Lattice studies on light hadrons in J/ψ radiative decays

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ToC

Motivations

The radiative decay of J/ψ is a good place to study light hadrons
For $J/\psi \to \gamma\eta$

1. Isoscalar η in $N_f = 2$ is the counterpart of the flavor singlet η_1 in $N_f = 3$;
2. Isoscalar η is the lightest isoscalar pesudoscalar in $N_f = 2$ and is stable
3. The production rates of pseudoscalars are usually large in J/ψ radiative decays

$\eta_1(1855)$ ($I^G J^{PC} = 0^+1^{-+}$) observed by BESIII (arXiv:2202.00621)

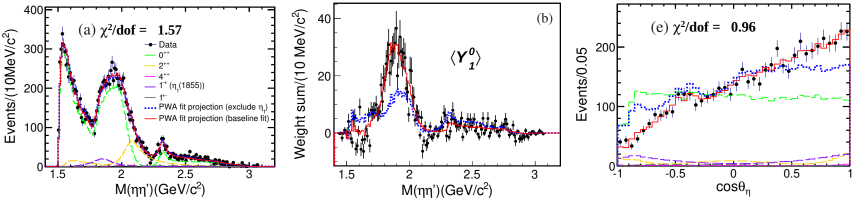


Figure: Blue dot lines: PWA fit excluding $\eta_1(1855)$

Resonance parametres of $\eta_1(1855)$: $m_{\eta_1} = 1855 \pm 9^{+6}_{-1}\mathrm{MeV}$,
$\Gamma_{\eta_1} = 188 \pm 18^{+3}_{-8}\mathrm{MeV}$
Combined branching ratio:
$\mathrm{Br}(J/\psi \to \gamma\eta_1 \to \gamma\eta\eta') = (2.70 \pm 0.41^{+0.16}_{-0.35}) \times 10^{-6}$
First candidate for isoscalar 1^{-+} hybrid.

Lattice setup

Table: Parametres of the gauge ensemble.

$L^3 \times T$	β	a_t^{-1}(GeV)	ξ	m_π(MeV)	$N_{\rm cfg}$
$16^3 \times 128$	2.0	$6.894(51)$	~ 5.3	$348.5(1.0)$	6991

1. Tadpole improved Symanzik's gauge action, $N_f = 2$ with degenerate u, d quarks;
2. A large statistics is mandatory for the study of J/ψ radiative decay into light hadrons;
3. Distillation method is used to calculate the relevant correlation functions.

Formalism and methodology

Formalism for calculating partial decay width of $J/\psi \to \gamma\eta$

1. Radiative decay width:$\Gamma(i \to \gamma f) = \frac{1}{2J_i+1}\frac{1}{32\pi^2}\int \mathrm{d}\Omega_q \frac{|\vec{q}|}{M_i^2}\sum |\mathcal{M}(r_i, r_f, r_\gamma)|^2$
2. Transition amplitude: $\mathcal{M}(r_i, r_f, r_\gamma) = \epsilon_\mu^*(p_\gamma, r_\gamma)\left\langle f(p_f, r_f)|j^\mu_{\mathrm{em}}|i(p_i, r_i)\right\rangle$
3. Multipole decomposition: $\left\langle f(p_f, r_f)|j^\mu_{\mathrm{em}}|i(p_i, r_i)\right\rangle = \sum_k \alpha_k^\mu(p_i, p_f, \epsilon_i, \epsilon_f)F_k(Q^2)$
4. Decay width in terms of form factors: $\Gamma(i \to \gamma f) \propto \sum_k F_k(0)$
5. The major task is thus to calculate the matrix elements

$$\mathcal{M}^{i\mu j}(\vec{p}) = \sum_{\lambda,\lambda'} \epsilon^i_{\lambda'}(\vec{0})\left\langle \eta_1(\vec{0}, \lambda')|j^\mu_{\mathrm{em}}(0)|J/\psi(\vec{q}, \lambda)\right\rangle \epsilon^{*j}_\lambda(\vec{p})$$

which can be extracted from the three point function

$$\Gamma_3^{i\mu j}(\vec{p}_f, \vec{q}, t_f, t) = \sum_y \left\langle 0|\mathcal{O}^i_f(\vec{p}_f, t_f) j^\mu_{\mathrm{em}} \mathcal{O}^j_i(\vec{p}_i, 0)|0\right\rangle, \mathcal{M}^{i\mu j} \sim \frac{\Gamma_3^{i\mu j}}{\Gamma_2^i \Gamma_2^j}$$

Multipole decompositions for $J/\psi \to \gamma\eta$ and $J/\psi \to \gamma\eta_1$

1. η:

$$\langle \eta(p_f) | j^{\mu}_{\mathrm{em}}(0) | J/\psi(p_i, \lambda) \rangle = M(Q^2) \epsilon^{\mu\nu\rho\sigma} p_{i,\nu} p_{f,\rho} \epsilon_{\sigma}(p_i, \lambda)$$

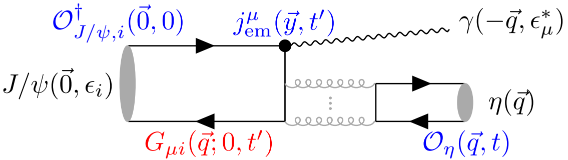
2. η_1:

$$\begin{aligned} \langle \eta_1(p_f, \lambda_f) | j^{\mu}_{\mathrm{em}}(0) | J/\psi(p_i, \lambda_i) \rangle &= A^{\mu}_1 M_1(Q^2) + A^{\mu}_2 E_2(Q^2) \\ &+ A^{\mu}_3 C_1(Q^2) + A^{\mu}_4 C_2(Q^2) \end{aligned}$$

Partial decay width of $J/\psi \to \gamma\eta$

Numerical setups:

1. Kinetic configuration: J/psi at rest and η moves with momemtum $\vec{q}$;
2. Operator $\mathcal{O}_\eta = \bar{u}\Gamma u + \bar{d}\Gamma d$ where Γ is a combination of $\gamma_4\gamma_5$, $\gamma_4\gamma_5\gamma_i\nabla_i$, $\epsilon_{ijk}\gamma_i\nabla_j\nabla_k$;
3. Three point function



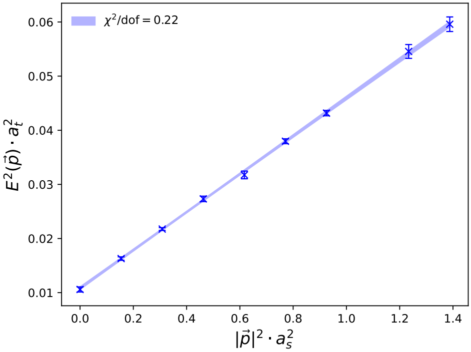
1. $\mathcal{O}_\eta$: using distillation method
2. $G_{\mu i}$: using wall source

Partial decay width of $J/\psi \to \gamma\eta$

Numerical results:

1. Dispersion relation of η:

$$E_\eta^2 a_t^2 = m_\eta^2 a_t^2 + \frac{1}{\xi^2} |\vec{p}|^2 a_s^2, \xi = 5.34(4), m_\eta = 717.4(8.4)\mathrm{MeV}$$

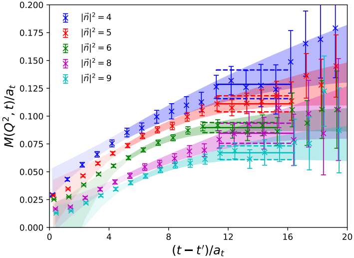


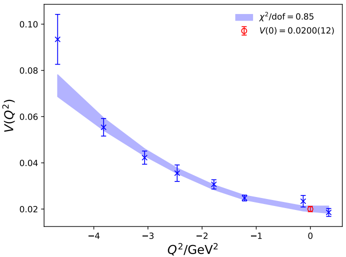
Partial decay width of $J/\psi \to \gamma\eta$

Numerical results:

1. Decay width given by $\Gamma(J/\psi \to \gamma\eta) = \frac{4\alpha}{27} |\vec{p}_\gamma|^3 |M(0)|^2$
2. We set $t' = 40$ to make sure η dominate;
3. $M(Q^2)$ still has $t - t'$ dependency;
4. Fit function for $M(Q^2)$:

$$M(Q^2) = M(0) + aQ^2 + bQ^4, M(0) = 0.01051(61)\text{GeV}^{-1}$$





Partial decay width of $J/\psi \to \gamma\eta$

Numerical results:

1. Decay width and branching ratio:

$$\Gamma(J/\psi \to \gamma\eta) = 0.385(45)\mathrm{keV}, \mathrm{Br}(J/\psi \to \gamma\eta) = 4.16 \times 10^{-3}$$

2. Comparable with experimental result

$$\mathrm{Br}(J/\psi \to \gamma\eta') = 5.25(7) \times 10^{-3}$$

Partial decay width of $J/\psi \to \gamma\eta_1$

Numeric details

1. Chosen operators: $\mathcal{O}^i_{J/\psi} = \gamma^i$, $\mathcal{O}^i_{\eta_1} = \frac{1}{\sqrt{2}}\epsilon^{ijk}\left(\bar{u}\gamma_j B_k u + \bar{d}\gamma_j B_k d\right)$ with $B_i = \epsilon_{ijk} D_j D_k$
2. Kinetic setup: η_1 at rest while J/ψ moves
3. J/ψ to γ and η_1 loop are calculated using distillation method, but the former is more challenging

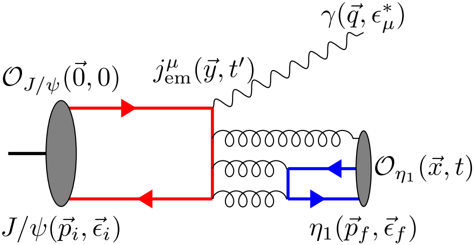
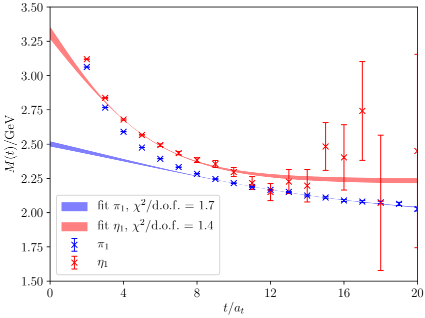


Figure: Schematic diagram of the process $J/\psi \to \gamma\eta_1$

Partial decay width of $J/\psi \to \gamma\eta_1$

Two point functions



$$m_{\eta_1} = 2.230(39)\text{GeV}, \qquad m_{\pi_1} = 1.950(28)\text{GeV} \tag{1}$$

These results are in agreement with a previous lattice study (arXiv:1309.2608)

Partial decay width of $J/\psi \to \gamma\eta_1$

Calculating the J/ψ to γ loop $G_i\mu$ is highly non-trivial:

1. Full distillation method smears the current j_{em}, making it non-local;
2. Point source method with $G_{i\mu}(\vec{0}, \vec{q}; t, t') = \sum_x e^{-i\vec{q}\cdot\vec{x}} \mathrm{Tr}\left[\gamma_5 S_c(\vec{x}, 0; t, t')\gamma_5\gamma^\mu S_c(\vec{x}, 0; t', t)\gamma_i\right]$ has no signal, for some unknown reason;
3. Momemtum source as used in arXiv:2206.02724 is too noisy;

We propose the "half perambulator" method:

$$
\begin{aligned}
G_{\mu i}(\vec{p}, \vec{q}; t, t') &= \sum_{\vec{x}} e^{-i\vec{q}\cdot\vec{x}} \mathrm{Tr}\left\{\gamma_5 [S_c V(t')]^\dagger(\vec{x}, t)\gamma_5\gamma^\mu \right. \\
&\left. \times \left[S_c V(t')\right](\vec{x}, t)[V^\dagger(t') D(\vec{p})\gamma_i V(t')]\right\}, \qquad (2)
\end{aligned}
$$

where $S_c V(t')$ may be calculated by inverting

$$
M S_c V(t') = V(t') \qquad (3)
$$

Partial decay width of $J/\psi \to \gamma\eta_1$

Numerical results

1. Decay width: $\Gamma(J/\psi \to \gamma\eta') = \frac{4\alpha}{27}\frac{|\vec{p}_\gamma|}{2m^2_{J/\psi}}\left[|M_1(0)|^2 + |E_2(0)|^2\right]$
2. $F_i(Q^2)$ are extracted using weighted average of matrix element for $t' \in [20, 40]$
3. Fit function for F_i:

$$F_i(Q^2) = v(Q^2)(a_i + b_i v^2(Q^2) + c_i v^4(Q^2))$$

$$v(Q^2) = \frac{\Omega(Q^2)}{m^2_{J/\psi} m^2_{\eta_1}} = \frac{|\vec{q}|}{m_{J/\psi}} (< 0.49)$$

$$\Omega(Q^2) = \frac{1}{4}\left[(m_{J/\psi} + m_{\eta_1})^2 + Q^2\right]\left[(m_{J/\psi} - m_{\eta_1})^2 + Q^2\right]$$

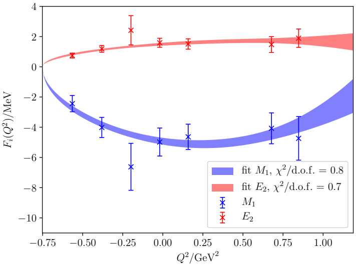


Figure: Extracted form factors

Partial decay width of $J/\psi \to \gamma\eta_1$

Numerical results

1. Extrapolating gives

$$\begin{aligned} M_1(0) &= -4.96(90)\text{MeV} \\ E_2(0) &= 1.41(26)\text{MeV}. \end{aligned} \tag{4}$$

2. Prediction of the decay width and branching fraction:

$$\Gamma(J/\psi \to \gamma\eta_1) = 2.29(77)\text{eV} \qquad \text{Br}(J/\psi \to \gamma\eta_1) = 2.47(83) \times 10^{-5}$$

3. Considering $m^2_{\eta_1(1855)}/m^2_{\eta_1} \approx 0.69$ and $(|p_\gamma(\eta_1(1855))|/|p_\gamma(\eta_1)|)^3 \approx 1.74$, Partial width for $\eta_1(1855)$ is predicted as

$$\begin{aligned} \text{Br}(\eta_1(1855) \to \eta\eta') &\approx 4.3(1.7)\%, \\ \Gamma(\eta_1(1855) \to \eta\eta') &\approx 8.1(3.3)\ \text{MeV}. \end{aligned} \tag{5}$$

4. Comparable to a phenomenological study (arXiv:2202.04918) predicts

$$\Gamma(\eta_1(1855) \to \eta\eta') \approx 11\ \text{MeV}. \tag{6}$$

Summary

1. J/ψ radiative decay is a good place to study the properties of light hadrons.
2. Lattice QCD is promising for this task;
3. $\mathrm{Br}(J/\psi \to \gamma\eta)$ is calculated on $N_f = 2$ lattice, and the result is in agreement with experimental value.
4. $\mathrm{Br}(J/\psi \to \gamma\eta')$ is predicted to be $6.1(2.2) \times 10^{-5}$ on $N_f = 2$ lattice.