2+1 味格点 QCD 中的 $D^{(*)}$, $D_s^{(*)}$ 介子衰变常数

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第二届中国格点量子色动力学研讨会，2022.10.09

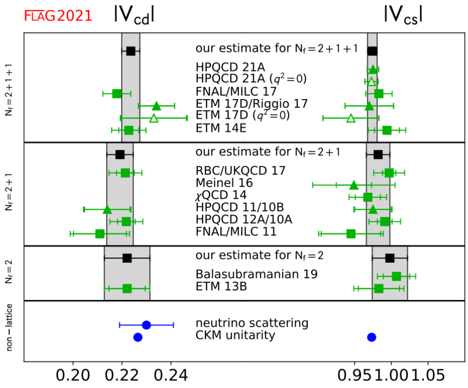
大纲

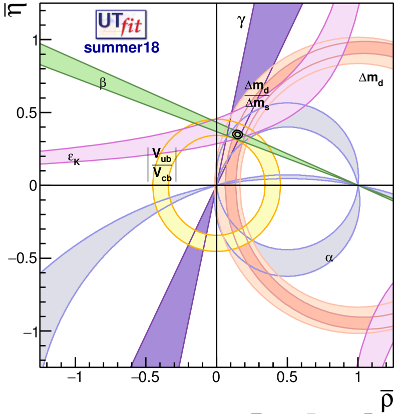
- 物理动机与格子基本信息
- 强子谱与两点函数
- 拟合结果

物理动机

物理意义

1. 精确测量 CKM 矩阵元，e.g. V_{cq}, $q = d, s$





物理动机

物理意义

1. 精确测量 CKM 矩阵元，e.g. V_{cq}, $q = d, s$

$$\mathcal{B}\left(D_{(s)} \to \ell\nu_\ell\right) = \frac{G_F^2 \left|V_{cq}\right|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2$$

$$\left\langle 0 \left| \bar{c}\gamma^\mu\gamma_5 q \right| D_q(p)\right\rangle = i f_{D_q} p^\mu_{D_q}, \qquad \left\langle 0 \left| \bar{c}\gamma^\mu q \right| D^*_q(p)\right\rangle = i f_{D^*_q} m_{D^*_q} \epsilon^\mu(p,\lambda)$$

2. 其他 QCD 理论计算的基本输入，e.g. $B \to D^{(*)}$，$B_s \to D_s^{(*)}$
3. 检验重夸克对称性的破坏程度，$f_{D^*_{(s)}}/f_{D_{(s)}}$

格子基本信息

基本信息[1]

1. 格子大小：$32^3 \times 64 \sim 2.6\text{fm}$，$a^{-1} = 2.383(9)\text{GeV} \sim 0.08\text{fm}$
2. 价夸克质量：$am_{u/d} = 0.0046 \sim 0.024$，$am_s = 0.037 \sim 0.043$，
   $am_c = 0.45 \sim 0.55$
   海夸克质量：$am_l = 0.004$，$am_s = 0.03$
3. 费米子作用量：domain wall（海夸克）+ overlap（价夸克）
4. 输入参数：$m_\pi^2 \Rightarrow m_{u/d}$，$m_{ss}^2 = 2m_K^2 - m_\pi^2 \Rightarrow m_s$，$m_{D_{(s)}} \Rightarrow m_c$

1 T. Blum *et al.* [RBC and UKQCD],PhysRevD.93.074505

强子谱与两点关联函数

两点函数

$$C(t,\vec{p}) = \sum_{\vec{x}} \left\langle 0 \left| \mathcal{O}(t,\vec{x}) \mathcal{O}^\dagger(0) \right| 0 \right\rangle e^{-i\vec{p}\cdot\vec{x}} \approx \sum_U \mathcal{O}[U,m]/N$$

$$= \sum_n \frac{1}{2E_n} \left| \langle 0 \left| \mathcal{O}(0) \right| n, \vec{p} \rangle \right|^2 e^{-E_n t}$$

$$\xrightarrow{t \gg 0, \vec{p}=0} \frac{1}{2M} |\langle 0 | \mathcal{O} | P/V \rangle|^2 e^{-Mt} \equiv A \cdot \left(e^{-Mt} + e^{-M(T-t)} \right)$$

其中，$\mathcal{O} = \bar{q}\gamma_5 c$，$\bar{q}\gamma_i c$，$\bar{q}\sigma_{0i} c$，$q = u/d$ or s

$$(m_q + m_c) \langle 0 \left| \bar{q}\gamma_5 c \right| D_q \rangle = m_{D_q}^2 f_{D_q}$$

拟合两点函数

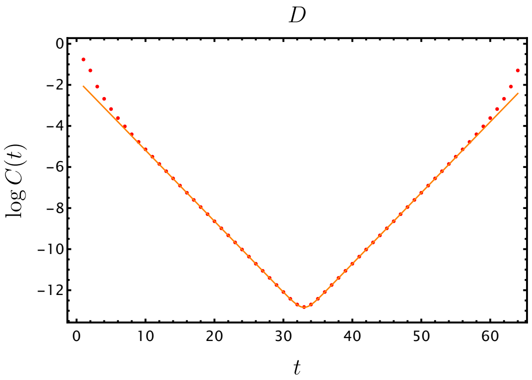
组态数：628

矢量粒子的两点函数做了三个方向的极化平均

$$a^3 C(t,\vec{p}) = a^3 A \cdot \left(e^{-aMt/a} + e^{-aM(T-t)/a}\right)$$

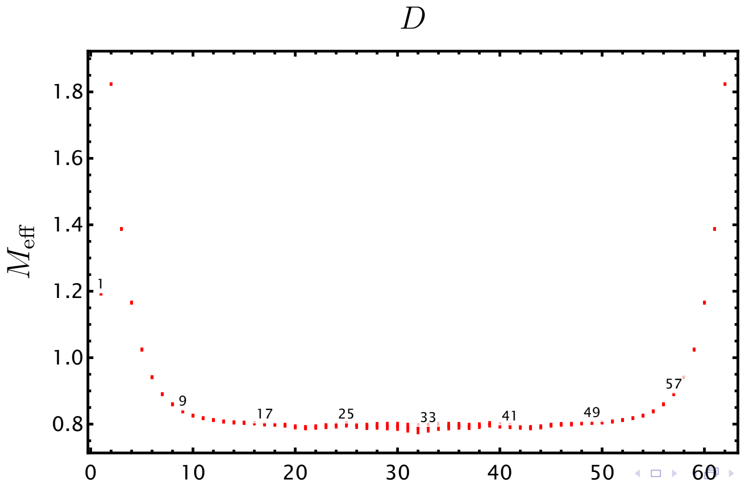
$$af_{D_q} = \sqrt{2a^3 A(am_q + am_c)/(aM)^3},$$

$$af_{D_q^*} = \sqrt{2a^3 A/aM}$$



强子谱与两点函数

$$\chi^2(\boldsymbol{\theta}) = (\boldsymbol{y} - \boldsymbol{\mu}(\boldsymbol{\theta}))^T V^{-1}(\boldsymbol{y} - \boldsymbol{\mu}(\boldsymbol{\theta})), \quad V_{ij} = \mathrm{cov}\,[y_i, y_j]$$



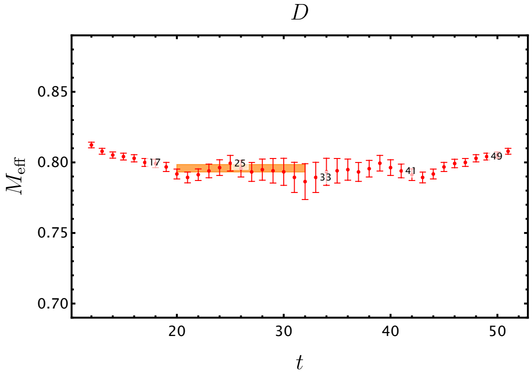
强子谱与两点函数

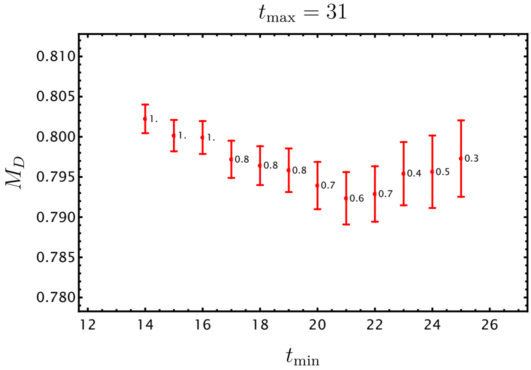
判断标准，

$$\chi^2_{min}/d.o.f \leq 1$$

拟合参数的误差，

$$\left(U^{-1}\right)_{ij} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial\theta_i\partial\theta_j}\bigg|_{\theta=\hat{\boldsymbol{\theta}}}$$





拟合结果

以 $D_{(s)}{}^*$ 为例

aM_{D^*}	am_q	0.0046	0.00585	...	0.018	0.024
am_c	0.45	0.8087(30)	0.8103(26)	...	0.8287(15)	0.8290(12)
	0.492	0.8502(31)	0.8518(27)	...	0.8644(15)	0.8705(13)
	0.5	0.8581(31)	0.8598(27)	...	0.8723(15)	0.8784(13)
	0.55	0.9076(32)	0.9093(28)	...	0.9219(16)	0.9279(14)

$aM_{D_s^*}$	am_s=0.037	am_s=0.04	am_s=0.043
am_c=0.45	0.8420(10)	0.8450(10)	0.8479(9)
am_c=0.492	0.8834(10)	0.8863(10)	0.8892(10)
am_c=0.50	0.8913(10)	0.8942(10)	0.8971(10)
am_c=0.55	0.9408(11)	0.9437(10)	0.9466(10)

拟合结果

af_{D^*}	am_q	0.0046	0.00585	…	0.018	0.024
am_c	0.45	0.0956(27)	0.0962(23)	…	0.1012(14)	0.1035(13)
	0.492	0.0953(26)	0.0959(24)	…	0.1010(15)	0.1034(13)
	0.5	0.0952(27)	0.0959(24)	…	0.1010(15)	0.1034(13)
	0.55	0.0948(28)	0.0955(25)	…	0.1008(16)	0.1033(14)

$af_{D_s^*}$	am_s=0.037	am_s=0.04	am_s=0.043
am_c=0.45	0.1079(11)	0.1089(10)	0.1099(10)
am_c=0.492	0.1079(11)	0.1090(11)	0.1100(10)
am_c=0.50	0.1079(11)	0.1090(11)	0.1100(10)
am_c=0.55	0.1080(12)	0.1090(11)	0.1101(11)

拟合结果

通过线性内插、外推

$$f_{D^*}(m_c, m_{u/d}) = f_{D^*}^{\text{phys}} + b_1(m_\pi^2 - m_\pi^2(\text{phys})) + b_2(M_D - M_D^{\text{phys}})$$
$$f_{D_s^*}(m_c, m_s) = f_{D_s^*}^{\text{phys}} + c_1(m_{ss}^2 - m_{ss}^2(\text{phys})) + c_2(M_{D_s} - M_{D_s}^{\text{phys}})$$

拟合物理点处的 $D^{(*)}$， $D_s^{(*)}$ 质量或衰变常数

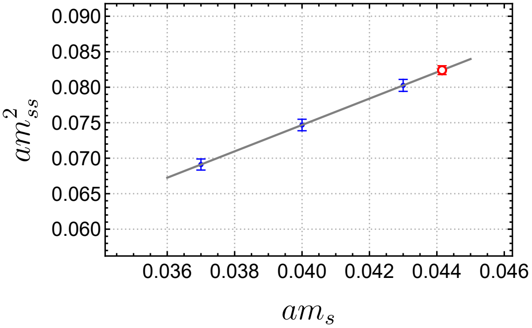
$$\chi^2 = \sum \frac{\mathcal{F}(am_c, am_q) - \mathcal{F}(\text{data})}{\sigma^2(\mathcal{F}(am_c, am_q)) + \sigma^2(\mathcal{F}(\text{data}))} \qquad q = u/d, s$$

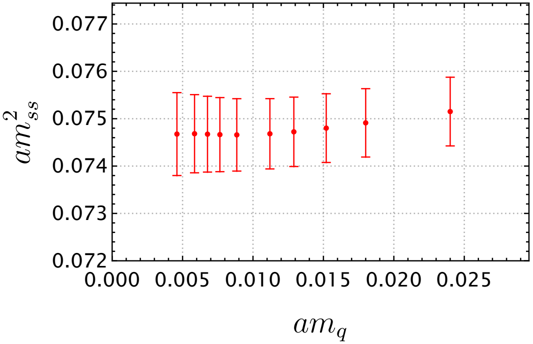
拟合结果

物理点：

$$am_{ss}^2 - am_{ss}^2(phys) = r\,(am_s - am_s(phys))$$

$$am_{ss}^2 = 2am_K^2 - am_\pi^2$$

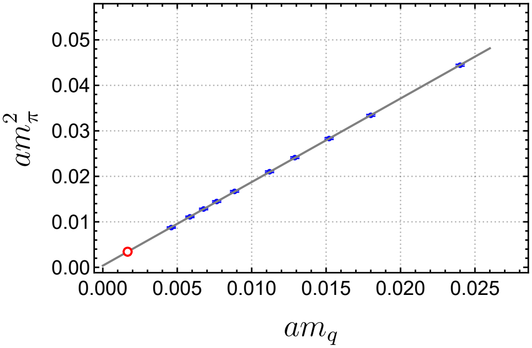


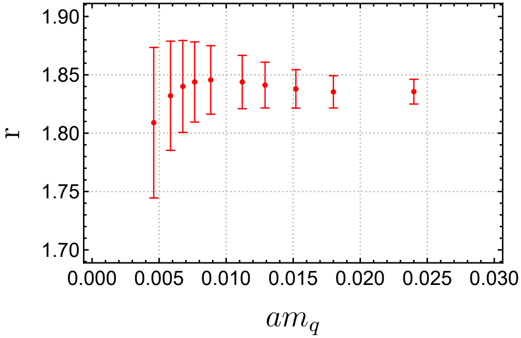


拟合结果

物理点:

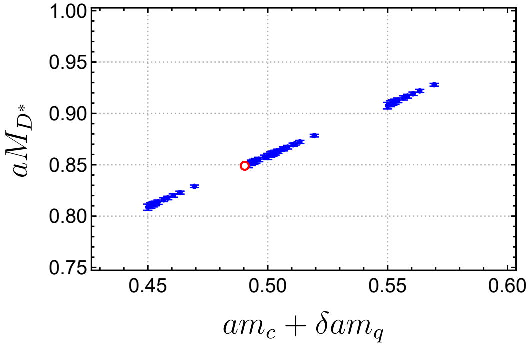
$$am_\pi^2 - am_\pi^2(phys) = r\left(am_q - am_q(phys)\right)$$



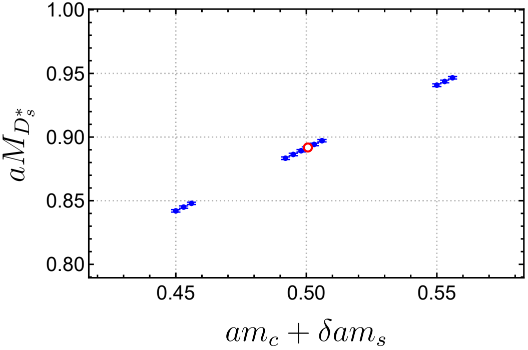


拟合结果

$\delta am_q = am_q - 0.0046$

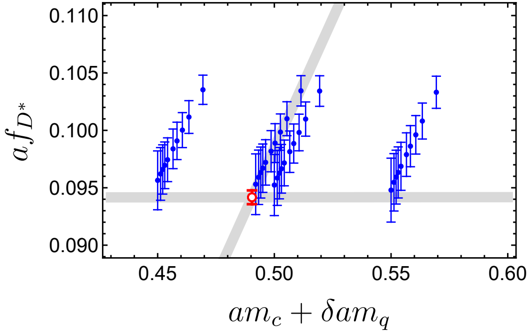


$\delta am_s = am_s - 0.037$

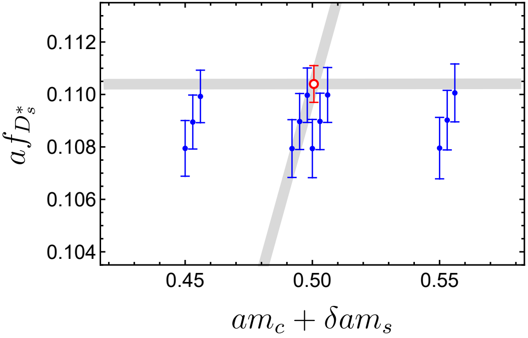


拟合结果

$\delta am_q = am_q - 0.0046$



$\delta am_s = am_s - 0.037$



拟合结果

重整化常数，2GeV

a^{-1}	$Z_A(Z_V)$	Z_T	Z_S
2.383(9)	1.0807(31)	1.1581(90)	1.011(15)

初步结果，单位:GeV

	M_D	M_{D^*}	M_{D_s}	$M_{D_s^*}$
Fit	1.872(7)	2.023(8)	1.967(8)	2.125(9)
PDG[1]	1.8695(4)	2.0101(4)	1.9690(14)	2.1066(34)

[1] R. L. Workman *et al.* [Particle Data Group] PTEP **2022** (2022), 083C01

拟合结果

初步结果，单位:GeV

a^{-1}	f_D	f_{D_s}	f_{D^*}	$f_{D_s^*}$
2.383(9)	0.2162(14)	0.2485(19)	0.2425(20)	0.2843(23)
1.730(4)[1]	0.213(2)(4)	0.249(5)	0.234(3)(5)	0.274(5)(5)
FLAG average[2]	0.2090(24)	0.2480(16)	-	-

a^{-1}	$f^T_{D_s^*}/f_{D_s^*}$	$f^T_{D^*}/f_{D^*}$	f_{D^*}/f_D	$f_{D_s^*}/f_{D_s}$	f_{D_s}/f_D	$f_{D_s^*}/f_{D^*}$
2.383(9)	0.91(2)	0.89(2)	1.12(2)	1.14(2)	1.15(2)	1.17(2)
1.730(4)[1]	0.92(4)	0.91(4)	1.10(3)	1.10(4)	1.16(3)	1.17(3)

1Y. Chen *et al.*,Chin. Phys. C **45** (2021) no.2, 023109

2Y. Aoki *et al.* [FLAG] Eur. Phys. J. C **82** (2022) no.10, 869

总结

1.overlap 费米子作用量具有格点上的手征对称性，不需要考虑 $f_{D_{(s)}}$ 的重整化。

2. 另两个海夸克质量点上的计算正在进行，最终外推到海夸克的手征极限。

3. 首次给出通过张量流定义的 $D^*_{(s)}$ 衰变常数的格点结果。