W-exchange contribution to the decays $\Xi_{cc}^{++} \to \Xi_c^{+(\prime)} \pi^+$ using light-cone sum rule

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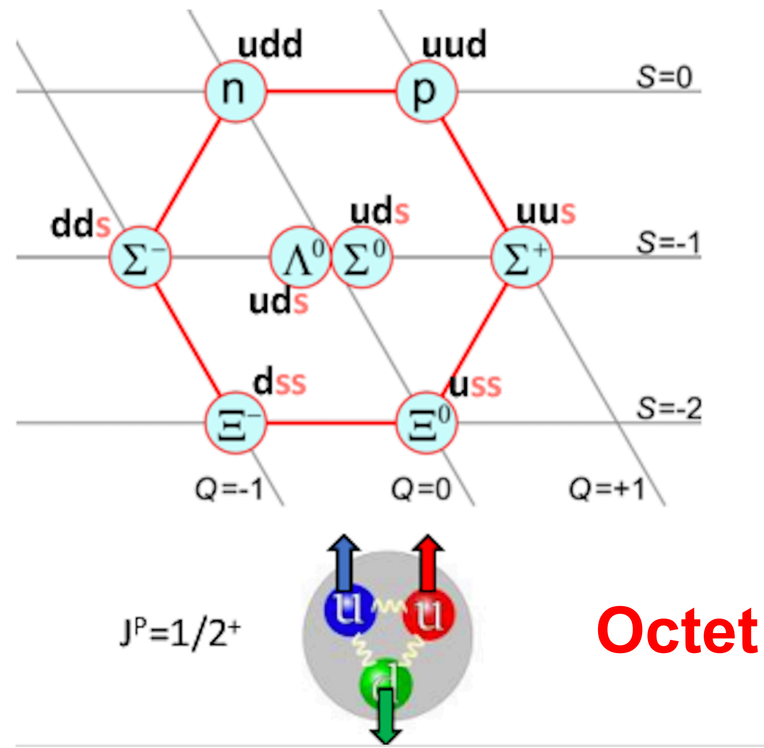
Outline

- Observation of doubly charmed baryon
- Status of the theoretical studies on $\Xi_{cc}^{++} \to \Xi_c^{+(\prime)} \pi^+$:
  W-emission and W-exchange contribution
- Light-cone sum rules calculation for the
  W-exchange diagram
- Numerical Results: Amplitudes and branching fraction

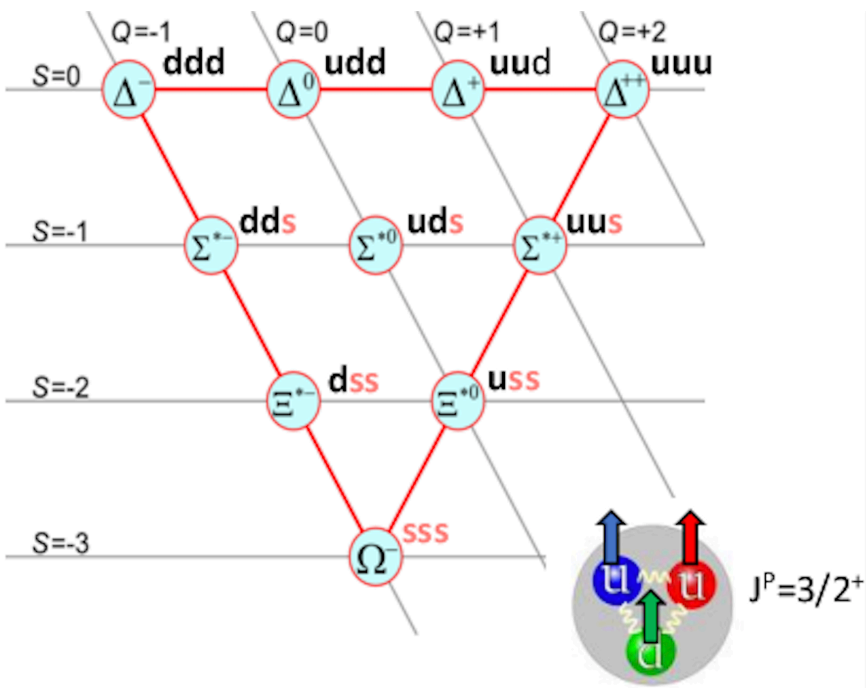
Motivation

Observation of doubly charmed baryon

Prediction from the quark model



Octet



Decuplet

Searching for the doubly charmed baryon

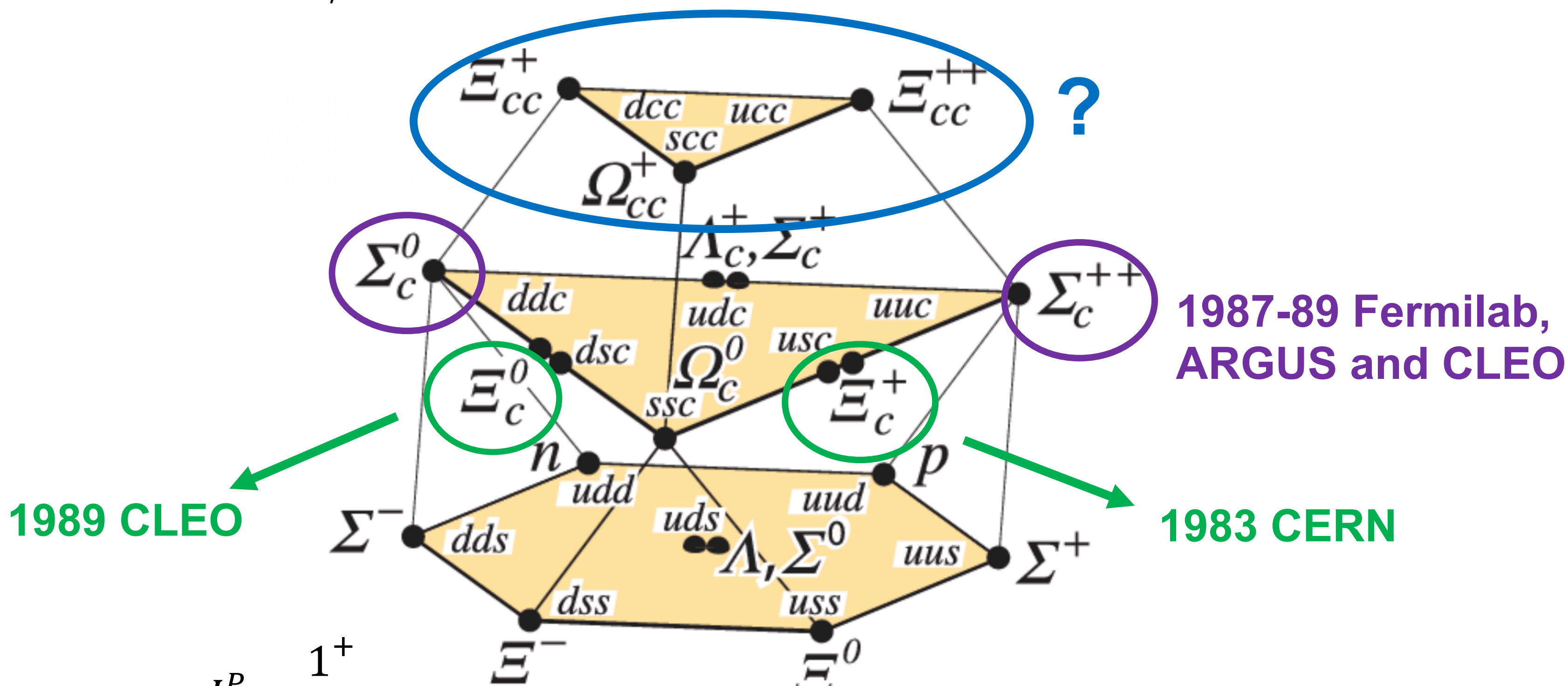
The SELEX collaboration: Phys. Rev. Lett. **89**, 112001 (2002) $\Xi_{cc}^{+} \to \Lambda_c^{+} K^{-} \pi^{+}$

Phys. Lett. B 628, 18 (2005) $\Xi_{cc}^{+} \to p D^{+} K^{-}$

Mass:

$3518.7 \pm 1.7\ \mathrm{MeV/c^2}$

Not confirmed by other experiments !



$$J^P = \frac{1}{2}^{+}$$

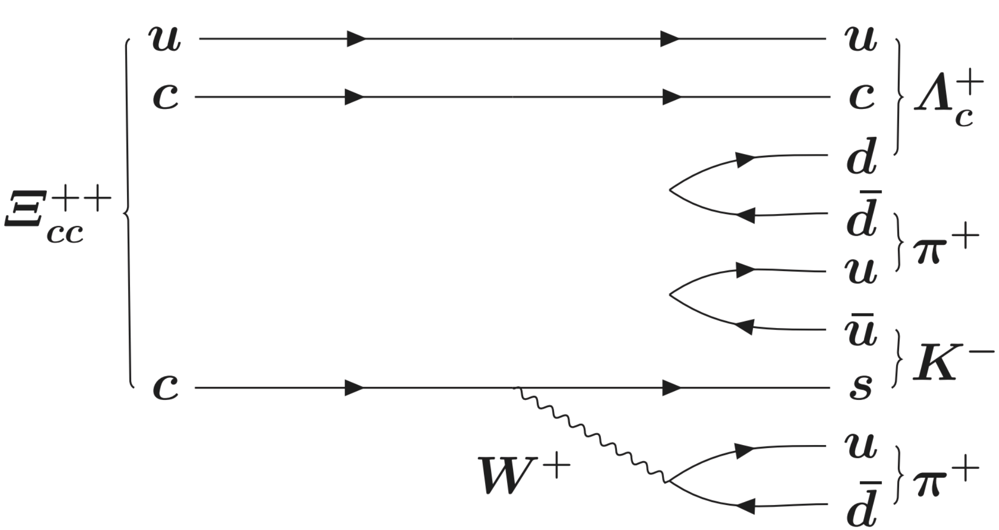
Searching for the doubly charmed baryon

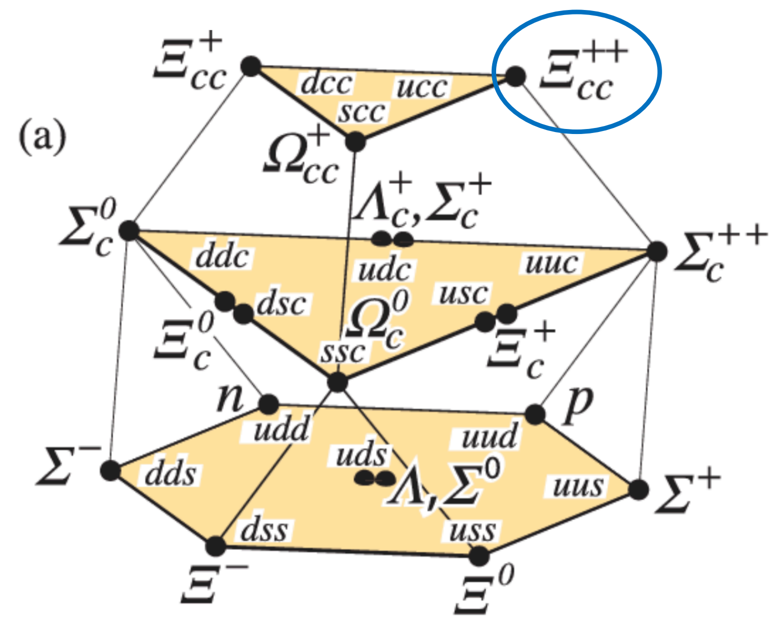
In 2017, the LHCb collaboration observed the Ξ_{cc}^{++} baryon.

Mass: $3620.6 \pm 1.5(\text{stat}) \pm 0.4(\text{syst}) \pm 0.3(\Xi_c^+)\,\text{MeV}/c^2$

Phys. Rev. Lett. 112001 (2017)

$$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$$





It is firstly predicted by: F.-S. Yu, H.-Y. Jiang, R.-H. Li, C.-D. Lü, W. Wang and Z.-X. Zhao, Chin. Phys. C 42, 051001 (2018)

Searching for the doubly charmed baryon

In 2018, the LHCb collaboration observed a two-body decay of Ξ_{cc}^{++} :

Phys. Rev. Lett. 121, 162002 (2018)

$$\Xi_{cc}^{++} \to \Xi_c^{+}\pi^{+}$$

$$\mathcal{R}(\mathcal{B}) \equiv \frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+}\pi^{+}) \times \mathcal{B}(\Xi_c^{+} \to pK^{-}\pi^{+})}{\mathcal{B}(\Xi_{cc}^{++} \to \Lambda_c^{+}K^{-}\pi^{+}\pi^{+}) \times \mathcal{B}(\Lambda_c^{+} \to pK^{-}\pi^{+})} = 0.035 \pm 0.009(\text{stat}) \pm 0.003(\text{syst})$$

Also predicted by:
Chin. Phys. C 42, 051001 (2018)

In 2022, a similar two-body decay of Ξ_{cc}^{++} was observed by LHCb:

JHEP 05 (2022) 038

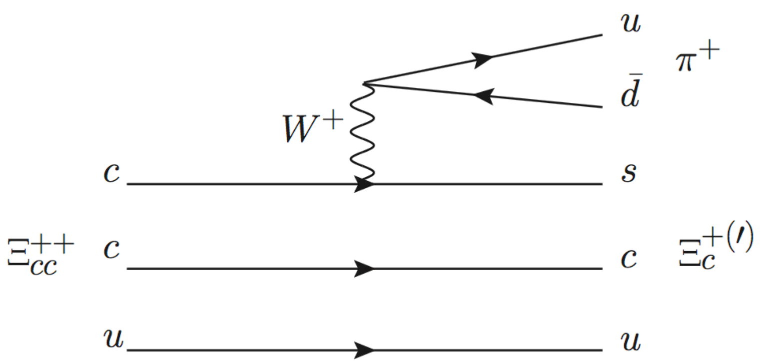
$$\Xi_{cc}^{++} \to \Xi_c^{+\prime}\pi^{+}$$

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+\prime}\pi^{+})}{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+}\pi^{+})} \equiv \frac{\mathcal{B}'}{\mathcal{B}} = 1.41 \pm 0.17 \pm 0.1$$

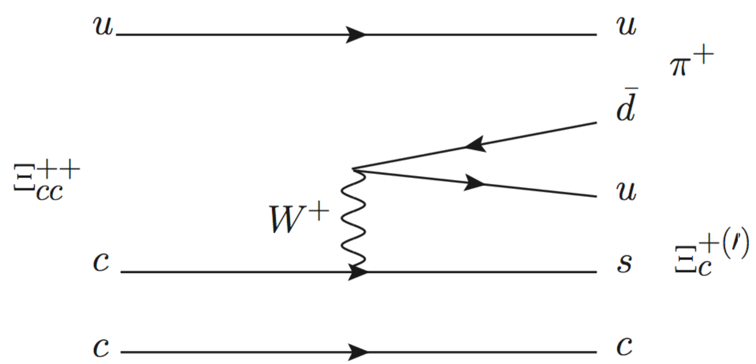
It is much larger than theoretical predictions

The weak decay of $\Xi_{cc}^{++} \longrightarrow \Xi_c^{+(\prime)}\pi^+$

$\Xi_{cc}^{++} \longrightarrow \Xi_c^{+(\prime)}\pi^+$ receives contributions from two topological diagrams: the **W-emission diagram (left)** and the **W-exchange diagram (right):**



Factorizable (fac)

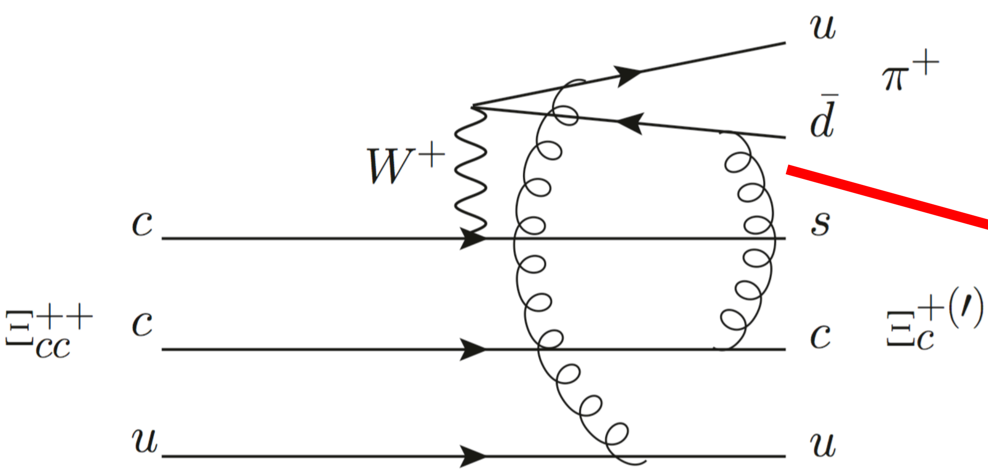


Non-factorizable (nf)

Recently, the theoretical prediction show that:

$$\text{fac} + \text{nf}: \quad \mathcal{B}'/\mathcal{B} \sim 0.8 < 1$$ **Too small**

The W-emission diagram



Color transparency:

The fast running pion can hardly be caught up by the soft gluons.

Naïve factorization: $\mathcal{A}_{\text{fac}} \propto \langle \pi^+ | \bar{u}\gamma_\mu(1-\gamma_5)d|0\rangle \langle \Xi_c^{+(\prime)} | \bar{s}\gamma^\mu(1-\gamma_5)c|\Xi_{cc}^{++}\rangle$

Pion decay constant **Form factors**

$$\langle \pi(q)|\bar{u}\gamma_\mu(1-\gamma_5)d|0\rangle = if_\pi q_\mu$$

$$\bar{u}_2\left[f_1(q^2)\gamma_\mu - f_2(q^2)i\sigma_{\mu\nu}\frac{q^\nu}{M} + f_3(q^2)\frac{q_\mu}{M} - \left(g_1(q^2)\gamma_\mu - g_2(q^2)i\sigma_{\mu\nu}\frac{q^\nu}{M} + g_3(q^2)\frac{q_\mu}{M}\right)\gamma_5\right]u_1$$

The W-emission diagram

$$\langle \Xi_c^{+(\prime)}(p-q)\pi^+(q)|\mathcal{H}_{\rm eff}(0)|\Xi_{cc}^{++}(p)\rangle_{\rm fac,nf} = i\ \bar{u}(p-q)\left[A^{(\prime){\rm fac,nf}} + B^{(\prime){\rm fac,nf}}\gamma_5\right]u(p)$$

$\Xi_{cc}^{++} \to \Xi_c^{+(\prime)}\pi^+$	$A^{\rm fac}$	$B^{\rm fac}$	$A^{\prime\rm fac}$	$B^{\prime\rm fac}$
QCDSR	-8.74	16.76	-3.55	34.13
LFQM	7.40	15.06	4.49	48.50
3LQM	-8.13	-12.97	-4.34	-37.59
NRQM	7.38	16.77	4.29	53.65
HQET	9.52	19.45	5.10	62.37

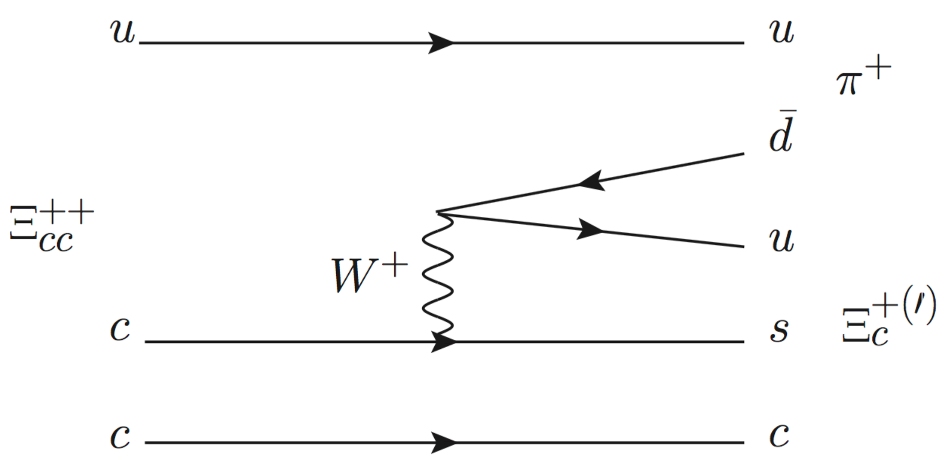
QCDSR: Y. J. Shi, W. Wang and Z. X. Zhao, Eur. Phys. J. C **80**, no.6, 568 (2020)

LFQM: H. Y. Cheng, G. Meng, F. Xu and J. Zou, Phys. Rev. D **101**, no.3, 034034 (2020)

3LQM: T. Gutsche et al. Phys. Rev. D **99**, no.5, 056013 (2019)

NRQM:, HQET: R. Dhir and N. Sharma, Eur. Phys. J. C **78**, no.9, 743 (2018)

The W-exchange diagram



It is non-factorizable and has only been evaluated by the Pole Model in the literature.

Pole Model:

$$A^{\rm pole} = - \sum_{B_n^*(1/2^-)} \left[\frac{g_{B_f B_n^* P}\, b_{n^* i}}{m_i - m_{n^*}} + \frac{b_{f n^*}\, g_{B_n^* B_i P}}{m_f - m_{n^*}} \right]$$

Strong couplings

$$B^{\rm pole} = \sum_{B_n} \left[\frac{g_{B_f B_n P}\, a_{ni}}{m_i - m_n} + \frac{a_{fn}\, g_{B_n B_i P}}{m_f - m_n} \right],$$

N. Sharma and R. Dhir, Phys. Rev. D **96**, no.11, 113006 (2017)

H. Y. Cheng, G. Meng, F. Xu and J. Zou, Phys. Rev. D **101**, no.3, 034034 (2020)

The W-exchange diagram

Method	$\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+)$	$\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+\prime} \pi^+)$	$\mathcal{B}'/\mathcal{B}$
LFQM+PM	0.69 %	4.65 %	6.74
3LCQM	0.71 %	3.39 %	4.77
HQET+PM	6.64 %	5.39 %	0.81
NRQM+PM	9.19 %	7.34 %	0.8

Experiment:

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+\prime} \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+)} \equiv \frac{\mathcal{B}'}{\mathcal{B}} = 1.41 \pm 0.17 \pm 0.1$$

This mismatching urges us to make a more in-depth study on the W-exchange contribution.

Theoretical Method

Light-cone sum rules

The framework of light-cone sum rules

To calculate a matrix element:

$$\langle \Xi_c^{+(\prime)}(p-q)\pi^+(q)|\mathcal{O}_i(0)|\Xi_{cc}^{++}(p)\rangle = i\ \bar{u}(p-q)(A^{(\prime)i} + B^{(\prime)i}\gamma_5)u(p)$$

$$\mathcal{O}_1 = \bar{s}\gamma_\mu(1-\gamma_5)c\ \bar{u}\gamma^\mu(1-\gamma_5)d,$$

$$\mathcal{O}_2 = \bar{s}_a\gamma_\mu(1-\gamma_5)c_b\ \bar{u}_b\gamma^\mu(1-\gamma_5)d_a,$$

We have to define a correlation function:

$$\Pi^{\mathcal{O}_i}(p,q,k) = i^2\int d^4x\ e^{-i(p-q)\cdot x}\int d^4y\ e^{i(p-k)\cdot y}\langle 0|T\left\{J_{\Xi_c^{(\prime)}}(y)\mathcal{O}_i(0)\bar{J}_{\Xi_{cc}}(x)\right\}|\pi^-(q)\rangle$$

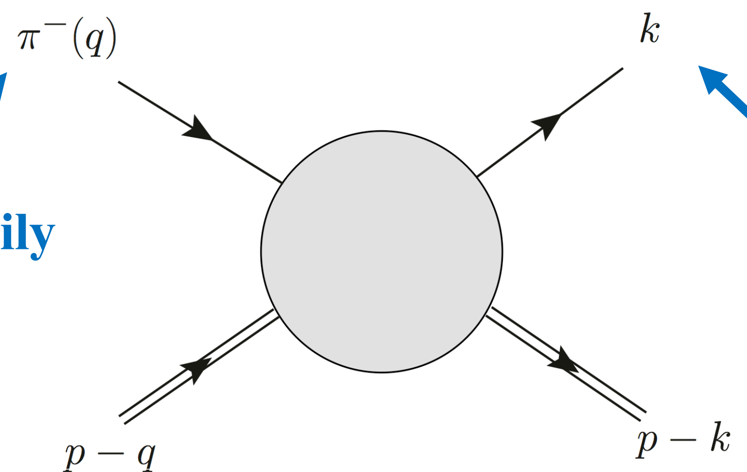
$$J_{\Xi_c} = \frac{1}{\sqrt{2}}\varepsilon_{abc}\left(u_a^T C\gamma_5 s_b - s_a^T C\gamma_5 u_b\right)Q_c,$$

$$J_{\Xi_c^\prime} = \frac{1}{\sqrt{2}}\varepsilon_{abc}\left(u_a^T C\gamma^\mu s_b + s_a^T C\gamma^\mu u_b\right)\gamma_\mu\gamma_5 Q_c,$$

$$J_{\Xi_{cc}} = \varepsilon_{abc}\left(Q_a^T C\gamma^\mu Q_b\right)\gamma_\mu\gamma_5 u_c,$$

Baryon currents:

The framework of light-cone sum rules



The pion is temporarily set as an initial state

k is an auxiliary momentum

$$\Pi^{\mathcal{O}_i}(p,q,k) = i^2 \int d^4x\, e^{-i(p-q)\cdot x} \int d^4y\, e^{i(p-k)\cdot y} \langle 0|T\left\{J_{\Xi_c^{(\prime)}}(y)\mathcal{O}_i(0)\bar{J}_{\Xi_{cc}}(x)\right\}|\pi^-(q)\rangle$$

Hadron level = **Quark-Gluon level**

Expressed by $A^{(\prime)i}, B^{(\prime)i}$

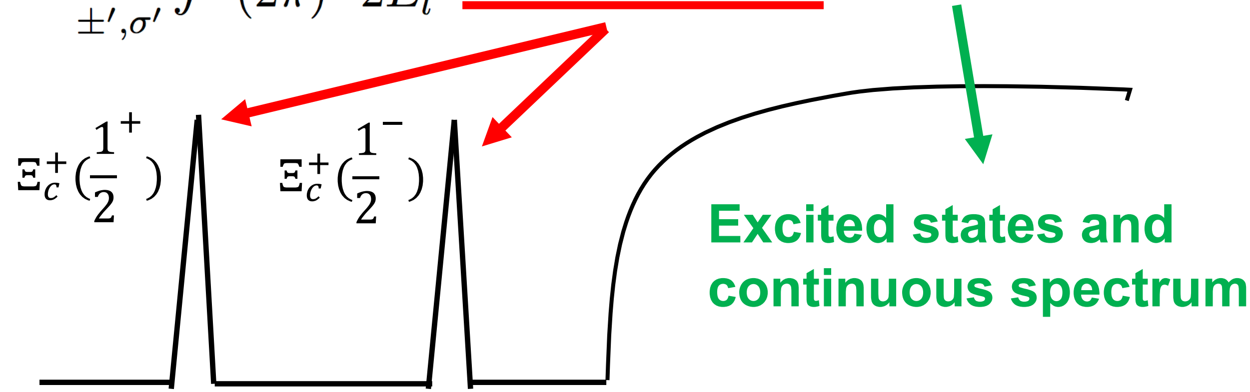
Operator product expansion (OPE)

Hadron Level: insert Ξ_c

$$i^2 \int d^4x\, e^{-i(p-q)\cdot x} \int d^4y\, e^{i(p-k)\cdot y} \langle 0 | T \left\{ J_{\Xi_c^{(\prime)}}(y) \mathcal{O}_i(0) \bar{J}_{\Xi_{cc}}(x) \right\} | \pi^-(q) \rangle$$

Insert

$$\sum_{\pm',\sigma'} \int \frac{d^3\vec{l}}{(2\pi)^3} \frac{1}{2E_l} |l, \sigma', \pm'\rangle\langle l, \sigma', \pm'| + \cdots$$



We can factorize out the matrix element:

$$\langle 0 | J_{\Xi_c}(y) | l, \sigma', + \rangle = \lambda^+_{\Xi_c} u(l, \sigma') e^{-il\cdot y},$$

$$\langle 0 | J_{\Xi_c}(y) | l, \sigma', - \rangle = \lambda^-_{\Xi_c} i\gamma_5 u(l, \sigma') e^{-il\cdot y},$$

If the pion is arranged as a final state at the beginning, this factorization cannot be realized.

Hadron Level: insert Ξ_c

$$i^2 \int d^4x d^4y\ e^{-i(p-q)\cdot x} e^{i(p-k)\cdot y}$$

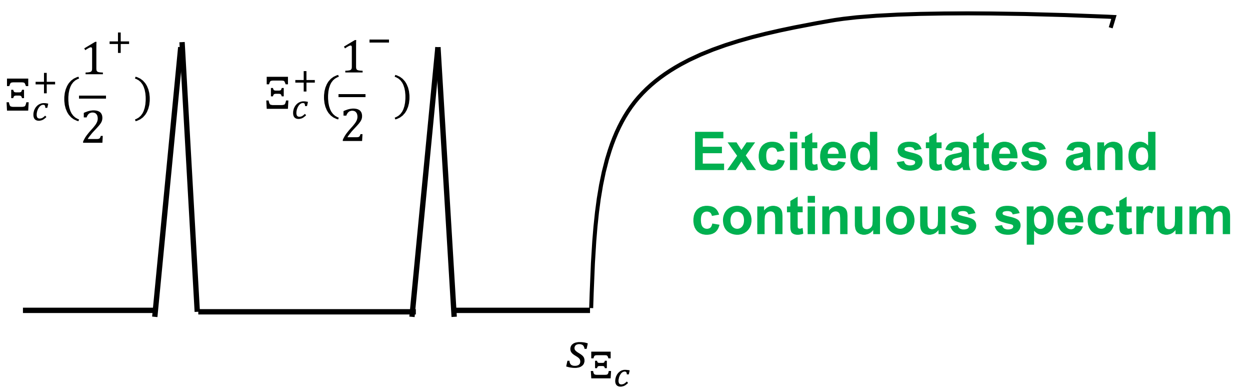
$$\times \sum_{\pm',\sigma'} \int \frac{d^3\vec{l}}{(2\pi)^3} \frac{1}{2E_l} \langle 0|J_{\Xi_c}(y)|l,\sigma',\pm'\rangle \langle l,\sigma',\pm'|\mathcal{O}_i(0)\bar{J}_{\Xi_{cc}}(x)|\pi^-(q)\rangle$$

$$+ \int_{s_{\Xi_c}}^{\infty} ds' \frac{\rho_{\Xi_c}(s',(p-q)^2,P^2)}{s'-(p-k)^2},$$

Depends on

$$p^2,\ q^2,\ k^2,\ (p-q)^2 \text{ and } P^2 = (p-k-q)^2.$$

There is no l^2 since it is on shell



Hadron Level: insert Ξ_c

$$\Pi_H^{\mathcal{O}_i}(p,q,k)_{\mathrm{WE}} = i^3 \int d^4x\, e^{-i(p-q)\cdot x} \sum_{\pm',\sigma'} \frac{1}{(p-k)^2 - m_{\Xi_c}^{\pm'2}}$$
$$\times \lambda_{\Xi_c}^{\pm'} u^{\pm'}(p-k,\sigma') \boxed{\langle p-k,\sigma',\pm'|\mathcal{O}_i(0)\bar{J}_{\Xi_{cc}}(x)|\pi^-(q)\rangle}$$
$$+ \int_{s_{\Xi_c}}^{\infty} ds' \frac{\rho_{\Xi_c}(s',(p-q)^2,P^2)}{s'-(p-k)^2},$$

Only depends on

$$(p-q)^2 \text{ and } P^2 = (p-k-q)^2$$

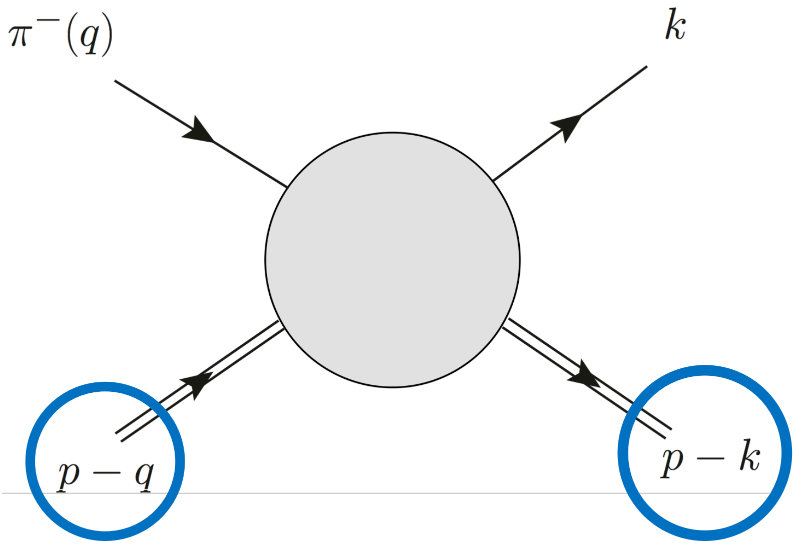
if for simplicity we require that:

$$p^2 = k^2 = 0 \qquad q^2 = m_\pi^2 \approx 0.$$

Hadron Level: Quark-hadron duality

Set the external momentums at the deep Euclidean region:

$$(p-k)^2 \sim (p-q)^2 \sim P^2 \ll 0$$



Calculated by OPE

$$\Pi_H^{\mathcal{O}_i}(p,q,k)_{\text{WE}} = \Pi_{QCD}^{\mathcal{O}_i}(p,q,k)_{\text{WE}} = \frac{1}{\pi}\int_{(m_c+m_s)^2}^{\infty} ds' \frac{\text{Im}\Pi_{QCD}^{\mathcal{O}_i}(s',(p-q)^2,P^2)_{\text{WE}}}{s'-(p-k)^2}$$

Threshold to produce Ξ_c

Expressed by dispersion integration

Hadron Level: Quark-hadron duality

Quark-Hadron duality

$$\Pi_H^{\mathcal{O}_i}(p,q,k)_{\rm WE} = \Pi_{QCD}^{\mathcal{O}_i}(p,q,k)_{\rm WE} = \frac{1}{\pi}\int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' \frac{{\rm Im}\Pi_{QCD}^{\mathcal{O}_i}(s',(p-q)^2,P^2)_{\rm WE}}{s'-(p-k)^2}$$

$$+\frac{1}{\pi}\int_{s_{\Xi_c}}^{\infty} ds' \frac{{\rm Im}\Pi_{QCD}^{\mathcal{O}_i}(s',(p-q)^2,P^2)_{\rm WE}}{s'-(p-k)^2}$$

$$\int_{s_{\Xi_c}}^{\infty} ds' \frac{\rho_{\Xi_c}(s',(p-q)^2,P^2)}{s'-(p-k)^2}$$

Canceled above s_{Ξ_c} $\quad s_{\Xi_c} > (m_c + m_s)^2$

Borel Transformation for $(p-k)^2$

$$\mathcal{B}_{T'^2}\left[(p-k)^{2n}\right] = 0$$

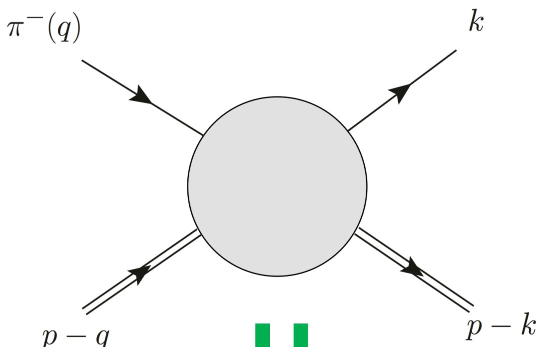
$$\mathcal{B}_{T'^2}\left[\frac{1}{[s'-(p-k)^2]^n}\right] = \frac{1}{(n-1)!}\frac{\exp[-s'/T'^2]}{(p-k)^{2(n-1)}}$$

Excited states (large s') can be suppressed

Hadron Level: Analytical continuity

We have to move the pion to the final state

$$\Pi_H^{\mathcal{O}_i}(p,q,k)_{\mathrm{WE}} = \frac{1}{\pi}\int_{(m_c+m_s)^2}^{\infty} ds' \frac{\mathrm{Im}\Pi_{QCD}^{\mathcal{O}_i}(s',(p-q)^2,P^2)_{\mathrm{WE}}}{s'-(p-k)^2}$$

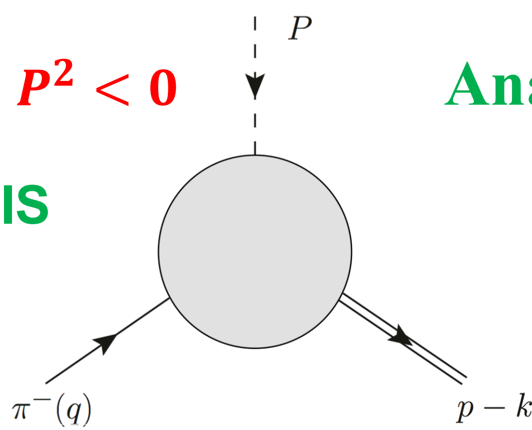


An analytical function of P^2

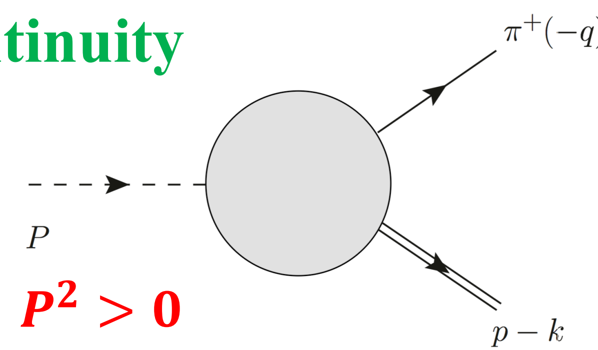
$=$

$P^2 < 0$

Like DIS



Analytical continuity



$P^2 > 0$

Hadron Level: insert Ξ_{cc}

The sum rules equation:

$$\sum_{\pm',\pm,\sigma',\sigma} e^{-m_{\Xi_c}^{\pm'2}/T'^2 - m_{\Xi_{cc}}^{\pm 2}/T^2} \lambda_{\Xi_c}^{\pm'} \lambda_{\Xi_{cc}}^{\pm}$$

$$\times\, u^{\pm'}(p-k,\sigma') \langle p-k,\sigma',\pm';\pi^+(-q)|\mathcal{O}_i(0)|p-q,\sigma,\pm\rangle_{\rm WE}\, \bar{u}^{\pm}(p-q,\sigma)$$

$$= \frac{1}{\pi^2}\int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' \int_{4m_c^2}^{s_{\Xi_{cc}}} ds\; e^{-s'/T'^2} e^{-s/T^2}\, {\rm Im}^2 \Pi_{QCD}^{\mathcal{O}_i}(s',s,P^2).$$

Only depends on P^2

Parameterizaed as:

$$i\,\bar{u}^{\pm'}(p-k,\sigma')\left[A_{1,i}^{\pm'\pm}(P^2) + B_{1,i}^{\pm'\pm}(P^2)\gamma_5 + A_{2,i}^{\pm'\pm}(P^2)\frac{\not{q}}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{\pm'\pm}(P^2)\frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}}\right] u^{\pm}(p-q,\sigma)$$

Two extra terms due to the non-vanishing k

Hadron Level: Extract amplitudes

2×4×2 = 16 structures in total:

$$i\sum_{\pm\pm'} e^{-m_{\Xi_c}^{\pm'2}/T'^2 - m_{\Xi_{cc}}^{\pm 2}/T^2}\lambda_{\Xi_c}^{\pm'}\lambda_{\Xi_{cc}}^{\pm}\underbrace{(\not p_2 + m_{\Xi_c}^{\pm'})}_{2}\underbrace{\left[A_{1,i}^{\pm'\pm} + B_{1,i}^{\pm'\pm}\gamma_5 + A_{2,i}^{\pm'\pm}\frac{\not q}{m_{\Xi_{cc}}^{\pm}} + B_{2,i}^{\pm'\pm}\frac{\not q\gamma_5}{m_{\Xi_{cc}}^{\pm}}\right]}_{4}\underbrace{(\not p_1 + m_{\Xi_{cc}})}_{2}$$

$$= \frac{1}{\pi^2}\int_{(m_c+m_s)^2}^{s_{\Xi_c}} ds' \int_{4m_c^2}^{s_{\Xi_{cc}}} ds\; e^{-s'/T'^2} e^{-s/T^2}\,\mathrm{Im}^2\Pi_{QCD}^{\mathcal{O}_i}(s', s, P^2)_{\mathrm{WE}},$$

The number of amplitudes is also 16

This enable us to solve out the 16 P^2 dependent amplitudes.

$k = 0$ and $P^2 = m_{\Xi_{cc}}^{+2}$

$\Xi_{cc}^{++}(\frac{1}{2}^+) \to \Xi_c^{+(\prime)}(\frac{1}{2}^+)\pi^+$

$$A_{\mathrm{WE}}^i = A_{1,i}^{++}(m_{\Xi_{cc}}^{+2}) + \left(1 - \frac{m_{\Xi_c}^+}{m_{\Xi_{cc}}^+}\right) A_{2,i}^{++}(m_{\Xi_{cc}}^{+2})$$

$$B_{\mathrm{WE}}^i = B_{1,i}^{++}(m_{\Xi_{cc}}^{+2}) - \left(1 + \frac{m_{\Xi_c}^+}{m_{\Xi_{cc}}^+}\right) B_{2,i}^{++}(m_{\Xi_{cc}}^{+2})$$

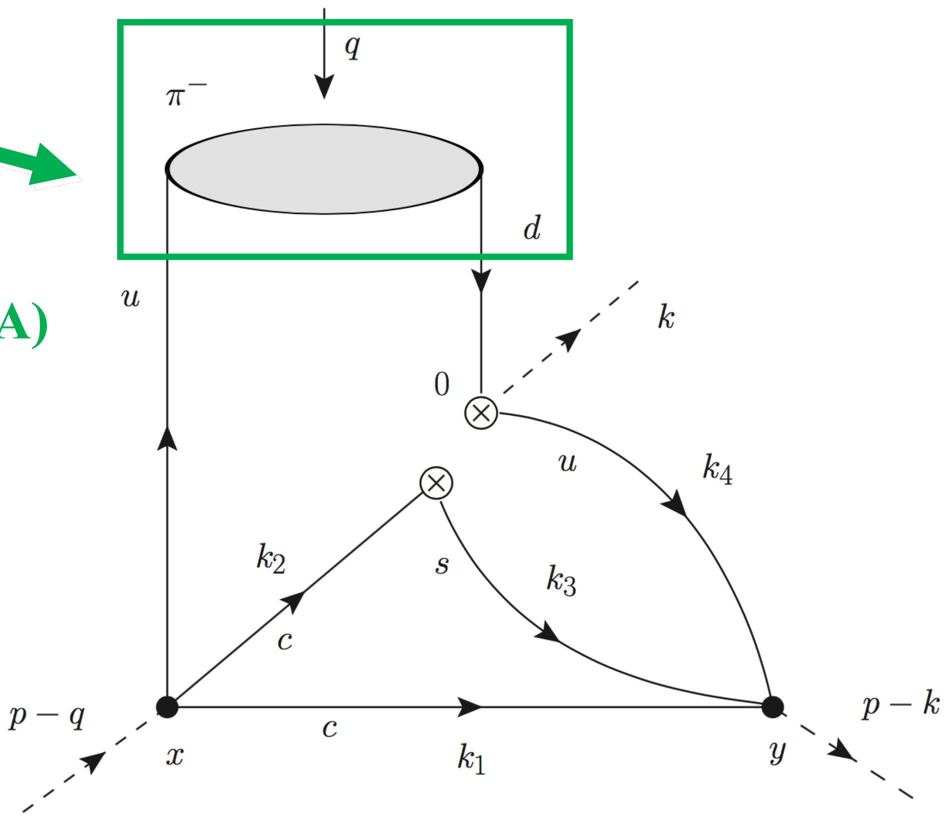
Quark-Gluon Level

$$-2\sqrt{2}\varepsilon_{abc}\varepsilon_{ebc}\int d^4x d^4y\ e^{-i(p-q)\cdot x}e^{i(p-k)\cdot y}$$

$$\times\left[S_Q(y-x)\gamma^\nu C S_Q^T(-x)C(1-\gamma_5)\gamma_\mu C S_s^T(y)C\gamma_5 S_u(y)\gamma^\mu(1-\gamma_5)\right]_{\alpha\beta}(\gamma_\nu\gamma_5)_{\rho\sigma}$$

$$\times\langle 0|\bar{u}_e^\rho(x)d_a^\beta(0)|\pi^-(q)\rangle$$

Parameterized by light-cone distribution functions (LCDA)



Quark-Gluon Level: LCDAs

Light-cone distribution amplitudes of pion:

$$\langle 0|\bar{u}_e^\rho(x) d_a^\beta(0)|\pi^-(q)\rangle = -\frac{i}{12}\delta_{ae} f_\pi \int_0^1 du\; e^{-i\bar{u}q\cdot x}\Big[(\not{q}\gamma_5)_{\beta\rho}\varphi_\pi(u) + (\gamma_5)_{\beta\rho}\mu_\pi \phi_{3\pi}^p(u) + \frac{1}{6}(\gamma_5\sigma_{\mu\nu})_{\beta\rho} q^\mu x^\nu \mu_\pi \phi_{3\pi}^\sigma(u)\Big],$$

Gegenbauer polynomials

Twist

2 $$\varphi_\pi(u) = 6u\bar{u}\left(1 + a_2 C_2^{3/2}(u-\bar{u}) + a_4 C_4^{3/2}(u-\bar{u})\right),$$

3p $$\phi_{3\pi}^p(u) = 1 + 30\frac{f_{3\pi}}{\mu_\pi f_\pi} C_2^{1/2}(u-\bar{u}) - 3\frac{f_{3\pi}\omega_{3\pi}}{\mu_\pi f_\pi} C_4^{1/2}(u-\bar{u}),$$

3σ $$\phi_{3\pi}^\sigma(u) = 6u(1-u)\left(1 + 5\frac{f_{3\pi}}{\mu_\pi f_\pi}\left(1 - \frac{\omega_{3\pi}}{10}\right) C_2^{3/2}(u-\bar{u})\right)$$

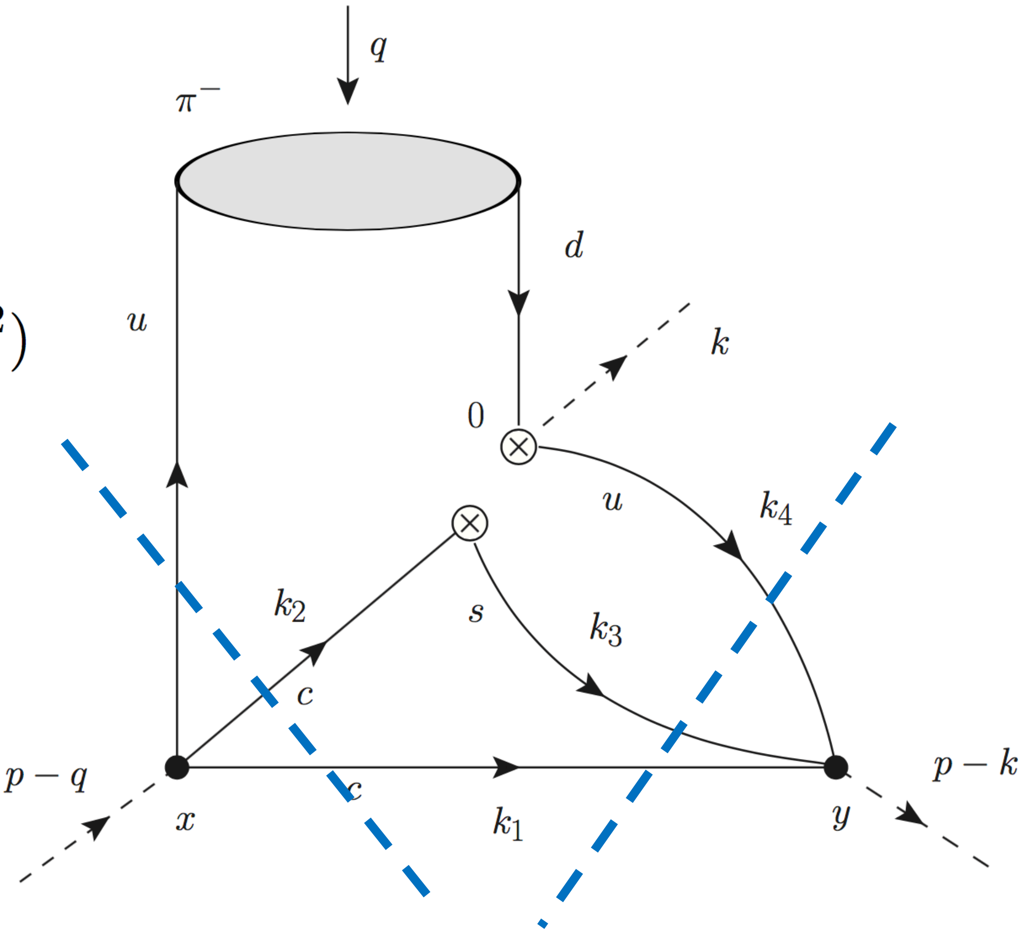
Quark-Gluon Level: Double imaginary part

$$\mathrm{Im}^2\Pi^{\mathcal{O}_i}_{QCD}(s', s, P^2)$$

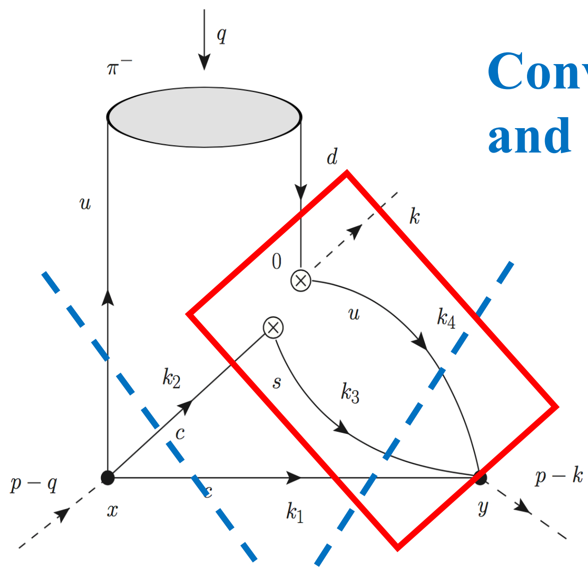
Cutting Rules

$$\frac{1}{l^2 - m^2} \to (-2\pi i)\delta(l^2 - m^2)$$

Each propagator is set on shell



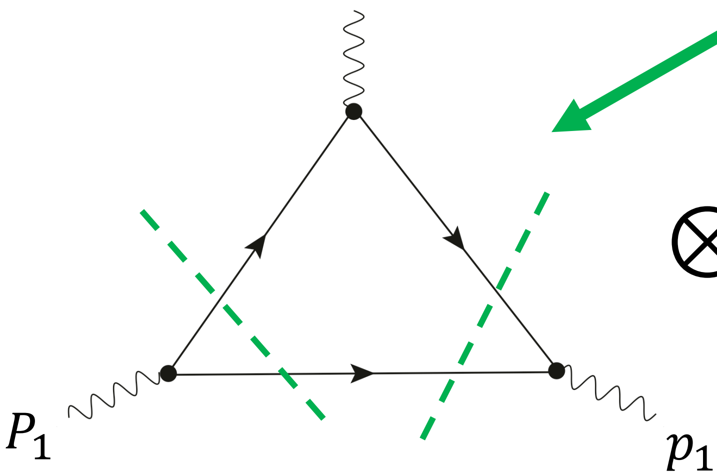
Quark-Gluon Level: Double imaginary part



Convolution of an on-shell triangle integration and a two-body phase integration:

$$\propto \int_0^1 du\ \varphi_\pi(u) \int dm_{34}^2 \int d\Phi_\Delta(P_1^2, p_2^2) \int d\Phi_2(m_{34}^2)$$

$$\times \cdots\cdots$$



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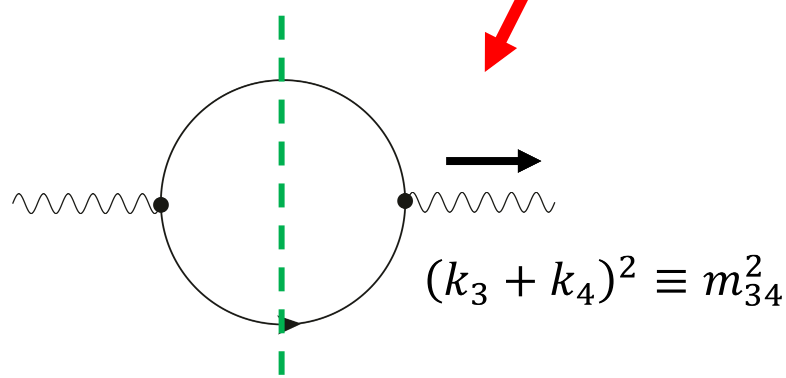


triangle integration

two-body phase space integration

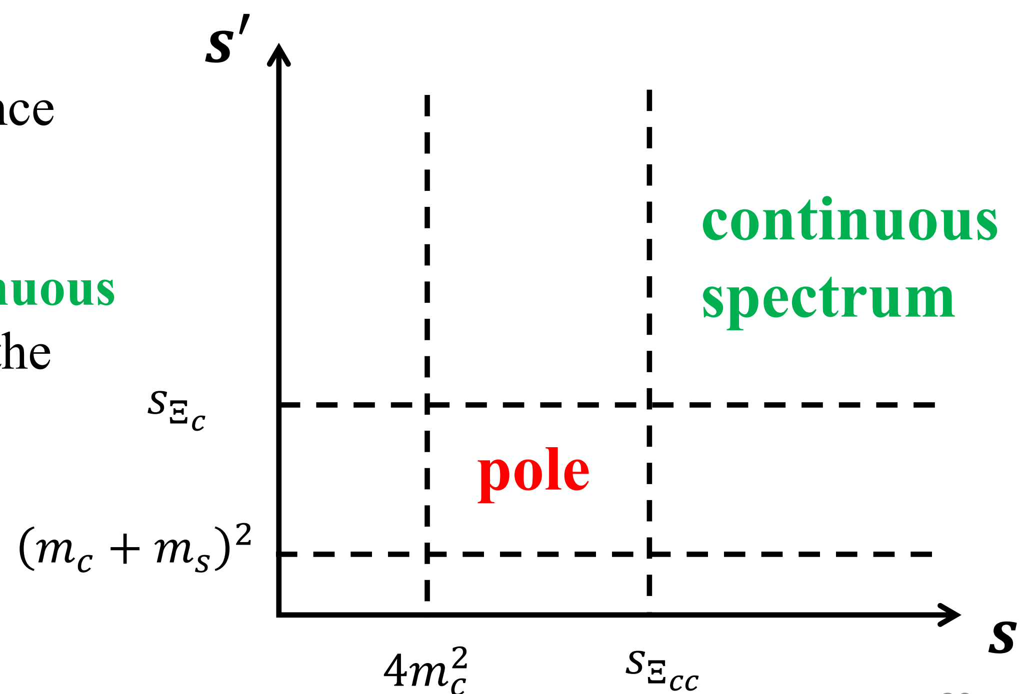
Numerical Results

Numerical Result: Borel parameters

$$i\sum_{\pm\pm'} e^{-m_{\Xi_c}^{\pm'2}/T'^2-m_{\Xi_{cc}}^{\pm 2}/T^2}\lambda_{\Xi_c}^{\pm'}\lambda_{\Xi_{cc}}^{\pm}(\not{p}_2+m_{\Xi_c}^{\pm'})\left[A_{1,i}^{\pm'\pm}+B_{1,i}^{\pm'\pm}\gamma_5+A_{2,i}^{\pm'\pm}\frac{\not{q}}{m_{\Xi_{cc}}^{\pm}}+B_{2,i}^{\pm'\pm}\frac{\not{q}\gamma_5}{m_{\Xi_{cc}}^{\pm}}\right](\not{p}_1+m_{\Xi_{cc}})$$
$$=\frac{1}{\pi^2}\int_{(m_c+m_s)^2}^{s_{\Xi_c}}ds'\int_{4m_c^2}^{s_{\Xi_{cc}}}ds\; e^{-s'/T'^2}e^{-s/T^2}\mathrm{Im}^2\Pi_{QCD}^{\mathcal{O}_i}(s',s,P^2)_{\mathrm{WE}},$$

The choice of Borel parameter T, T' must satisfy:

1. The result has little dependence on T, T'
2. The contribution from **continuous spectrum** must be less than the **pole** contribution



Numerical Result: Borel parameters

Two assumptions :

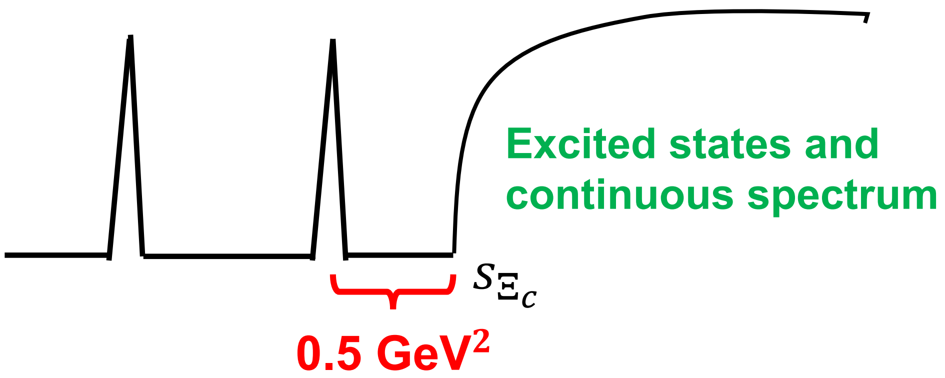
(1) $$\frac{T^2}{T'^2} \approx \frac{M_1^2 - m_1^2}{M_2^2 - m_2}$$

$M_{1(2)}$ is the mass of the initial (final) baryon and $m_{1(2)}$ is the mass of the quark before(after) the weak decay.

Used in the study of D meson decays

P. Ball, V. M. Braun and H. G. Dosch, Phys. Rev. D **44**, 3567-3581 (1991)

(2)
$$s_{\Xi_{cc}} = (4.1 \pm 0.1)^2 \ \text{GeV}^2$$
$$s_{\Xi_c} = (3.2 \pm 0.1)^2 \ \text{GeV}^2$$
$$s_{\Xi_c'} = (3.3 \pm 0.1)^2 \ \text{GeV}^2$$



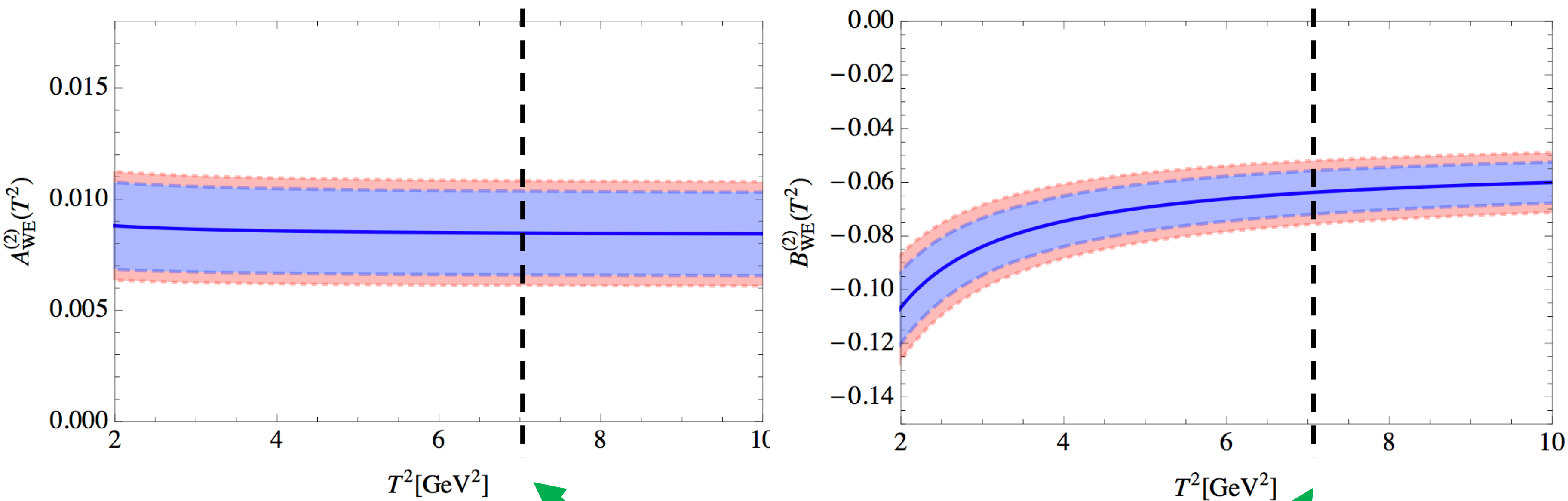
An empirical value from the study of B_c decays

Z. G. Wang, Eur. Phys. J. A **49**, 131 (2013)

Numerical Result: Borel parameters

Blue band: uncertainty of the $s_{\Xi_{cc}}$ and s_{Ξ_c}

Red band: uncertainty of MC integration



Use " Pole > Continuous" to determine the upper bound.

Numerical Result: W-exchange Amplitudes

$$6 < T^2 < 8\ \text{GeV}^2 \text{ for } \Xi_{cc}^{++} \to \Xi_c^+ \pi^+$$

$$5 < T^2 < 7\ \text{GeV}^2 \text{ for } \Xi_{cc}^{++} \to \Xi_c^{+\prime} \pi^+$$

$$s_{\Xi_{cc}} = (4.1 \pm 0.1)^2\ \text{GeV}^2$$

$$s_{\Xi_c} = (3.2 \pm 0.1)^2\ \text{GeV}^2$$

$$s_{\Xi_c'} = (3.3 \pm 0.1)^2\ \text{GeV}^2$$

The uncertainties are used to evaluate the numerical error.

$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$	Twist-2	Twist-3p	Twist-3σ	Total
A_{WE}	0.0084 ± 0.0024	-0.077 ± 0.01	-0.056 ± 0.002	-0.124 ± 0.011
B_{WE}	-0.064 ± 0.01	0.052 ± 0.01	0.165 ± 0.025	0.153 ± 0.029

$\Xi_{cc}^{++} \to \Xi_c^{+\prime} \pi^+$	Twist-2	Twist-3p	Twist-3σ	Total
A'_{WE}	0.0027 ± 0.0005	0.0089 ± 0.002	-0.018 ± 0.0003	-0.0062 ± 0.002
B'_{WE}	0.0023 ± 0.0006	0.052 ± 0.016	0.011 ± 0.003	0.066 ± 0.016

Numerical Result: W-exchange Amplitudes

$$\langle \Xi_c^{+(\prime)}(p-q)\pi^+(q)|\mathcal{H}_{\rm eff}(0)|\Xi_{cc}^{++}(p)\rangle_{\rm fac,nf} = i\ \bar{u}(p-q)\left[A^{(\prime){\rm fac,nf}} + B^{(\prime){\rm fac,nf}}\gamma_5\right]u(p)$$

fac+nf

$\Xi_{cc}^{++}\to\Xi_c^+\pi^+$	$A^{\rm fac}$	$A^{\rm nf}$	$A^{\rm tot}$	$B^{\rm fac}$	$B^{\rm nf}$	$B^{\rm tot}$
This work	$--$	-16.67 ± 1.41	$--$	$--$	20.47 ± 3.89	$--$
QCDSR	-8.74 ± 2.91	$--$	$--$	16.76 ± 5.36	$--$	$--$
LFQM + PM	7.40	-10.79	-3.38	15.06	-18.91	-3.85
3LQM	-8.13	10.50	3.37	-12.97	18.53	5.56
NRQM + PM	7.38	0	7.38	16.77	24.95	41.72
HQET + PM	9.52	0	9.52	19.45	24.95	44.40

$\Xi_{cc}^{++}\to\Xi_c^{+\prime}\pi^+$	$A^{\prime\rm fac}$	$A^{\prime\rm nf}$	$A^{\prime\rm tot}$	$B^{\prime\rm fac}$	$B^{\prime\rm nf}$	$B^{\prime\rm tot}$
This work	$--$	-0.83 ± 0.28	$--$	$--$	8.86 ± 2.16	$--$
QCDSR	-3.55 ± 0.68	$--$	$--$	34.13 ± 11.6	$--$	$--$
LFQM + PM	4.49	-0.04	4.45	48.50	-0.06	48.44
3LQM	-4.34	-0.11	-4.45	-37.59	-1.37	-38.96
NRQM + PM	4.29	0	4.29	53.65	0	53.65
HQET + PM	5.10	0	5.10	62.37	0	62.37

in unit $10^{-2}G_F\ \mathrm{GeV}^2$

Numerical Result: Branching fraction

"fac" from literature and "nf" from this work

Method	A^{tot}	B^{tot}	$\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+)$	A'^{tot}	B'^{tot}	$\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+\prime} \pi^+)$	$\mathcal{B}'/\mathcal{B}$
QCDSR+LCSR	-25.4 ± 4.32	37.23 ± 9.25	40 ± 14 %	-4.38 ± 0.96	42.99 ± 13.76	3.91 ± 2.5 %	0.098 ± 0.14
LFQM+LCSR	-9.27 ± 1.41	35.53 ± 3.89	7.54 ± 2.22 %	3.66 ± 0.28	57.36 ± 2.16	5.83 ± 0.5 %	0.77 ± 0.42
3LCQM+LCSR	-24.8 ± 1.41	7.5 ± 3.89	35.55 ± 4.29 %	-5.17 ± 0.28	-28.73 ± 2.16	2.75 ± 0.35 %	0.08 ± 0.02
NRQM+LCSR	-9.29 ± 1.41	37.24 ± 3.89	7.82 ± 2.25 %	3.46 ± 0.28	62.51 ± 2.16	6.70 ± 0.54 %	0.85 ± 0.44
HQET+LCSR	-7.18 ± 1.41	39.92 ± 3.89	6.22 ± 1.94 %	4.27 ± 0.28	71.23 ± 2.16	8.85 ± 0.62 %	1.42 ± 0.78
LFQM+PM	-3.83	3.85	0.69 %	4.45	48.44	4.65 %	6.74
3LCQM	3.37	5.56	0.71 %	-4.45	-38.96	3.39 %	4.77
HQET+PM	7.38	41.72	6.64 %	4.29	53.65	5.39 %	0.81
NRQM+PM	9.52	44.40	9.19 %	5.1	62.37	7.34 %	0.8
FSR($\eta = 1.0$)	−−	−−	7.11%	−−	−−	4.72 %	0.66
FSR($\eta = 1.5$)	−−	−−	8.48%	−−	−−	4.72 %	0.56
FSR($\eta = 2.0$)	−−	−−	10.75%	−−	−−	4.74 %	0.44

Final state rescattering

Chin. Phys. C **45**, no.5, 053105 (2021)

Consistent with experiment

$$(\mathcal{B}'/\mathcal{B})_{\text{expt}} = 1.41 \pm 0.17 \pm 0.1$$

The interference between B^{fac} and B^{nf} tends to be constructive

Summary

- The W-exchange amplitudes of $\Xi_{cc}^{++} \to \Xi_c^{+(\prime)} \pi^+$ are calculated by light-cone sum rules.

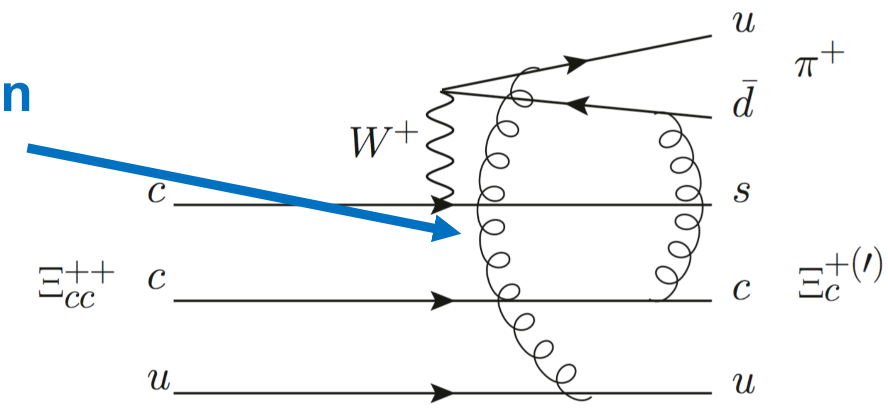
- The possible branching fractions are obtained by combining our W-exchange amplitudes with the factorizable amplitudes from various theoretical works in the literature.

- One of the possible branching fractions is consistent with the experimental result.

Outlook

- Further studies on the W-emission contribution are required to determine the sign of the amplitudes

Effects of soft gluon exchange?



- Contributions from higher twist LCDAs for the calculation of the W-exchange diagram.

Thank you for your attention !