Spin decomposition in Charmonium

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Outline

① **Background and introduction**

② **Lattice calculation of quark spin and gluon total angular momentum**

③ **Summary**

Spin decomposition in QCD

- **Jaffe-Manohar decomposition** R. Jaffe and A. Manohar, NPB 337, 509 (1990)

Gauge dependent

$$J = \frac{1}{2}\sum_q \Delta_q + \sum_q \mathscr{L}_q + \Delta_G + L_G$$

quark spin

quark orbital angular momentum (OAM)

Gluon spin

Gluon OAM

Gauge indenpendent gluon spin

X. Chen, X. Lv, W. Sun, F. Wang and T. Goldman PRL 100, (2008)
X. Ji, J. Zhang and Y. Zhao, PRL, 111 (2013)
Y. Yang, et. al (χQCD Collaboration), PRL. 118 (2017)

- **Ji's decomposition** X. Ji, Phys. Rev. Lett. 78, 610 (1997)

$$J = \frac{1}{2}\sum_q \Delta_q + \sum_q L_q + J_g$$

Gauge invariant

quark spin

quark orbital angular momentum (OAM)

Gluon spin+OAM

Nucleon spin decomposition

C. Alexandrou, et. al ,PRL. 119 (2017)
G. Wang, et. al, (χQCD Collaboration), *PRD* 106 (2022)

Naive spin decomposition of charmonium

- **The quantum number of charmonium (J^{PC})**

exotic state

J=1	1^{--}	1^{+-}	1^{++}	1^{-+}
J=2	2^{--}	2^{-+}	2^{++}	2^{+-}

$\bar{Q}Q$

$P = (-1)^{L+1}$

$C = (-1)^{L+S}$

- **The naive spin decomposition in quark model**

1^{--}	S=1	L=0
1^{+-}	S=0	L=1
1^{++}	S=1	L=1

1.Are the predictions of quark model comparable with QCD?

2.How about the contribution of gluon?

3.What is the spin structure of exotic states?

Outline

① **Background and introduction**

② **Lattice calculation of quark spin and gluon total angular momentum**

ensemble	$L^3 \times T$	a (fm)	m_π (MeV)	$m_c a$	$N_{\rm cfg}$
32I	$32^3 \times 64$	0.0828(3)	300	0.493	305

③ **Summary**

Overlap Fermion

- **The definition of Overlap operator**

$$D_{ov} = \rho(1 + \gamma_5 \epsilon_{ov}(\gamma_5 D_w))$$

Where $\epsilon_{ov}(\gamma_5 D_w)$ is the sign function of Wilson operator D_w

- **Overlap operator satisfies Ginsparg-Wilson Relation**

$$D_{ov}\gamma_5 + \gamma_5 D_{ov} = \frac{1}{\rho} D_{ov}\gamma_5 D_{ov}$$

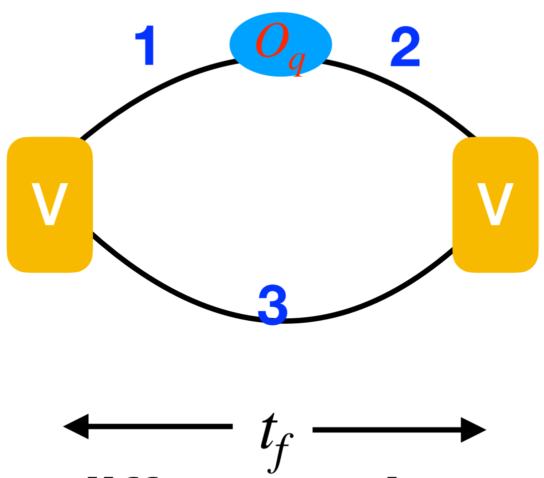
- **We can change D_{ov} to be $D_c = \dfrac{D_{ov}}{1 - \frac{1}{2\rho}D_{ov}}$, then D_c satisfies**

$$D_c\gamma_5 + \gamma_5 D_c = 0$$

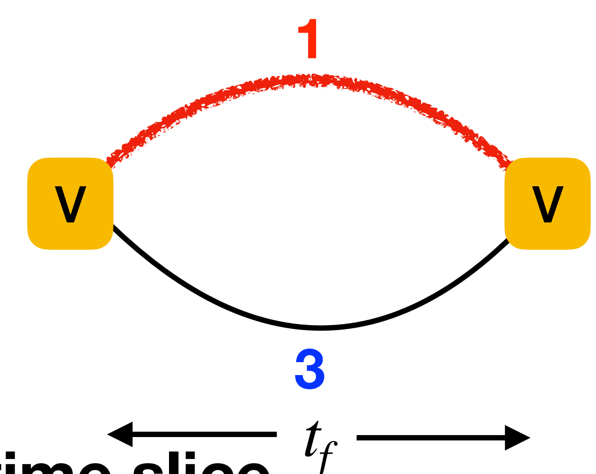
Calculation of the quark spin in the hardon

- The quark spin contribution can be obtained by the ratio of connected 3pt correlation function to 2pt correlation function. Since we need calculate the quark spin in different hadron states (1^{--},1^{+-}...), a better choice is using summed current sequential method

C. Bouchard, et al., PRD96(2017)
C.C. Chang et,al. Nature(2018),558



Sum the quark mass operator over all the lattice size($L^3 \times T$)



$= SC_3(t_f, V)$

- The difference between the ratio at adjacent time slice

$$R(t_f, O) = \frac{\langle SC_3(t_f, O)\rangle}{\langle C_2(t_f)\rangle} - \frac{\langle SC_3(t_f-1, O)\rangle}{\langle C_2(t_f-1)\rangle} = \langle H|O|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Matrix element at the ground state

The contamination of excited state

Quark spin in charmonium (spin one)

- **The quark spin operator sandwiched by different hadron external state.**

Quark spin operator $O_{\Sigma_q} = \sum_x \bar{q}\gamma_z\gamma_5 q(x)$

Spin basis

$$V_+ = \frac{1}{\sqrt{2}}(V_x + iV_y)$$

$$V_0 = V_z$$

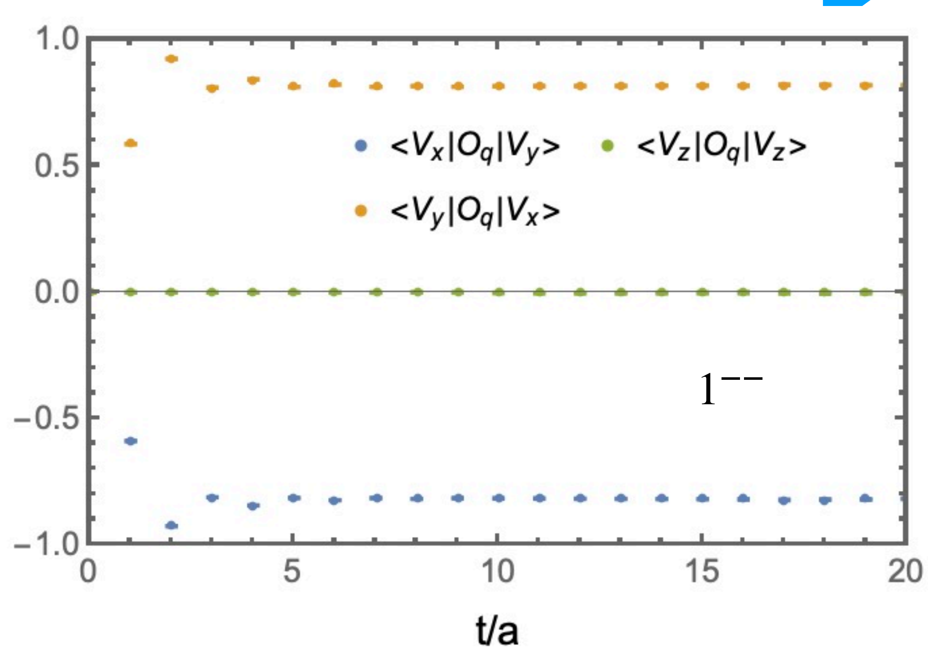
$$V_- = \frac{1}{\sqrt{2}}(V_x - iV_y)$$

$$\langle V_+ | O_{\Sigma_q} | V_+ \rangle = -\langle V_- | O_{\Sigma_q} | V_- \rangle \neq 0$$

$$\langle V_0 | O_{\Sigma_q} | V_0 \rangle = 0$$

$$\langle V_x | \bar{q}\gamma_z\gamma_5 q | V_y \rangle \neq 0$$

$$\langle V_z | \bar{q}\gamma_z\gamma_5 q | V_z \rangle = 0$$



Quark spin in charmonium (spin two)

- **The irreducible representation of of spin-2 charmonium can be converted to the spin basis** $S_M^{J=2} = \langle 1, m_1; 1, m_2 | J, M \rangle \bar{\psi} \gamma_{m_1} D_{m_2} \psi$

Irreducible representation (2^{++})

J. Dudek, et. al, PRD 77 (2008) 034501

T_2^x	$\lvert \epsilon_{1jk} \rvert \gamma^j D^k / \sqrt{2}$
T_2^y	$\lvert \epsilon_{2jk} \rvert \gamma^j D^k / \sqrt{2}$
T_2^z	$\lvert \epsilon_{3jk} \rvert \gamma^j D^k / \sqrt{2}$
E^x	$Q_{1jk} \gamma^j D^k$
E^y	$Q_{2jk} \gamma^j D^k$

$\mathbb{Q}_{111} = \frac{1}{\sqrt{2}}; \ \mathbb{Q}_{122} = -\frac{1}{\sqrt{2}}; \ \mathbb{Q}_{211} = -\frac{1}{\sqrt{6}};$

$\mathbb{Q}_{222} = -\frac{1}{\sqrt{6}}; \ \mathbb{Q}_{233} = \frac{2}{\sqrt{3}}.$

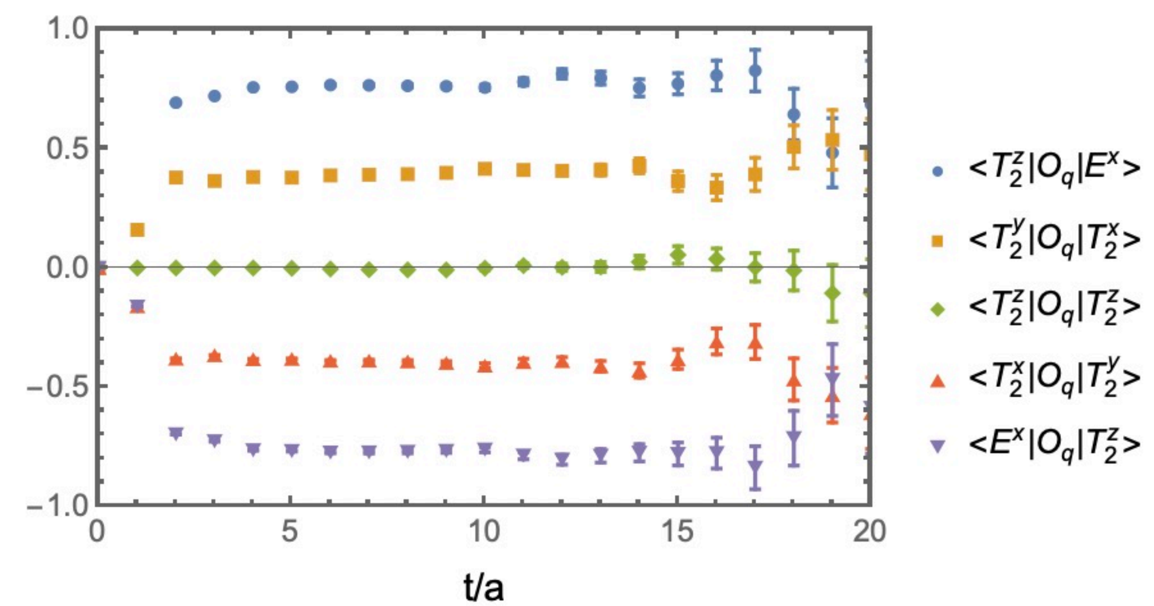
Spin basis

V_{2+}	$(iT_2^z + E^x)/\sqrt{2}$
V_{1+}	$(iT_2^y + T_2^x)/\sqrt{2}$
V_0	E^y
V_{1-}	$(-iT_2^y + T_2^x)/\sqrt{2}$
V_{2-}	$(-iT_2^z + E^x)/\sqrt{2}$

$$\langle V_{2+} | O_{\Sigma_q} | V_{2+} \rangle = 2 \langle V_{1+} | O_{\Sigma_q} | V_{1+} \rangle$$

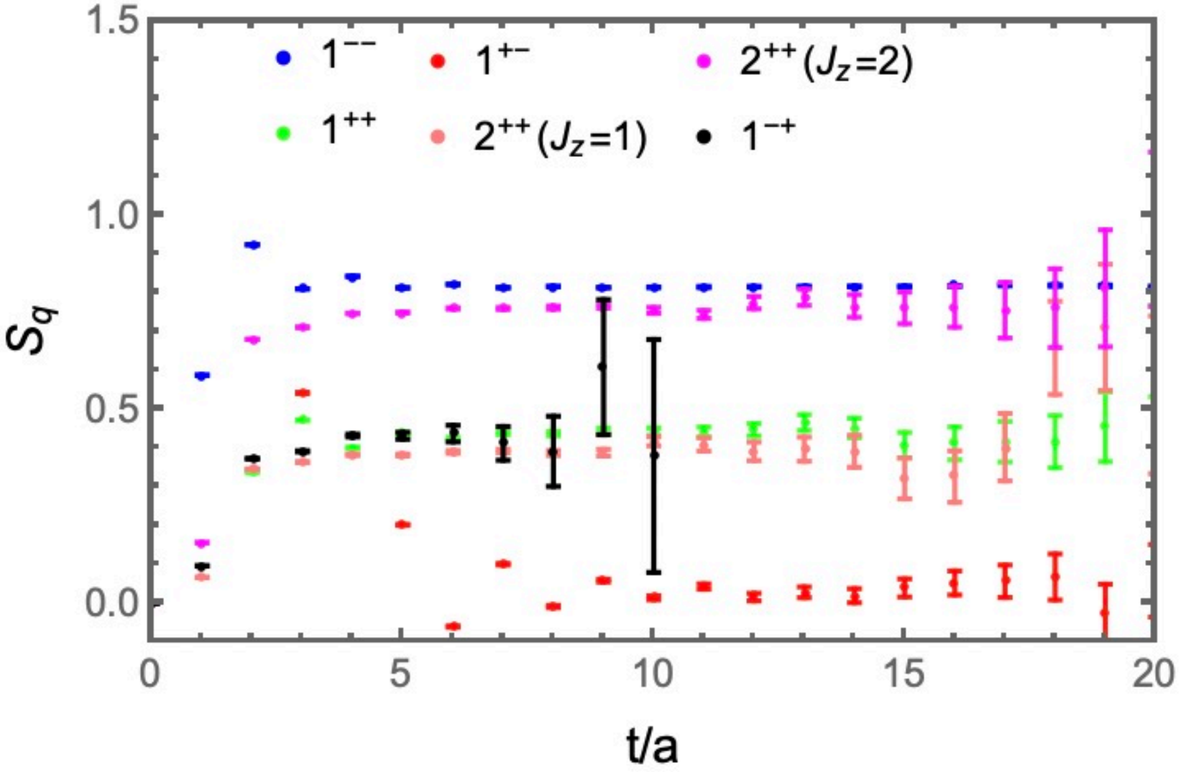
$$\langle E^x | O_{\Sigma_q} | T_2^z \rangle = 2 \langle T_2^x | O_{\Sigma_q} | T_2^y \rangle$$

Matrix elements of 2^{++} channel



Quark spin in different charmonium

- **The contribution of quark spin to the different charmonium spin**



Comparison with the prediction of quark model

1.The contribution of quark spin is very small in 1^{+-} (p wave), but is dominantly contributes to the spin of 1^{--} (s wave) .

2. The quark spin in 1^{++} and $2^{++}(J_z = 1)$ are very close since their spin structure same (L=1, S=1).

3. The quark spin in 1^{--} and $2^{++}(J_z = 2)$ are also similar.

4. The quark spin contributes half spin of 1^{-+} exotic state.

Total angular momentum of gluon

- **The gravitational form factor of vector meson (moving frame)**

M. Polyakov and B. Sun, PRD 100 (2019)

$$\langle p',\sigma'|\hat{T}^a_{\mu\nu}(x)|p,\sigma\rangle = \Big[2P_\mu P_\nu\Big(-\epsilon'^*\cdot\epsilon\, A_0^a(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}A_1^a(t)\Big)$$
$$+2\left[P_\mu(\epsilon'^*_\nu\,\epsilon\cdot P + \epsilon_\nu\,\epsilon'^*\cdot P) + P_\nu(\epsilon'^*_\mu\,\epsilon\cdot P + \epsilon_\mu\,\epsilon'^*\cdot P)\right]\,J^a(t)$$
$$+\frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\Big(\epsilon'^*\cdot\epsilon\,D_0^a(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}D_1^a(t)\Big)$$
$$+\ldots$$

$$P = \frac{p+p'}{2} \qquad \Delta = p'-p$$

Angular momentum (t=0)

The GFFs of spin-2 particle should be more complicated

- **Calculating angular momentum in the rest frame**

EMT

$$T_g^{\mu\nu} = F^{a,\mu\eta}F^{a,}{}_\eta{}^\nu + \frac{1}{4}g^{\mu\nu}F^{a,\kappa\eta}F^{a,}{}_{\kappa\eta}\,.$$

$$J_i = \int d^3\vec{r}\,\epsilon_{ijk}r_jT^{4k}(\vec{r}) = \int d^3\vec{r}\;\vec{r}\times(\vec{E}\times\vec{B})$$

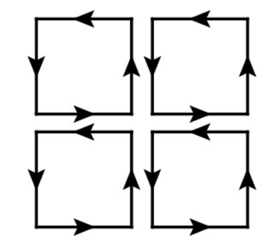
The spatial separation between sink and gluon operator.

three point correlation function

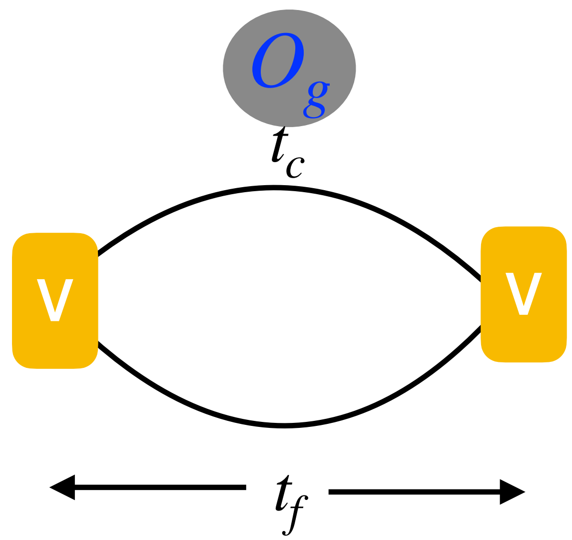
$$C_3 = \frac{1}{V}\sum_{\vec{x}}\sum_{\vec{y}}\epsilon_{ijk}\,(x_i-y_i)\,\langle H(t_f,y)T^{4k}(t_g,x)H_w(0)\rangle$$

Calculation of the gluon total angular momentum (AM)

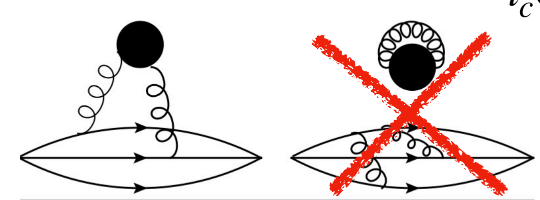
- **For the gluon AM, the 3pt correlation function can be described as**

$F^{\mu\nu} \propto$ 

1 hyp smearing

$C_3(t_f, t) =$ 

$$= \left(C_2(t_f) - \langle C_2(t_f)\rangle\right)\left(\sum_{t_c \in (o, t_f)} O_g(t_c) - \langle O_g(t_c)\rangle\right)$$



t_c: timeslice of O_g

- **We can extract the matrix element of gluon condensate at the ground state through difference between the ratio at adjacent time slice**

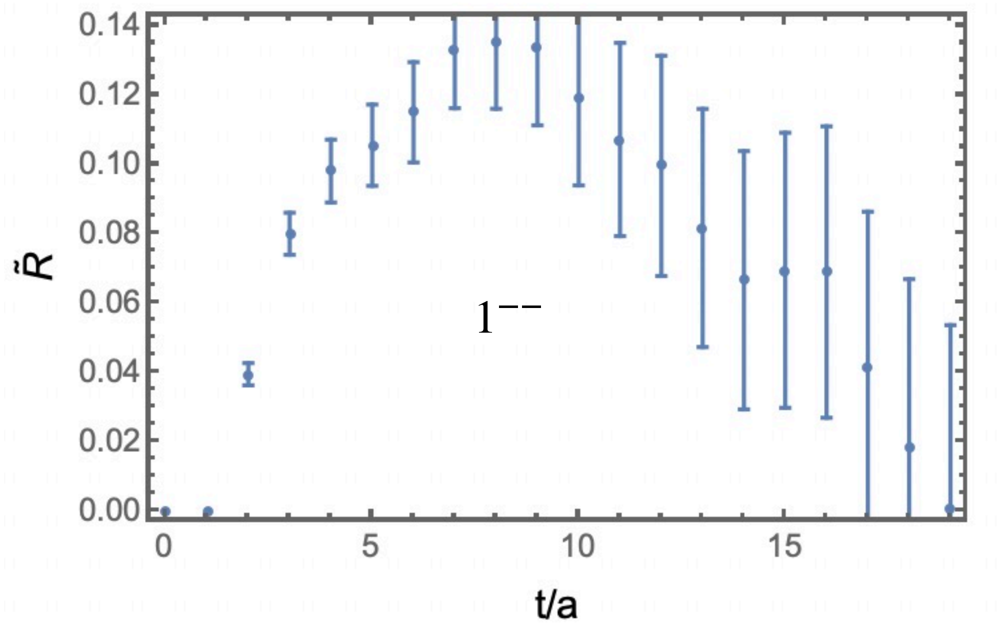
$$\tilde{R}(t_f, \tilde{O}) = \frac{\sum_{t_f > t > 0}\langle C_3(t_f, t, \tilde{O})\rangle}{\langle C_2(t_f)\rangle} - \frac{\sum_{t_f - 1 > t > 0}\langle C_3(t_f - 1, \tilde{O})\rangle}{\langle C_2(t_f - 1)\rangle} = \langle H|\tilde{O}|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

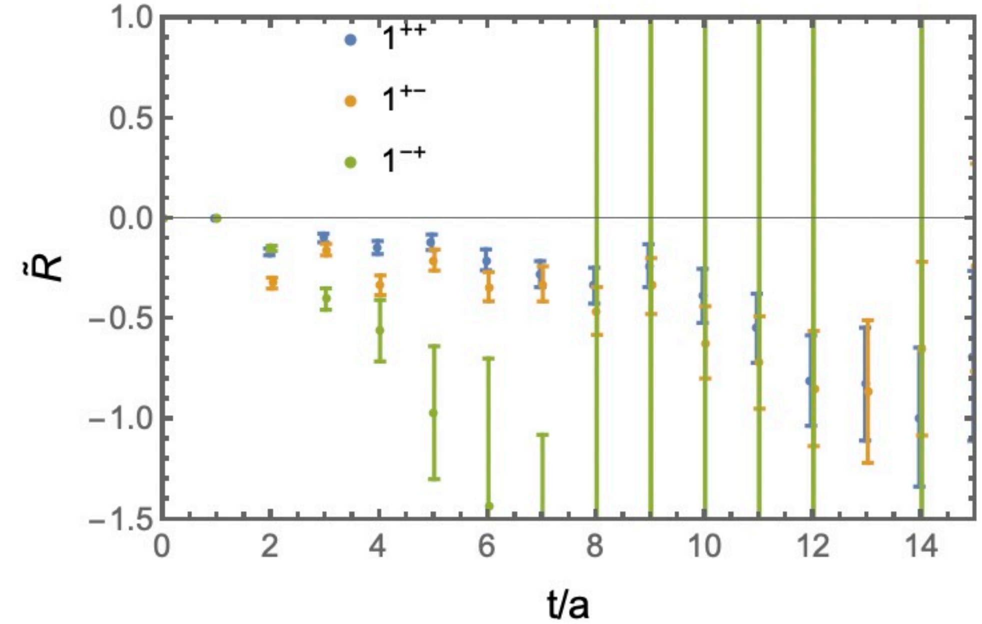
Matrix element at the ground state

The contamination of excited state

Numerical results of ratio of gluon angular momentum (spin 1)

- **The results of gluon angular momentum operator 3pt to 2pt**





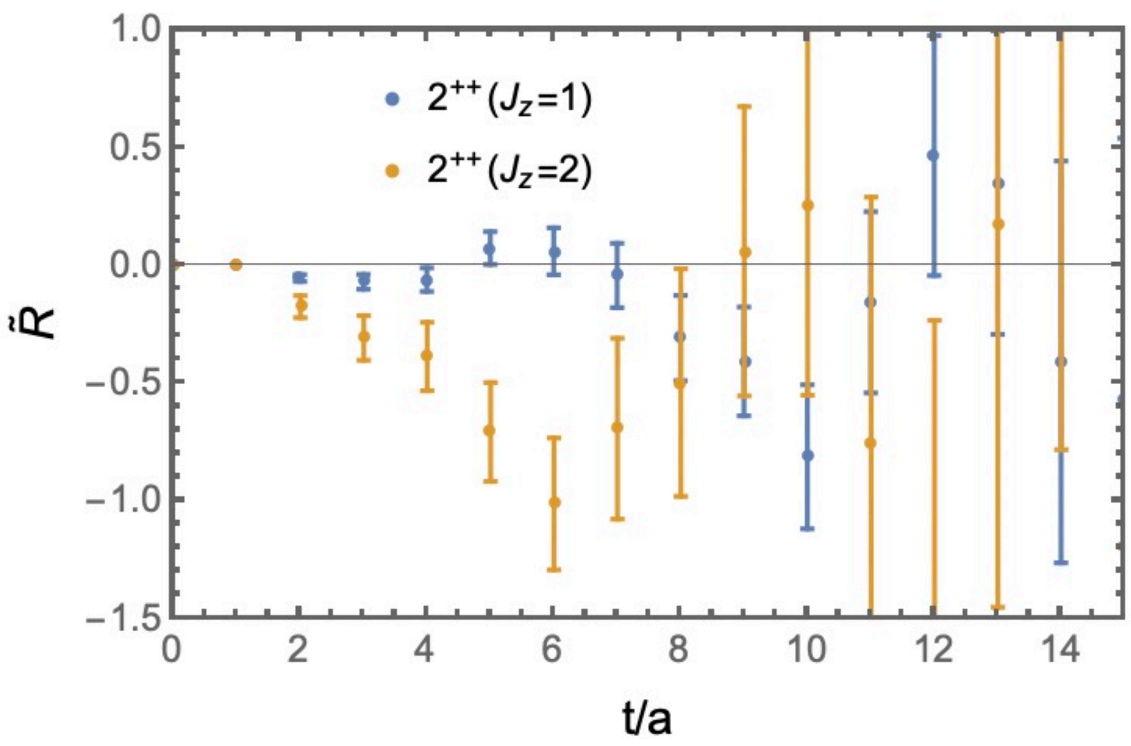
The gluon spin in 1^{--} is small

The gluon spin in 1^{++} and 1^{+-} is similar

Large uncertainty for 1^{-+} channel

Numerical results (spin 2)

- **The results of gluon angular momentum operator 3pt to 2pt**



The gluon spin in $J_z = 1$ channel is small but it has large contribution to the spin of $J_z = 2$ channel

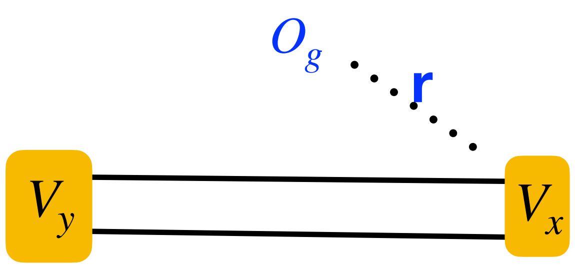
Large uncertainly

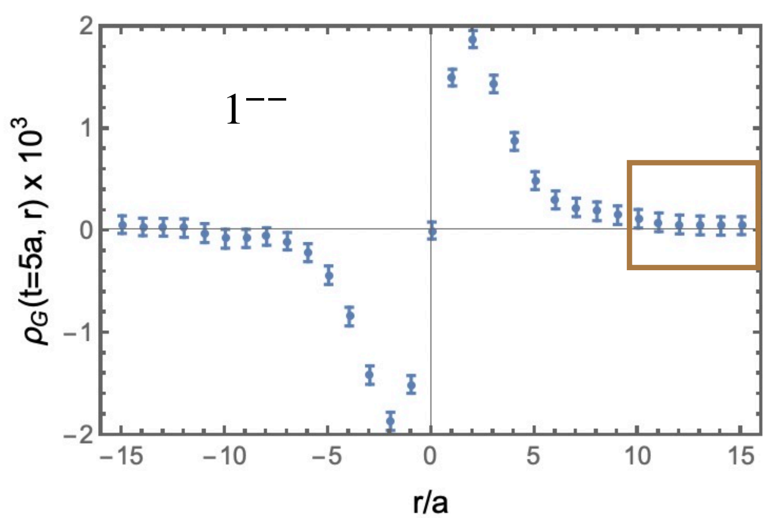
Density of gluon operator

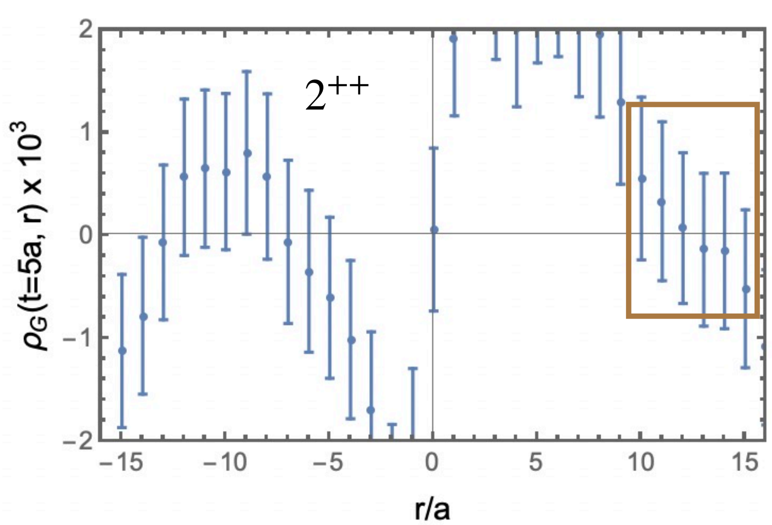
- **The density of gluon AM operator**

3pt for density calculation $$C_3(r,t) = \frac{1}{V}\sum_{\vec{y}}\sum_{\vec{z}} \delta(z_2 - r)\ \langle H(t_f, y+z; \epsilon_y) T^{4k}(t_g, y) H_w(0; \epsilon_x)\rangle$$

$$\rho_G(t,r) = \frac{C_3(r,t)}{C_2(t)} - \frac{C_3(r,t-1)}{C_2(t-1)}$$







1.Finite volume effect is negligible

2. The center value decay with r but the error does not

Drop these points can improve the signal (on-going)

Results

- **The contributions of quark spin and gluon angular momentum in different charmonium states**

The off-diagonal terms are assumed to be zero.

$$\begin{pmatrix} S_q \\ J_g \end{pmatrix}^R = \begin{pmatrix} Z_A & 0 \\ 0 & Z_{gg} \end{pmatrix} \begin{pmatrix} S_q \\ J_g \end{pmatrix}^B$$

Renormalization constants are from

Y. Yang, et. al (χQCD Collaboration), PRL. 121 (2018)
F. He, et. al (χQCD Collaboration), 2204.09246

	(Quark Spin)Sq	(Gluon AM)Jg
1^{--}	0.88(2)	0.13(2)
1^{+-}	0.04(4)	-0.53(11)
1^{++}	0.49(5)	-0.40(8)
1^{-+}	0.46(11)	?
$2^{++}(J_z = 2)$	0.85(8)	?
$2^{++}(J_z = 1)$	0.46(8)	?

Summary

- **We studied spin decomposition in different charmonium state. The contribution of quark spin is compatible with the prediction of quark model.**

- **Despite the large large uncertainty, the gluon total angular momentum contributes negative value to most of channel (1^{++}, 1^{+-}...), then the quark QAM should be large enough to make the total spin close to one.**

- **The quark spin contributes to half of spin of exotic 1^{-+} state.**

Thank you for your attention!