中国科学技术大学
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Q-ball Superradiance

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Saffin, Xie, **SYZ**, 2212.03269 [hep-th]

Non-perturbative field configurations

QFT is more than just Feynman diagrams!

- Topological solitons (topological defects)
  - (often) static configuration, stable due to topological reasons

monopoles, cosmic strings, domain walls, kinks, instantons, p-branes, ...

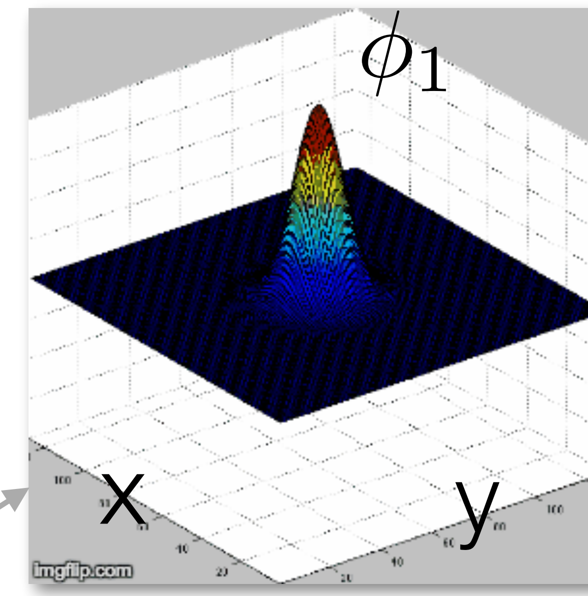
- Non-topological solitons (Q-balls)
  - time-periodic configuration, stable due to Noether charge

What is a Q-ball?

Friedberg, T.D. Lee & Sirlin 1976
Coleman 1985

- A Q-ball is:

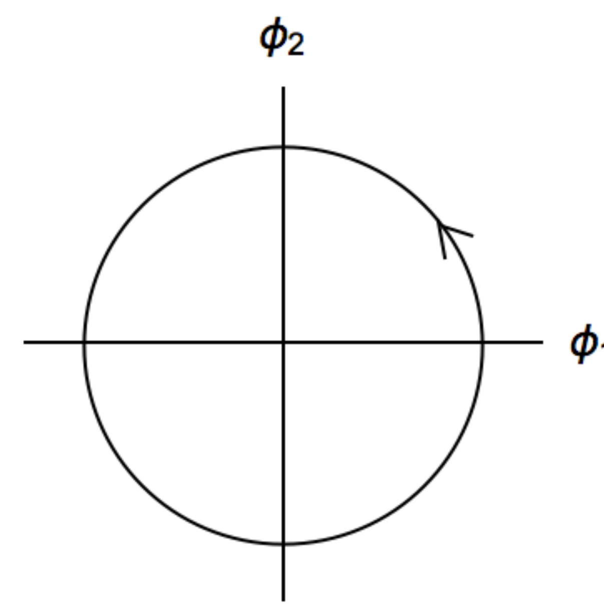
  a stable, localized, time-dependent field configuration with a Noether charge



- Simplest case: U(1) symmetric scalar field

$$\mathcal{L} = -\partial_\mu \Phi \partial^\mu \bar{\Phi} - V(|\Phi|)$$

$$\Phi = \phi_1 + i\phi_2$$



Q-ball solution:

$$\Phi_Q = \frac{1}{\sqrt{2}} f(r) e^{i w_Q t}, \qquad w_Q = const$$

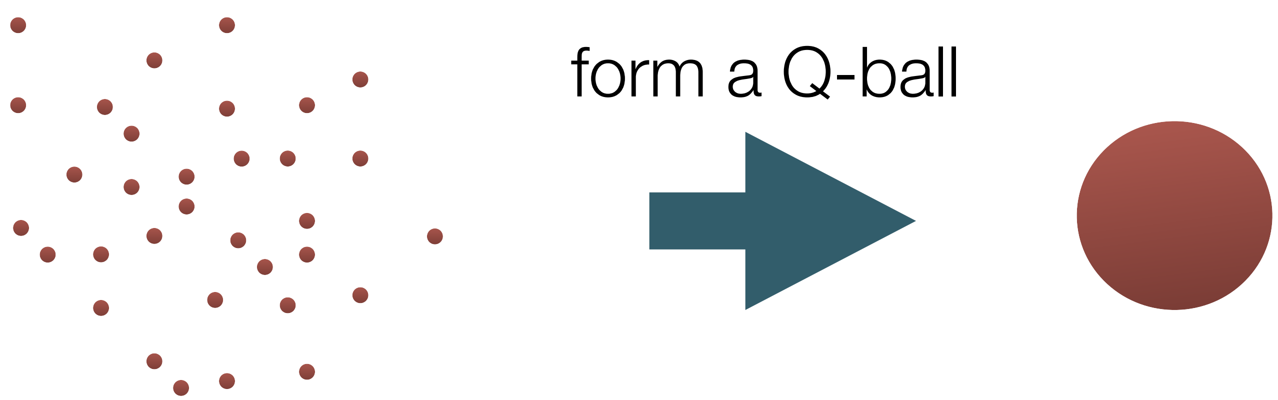
Q-ball configuration is min of energy functional for given charge

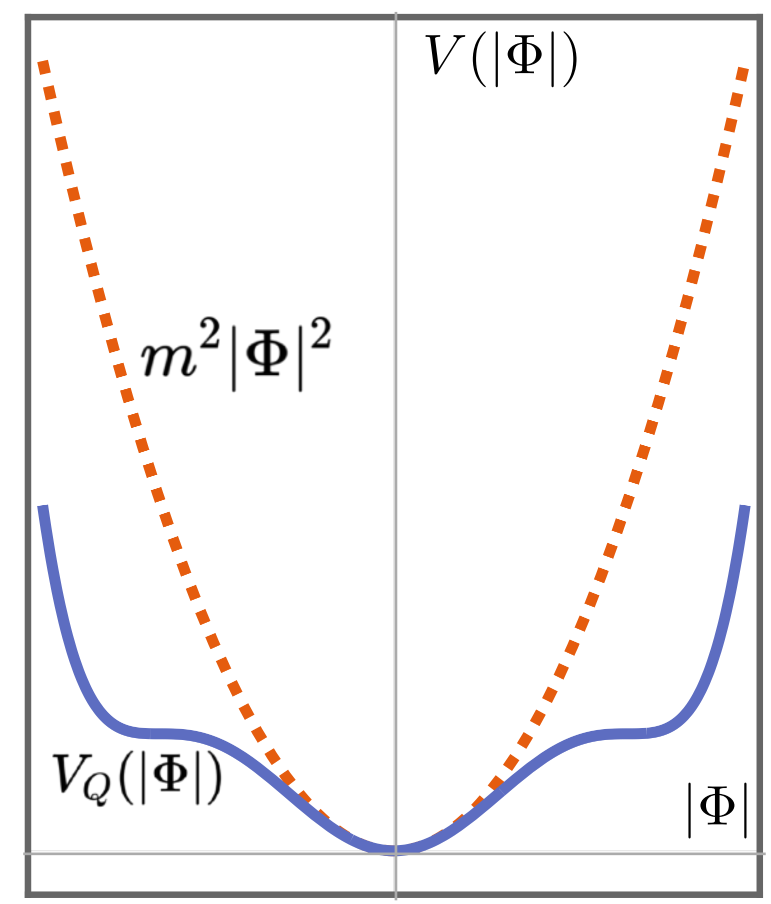
When can Q-balls form?

Q-balls can form when the potential is attractive.

when potential is shallower than $|\Phi|^2$

eg: $V_Q(|\Phi|) = m^2|\Phi|^2 - \lambda|\Phi|^4 + g|\Phi|^6$





A Q-ball is a Bose-Einstein condensate.

Within Q-ball

energy per particle $<$ particle mass (m)

A Q-ball can also spin

In 2+1D, spinning Q-ball is ring-shaped

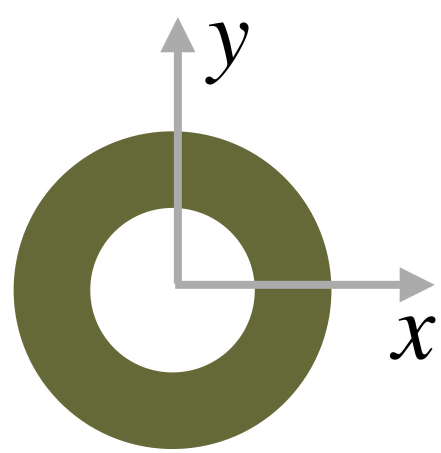
$$\Phi_Q(t, r, \varphi) = \frac{1}{\sqrt{2}} f(r) e^{-i\omega_Q t + i m_Q \varphi}$$

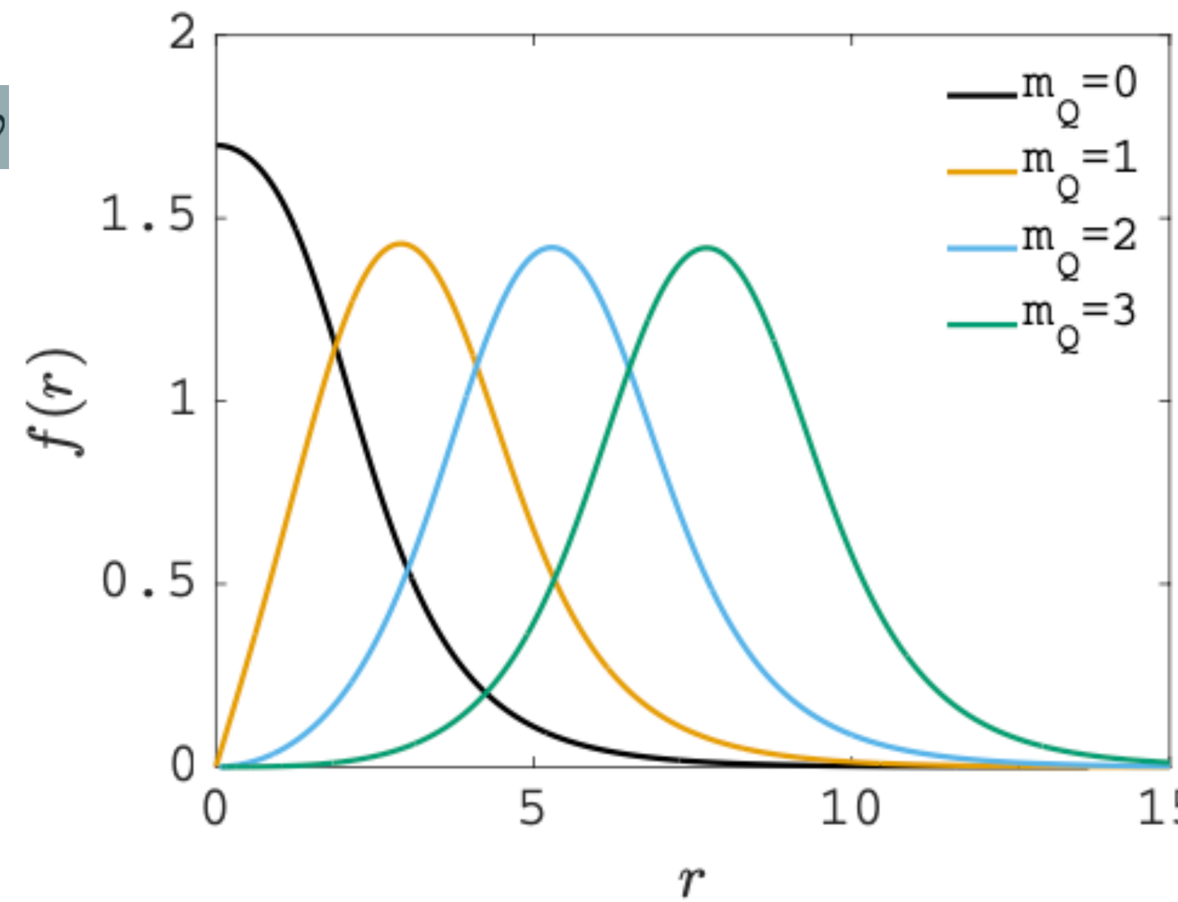
Angular phase velocity

$$\Omega_Q = \omega_Q / m_Q$$

Angular momentum

$$L = m_Q Q$$





In 3+1D, spinning Q-ball

Volkov & Wohnert hep-th/0205157

$$\Phi_Q(t, r, \varphi) = \frac{1}{\sqrt{2}} f(r, \theta) e^{-i\omega_Q t + i m_Q \varphi}$$

What roles Q-balls may play?

- Q-balls in particle physics and cosmology
  - Q-balls from Affleck-Dine condensate decay — Kusenko & Shaposhnikov 1997

    many flat directions in SUSY potentials — Dine, Randall & Thomas 1995
  - Q-ball dark matter — Kusenko & Shaposhnikov 1997
- Boson stars (cousins of Q-balls, when gravity is strong)

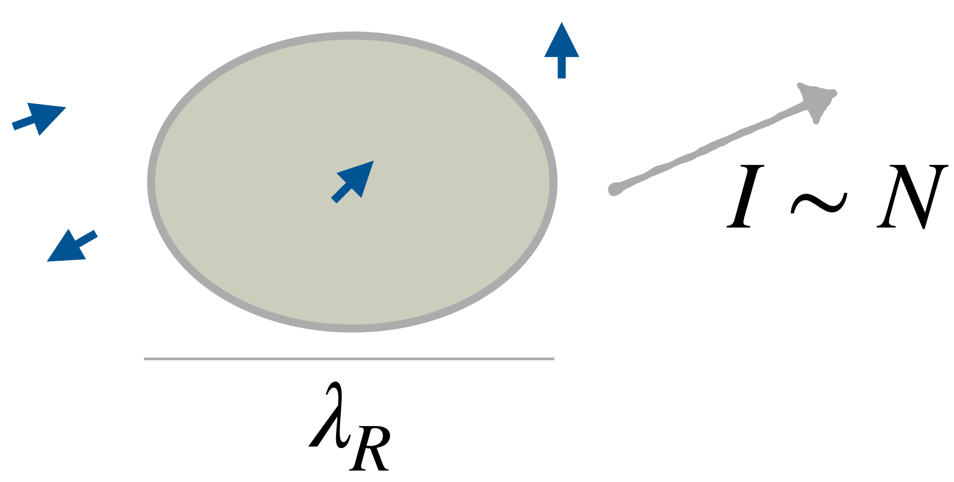
  Kaup 1968
- Q-balls in labs
  - realized experimentally in superfluid He3-B — Bunkov & Volovik 2007

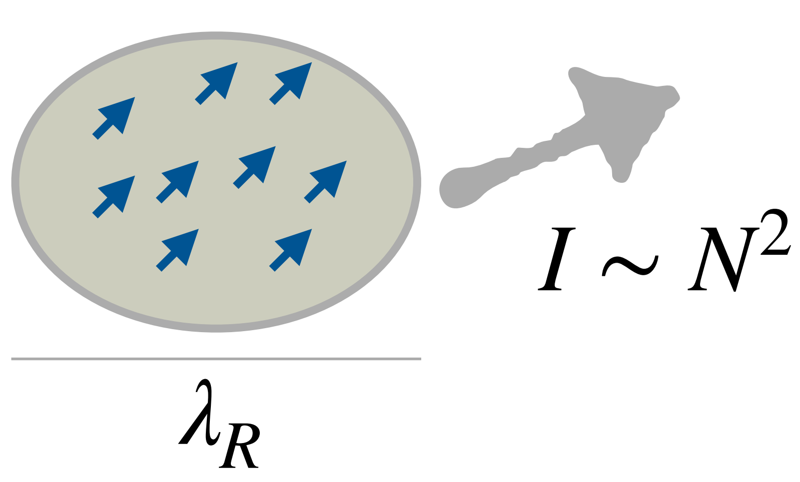
What is superradiance?

Superradiance = Radiation amplification

A collection of phenomena:

- Inertial motion superradiance — Ginzburg, Frank 1947
  - Cherenkov radiation
  - Mach shocks
  - Critical speed for superfluidity
- Coherent superradiance — Dicke 1954

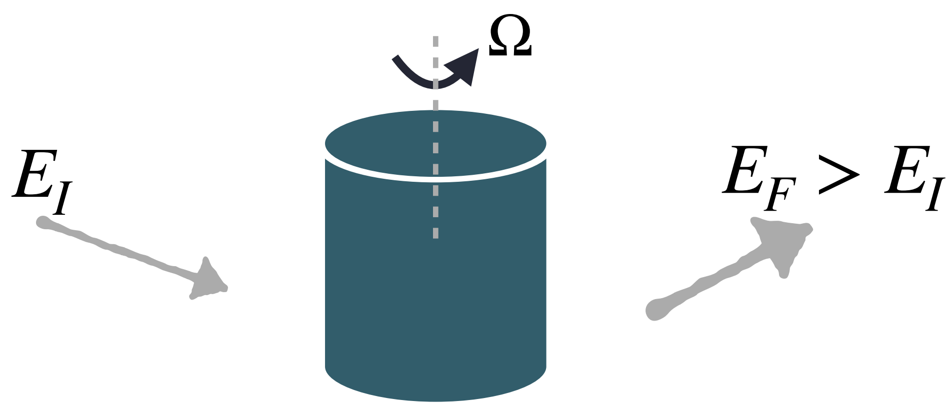


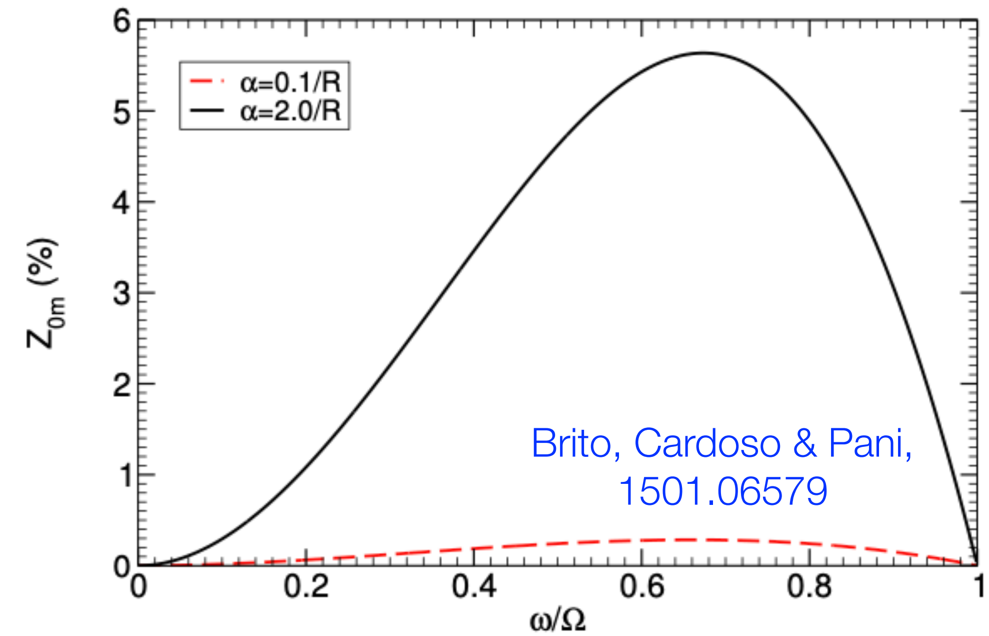


- Rotational Superradiance

Zel'dovich 1971; Misner 1972

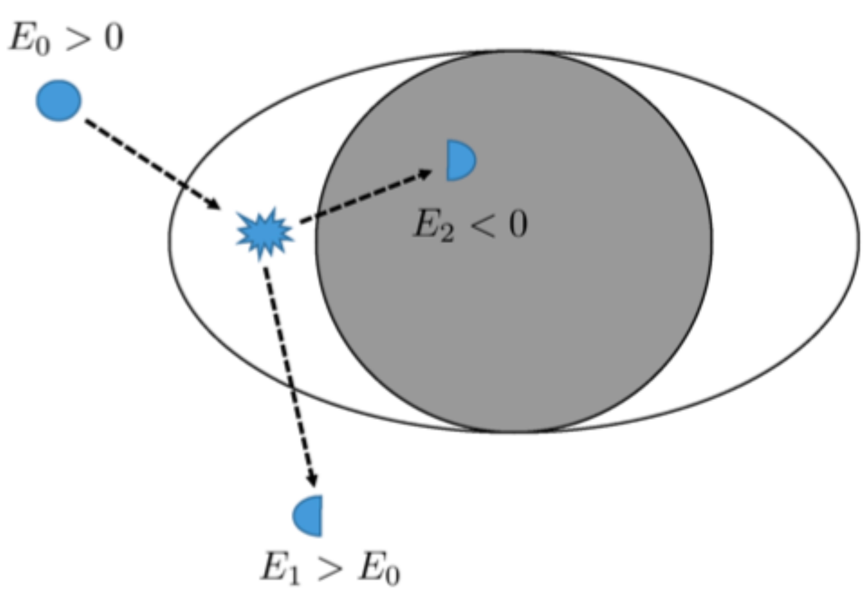
  - Rotating cylinder

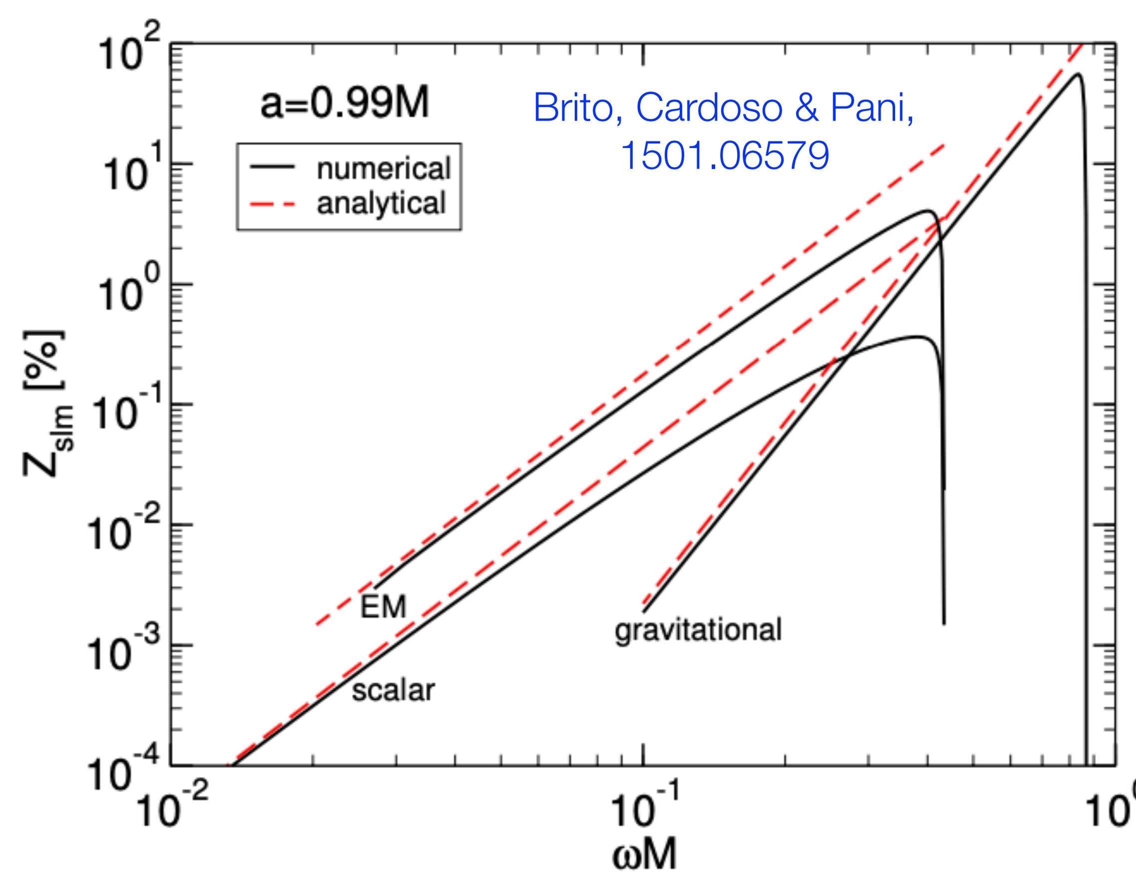




Zel'Dovich criterion: $\omega < m\Omega$

  - Kerr black hole (Penrose process for radiation)





Zel'Dovich criterion: $\omega < m\Omega$

Waves scattering off a Q-ball

Klein-Gordon equation

$$\Box\Phi - \frac{\partial V(|\Phi|)}{\partial \Phi^\dagger} = 0$$

Q-ball solution

$$\Phi_Q(t,r,\varphi) = \frac{1}{\sqrt{2}} f(r) e^{-i\omega_Q t + i m_Q \varphi}$$

Perturbation EoM $\Phi = \Phi_Q + \phi(t,r,\varphi)$

coupled

$$\Box\phi - U(r)\phi - e^{-2i(\omega_Q t - m_Q\varphi)} W(r)\phi^* = 0$$

$U = \frac{1}{2}\left(\frac{\mathrm{d}^2 V}{\mathrm{d}f^2} + \frac{1}{f}\frac{\mathrm{d}V}{\mathrm{d}f}\right)$ $\quad W = \frac{1}{2}\left(\frac{\mathrm{d}^2 V}{\mathrm{d}f^2} - \frac{1}{f}\frac{\mathrm{d}V}{\mathrm{d}f}\right)$

coherent background oscillation

Minimal scattering waves

$$\phi = \eta_+(\omega, m, r)e^{-i\omega_+ t + im_+\varphi} + \eta_-(\omega, m, r)e^{-i\omega_- t + im_-\varphi}$$

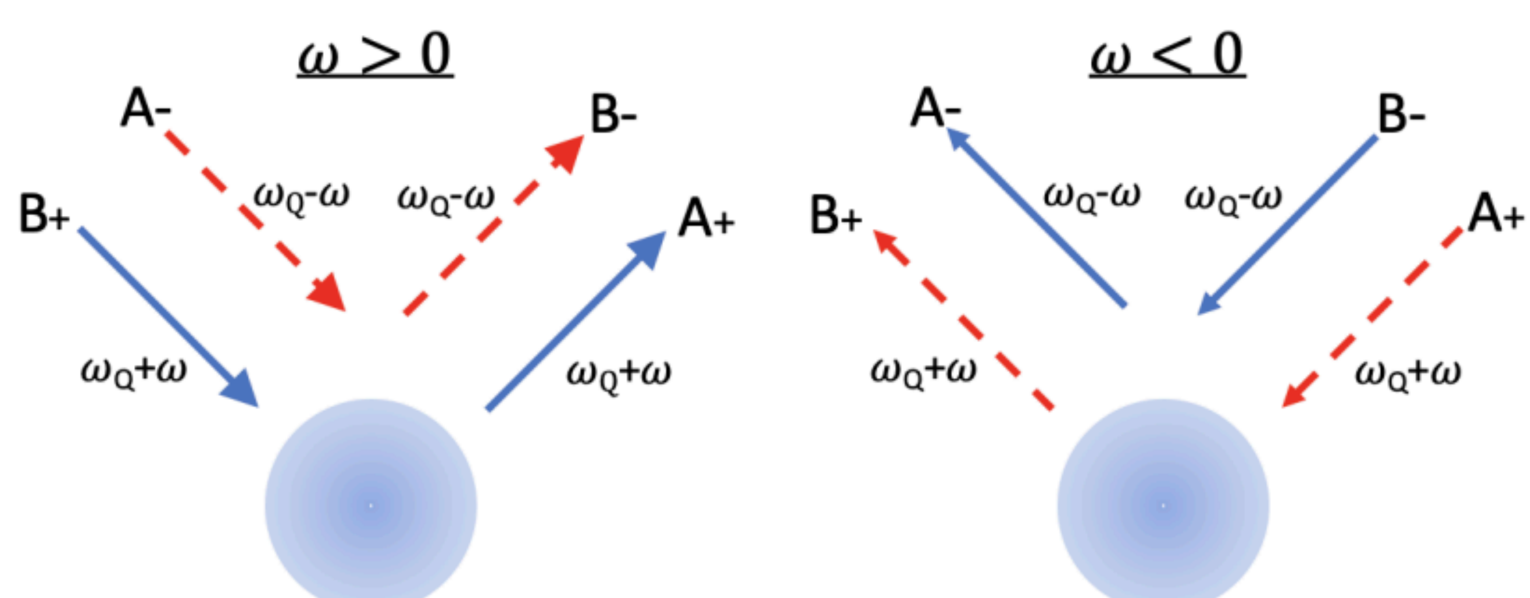
$$\omega_\pm = \omega_Q \pm \omega \qquad m_\pm = m_Q \pm m$$

EoMs for mode functions

$$\eta''_\pm + \frac{1}{r}\eta'_\pm + \left(\omega_\pm^2 - U - \frac{m_\pm^2}{r^2}\right)\eta_\pm - W\eta^*_\mp = 0$$

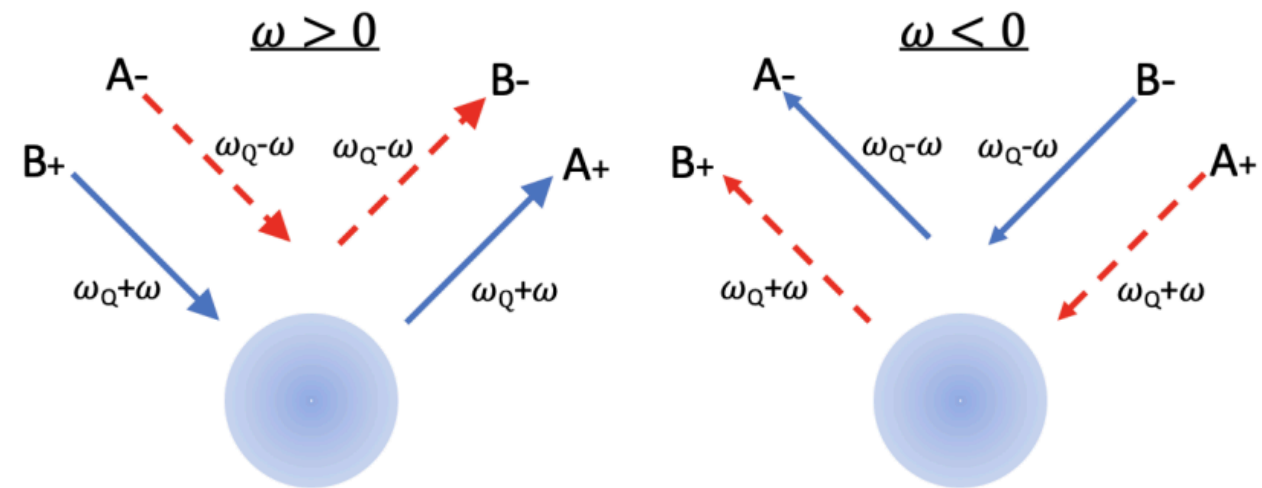
In and out waves

$$\eta_\pm(\omega, m, r \to \infty) \to \frac{A_\pm}{\sqrt{k_\pm r}}e^{ik_\pm r} + \frac{B_\pm}{\sqrt{k_\pm r}}e^{-ik_\pm r}$$



Particle number conservation

$$|A_+|^2 + |B_-|^2 = |B_+|^2 + |A_-|^2$$



Energy amplification

$$\mathcal{A}_E = \left(\frac{\frac{\omega_+^2}{k_+}|A_+|^2 + \frac{\omega_-^2}{k_-}|B_-|^2}{\frac{\omega_+^2}{k_+}|B_+|^2 + \frac{\omega_-^2}{k_-}|A_-|^2} \right)^{\mathrm{sign}(\omega)}$$

$$\omega_\pm = \omega_Q \pm \omega$$
$$m_\pm = m_Q \pm m$$

Superradiance criterion:

$$\frac{(\omega_Q + \omega_E)^2}{\sqrt{(\omega_Q + \omega_E)^2 - 1}} = \frac{(\omega_Q - \omega_E)^2}{\sqrt{(\omega_Q - \omega_E)^2 - 1}}.$$

Angular momentum amplification

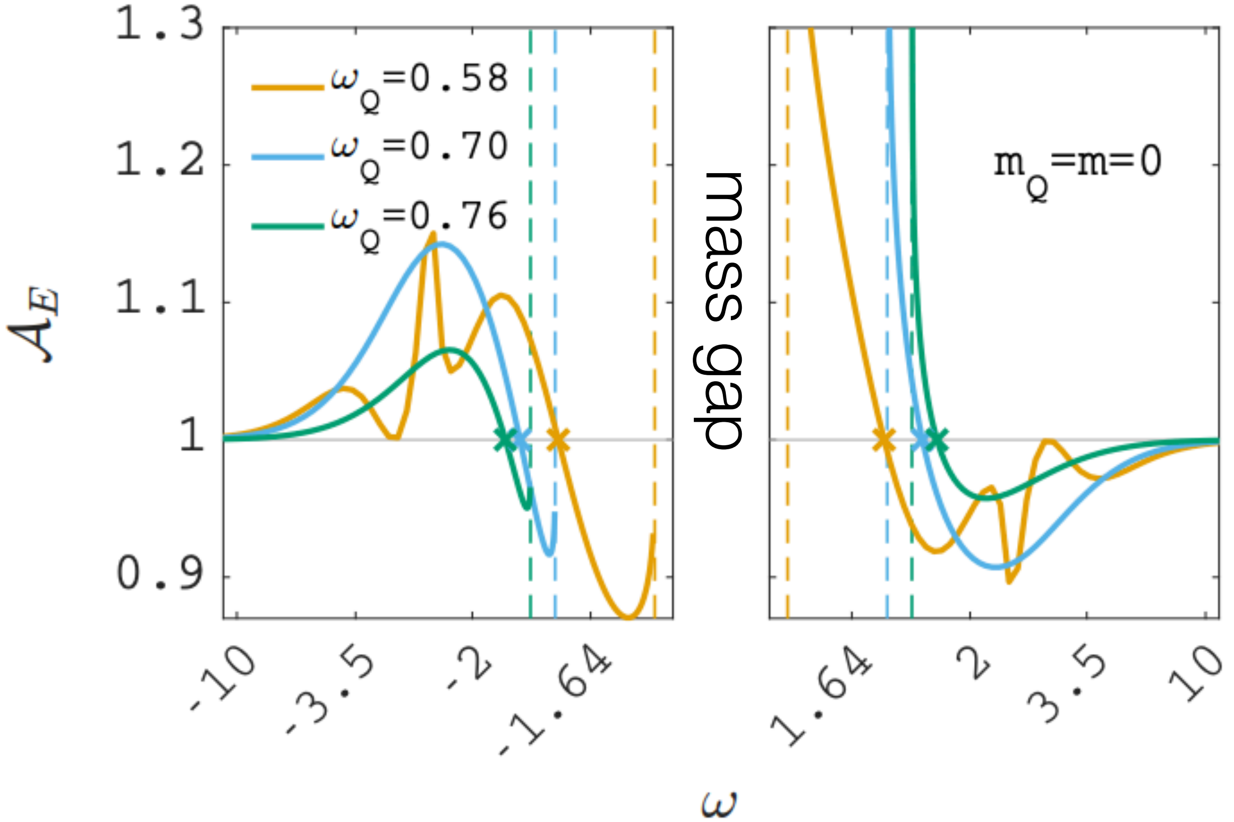
$$\mathcal{A}_L = \left(\frac{\frac{\omega_+ m_+}{k_+}|A_+|^2 + \frac{\omega_- m_-}{k_-}|B_-|^2}{\frac{\omega_+ m_+}{k_+}|B_+|^2 + \frac{\omega_- m_-}{k_-}|A_-|^2} \right)^{\mathrm{sign}(\omega)}$$

Superradiance criterion:

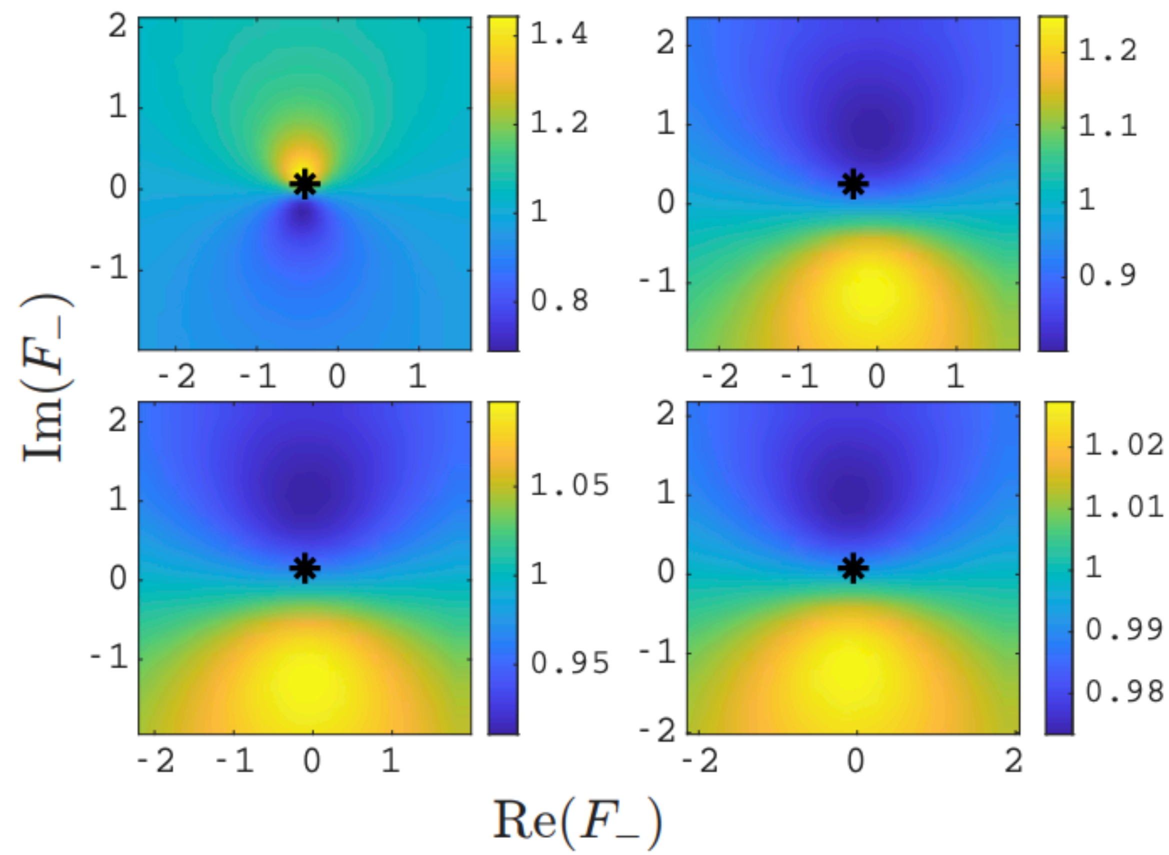
$$\frac{(\omega_Q + \omega_L)(m_Q + m_L)}{\sqrt{(\omega_Q + \omega_L)^2 - 1}} = \frac{(\omega_Q - \omega_L)(m_Q - m_L)}{\sqrt{(\omega_Q - \omega_L)^2 - 1}}.$$

Superradiance: non-spinning Q-ball

Only η_+ as ingoing wave



Both η_+ and η_- as ingoing wave



- matchs superradiance criterion well
- biggest enhancement if $\mathrm{sign}(\omega_+) = \mathrm{sign}(\omega_Q)$
- multi-peak spectra

Both modes ingoing can enhance superradiance

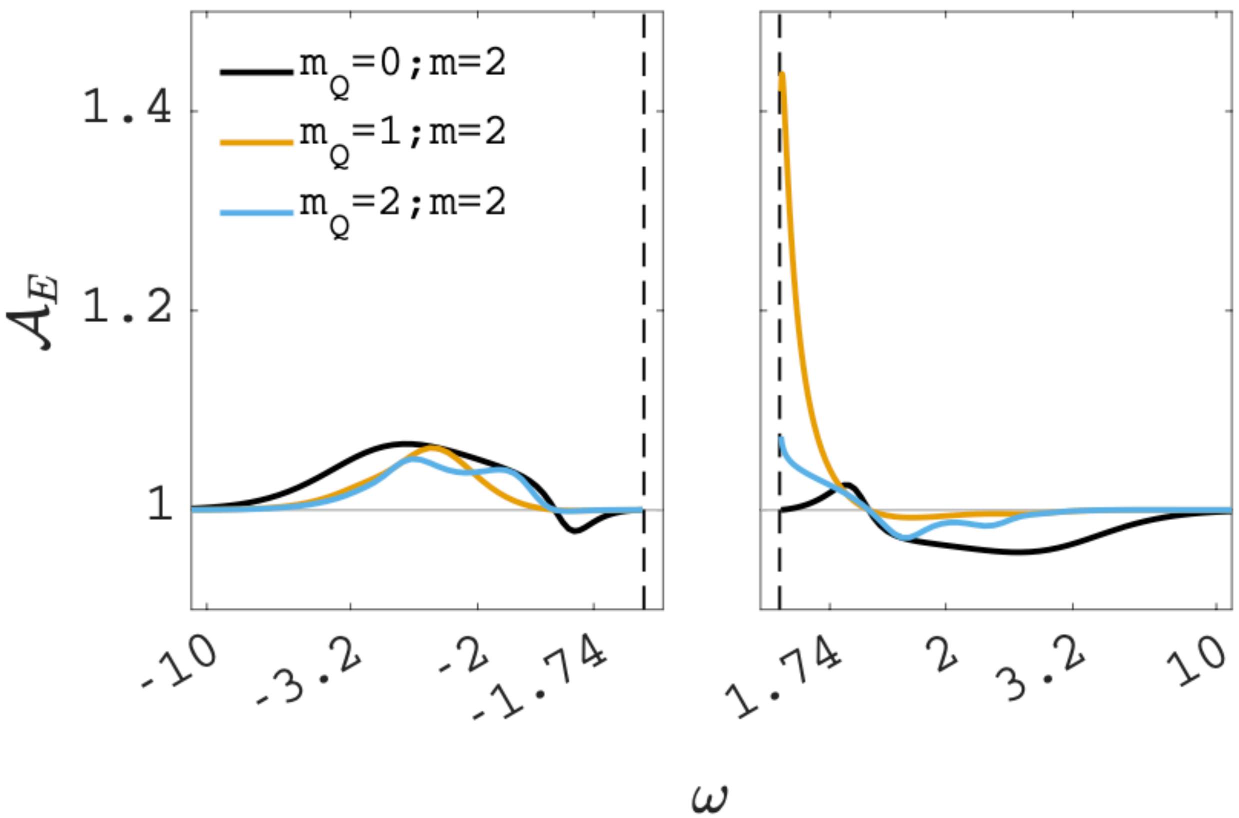
Q-ball's coherent rotation in field space can induce superradiance!

similar to Dicke superradiance in spirit

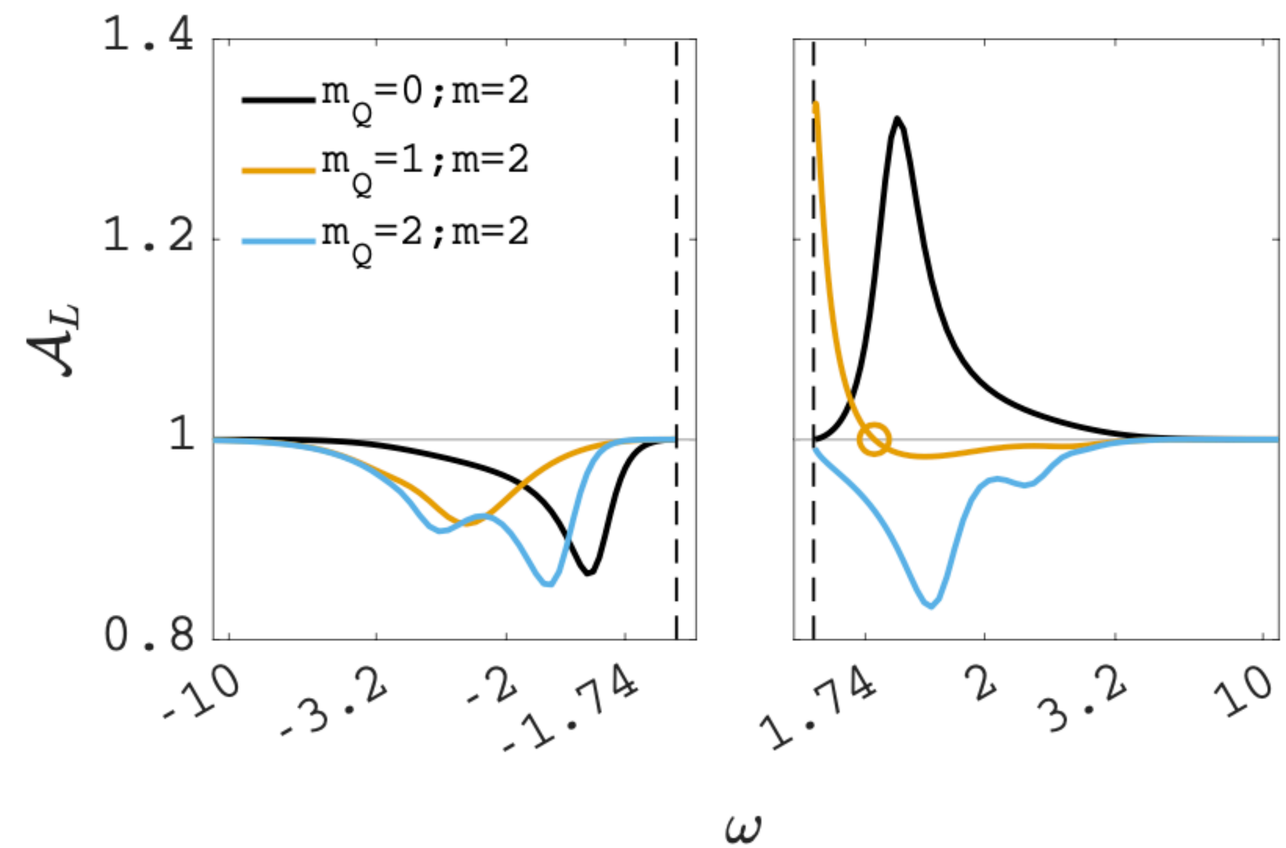
Superradiance: spinning Q-ball

only η_+ as ingoing wave

Energy superradiance



Angular momentum superradiance



- energy superradiance criterion unchanged
- **field + real space** rotational superradiance
- enhancement if $\mathrm{sign}(\omega_+/m_+) = \mathrm{sign}(\omega_Q/m_Q)$

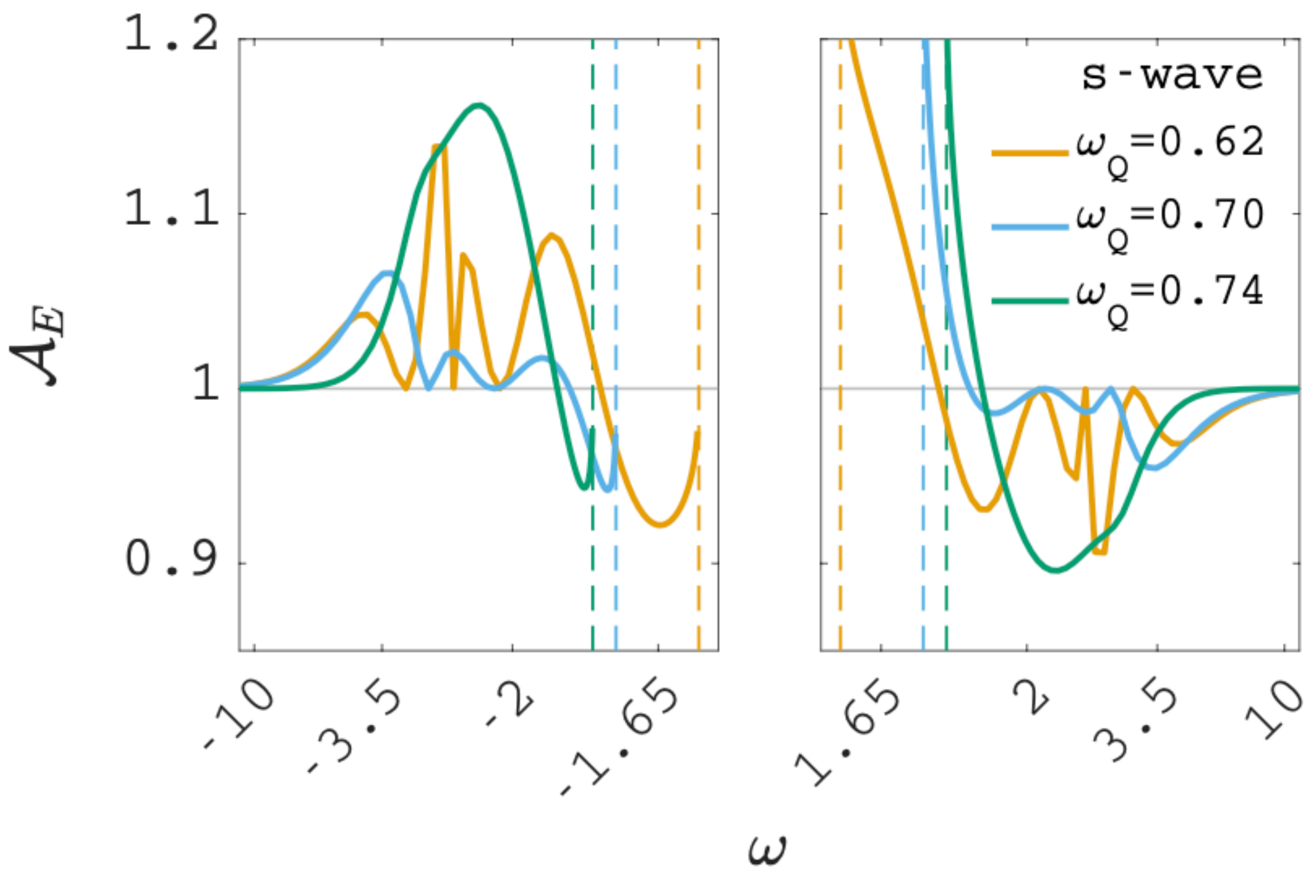
- matchs superradiance criterion well
- pattern is different from energy's
- violate Zel'dovich's criterion $\omega < m\Omega_Q$

Q-ball's rotation in real space can further enhance superradiance!

Q-ball superradiance in 3+1D

only η_+ as ingoing wave

Energy superradiance in spherical symmetry



- very similar to the 2+1D case
- more multi-peak structure

Outlook

- Q-balls are stable in isolation, but Q-ball superradiance implies they are **unstable in “dirty” environments.**
- Q-balls are a dark matter candidate and play other interesting roles in cosmology, for which longevity is a prerequisite. How are these scenarios affected?
- Boson stars are essentially Q-balls with gravity. Our results hints that boson stars can superradiate. What are the implications for gravitational wave physics?