - z_s),$$

Hybrid-Ratio to MS-bar Matching



Outlook

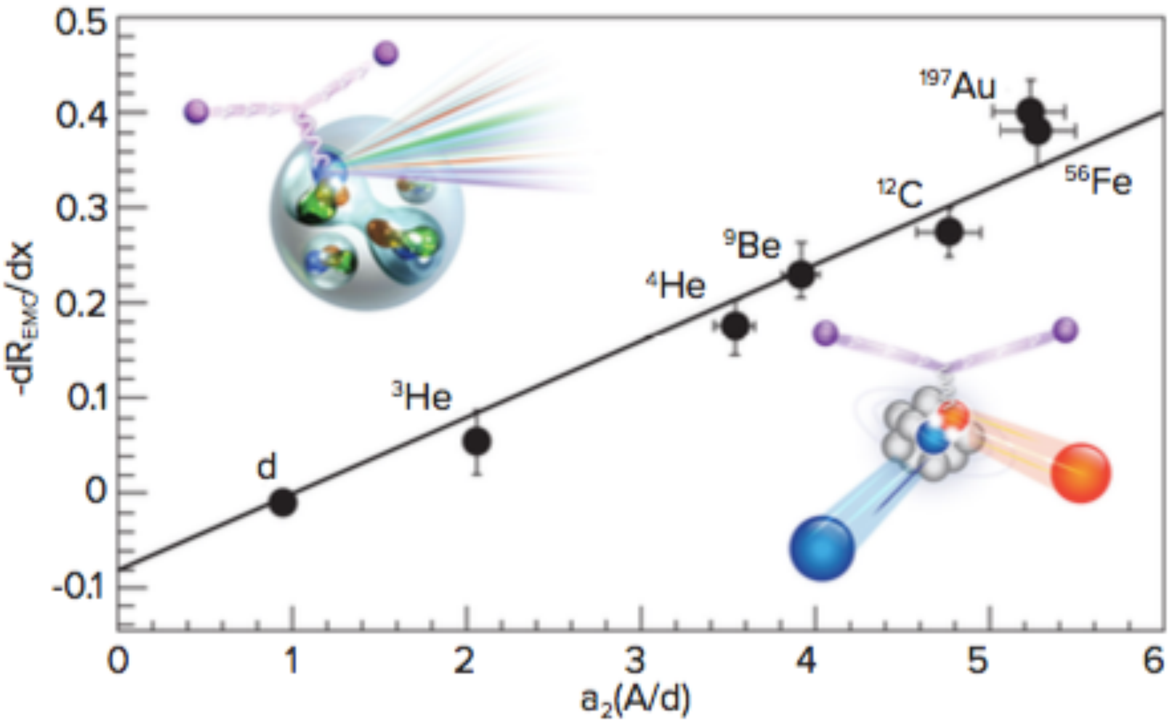
- Rapid progress made since 2013
- Further error study (non-singlet)
- Singlet PDF's: s, c, b and gluons
- If it works, complimentary to exp.: PDF (sea asymmetry, small and large x's, non-valence partons), DA, GPD, TMD, Wigner distributions … and one more thing…



Higinbotham, Miller, Hen, Rith, CERN Cour. 53N4 (2013) 24

An Astonishing Empirical Result!

Weinstein et al., PRL106, 052301 (2011)



EFT w/ Detmold, Lynn, Schwenk, PRL 119 (2017) 262502 :

- EMC-SRC linear relation reproduced
- Some a_2 reproduced ab initioly
- Remaining problem: EMC slope from LQCD (only need deuteron)

Backup slides

Matching Kernel

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots$$

- 1^{st} calculation (cut-off scheme): Xiong, Ji, Zhang, Zhao
- Factorization: Ma, Qiu; Li; Izubuchi, Ji, Jin, Stewart, Zhao
- Multiplictive Renormalizability: Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida; Green, Jansen, Steffens; Zhang, Ji, Schäfer, Wang, Zhao; Li, Ma, Qiu
- Linear divergence: Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang;
- LPT: Xiong, Luu, Meissner; Constantinou et al.
- RI: Monahan & Orginos; Yong & Stewart; Constantinou et al.; LP3)
- NPR: Constantinou et al.; LP3.

More Systematics Studies

- We need

$$\frac{\pi}{a} \gg P_z \gg \frac{1}{z_{max}} \gg \Lambda_{QCD}, m_\pi \gg \frac{\pi}{L}$$

Now we have

6.8 > 3 >> 0.15 ~ 0.2,0.14 > 0.1 (GeV)

More Systematics Studies

- We need

$$\frac{\pi}{a} \gg P_z \gg \frac{1}{z_{max}} \gg \Lambda_{QCD}, m_\pi \gg \frac{\pi}{L}$$

Now we have

6.8 > 3 >> 0.15 ~ 0.2,0.14 > 0.1 (GeV)

- Finite volume effect: ChPT (w/ Wei-Yang Liu 2011.13536). Less than 1% when $P_z/M \geq 1$ and $m_\pi L \geq 3$.

(JWC, Ji, **PLB**523 (2001) 107; **PRL** 87 (2001) 152002; **PRL** 88 (2002) 052003; JWC, Stewart, **PRL** 92 (2004) 202001; Arndt, Savage, **NPA**697 (2002) 429)

More Systematics Studies

- We need

$$\frac{\pi}{a} \gg P_z \gg \frac{1}{z_{max}} \gg \Lambda_{QCD}, m_\pi \gg \frac{\pi}{L}$$

Now we have

$6.8 > 3 >> 0.15 \sim 0.2, 0.14 > 0.1$ (GeV)

- long tail and NPR: Hybrid (2008.03886) or self-renormalization (2103.02965) (LPC)

More Systematics Studies

- We need

$$\frac{\pi}{a} \gg P_z \gg \frac{1}{z_{max}} \gg \Lambda_{QCD}, m_\pi \gg \frac{\pi}{L}$$

Now we have

6.8 > 3 >> 0.15 ~ 0.2,0.14 > 0.1 (GeV)

- Continuum limit: Residual divergence seen in RI/MOM for a rest pion state (χQCD 2012.05448).

Lattice Setup (isovector proton PDF)

- Lattice: $64^3 \times 96$

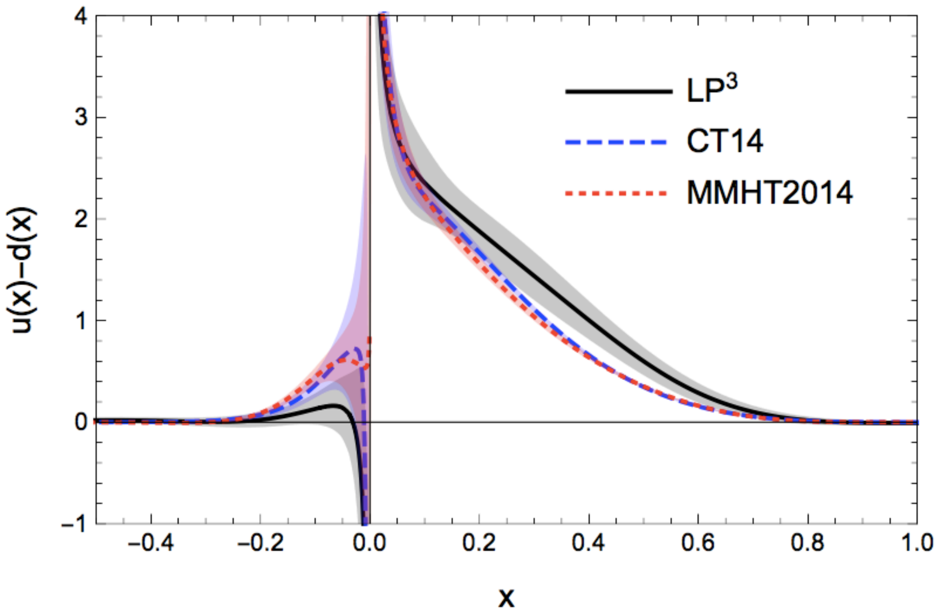
  $a = 0.09 \text{ fm} \qquad L \approx 5.8 \text{ fm}$
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

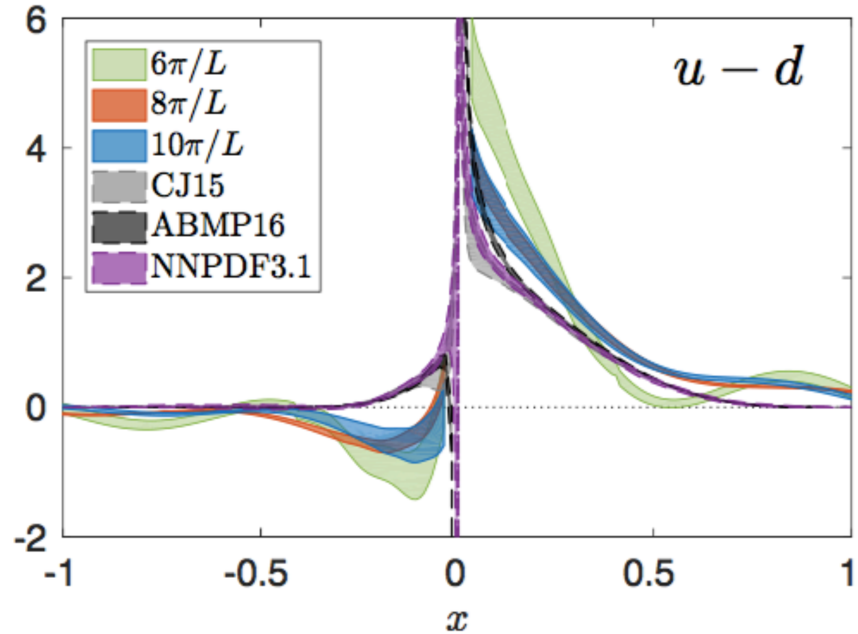
  $N_f = 2+1+1 \qquad M_\pi \approx 135 \text{ MeV}$
- Gauge fields/links: hypercubic (HYP) smearing (one step), 884 config.

- $P^z = n\frac{2\pi}{L}$ = 2.2, 2.4, 3.0 GeV (n = 10,12,14)

(high momentum smearing: Bali, Lang, Musch, Schafer; smaller energy gap)

Parton Density

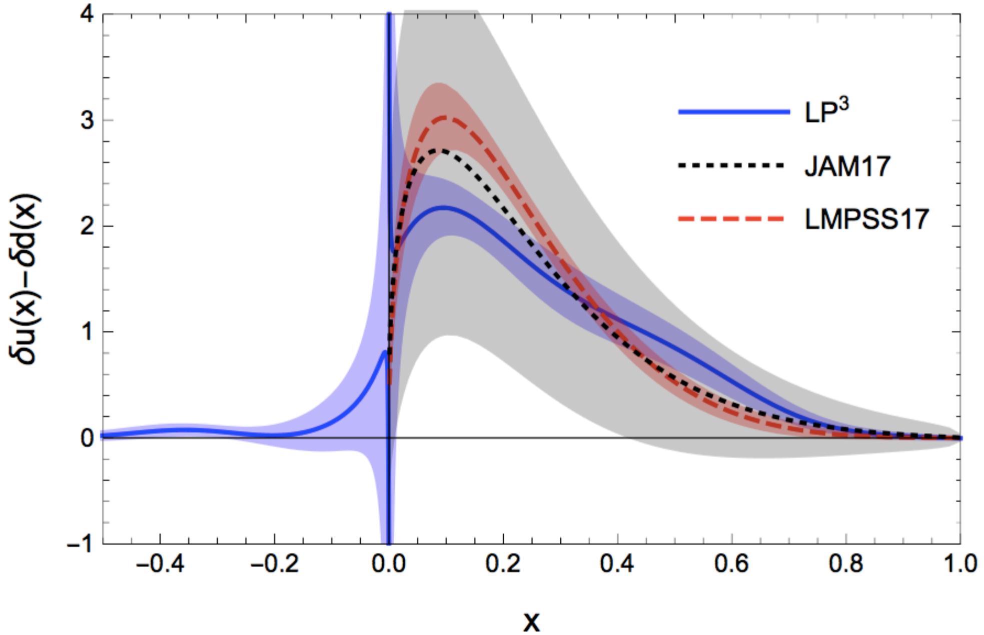




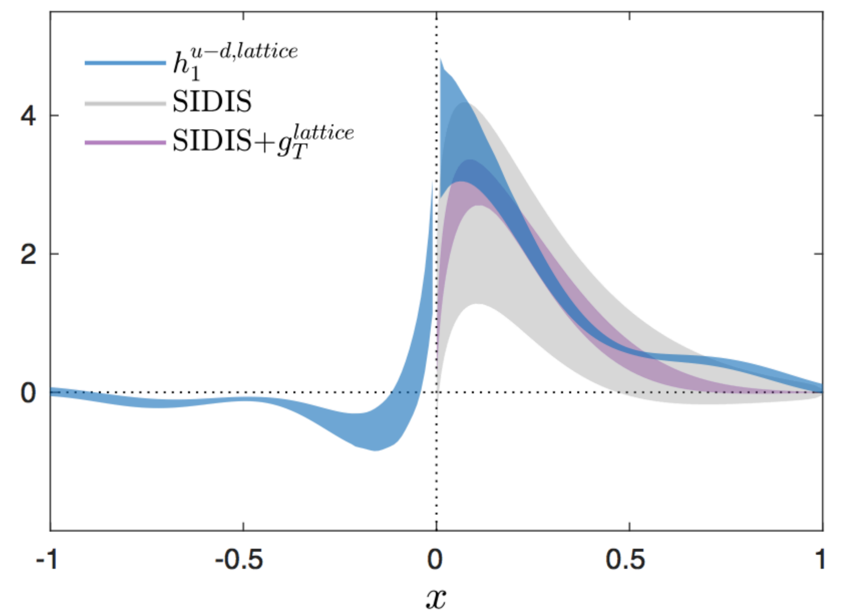
LP3(1803.04393)

ETMC(1803.02685)

Transversity



LP3 (1810.05043)



ETMC(1803.02685)

First (isovector) LPDF Computation

- Lattice: $24^3 \times 64$

  $a \approx 0.12 \ \text{fm} \quad L \approx 3 \ \text{fm}$
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

  $N_f = 2 + 1 + 1 \qquad M_\pi \approx 310 \ \text{MeV}$
- Gauge fields/links: hypercubic (HYP) smearing, 461 config.
- $P^z = \frac{2\pi}{L} n = n \times 0.43 \ GeV$ n = 1,2,3…

Review: Ji's LPDF (LaMET)

$$\widetilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{-ixzP^z} \langle P | \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) | P\rangle$$

$$\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z)P^z$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^n\psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

op. symmetric but not traceless

$$\left(\lambda_{\mu_1}\lambda_{\mu_2} - g_{\mu_1\mu_2}\lambda^2/4\right)$$

Review: Ji's LPDF (LaMET)

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

- LHS: trace, twist-4 $\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots$$

Non-Perturbative Renormalization + Matching

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots$$

- NPR (RI/MOM scheme), γ_t Landau gauge $\quad p^2 = -\mu_R^2 \quad p_z = p_z^R$
- RI/MOM to $\overline{\mathrm{MS}}$ performed at one loop

Rossi & Testa's criticism

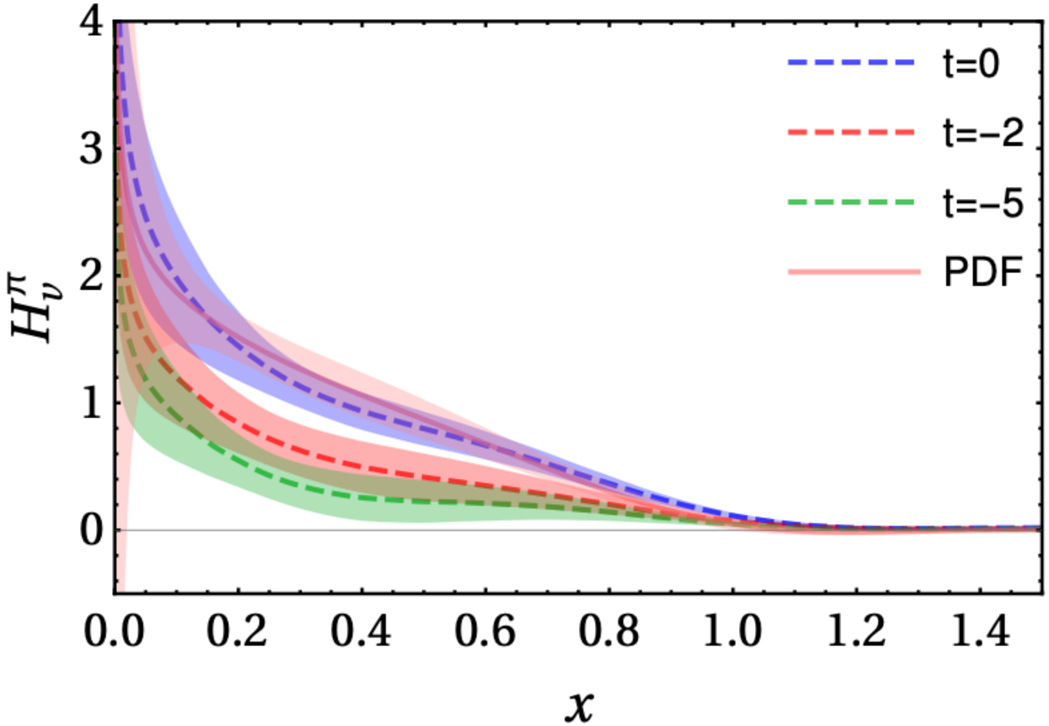
(1706.04428,1806.00808)

- Criticism: The twist-4 effect is $\mathcal{O}(1/(aP_z)^2)$ from dimensional analysis instead of $\mathcal{O}(\Lambda_{QCD}^2/P_z^2)$
- This can be avoided by renormalizing the quark bilinear operators non-perturbatively such that one can go to continuum limit where the lattice spacing dependence disappears.
- The matching formula should be between the renormalized quasi-PDF and PDF, not between bare quasi-PDF and PDF as in earlier versions.

Advantages of RI/MOM

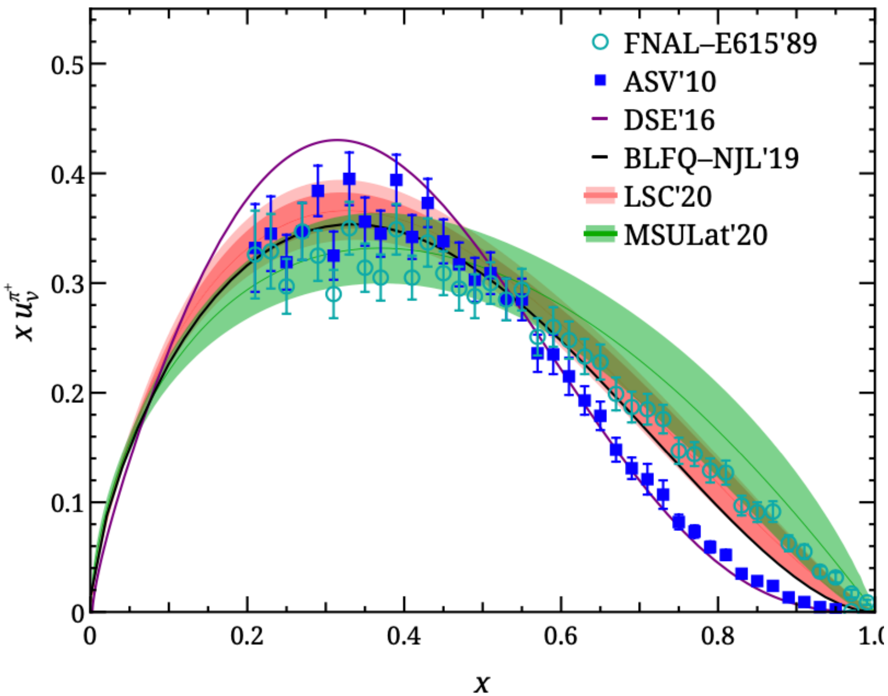
- RI/MOM: Quasi-PDF is renormalized non-perturbatively by performing an off-shell subtraction. Continuum limit can be taken afterwards to recover rotation symmetry, s.t.
- (1) power divergent mixing to lower moments removed
- (2) power divergent mixing with higher twist (same dim. different spin) also removed (Rossi and Testa problem)

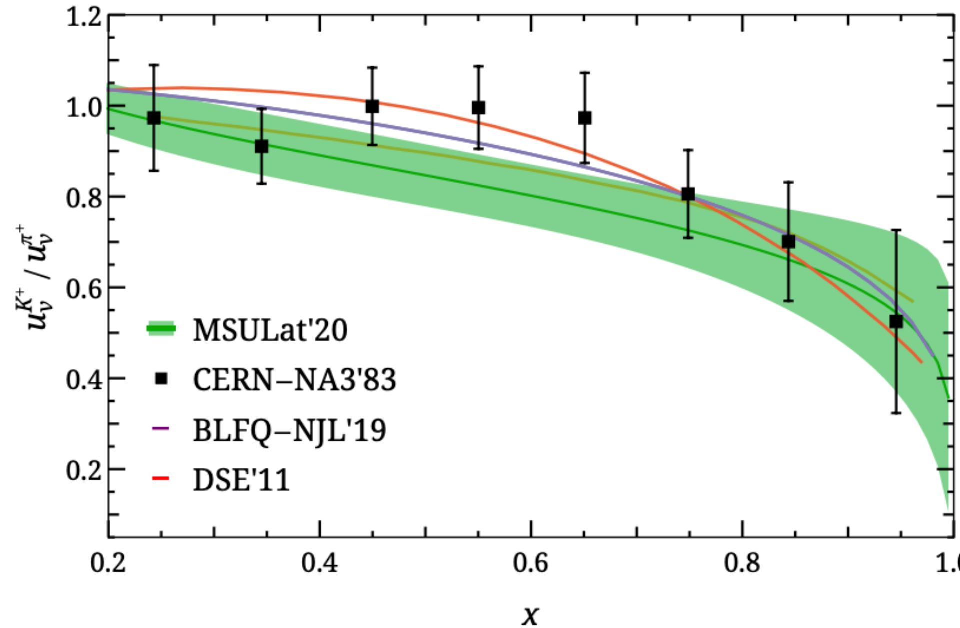
Generalized Parton Distributions



JWC, HW Lin, JH Zhang (1904.12376)

Meson Valence Quark Distributions





HW Lin, JWC, Z Fan, JH Zhang, R Zhang (2003.14128)