Finite temperature QCD phase transition with 3 flavors of Mobius domain wall fermions

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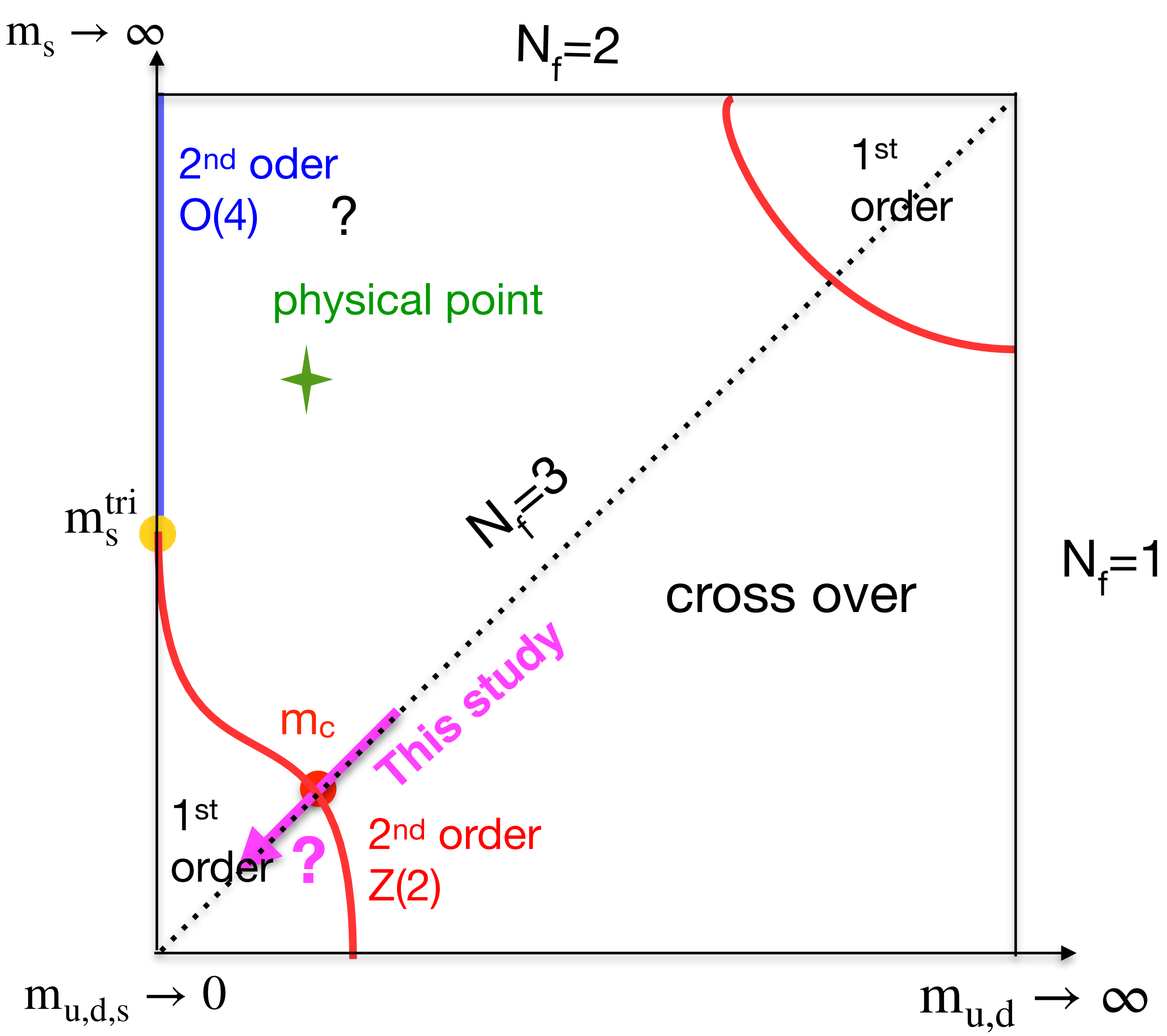
Outline

- Background & Motivation
- Previous lattice studies
- Lattice setup
- Results
- Summary & Outlook

The nature of QCD phase transition at $\mu_B = 0$

Columbia plot



Order of phase transition depends on m_l, m_s & N_f

Sigma model: 1st order phase transition in the chiral limit for $N_f = 3$

[Pisarski, Wilczek PRD 84]

This work:

➡ Explore $N_f = 3$ chiral region using first-principle lattice QCD

Previous Nf=3 lattice QCD studies

- 1st order phase transition is found at non-zero quark masses using O(a) improved Wilson fermion and $m_\pi^c \lesssim 110$ MeV [Kuramashi et al., PRD 20]
- No direct evidence of a 1st order region was found using HISQ on Nt = 6 and $m_\pi^c \lesssim 50$ MeV [A. Bazavov et al., PRD 17]
- 2nd order phase transition was found in the continuum limit with staggered [Francesca Cuteri et al., JHEP 21]
- 2nd order phase transition was found in the chiral limit on Nt=8 using HISQ [Lorenzo Dini et al., PRD 22]

Consensus: 1st order region shrinks for both fermions [de Forcrand, O.P. PoS LAT 07; Jin et al. PRD 15,17]

Our aim is to investigate $N_f = 3$ QCD phase structure with Mobius Domain Wall Fermion

Why MDWF

- Exact chiral symmetry at finite a for infinite Ls
- Reduced χ_{SB} parameterized by residual mass when Ls is finite

Lattice Setup

- N_f=3 Mobius Domain Wall Fermion with L_s=16
- Symanzik gauge action at $\beta = 4.0$ (a=0.1361(20) fm)
- Using Wilson flow t_0 to set the scale and matching with N_f=2+1 physical point
  $\sqrt{t_0}^{phys} = 0.1465(21)(13)$ fm [S.Borsanyi et al., JHEP 2012]

☆T > 0:

- ☑ N_t=8 (T=181.1(2.6) MeV): N_s=16, $0 \le m_l \le 0.2$
  N_s=24, $0 \le m_l \le 0.14$
- ☑ N_t=12 (T=120.8(1.8) MeV): N_s=24, $-0.006 \le m_l \le 0.1$

☆T = 0:

N_s=24, N_t=48, $0.02 \le m_l \le 0.045$

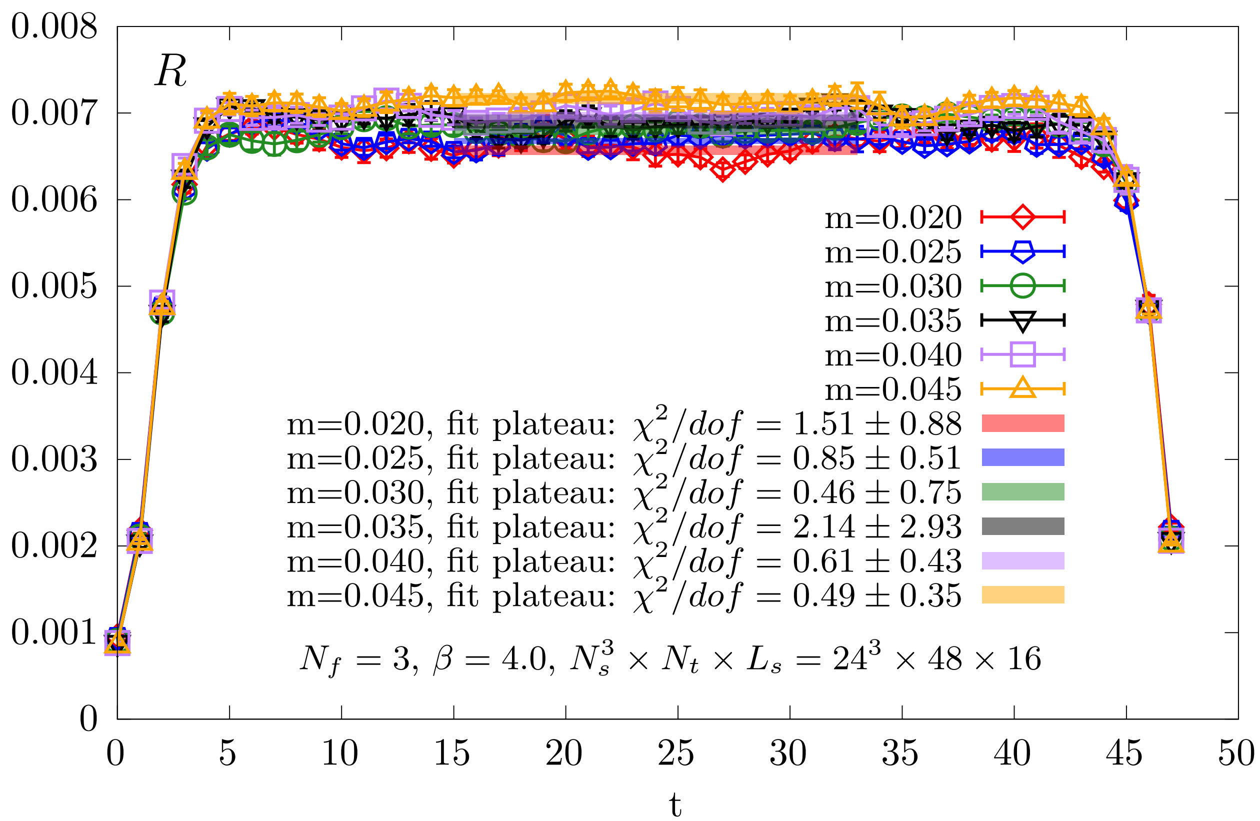
- Measured: residual mass, chiral condensate, disconnected chiral susceptibility & Binder cumulant
- Codes: Grid & Hardons
- Resources: Supercomputer Fugaku

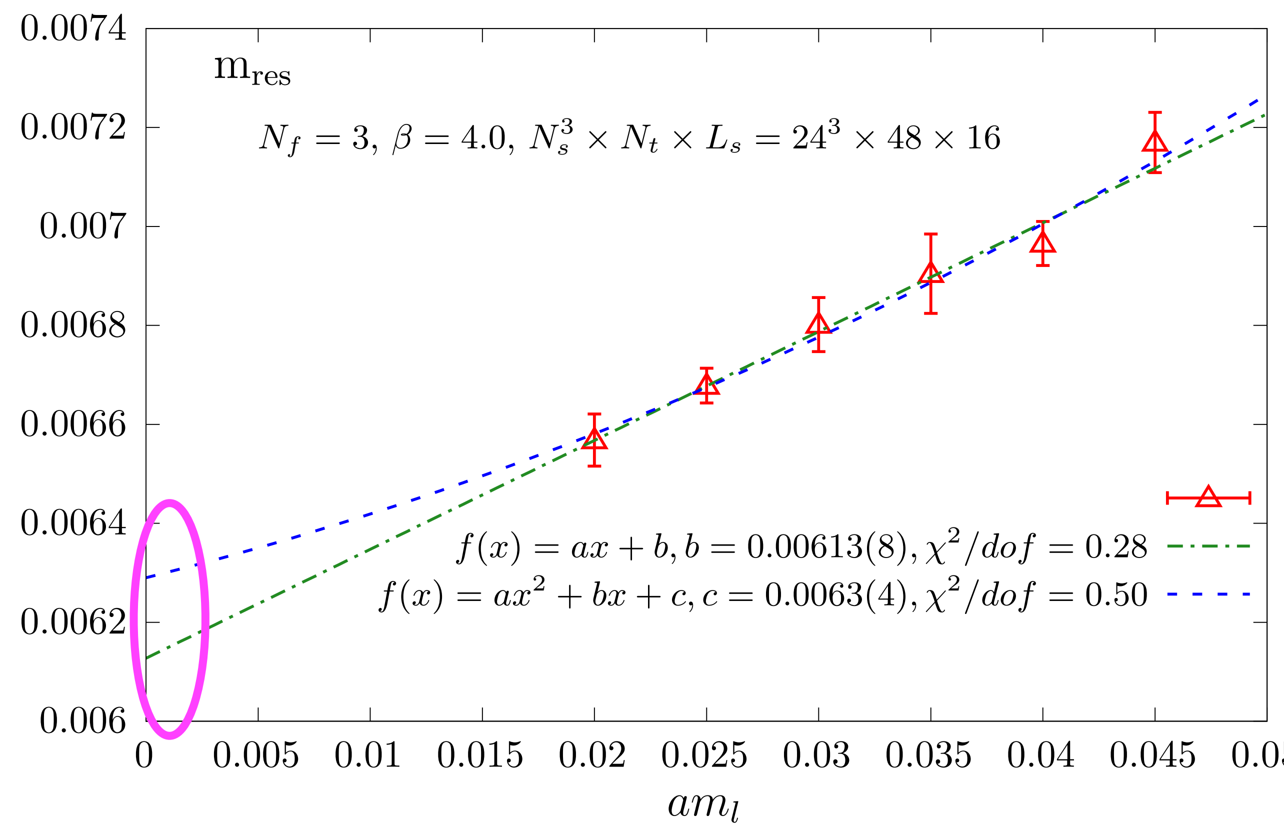
Residual chiral symmetry breaking

- For finite Ls chiral symmetry is broken, leading to an **additive renormalization of the quark mass** $m_l \to m_l + m_{\rm res}$
- $m_{\rm res} \to 0$ as $L_s \to \infty$**,** cost is high when increase Ls, practical simulation: Ls=16
- Measure the ratio of midpoint correlator to the pion correlator evaluated at large distance

$$m_{\rm res} = R(t) = \frac{\left\langle \sum_{\vec{x}} J^a_{5q}(\vec{x}, t)\pi^a(\vec{0},0) \right\rangle}{\left\langle \sum_{\vec{x}} J^a_{5}(\vec{x}, t)\pi^a(\vec{0},0) \right\rangle}$$

Smallest input m_l is -0.006

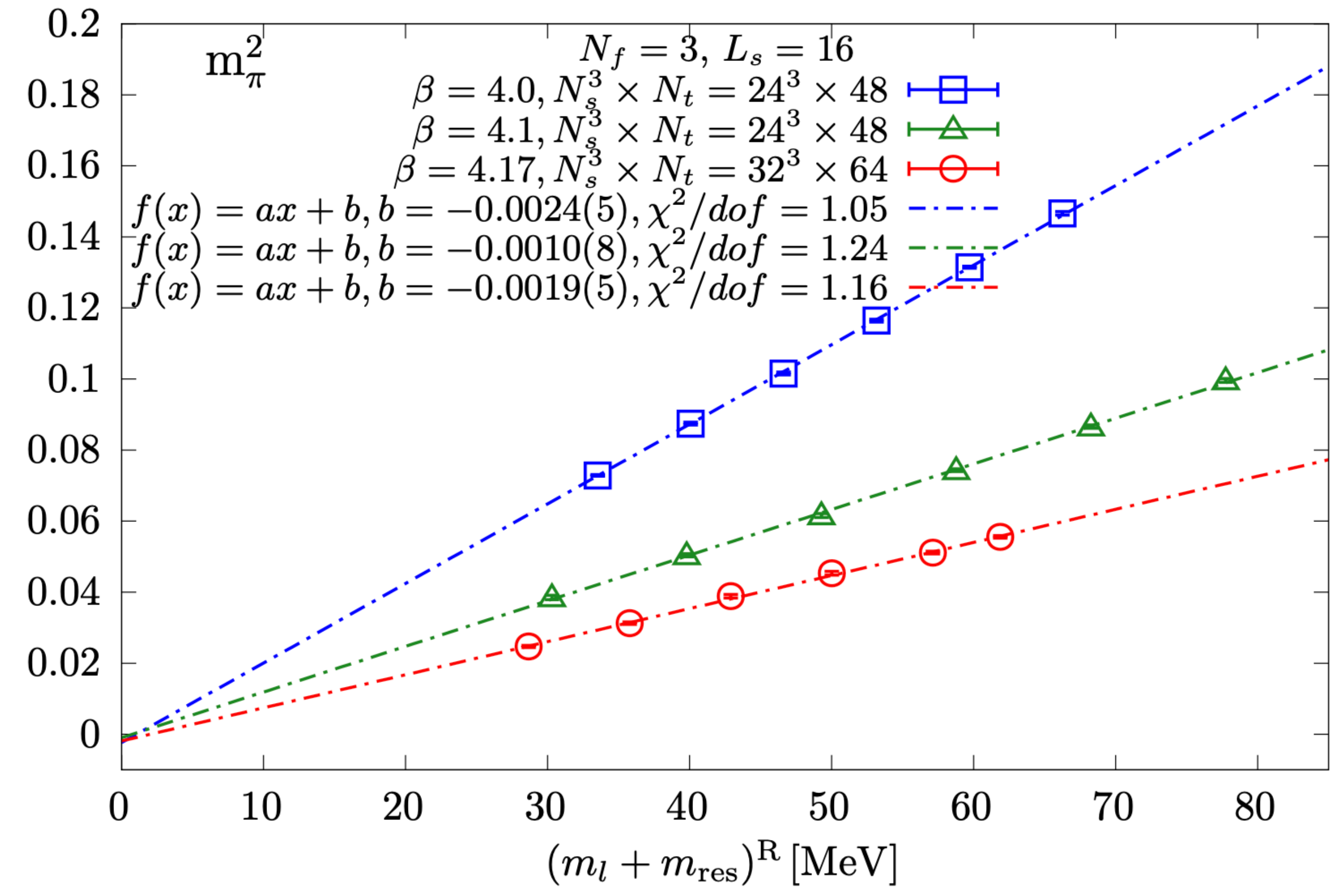




Residual chiral symmetry breaking on chiral condensate

- From low energy effective QCD $\mathscr{L}$, the effect of mixing between chiral walls for long-distance quantities will result in $m_l \to m_l + m_{\rm res}$. e.g. $m_\pi^2 \propto m_l + m_{\rm res}$

- For quantities whose sensitivity to χ_{SB} effects extends up to the cutoff scale, the above argument doesn't go through. e.g.$\langle\bar{\psi}\psi\rangle$



$$\langle\bar{\psi}\psi\rangle\,|_{DWF} \sim \frac{m_l + x m_{res}}{a^2} + \langle\bar{\psi}\psi\rangle\,|_{\rm cont.} + \ldots$$

x is not known , expected $x = \mathcal{O}(1)$

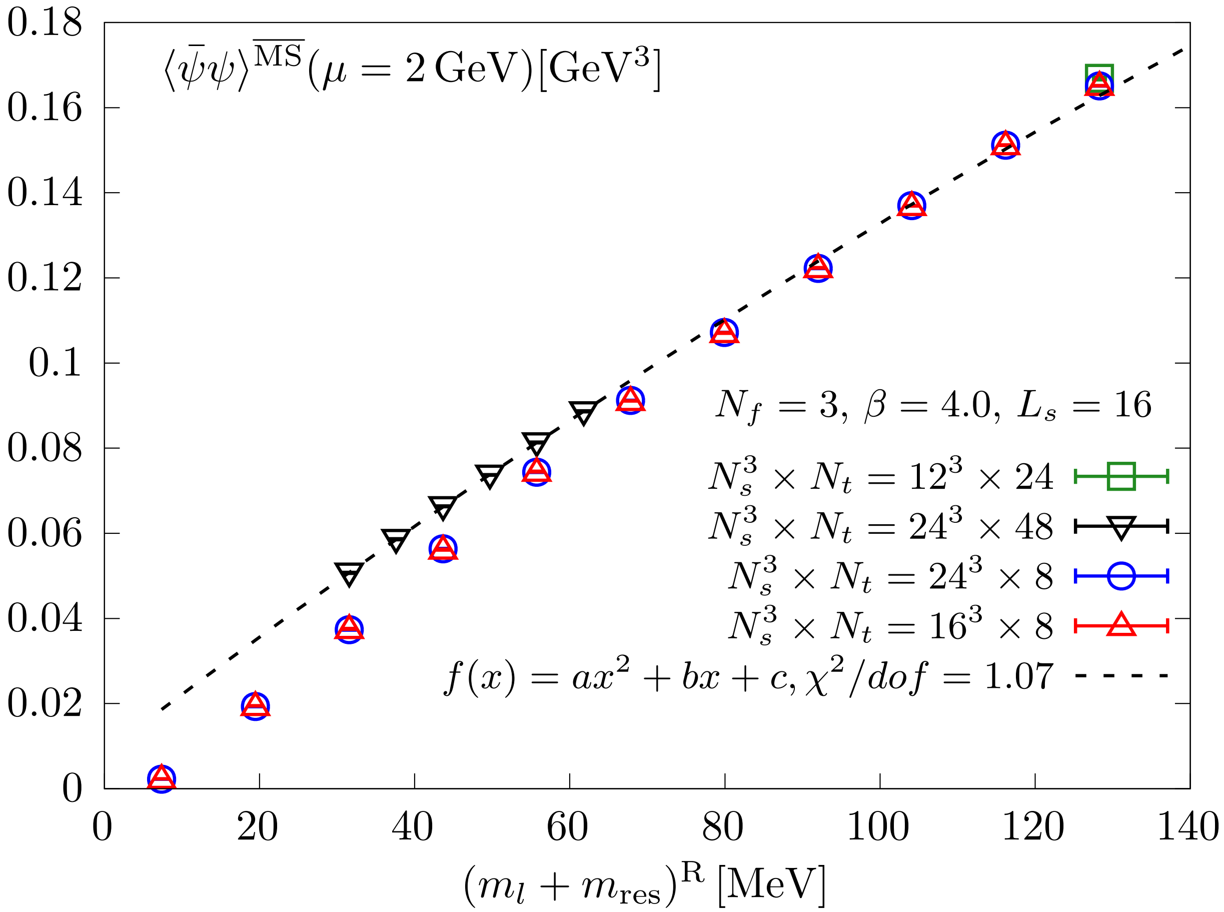
[S. Sharpe, arXiv: 0706.0218]

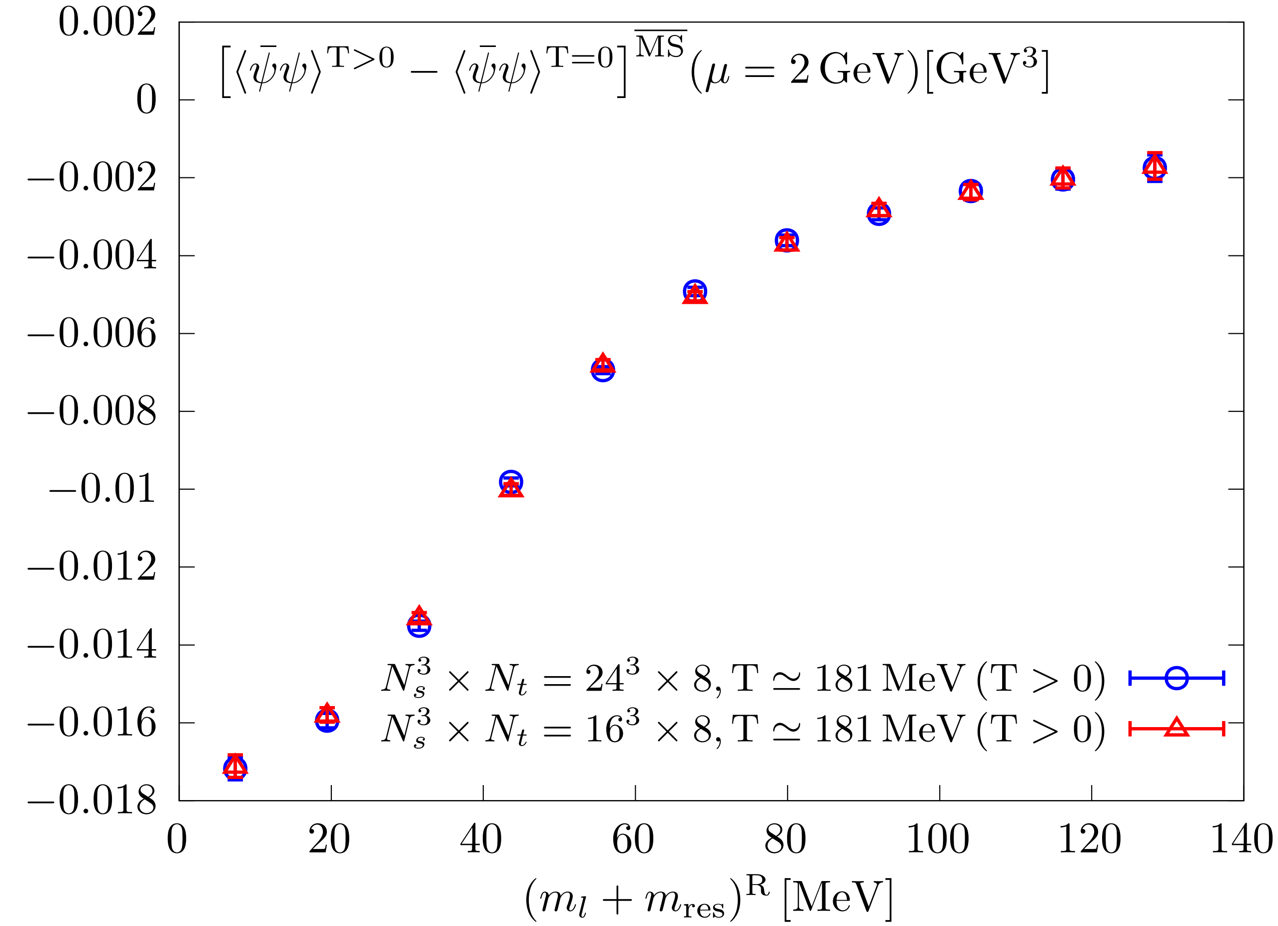
Power divergence remains if one extrapolates to $m_q = m_l + m_{res} = 0$

$$\Rightarrow \lim_{m_q\to 0}\lim_{L\to 0}\langle\bar{\psi}\psi\rangle\,|_{DWF} \sim \langle\bar{\psi}\psi\rangle\,|_{\rm cont.} + (x-1)\frac{m_{res}}{a^2}\ldots$$

Chiral condensate at T= 181.1(2.6) MeV

Order parameter for χ_{SB}: $\langle \bar{\psi}\psi \rangle = \frac{1}{N_s^3 N_t} \mathrm{Tr} \langle M^{-1} \rangle$





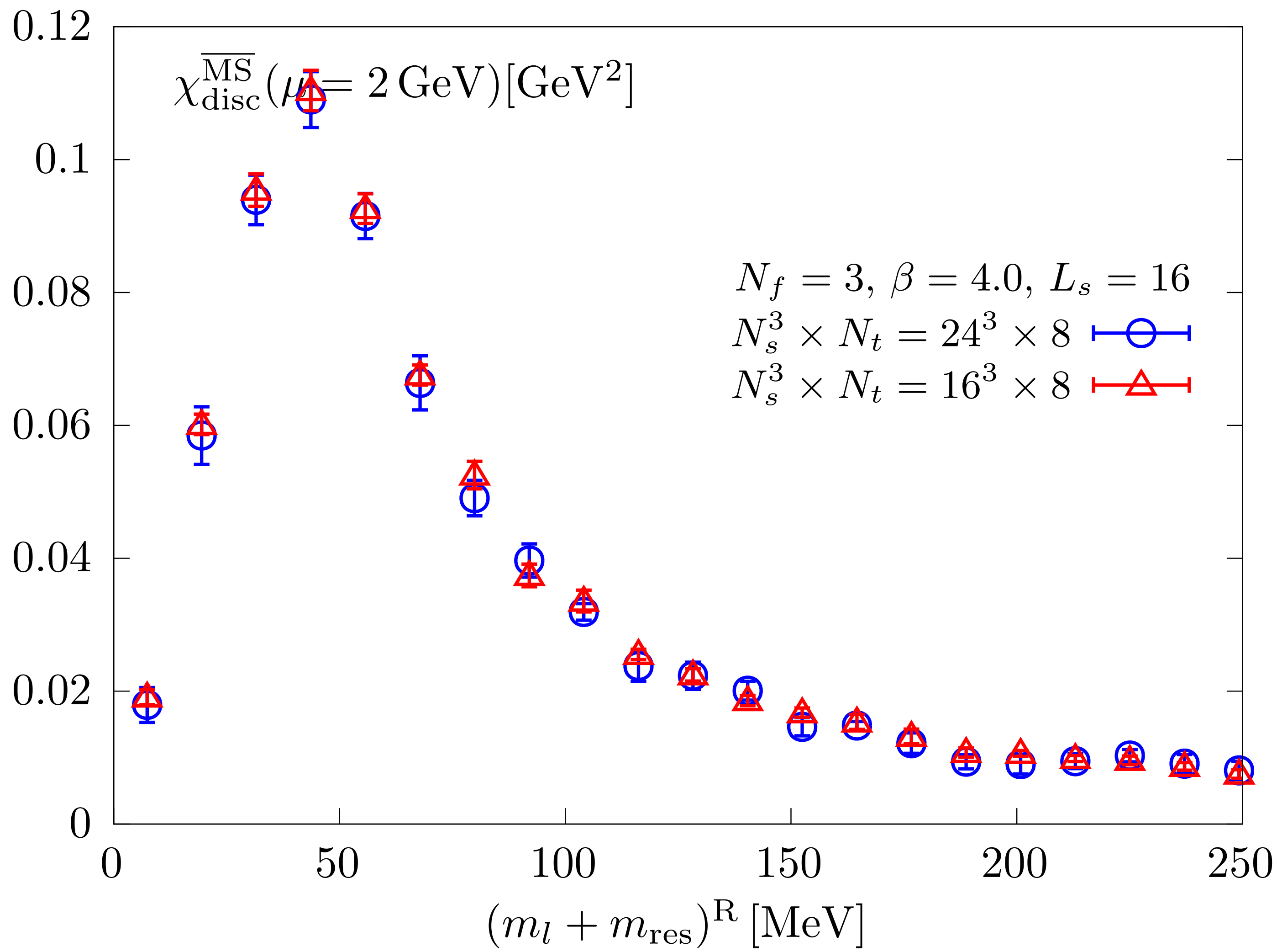
- ☑ Quark mass in $\overline{\mathrm{MS}}$ scheme: $m^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = Z_m^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) a^{-1}(am)$
- ☑ Multiplicative renormalization: $\langle \bar{\psi}\psi \rangle^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = \frac{a^{-3}(a^3 \langle \bar{\psi}\psi \rangle)}{Z_m^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})}$
- ☑ Remove additive divergence $\frac{m_l + x m_{res}}{a^2}$ by $\langle \bar{\psi}\psi \rangle^{\mathrm{T}>0} - \langle \bar{\psi}\psi \rangle^{\mathrm{T}=0}$
- a crossover

Disconnected chiral susceptibility at 181.1(2.6) MeV

$$\chi_{\rm disc} = \frac{1}{N_s^3 N_t}\left(\left\langle \left(\mathrm{Tr}\, M^{-1}\right)^2 \right\rangle - \left\langle \mathrm{Tr}\, M^{-1} \right\rangle^2\right)$$

Renormalized to $\overline{\rm MS}(\mu = 2\,\mathrm{GeV})$ with $(Z_{\rm m}^{\overline{\rm MS}})^{-2}$ **:** $\chi_{\rm disc}^{\overline{\rm MS}}(\mu = 2\,\mathrm{GeV})[\mathrm{GeV}^2] = \left(\frac{1}{Z_m^{\overline{\rm MS}}}\right)^2 \chi_{\rm disc}^{\rm bare}\ \left(a^{-2}[\mathrm{GeV}^2]\right)$

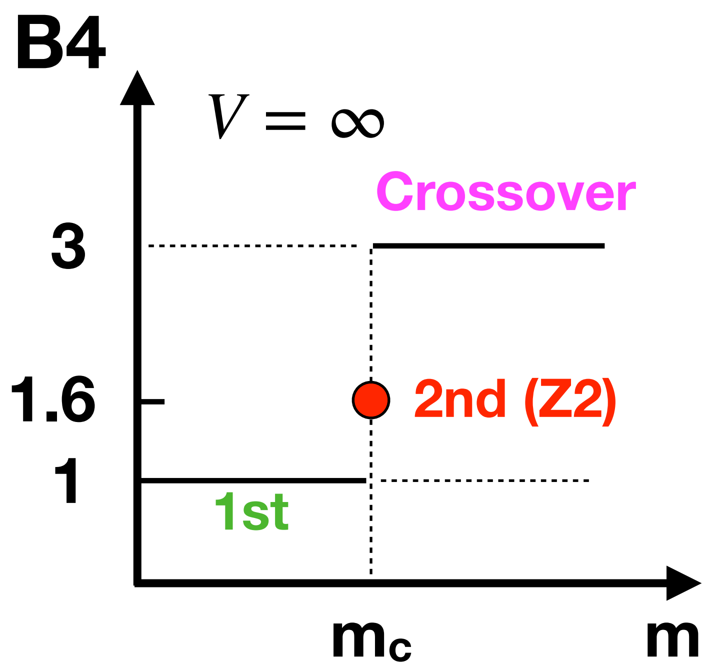
Describes fluctuations of the chiral condensate & Peak at transition point

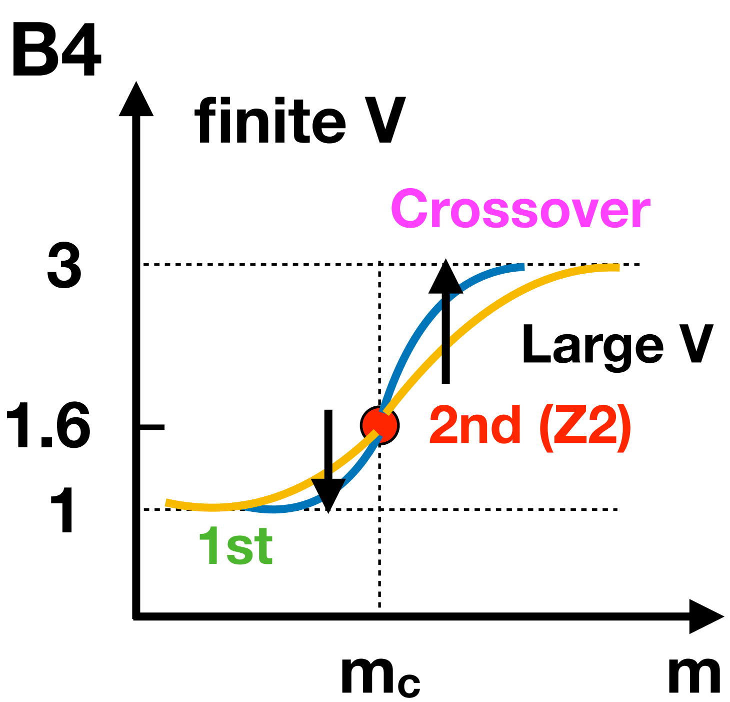


- **Pseudo critical mass is around 44 MeV**

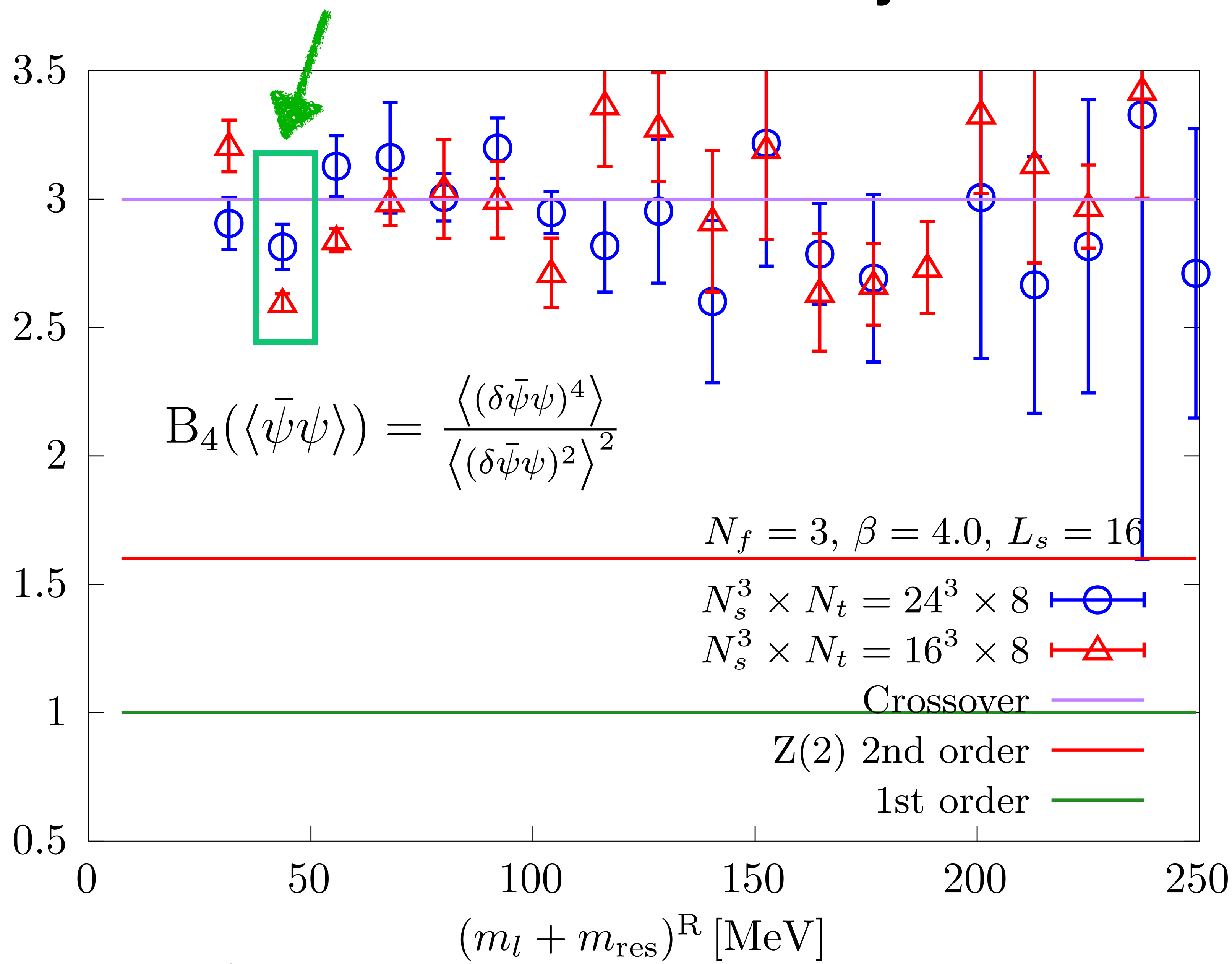
Binder cumulant of chiral condensate at 181.1(2.6) MeV

$$B_4(\bar\psi\psi) = \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2}, \quad \delta\bar\psi\psi = \bar\psi\psi - \langle\bar\psi\psi\rangle$$

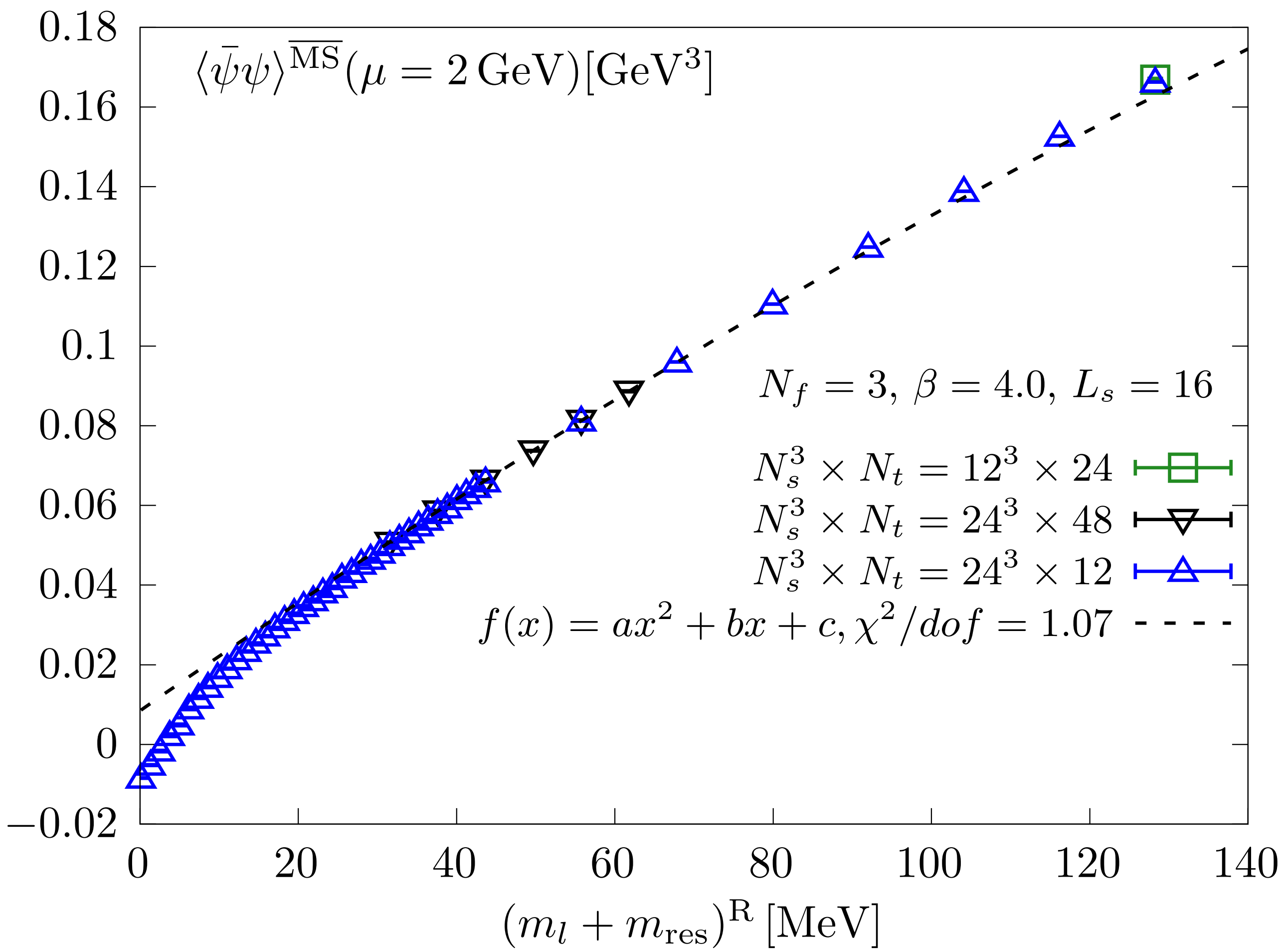


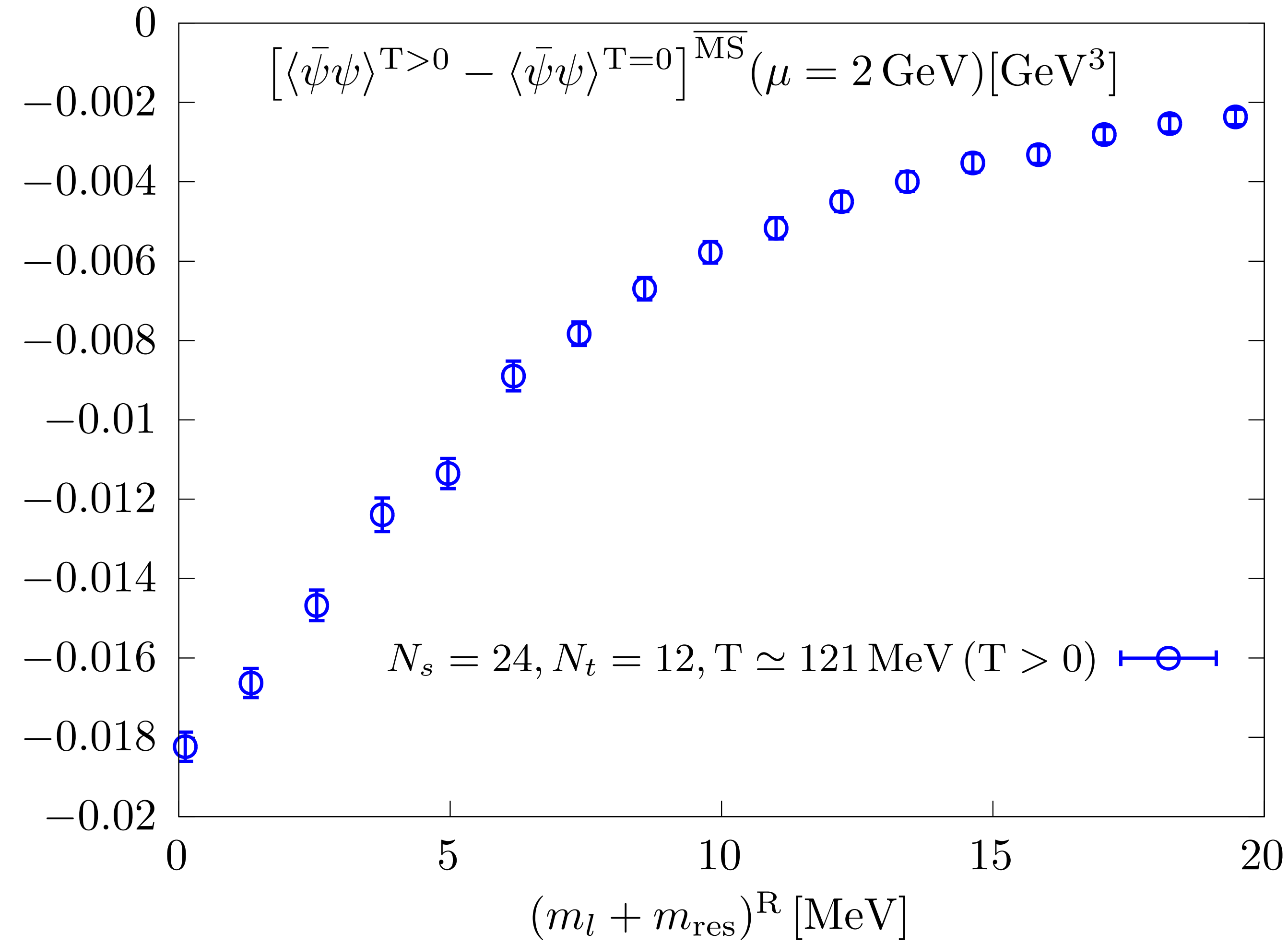


B4 at transition point seems to be a crossover in the thermodynamic limit.



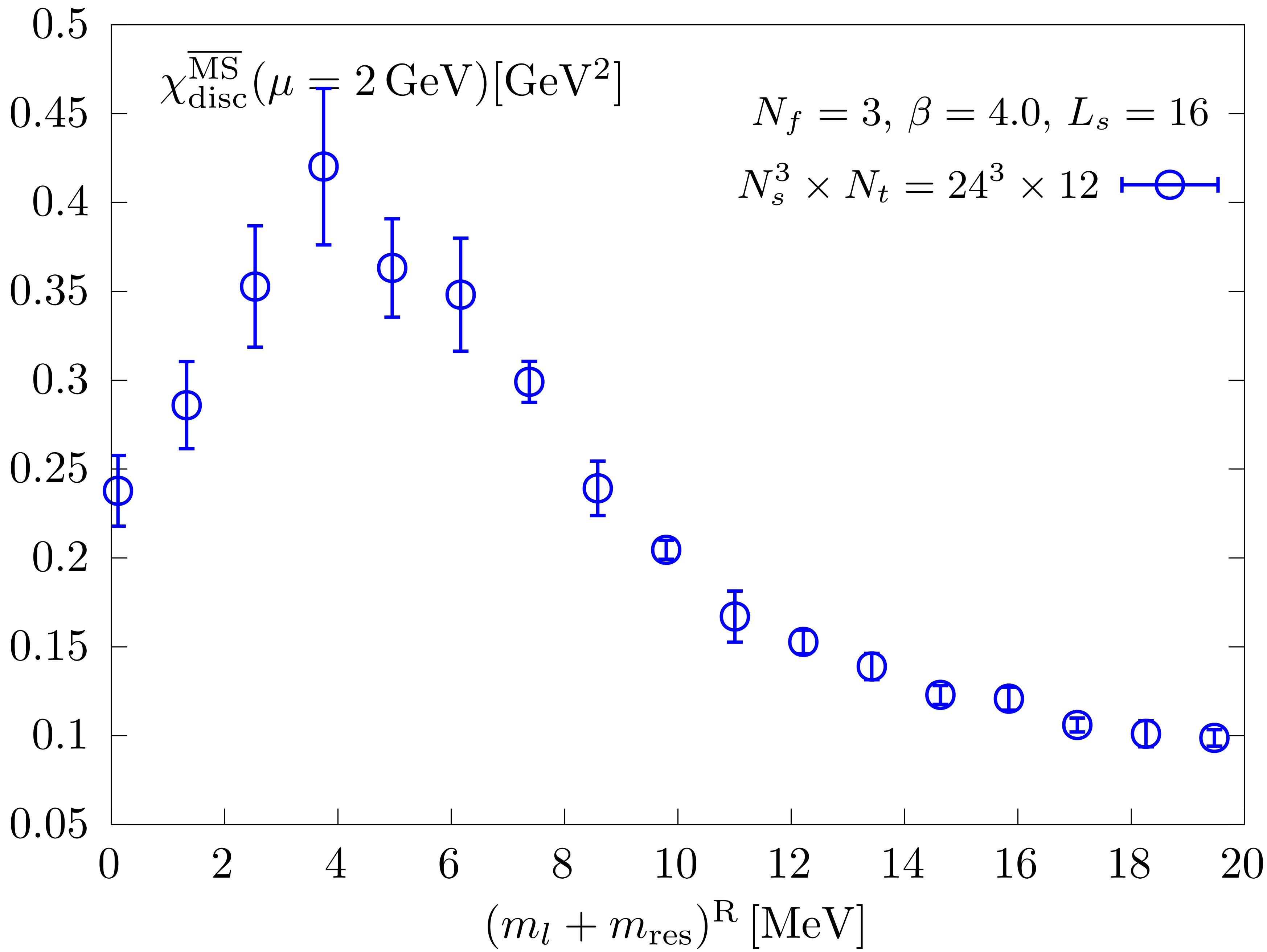
Chiral condensate at T~120.8(1.8) MeV





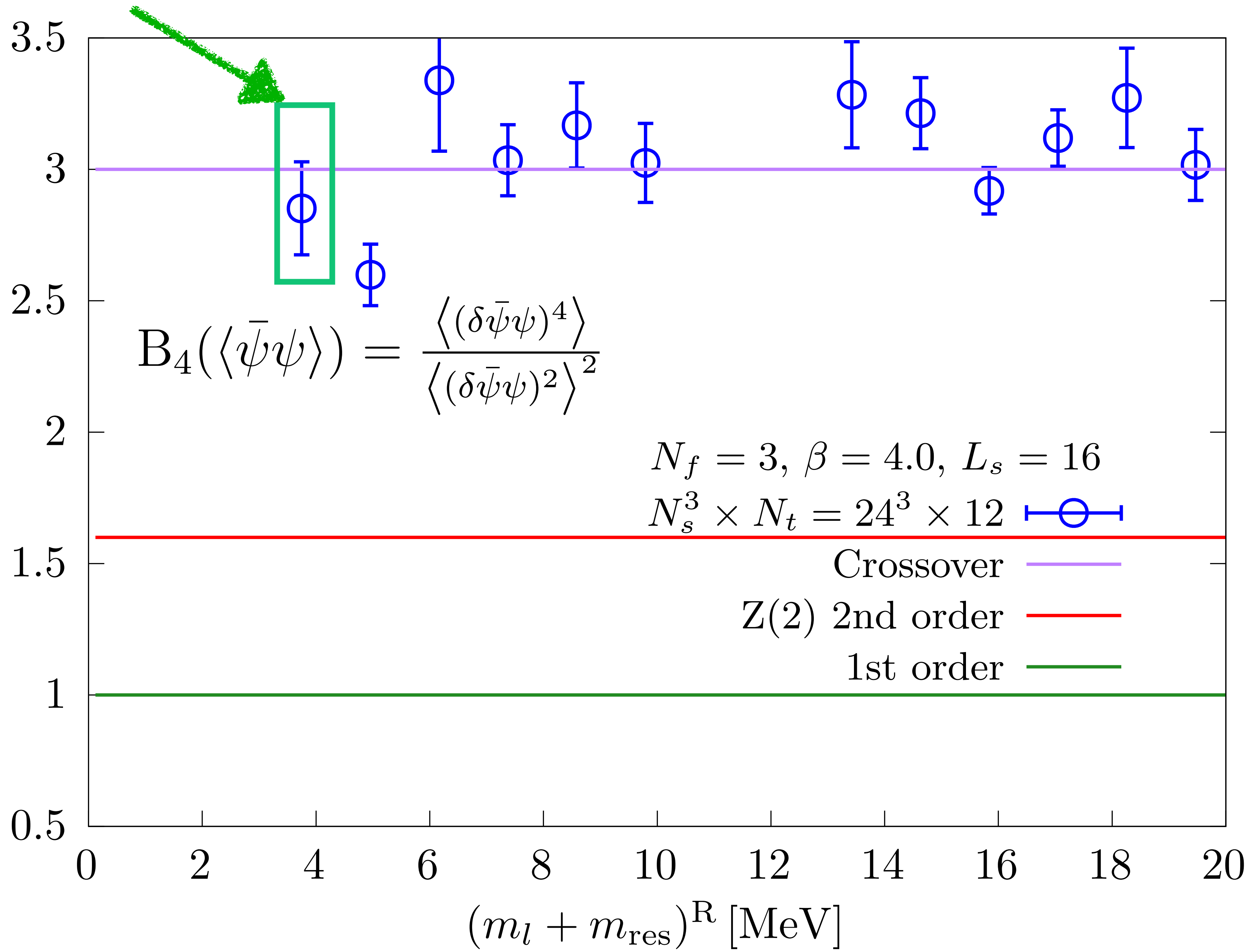
- **Residual additive divergence remains**
- **Additive & multiplicative divergence has been removed**

Disconnected chiral susceptibility at 120.8(1.8) MeV



- **Transition point is around 3.7 MeV**

Binder cumulant of chiral condensate at 120.8(1.8) MeV



- **Suggests a crossover, though another volume would be important to confirm**

Summary and outlook

Summary:

- ☑ For $\langle \bar{\psi}\psi \rangle$, the explicit χ_{SB} effect due to finite Ls is more complicated than a simple additive shift of the input quark mass by $m_{\rm res}$
- ☑ It is a crossover at T~181.1(2.6) MeV, pseudo critical quark mass is around 44 MeV
- ☑ Data suggest a crossover at T~120.8(1.8) MeV and pseudo critical quark mass is around 3.7 MeV, need another volume to confirm

Outlook:

- ☐ Add another larger volume $36^3 \times 12$ for T~120.8(1.8) MeV
- ☐ Investigate the Ls dependence to check whether our chiral symmetry is ok

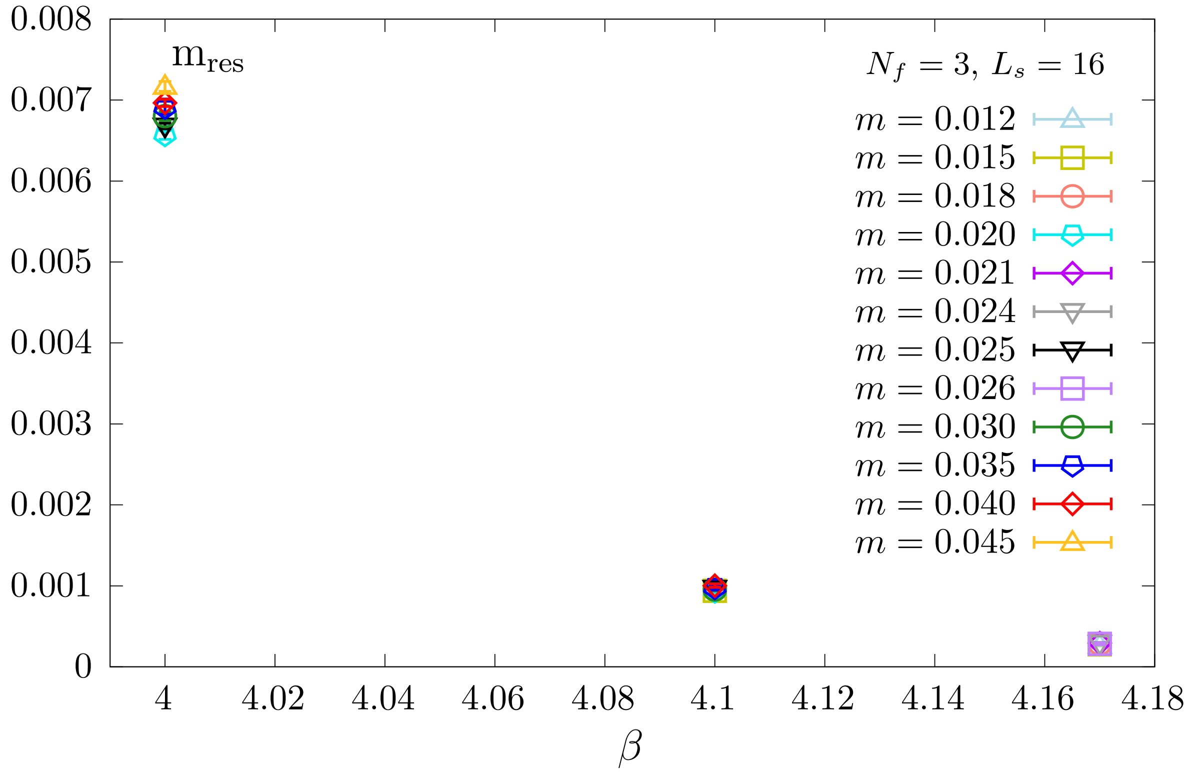
Acknowledgements

- Codes
  - HMC
    - Grid (Regensburg branch)
  - Measurements
    - Bridge++
    - Hadrons / Grid

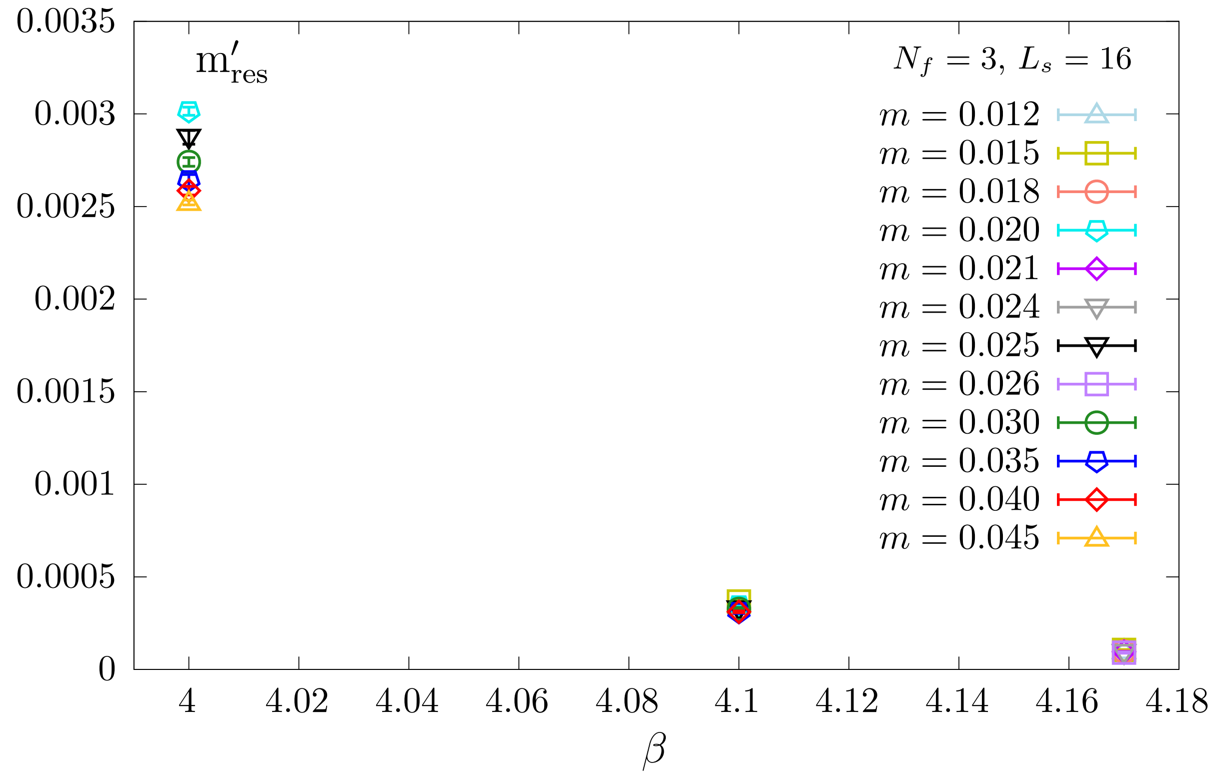
- Computers
  - Supercomputer Fugaku provided by the RIKEN Center for Computational Science through HPCI project #hp210032 and Usability Research ra000001.
  - Wisteria/BDEC-01 Oddysey at Univ. Tokyo/JCAHPC through HPCI project #hp220108 Ito supercomputer at Kyushu University through HPCI project #hp190124 and hp200050
  - Hokusai BigWaterfall at RIKEN

- Grants
  - JSPS Kakenhi (20H01907)

Backup



m_{res}
$N_f = 3,\ L_s = 16$
$m = 0.012$
$m = 0.015$
$m = 0.018$
$m = 0.020$
$m = 0.021$
$m = 0.024$
$m = 0.025$
$m = 0.026$
$m = 0.030$
$m = 0.035$
$m = 0.040$
$m = 0.045$
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
4
4.02
4.04
4.06
4.08
4.1
4.12
4.14
4.16
4.18
β



m'_{res}
$N_f = 3,\ L_s = 16$
$m = 0.012$
$m = 0.015$
$m = 0.018$
$m = 0.020$
$m = 0.021$
$m = 0.024$
$m = 0.025$
$m = 0.026$
$m = 0.030$
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$m = 0.040$
$m = 0.045$
0.0035
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4.1
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4.14
4.16
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β