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# $N\phi$ interaction from lattice QCD


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- Introduction
- Theoretical framework
- Results and discussions
- Summary and outlook

# $N\phi$ interaction

- Fundamental inputs for studying

R. S. Hayano and T. Hatsuda, Rev. Mod. Phys. **82**, 2949 (2010)

  - meson properties in nuclear matter
  - modification of QCD condensates relevant to chiral symmetry

- Experimental studies

L. Tolos and L. Fabbietti, Prog. Part. Nucl. Phys. 112, 103770 (2020); ALICE Coll, Phys. Rev. Lett. 127, 172301 (2021)
I. Strakovsky, L. Pentchev, and A. Titov, Phys. Rev. C 101, 045201 (2020)

  - production and absorption reactions with various nuclear targets
  - low-energy properties are little known

| **Reference** | **Spin-averaged $a_0$ [fm]** |
|---|---|
| Strakovsky *et al.* 2020 | $\pm 0.063(10)$ |
| ALICE 2021 | $-0.85(34)_{\text{stat.}}(14)_{\text{syst.}}$ |

! huge discrepancy !

- Theoretical studies

P. Gubler and D. Satow, Prog. Part. Nucl. Phys. 106, 1 (2019); T. Hatsuda and S. H. Lee, Phys. Rev. C 46, R34 (1992)
J. Castella and G. Krein, Phys. Rev. D 98, 014029 (2018)

  - $\phi$ in-medium property $\leftrightarrow$ $\bar{s}s$ condensate
  - open questions on $N\phi$: microscopic origin, the role of dynamical $\pi$

- A *first principle study* is highly needed $\rightarrow$ LQCD

# Lattice calculation of $N\phi$ interaction

- For exploring the role of dynamical pion, we need
  - dynamical light quarks near the physical point
  - large lattice volume to accommodate propagating of pion

- Highest spin channel, $N\phi(\,^4S_{3/2})$, is easy to handle, due to small open channel effects
  - two-body channels $\Lambda K, \Sigma K$ ( $^2D_{3/2}$) are kinematically suppressed
  - (phase space suppressed) multi-particle channels including: $N\phi \to \Sigma^* K \to \Lambda\pi K \ldots$

- In this talk: $N\phi$ system in a large volume with near the physical quark masses
  - interaction potential by HAL QCD method
  - role of dynamical pion
  - scattering properties

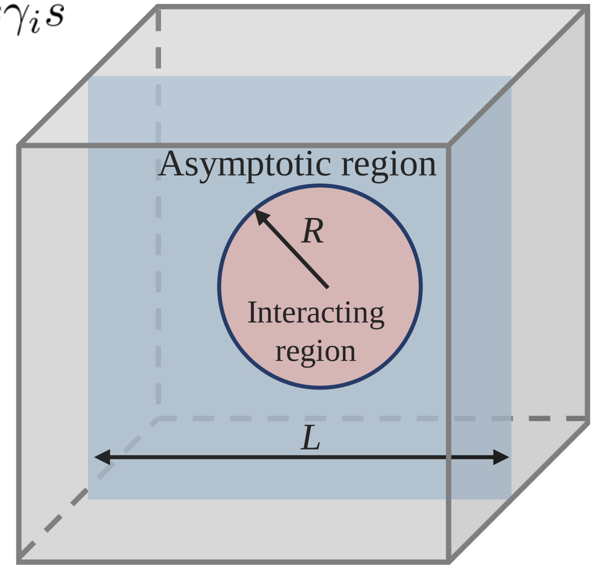
# HAL QCD method

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99,** 022001 (2007)

- Nambu-Bethe-Salpeter (NBS) amplitude

$$\psi^k(\boldsymbol{r},t) = \langle 0|N(\boldsymbol{r},0)\phi(\boldsymbol{0},0)|N(\boldsymbol{k})\phi(-\boldsymbol{k});E\rangle$$


$N_\alpha = (u^T C\gamma_5 d)u_\alpha$ $\phi_i = \bar{s}\gamma_i s$




- asymptotic region
  - ✓ Helmholtz eq. $(\nabla^2 + k^2)\psi(r) \simeq 0$
  - ✓ scattering phase shifts $\psi^k(r) \simeq A\frac{\sin(kr+\delta(k))}{kr}$
- "interacting" region

$$(\nabla^2 + k^2)\psi^k(\boldsymbol{r}) = m \times \int d^3r' U(\boldsymbol{r},\boldsymbol{r}')\psi^k(\boldsymbol{r}')$$

  - ✓ non-local energy-independent kernel $U(\boldsymbol{r},\boldsymbol{r}')$
  - ✓ derivative expansion $U(\boldsymbol{r},\boldsymbol{r}') = V(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}') + \sum V_i(\boldsymbol{r})\nabla^i\delta(\boldsymbol{r}-\boldsymbol{r}')$

Check on convergence: Iritani *et al.*, JHEP 03, 007 (2019); YL *et al.*, PRD 105, 074512 (2022)

- In practice, it is exponentially difficult to obtain $\psi^k(r)$ for baryonic system

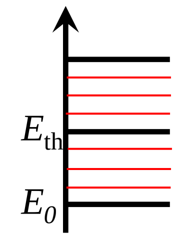
G. P. Lepage, in From Actions to Answers: Proceedings of the TASI 1989

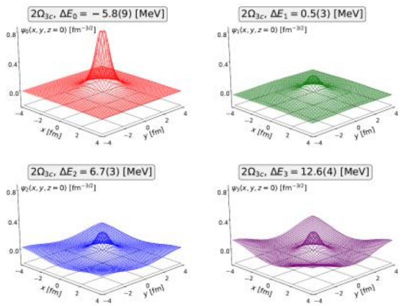
# Time-dependent HAL QCD method

N. Ishii, *et al.* (HAL QCD Collaboration), Phys. Lett. B **712,** 437 (2012)

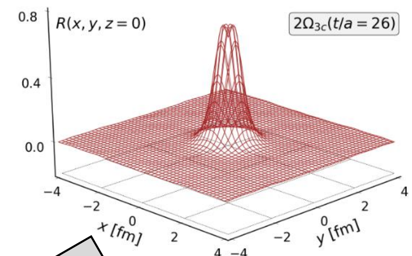
- Define R-correlator as a summation over all elastic states

$$R(\boldsymbol{r},t) = \sum_n a_n \psi_n(\boldsymbol{r}) e^{-E_n t} \xrightarrow{t \gg \frac{1}{E_{\rm th}-E_0}} \sum_{n<N_{\rm th}} a_n \psi_n(\boldsymbol{r}) e^{-E_n t}$$

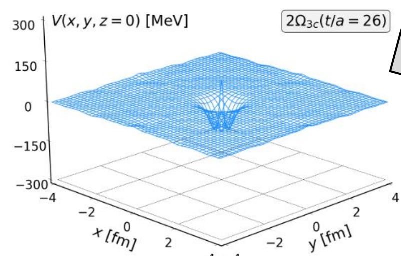



eigenstates




Rcorr.




potential

- Master equation

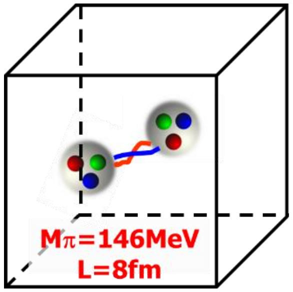
$\sim k^2/(2\mu)$

$$\left(\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)$$

- $\delta = \frac{m_N - m_\phi}{m_N + m_\phi}$ ; $\mu = \frac{m_N m_\phi}{m_N + m_\phi}$

# Lattice setup

- (2+1)-flavor configuration near the physical point
  - Iwasaki gauge action and $O(a)$-improved Wilson quark action




| $L \times T$ | $a$ [fm] | $La$ [fm] | $m_\pi$ [MeV] | $m_K$ [MeV] |
|---|---|---|---|---|
| $96^3 \times 96$ | 0.0846(7) | 8.1 | 146 | 525 |

**K.-I. Ishikawa, *et al.*** (PACS Collaboration), *Proc. Sci.*, LATTICE2015 (2016) 075

- Relevant hadron masses

| Hadron | Lattice Mass [MeV] | Physical Mass[MeV] |
|---|---|---|
| $N$ | **954.0(2.9)** | 938.9 |
| $\phi$ | **1048.0(4)** | 1019.5 |

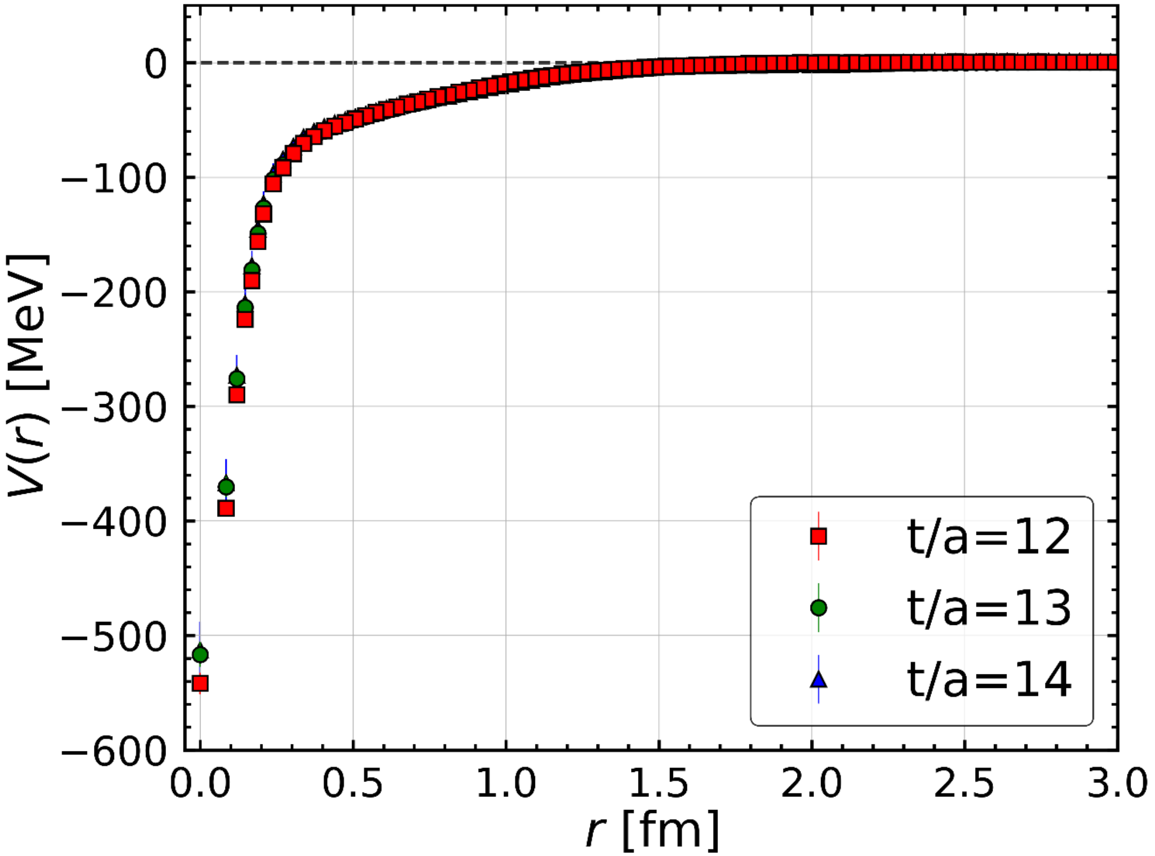
  - Note that $\phi \to K\bar{K}$ is prohibited due to $m_\phi < 2m_K$

- Statistics

$$200_{\text{conf.}} \times 80_{\text{src}} \times 4_{\text{rot.}} \times 2_{\text{b.f.}} = 128000$$

# Potential

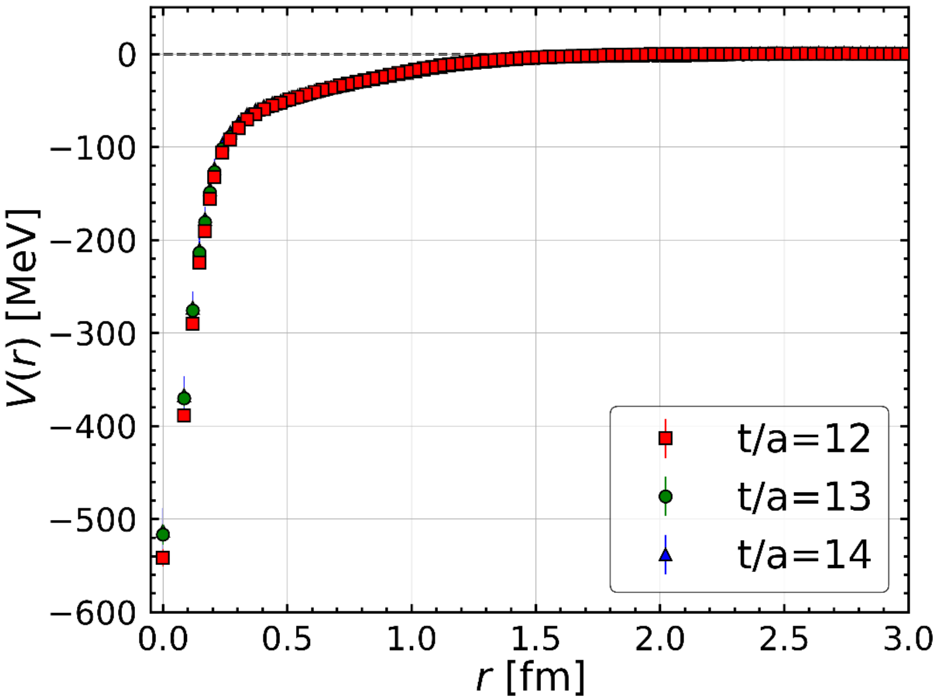
- $N\phi(\,^4S_{3/2})$ potential at Euclidean time 12, 13 and 14



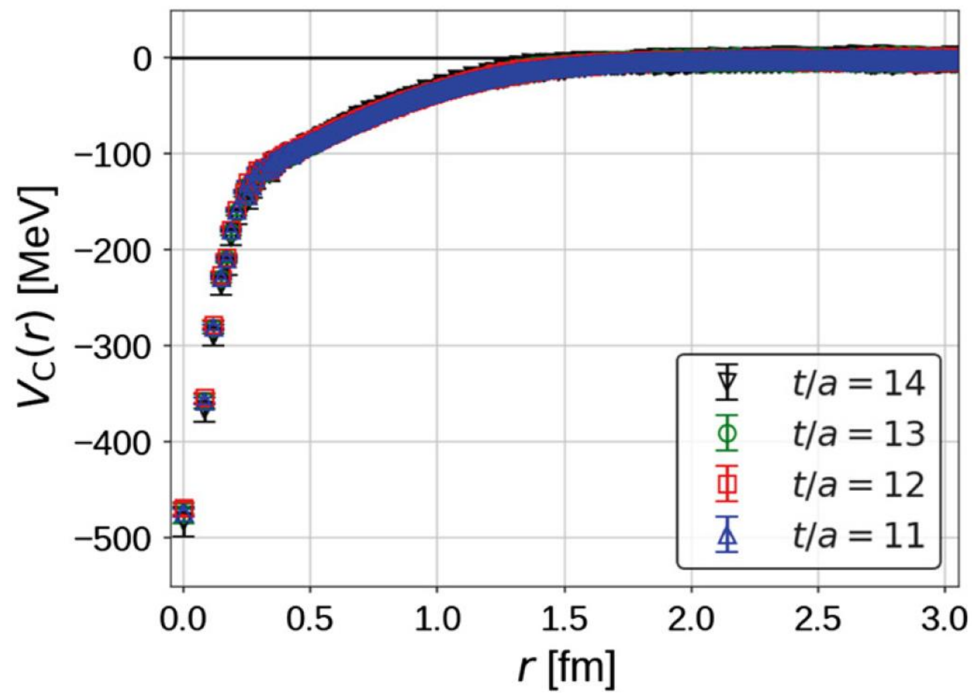

- attractive core, Pauli exclusion does not operate due to no common quarks
- long-ranged attractive tail, hints of pion dynamics
- weak $t$ dependence

# $N\phi$ v.s. $N\Omega$

➢ $N\phi(\,^4S_{3/2})$ potential




➢ $N\Omega(\,^5S_2)$ potential




T. Iritani, *et al.* (HAL QCD Coll.), Phys. Lett. B 792, 284 (2019)

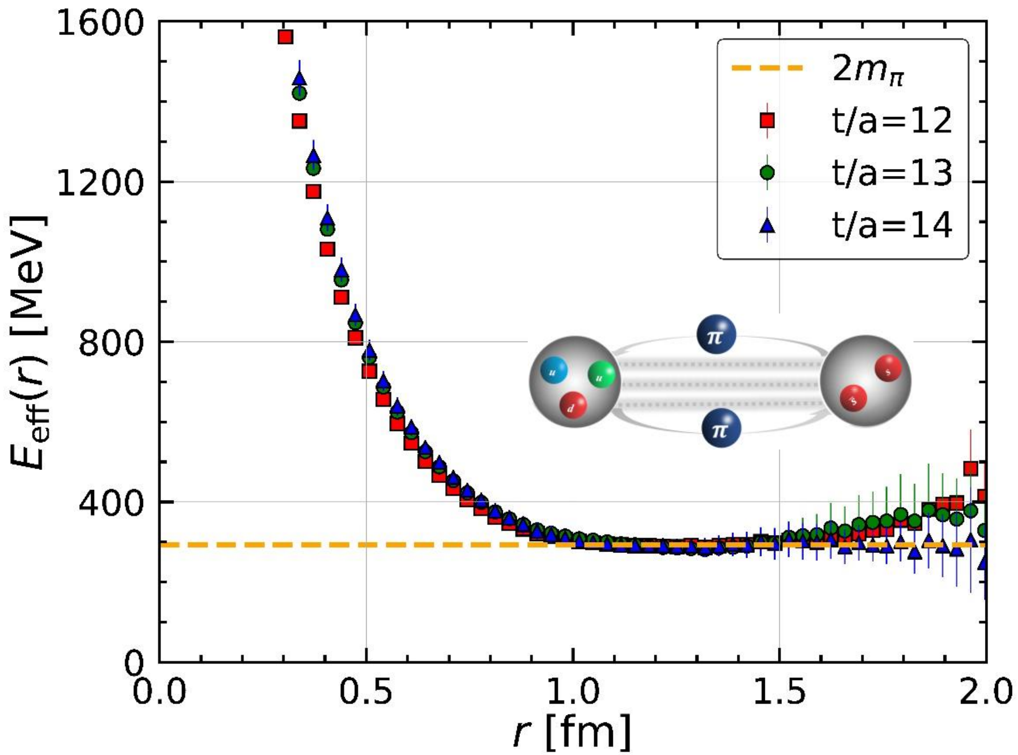
- universality of pion dynamics (?)

# Two-pion-exchange tail

- Interaction between nucleon and heavy quarkonium at large distances

J. Castella and G. Krein, Phys. Rev. D **98**, 014029 (2018);H. Fujii and D. Kharzeev, Phys. Rev. D **60**, 114039 (1999)
G. Bhanot and M. E. Peskin, Nucl. Phys. B **156**, 391(1979)

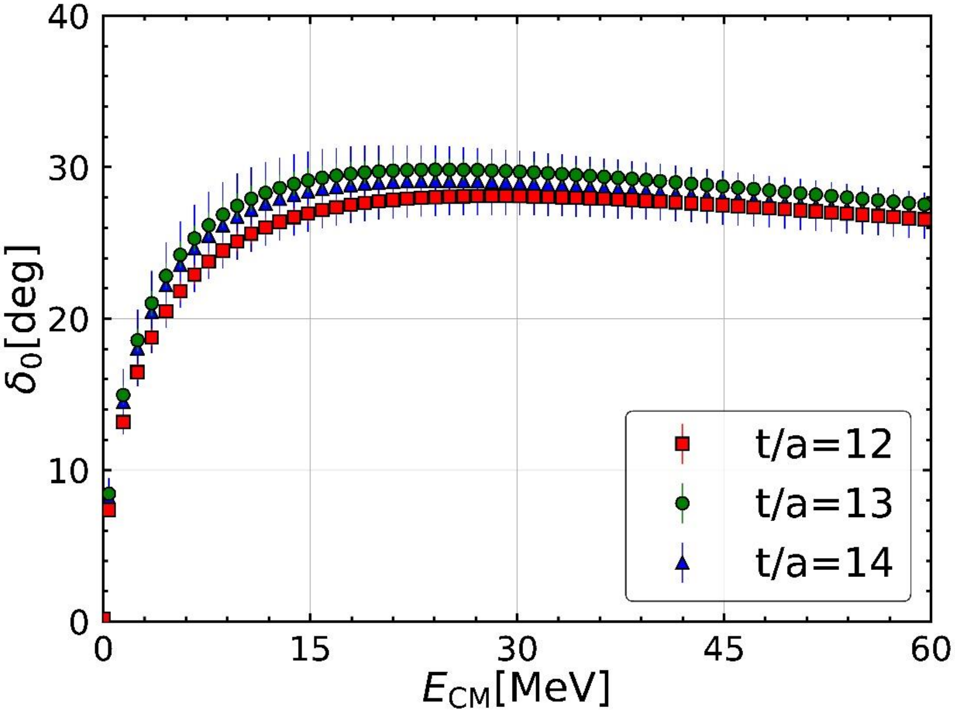
$$V(r) = -\alpha \frac{e^{-2m_\pi r}}{r^2} \quad \Rightarrow \quad E_{\text{eff}}(r) = -\frac{\ln\left[-V(r)r^2/\alpha\right]}{r}$$



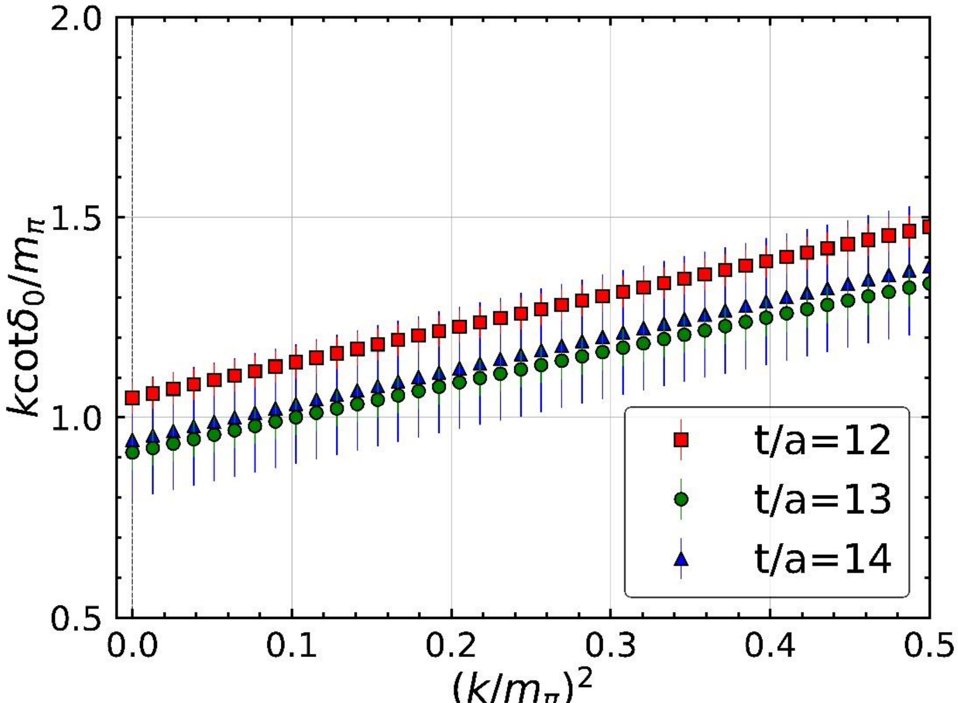

- long-range ($r > 1$ fm) potential is indeed dominated by the TPE

# Physical observables

- Scattering phase shifts




- Effective range expansion




$$k \cot \delta_0 = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + O(k^2)$$

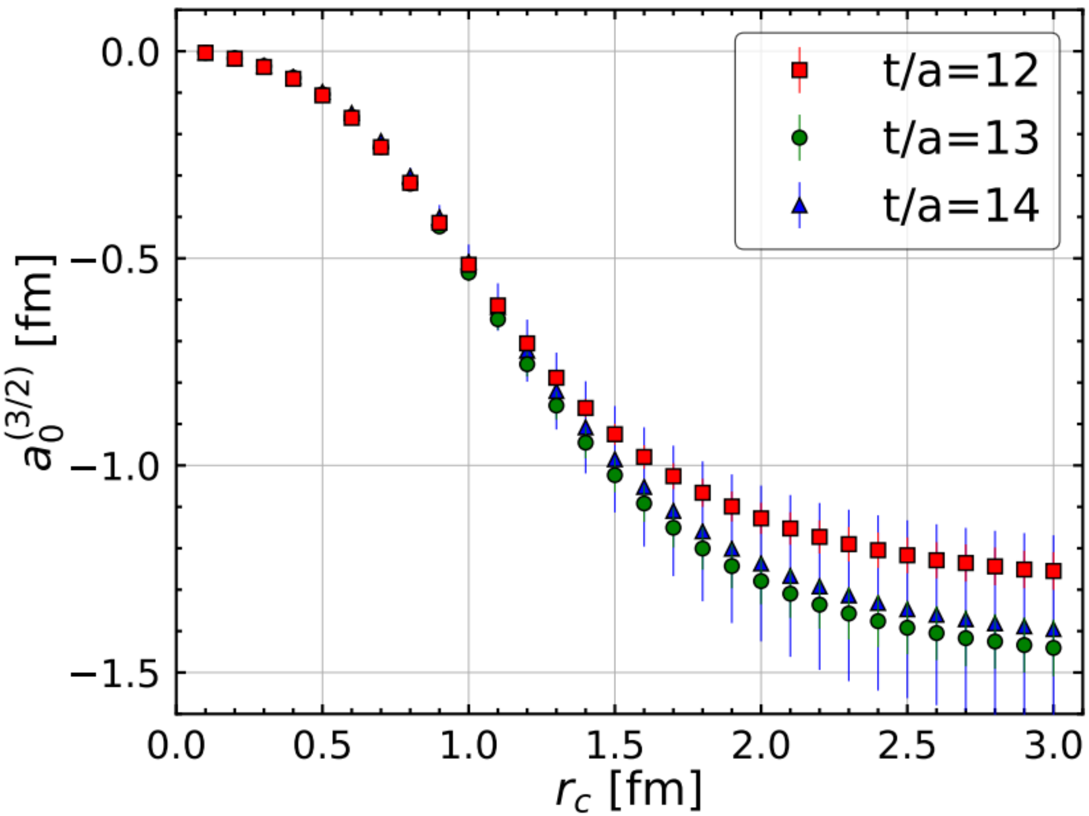
| $m_\pi$ [MeV] | $a_0^{(3/2)}$ [fm] | $r_{\rm eff}^{(3/2)}$ [fm] |
|---|---|---|
| 146.4 | $-1.43(23)_{\rm stat.}\left(^{+36}_{-06}\right)_{\rm syst.}$ | $2.36(10)_{\rm stat.}\left(^{+02}_{-48}\right)_{\rm syst.}$ |

consistent w/ ALICE Exp.

# TPE contribution to scattering length

- Scattering length from potential with long-range cutoff

$$V(r; r_c) = \begin{cases} V_{\rm fit}(r), & r \leq r_c \\ 0, & r > r_c \end{cases}$$




- long-range potential significantly contributes to scattering length

# Other systematic errors

- Slightly heavy quark masses
  - TPE becomes slightly long-ranged with weaker strength $m_\pi(\text{Lat.}) \to m_\pi(\text{Phy.})$
  - hadron masses decrease $m_{\phi,N}(\text{Lat.}) \to m_{\phi,N}(\text{Phy.})$
  - towards physical point

    physical $m_{\phi,N}$, change $m_\pi$ to 138.0 MeV for TPE tail

| $m_\pi$ [MeV] | $a_0^{(3/2)}$ [fm] | $r_{\text{eff}}^{(3/2)}$ [fm] |
|---|---|---|
| 146.4 | $-1.43(23)_{\text{stat.}} \left({}^{+36}_{-06}\right)_{\text{syst.}}$ | $2.36(10)_{\text{stat.}} \left({}^{+02}_{-48}\right)_{\text{syst.}}$ |
| 138.0 | $\simeq -1.25$ | $\simeq 2.49$ |

- Finite volume
  - expected to be as small as $\exp(-2m_\pi L/2) \simeq 0.3\%$

- Finite cutoff
  - non-perturbative $O(a)$-improvement for $u, d, s$ quark, $O\left(\left(a\Lambda_{\text{QCD}}\right)^2\right) \sim O(1)\%$
  - even completely cut $V(r)$ at $r < 0.1$ fm $\to 2\%$

# Summary and outlook

- **Summary:** $N\phi(\,^4S_{3/2})$ interaction is studied from LQCD for the first time w/ nearly physical quark masses and large volume
  - Extract potential from spacetime correlation function
  - Potential: attractive; TPE tail
  - Scattering properties

- **Outlook:**
  - $N\phi(\,^2S_{1/2})$ interaction from a coupled-channel analysis $\Lambda K - \Sigma K - p\phi$
  - Physical point simulations ($\phi \to K\bar{K}$)

Thanks for your attention