Lattice calculation of the $\eta_c\eta_c$ and $J/\psi J/\psi$ scattering length

Lattice China 2022

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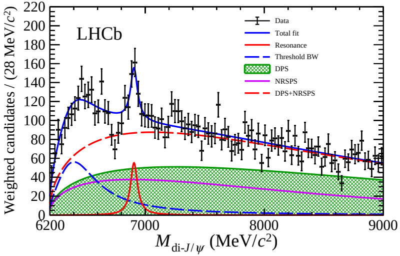
Background

- LHCb discovered a new state $X(6900)$ that could be **fully charmed tetraquark state** $T_{cc\bar{c}\bar{c}}$[1]



- Later CMS reported $X(6600), X(6900)$ and $X(7200)$, ATLAS reported $X(6900)$ and $X(7200)$[2]
- A possible $X(6200)$ near the physical di-J/ψ threshold is predicted on LHCb data[3]
- Possible $J^{pc} = 0^{++}, 2^{++}$
- As a preliminary work, we study $\eta_c\eta_c$ **and** $J/\psi J/\psi$ scattering

[1]Sci.Bull. 65 (2020) 23
[2]ICHEP (2022)
[3]PRL 126 (2021) 132001

Operator construction

The interpolators for J/ψ and η_c are

$$\mathcal{V}_i(t) = \bar{c}(t)\gamma_i c(t),\, \mathcal{P} = \bar{c}(t)\gamma_5 c(t) \tag{1}$$

We consider A_1, E, T_2 representations of the O_h group[4]
The zero-momentum operators are chosen as

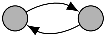
$$\begin{aligned}
\mathcal{O}^{A_1}_{\mathrm{di}-\eta_c}(t) &= \mathcal{P}(t)\mathcal{P}(t) \\
\mathcal{O}^{E}_{\mathrm{di}-J/\psi}(t) &= \begin{cases} \frac{1}{\sqrt{2}}\left[\mathcal{V}_1(t)\mathcal{V}_1(t) - \mathcal{V}_2(t)\mathcal{V}_2(t)\right] \\ \frac{1}{\sqrt{2}}\left[\mathcal{V}_2(t)\mathcal{V}_2(t) - \mathcal{V}_3(t)\mathcal{V}_3(t)\right] \end{cases} \\
\mathcal{O}^{T_2}_{\mathrm{di}-J/\psi}(t) &= \begin{cases} \mathcal{V}_2(t)\mathcal{V}_3(t) \\ \mathcal{V}_3(t)\mathcal{V}_1(t) \\ \mathcal{V}_1(t)\mathcal{V}_2(t) \end{cases}
\end{aligned} \tag{2}$$

The Z_4 -stochastic wall source is adopted

[4]The $O(3)$ symmetry is broken into O_h symmetry on the lattice.

Correlation function

Construct the correlation function, *e.g.* the simplest being



$$
\begin{aligned}
C_{\mathcal{P}}(\vec{p}, t) &= \langle \mathcal{P}(\vec{p}, t) \mathcal{P}^{\dagger}(\vec{p}, t) \rangle \\
&= \sum_{\vec{x}, \vec{y}} \mathrm{e}^{i\vec{p}\cdot(\vec{x}-\vec{y})} \langle \langle \mathcal{P}(\vec{y}, t) \mathcal{P}(\vec{x}, t) \rangle_F \rangle_G \\
&= \sum_{\vec{x}, \vec{y}} \mathrm{e}^{i\vec{p}\cdot(\vec{x}-\vec{y})} \langle \langle \bar{c}(\vec{y}, t) \gamma_5 c(\vec{y}, t) \bar{c}(\vec{x}, 0) \gamma_5 c(\vec{x}, 0) \rangle_F \rangle_G \\
&= -\sum_{\vec{y}} \mathrm{e}^{-i\vec{p}\cdot\vec{y}} \langle \mathrm{tr}[S^{\dagger}(\vec{y}, t; \vec{0}, 0) S(\vec{y}, t; \vec{p}, 0)] \rangle_G
\end{aligned} \tag{3}
$$

Other correlators are similarly put into this propagator form, by Wick's theorem
In this single-channel study, only $\vec{p} = 0$ is considered

Correlation function

The $\eta_c\eta_c$ two-particle correlator:

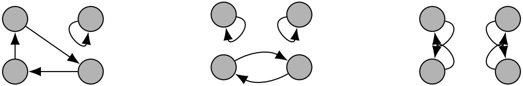
$$
\begin{aligned}
C^{\beta\alpha}_{\eta_c\eta_c} &= \langle \mathcal{O}^{\beta}_{\eta_c\eta_c}(t)\mathcal{O}^{\alpha\dagger}_{\eta_c\eta_c}(0)\rangle \\
&= \langle \mathcal{P}(\vec{p}_\beta, t)\mathcal{P}(-\vec{p}_\beta, t)\mathcal{P}(-\vec{p}_\alpha, 0)\mathcal{P}(\vec{p}_\alpha, 0)\rangle \\
&= \sum_{\vec{x},\vec{y},\vec{z},\vec{w}} \mathrm{e}^{i\vec{p}_\beta\cdot(-\vec{w}+\vec{z})+i\vec{p}_\alpha\cdot(\vec{y}-\vec{x})}\langle \mathcal{P}(\vec{w}, t)\mathcal{P}(\vec{z}, t)\mathcal{P}(\vec{y}, 0)\mathcal{P}(\vec{x}, 0)\rangle \\
&= \sum_{\vec{x},\vec{y},\vec{z},\vec{w}} \mathrm{e}^{i\vec{p}_\beta\cdot(-\vec{w}+\vec{z})+i\vec{p}_\alpha\cdot(\vec{y}-\vec{x})}\langle \bar{c}\gamma_5 c(\vec{w}, t)\bar{c}\gamma_5 c(\vec{z}, t)\bar{c}\gamma_5 c(\vec{y}, 0)\bar{c}\gamma_5 c(\vec{x}, 0)\rangle \\
&= \sum_{\vec{z},\vec{w}} \mathrm{e}^{i\vec{p}_\beta\cdot(\vec{z}-\vec{w})}\cdot\langle \\
&\quad + \mathrm{tr}[S^\dagger(\vec{z}, t; \vec{p}_\alpha, 0)S(\vec{z}, t; \vec{0}, 0)]\cdot\mathrm{tr}[S^\dagger(\vec{w}, t; \vec{0}, 0)S(\vec{w}, t; \vec{p}_\alpha, 0)] \\
&\quad + \mathrm{tr}[S^\dagger(\vec{w}, t; \vec{p}_\alpha, 0)S(\vec{w}, t; \vec{0}, 0)]\cdot\mathrm{tr}[S^\dagger(\vec{z}, t; \vec{0}, 0)S(\vec{z}, t; \vec{p}_\alpha, 0)] \\
&\quad - \mathrm{tr}[S^\dagger(\vec{w}, t; \vec{p}_\alpha, 0)S(\vec{w}, t; \vec{0}, 0)S^\dagger(\vec{z}, t; \vec{p}_\alpha, 0)S(\vec{z}, t; \vec{0}, 0)] \\
&\quad - \mathrm{tr}[S^\dagger(\vec{w}, t; \vec{p}_\alpha, 0)S(\vec{w}, t; \vec{p}_\alpha, 0)S^\dagger(\vec{z}, t; \vec{0}, 0)S(\vec{z}, t; \vec{0}, 0)]\rangle,
\end{aligned}
\tag{4}
$$

We still take $\vec{p}_\alpha = \vec{p}_\beta = 0$

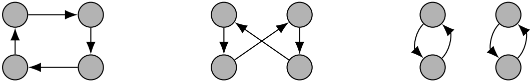
Diagrams

Among those Wick contractions, the following diagrams are considered:

The disconnected diagrams are suppressed, *e.g.*



The following annihilation diagrams are also suppressed either by OZI rule[5] or the similar mechanism (scale like $\alpha_s(m_c)$, also consider the large-N_c limit)



[5] Any strongly occurring process will be suppressed if, through only the removal of internal gluon lines, its Feynman diagram can be separated into two disconnected diagrams.

Ratio

The correlators are related to the energy levels:

$$
\begin{aligned}
C_h^{(2)}(t) &= A_h \left[e^{-m_h t} + e^{-m_h (T-t)} \right] + \dots \\
C_\Gamma^{(4)}(t) &= A_\Gamma \left[e^{-E^\Gamma t} + e^{-E^\Gamma (T-t)} + 2 e^{-m_h T} \right] + \dots
\end{aligned} \tag{5}
$$

To deal with the constant thermal pollution and systematic error, construct a ratio

$$
\begin{aligned}
R^\Gamma(t) &= \frac{C_\Gamma^{(4)}(t) - C_\Gamma^{(4)}(t+1)}{(C_h^{(2)}(t))^2 - (C_h^{(2)}(t+1))^2} \\
&\xrightarrow{t \gg 1} A_R \left[\cosh\left(\delta E^\Gamma t'\right) + \sinh\left(\delta E^\Gamma t'\right) \coth\left(2 m_h t'\right) \right] \\
&\xrightarrow[\delta E / 2 m_{c\bar{c}} \ll 1]{T \gg t} e^{-\delta E^\Gamma t},
\end{aligned} \tag{6}
$$

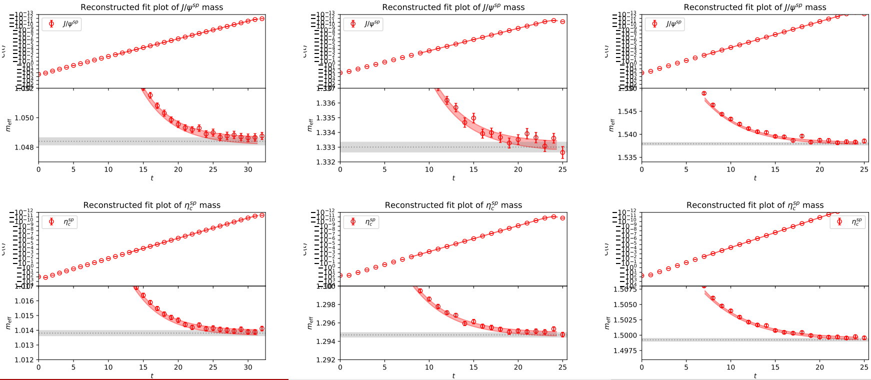
where $t' = t + 1/2 - T/2$ and $\delta E^\Gamma = E^\Gamma - 2 m_h$

Single-particle analysis

Ensembles used in this work:

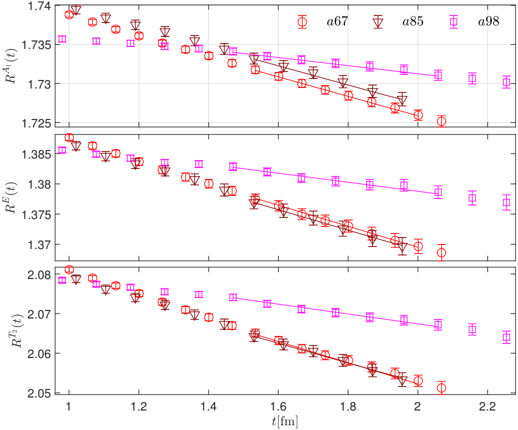
Ensemble	a (fm)	$a\mu$	$L^3 \times T$	$N_{\rm conf} \times T$	m_π(MeV)
a67	0.0667(20)	0.0030	$32^3 \times 64$	200×64	300
a85	0.085(2)	0.0040	$24^3 \times 48$	200×48	315
a98	0.098(3)	0.0060	$24^3 \times 48$	236×48	365

Effective mass extraction:



Two-particle analysis

The ratio "correlator"



This behavior indicates positive energy shifts (or negative scattering lengths)

Two-particle analysis

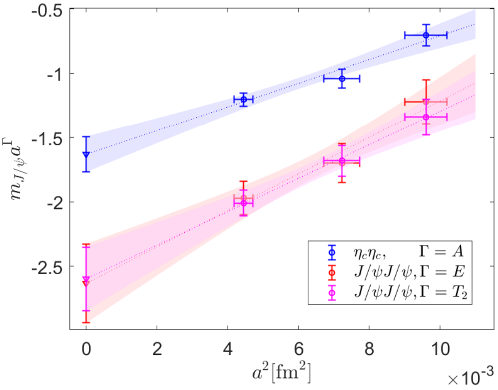
For this single-channel scattering, the scattering lengths can be extracted either from Lüscher finite size method or the effective range expansion
We report the former as the results

$$\delta E^{\Gamma} = -\frac{4\pi a^{\Gamma}}{mL^3}\left[1 + c_1\frac{a^{\Gamma}}{L} + c_2\left(\frac{a^{\Gamma}}{L}\right)^2 + \mathcal{O}(L^{-3})\right] \tag{7}$$

Ensemble	Γ	A_1	E	T_2
a98	$a\delta E^{\Gamma} \times 10^4$	2.94(35)	5.31(83)	5.88(68)
a85	$a\delta E^{\Gamma} \times 10^4$	6.03(46)	10.47(1.09)	10.29(87)
a67	$a\delta E^{\Gamma} \times 10^4$	4.81(23)	8.44(66)	8.67(52)
a98	$m_{J/\psi}a^{\Gamma}$	-0.705(81)	-1.22(17)	-1.34(14)
a85	$m_{J/\psi}a^{\Gamma}$	-1.042(72)	-1.70(15)	-1.68(12)
a67	$m_{J/\psi}a^{\Gamma}$	-1.202(51)	-1.97(13)	-2.01(10)
Cont.Limit	a^{Γ}[fm]	-0.104(09)	-0.168(20)	-0.165(16)

Extrapolation

Twisted mass fermion has Automatic $\mathcal{O}(a)$ improvement



a^E and a^{T_2} are consistent at the continuum limit, we take the result of $\Gamma = T_2$ as final estimation

$$a^{0^+}_{\eta_c\eta_c} = -0.104(09)\ \mathrm{fm}$$
$$a^{2^+}_{J/\psi J/\psi} = -0.165(16)\ \mathrm{fm} \tag{8}$$

Summary

In this work, we have

- performed the calculation of s-wave J/ψ and η_c scattering energy shifts
- extract the scattering length and extrapolate to the continuum limit
- observed weakly repulsive interaction between the two charmonia systems (see e.g. PRD 97 (2018) 054505 for b system)

Possible improvements:

- takes care of the coupled channel effects
- takes care of the OZI-suppressed diagrams

Thank you!