NuclearScience
Computing Center at CCNU

Lattice QCD prediction of kaon form factor at large Q^2 up to 28 GeV^2

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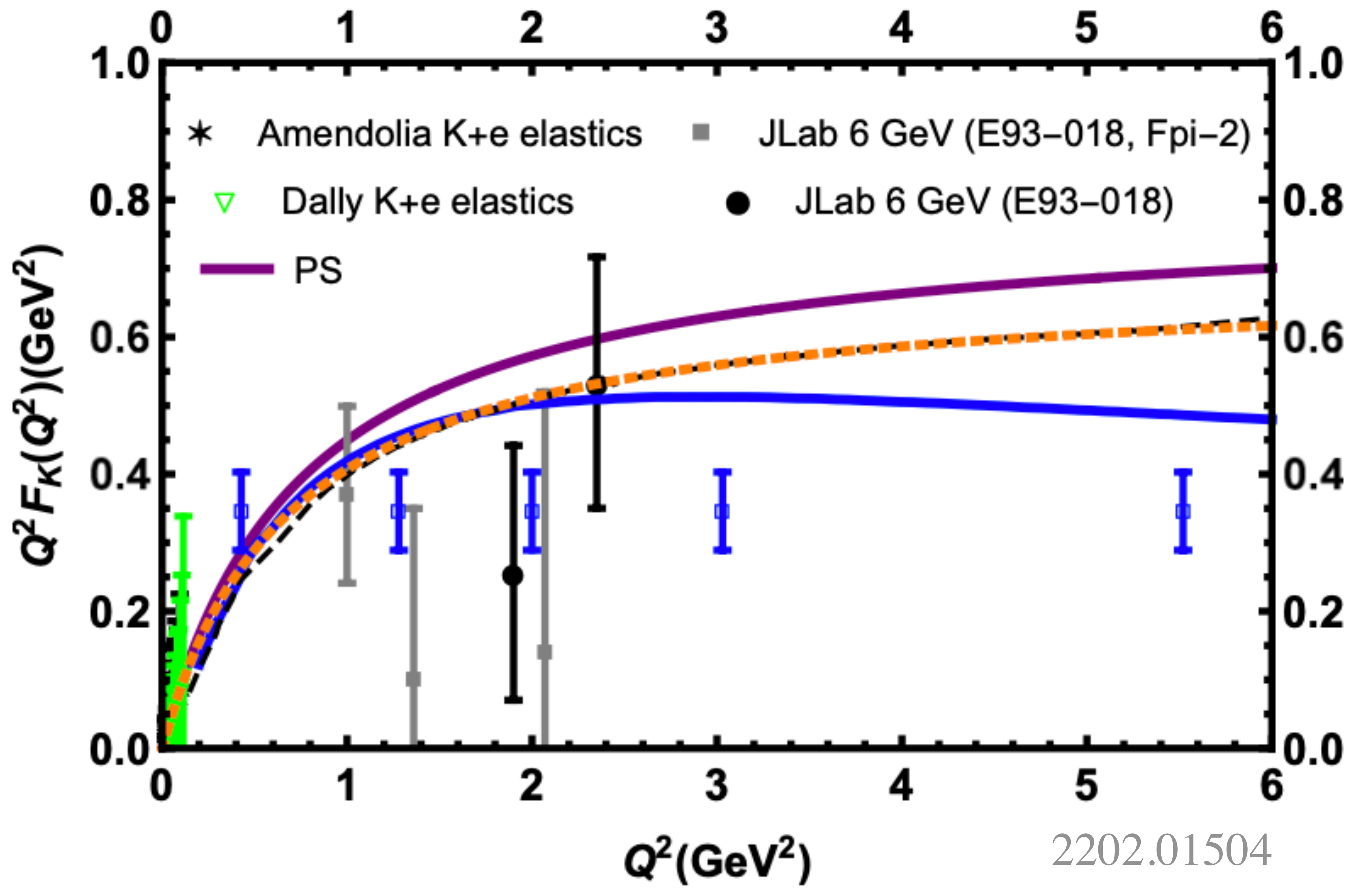
Outline

- Motivation
- Lattice Setup
- Compute form factor on the lattice
- Results & Summary

Motivation

☞ Form factor: the Fourier transform of electromagnetic current distribution in space

- The hadron structure
- The interplay between the emergent hadron mass (EHM) & the Higgs-mass mechanism
- FF + PDF -> GPD, three-dimensional image of the hadron
- Nucleon & Pion & **Kaon**



2202.01504

☞ Kaon:

- Experiment: JLab, EIC, EicC … (2102.11788, 2102.09222)
- Effective theories: QCD sum rules, DSE … (1703.04875)
- **Lattice QCD: from first principle**
  - ❍ **State-of-the-art: $Q^2 \leq$ 3 GeV^2** (2111.08135)
  - ❍ **This work: Q^2 up to 28 GeV^2**

Lattice setup:

- Lattice size: $N_s^3 \times N_t = 64^3 \times 64$, lattice spacing: $a = 0.076, 0.04$ fm
- Sea quark: Highly Improved Staggered Quark (HISQ) action

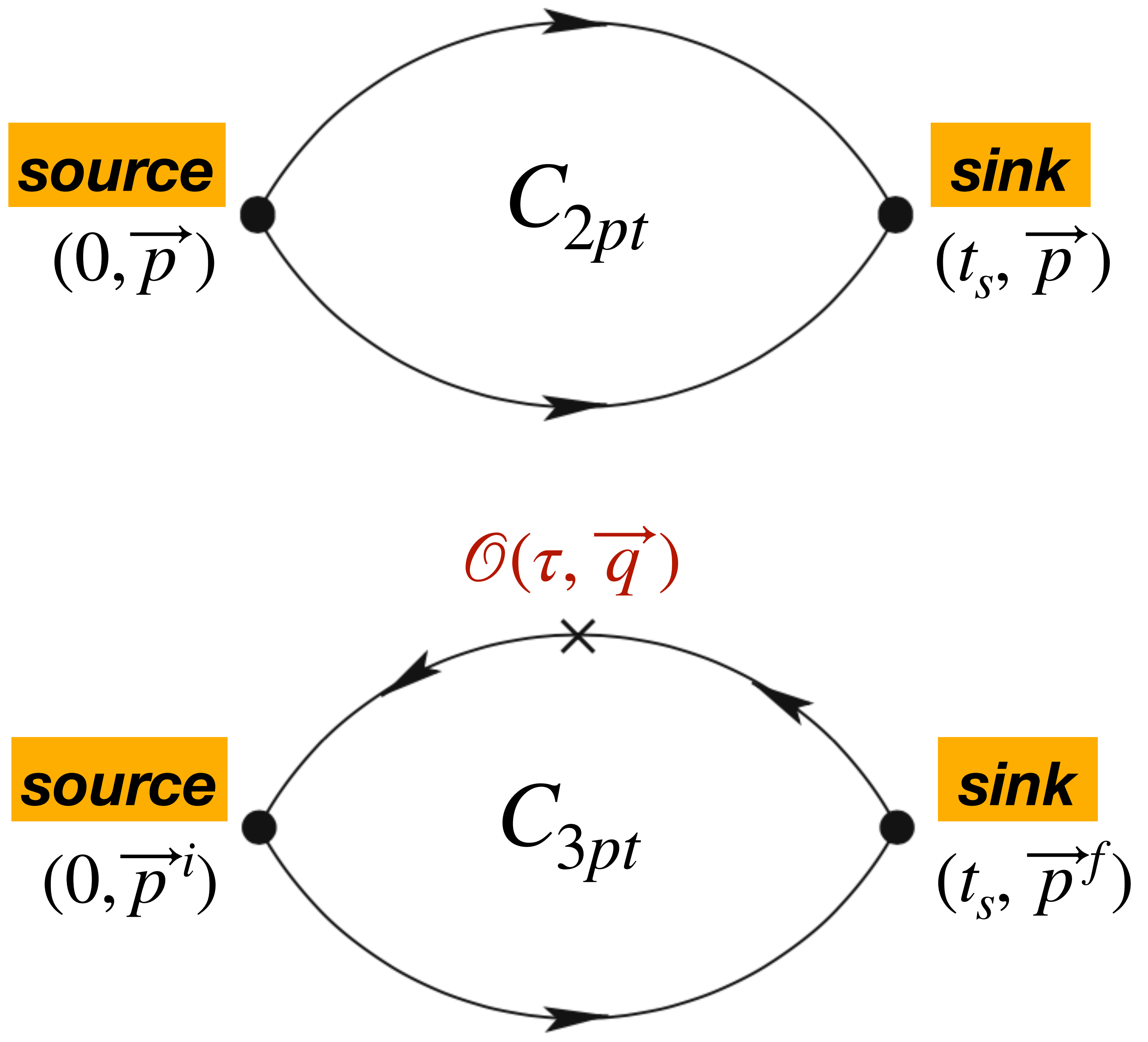
  Valence quark: Wilson-Clover action

  $\Rightarrow m_{\pi^+} = 140$ MeV, $m_{K^+} = 497$ MeV **at the physical point**
- Use boost smearing with the corresponding signs of the quark momenta at source & sink to improve the signal

  $\Rightarrow$ The wide range of Q^2 is **0~28 GeV**^{2}

Compute form factor on the Lattice

Construct a hadron state: $C_{2pt}(t, \vec{p}) = \left\langle H(t_s, \vec{p}) H^{\dagger}(0, \vec{p}) \right\rangle$



Insert an electromagnetic current $\mathcal{O}(\tau, \vec{q})$ to probe the hadron

$$\vec{p}^{\,f} = \vec{p}^{\,i} + \vec{q}$$

$$C_{3pt}(\tau, t_s; \vec{p}^{\,i}, \vec{p}^{\,f}) = \left\langle H(t_s, \vec{p}^{\,f}) \hat{\mathcal{O}}_{\gamma^\mu}(\tau, \vec{q}) H^{\dagger}(0, \vec{p}^{\,i}) \right\rangle$$

Extract the bare form factor:

$$F^B = \left\langle E_0, \vec{p}^{\,f} \,|\, \hat{\mathcal{O}}_{\gamma^\mu}(\tau, \vec{q}) \,|\, E_0, \vec{p}^{\,i} \right\rangle$$

from ~ C_{3pt} / C_{2pt}

Renormalization →

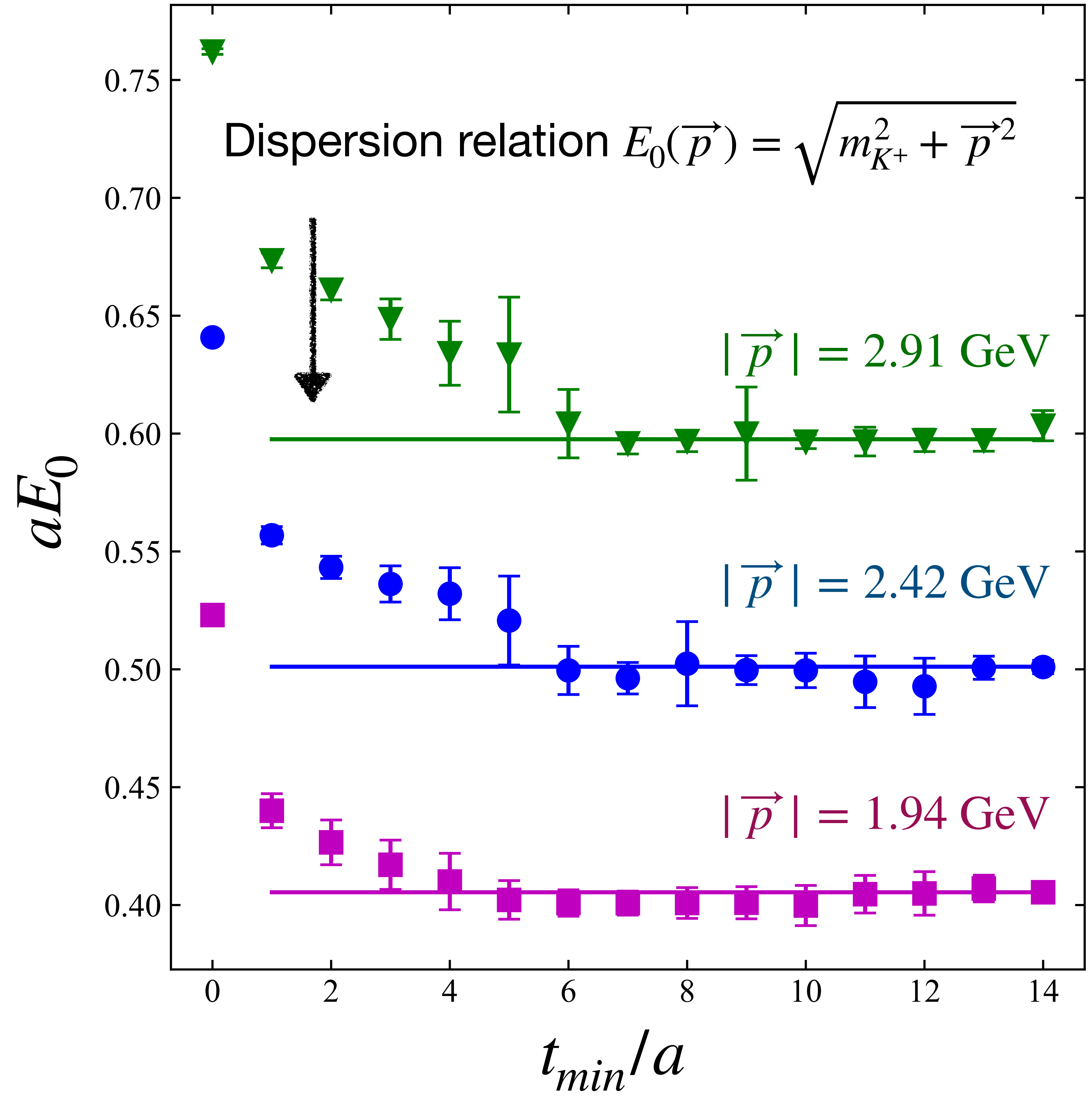
Form factor: $F(Q^2) = F^B \times Z_V$

with $Q^2 = -q^2$

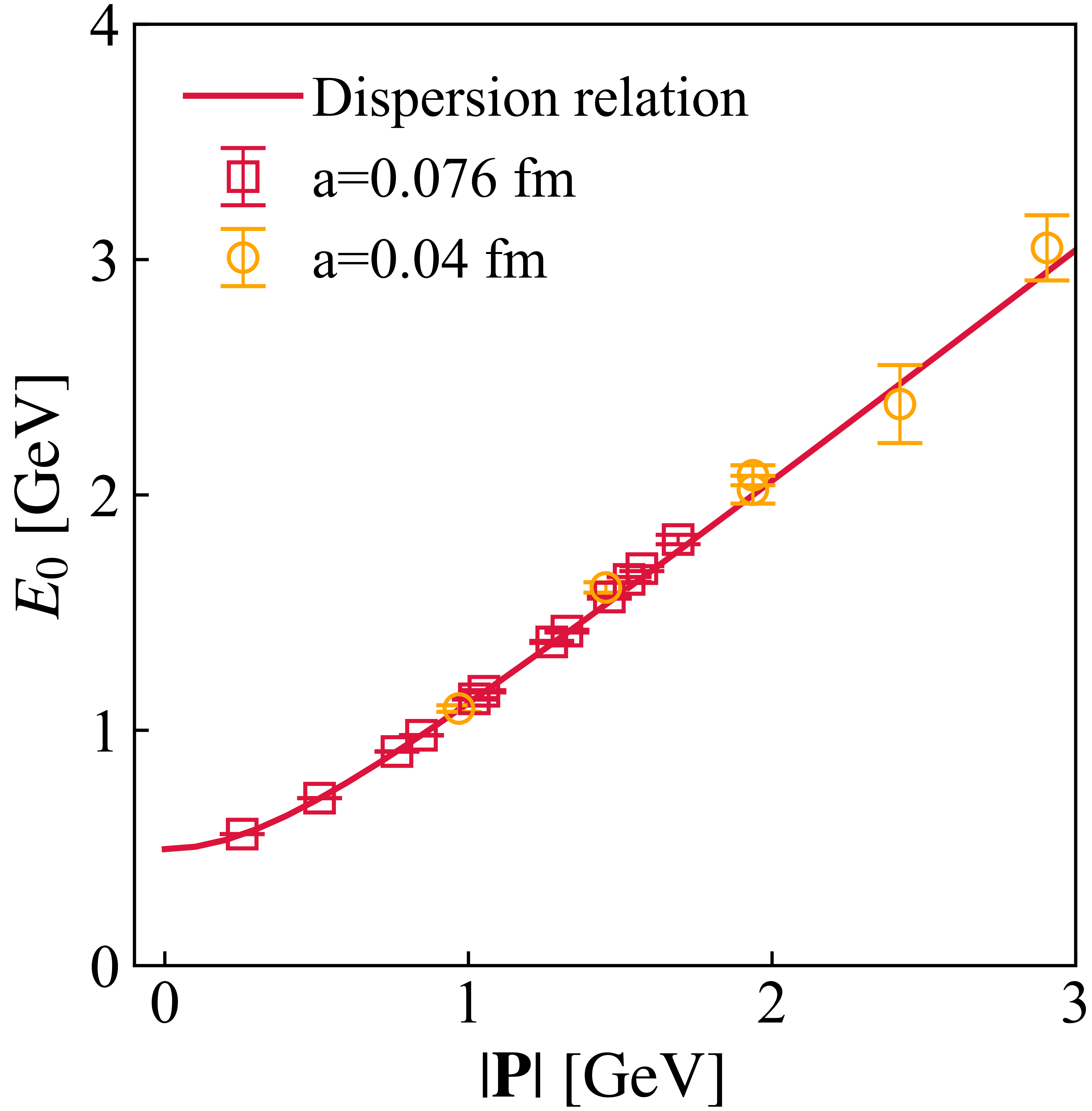
1. Construct the hadron state C_{2pt}

$$C_{2pt}(t_s) = \sum_{k=0}^{N_{state}-1} A_k \left[e^{-E_k t_s} + e^{-E_k(aN_t - t_s)}\right]$$

Fit with $N_{state} = 2$ → $E_0, E_1;\ A_0, A_1$



Fit with different range of $t_s/a \in [t_{min}/a, N_t/2]$



All results of E_0

2. Extract the bare form factor

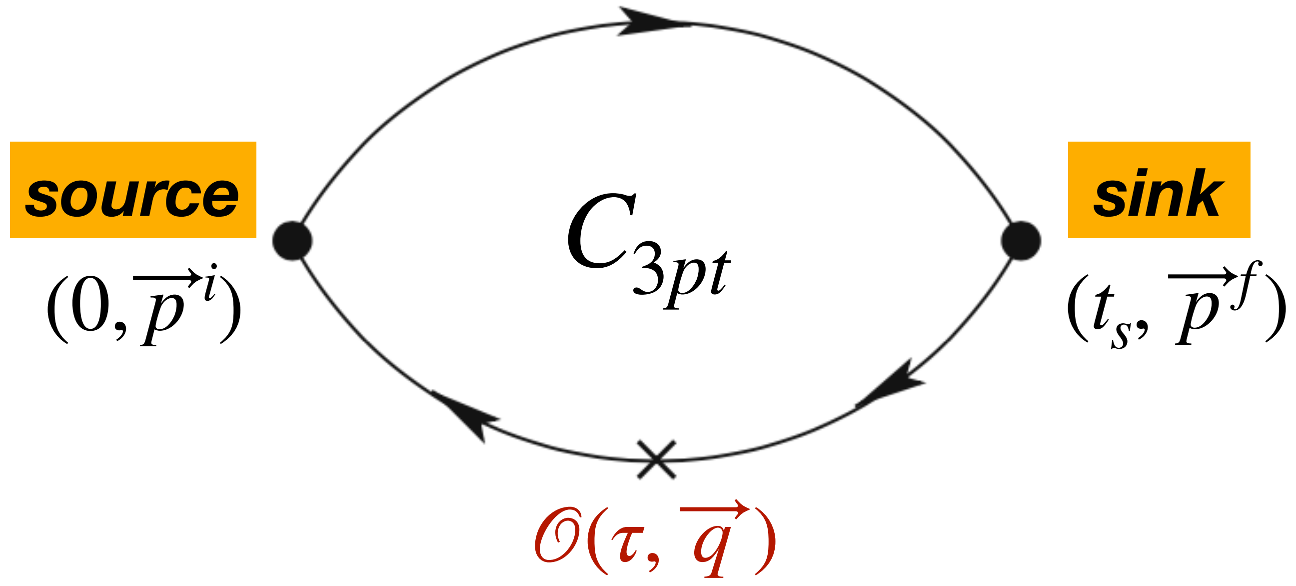
$$C_{3pt}(\vec{p}^{f}, t_s; \vec{q}, \tau; \vec{p}^{i}, 0) = \sum_{n,k=0}^{N_{state}-1} \underbrace{\left\langle \Omega \left| H(\vec{p}^{f}) \right| n \right\rangle \left\langle k \left| H^{\dagger}(\vec{p}^{i}) \right| \Omega \right\rangle e^{-(E_k^{\vec{p}^{i}} - E_n^{\vec{p}^{f}})\tau} e^{-E_n^{\vec{p}^{f}} t_s}} \times \left\langle n \left| \hat{\mathcal{O}}_{\Gamma}(\tau) \right| k \right\rangle$$

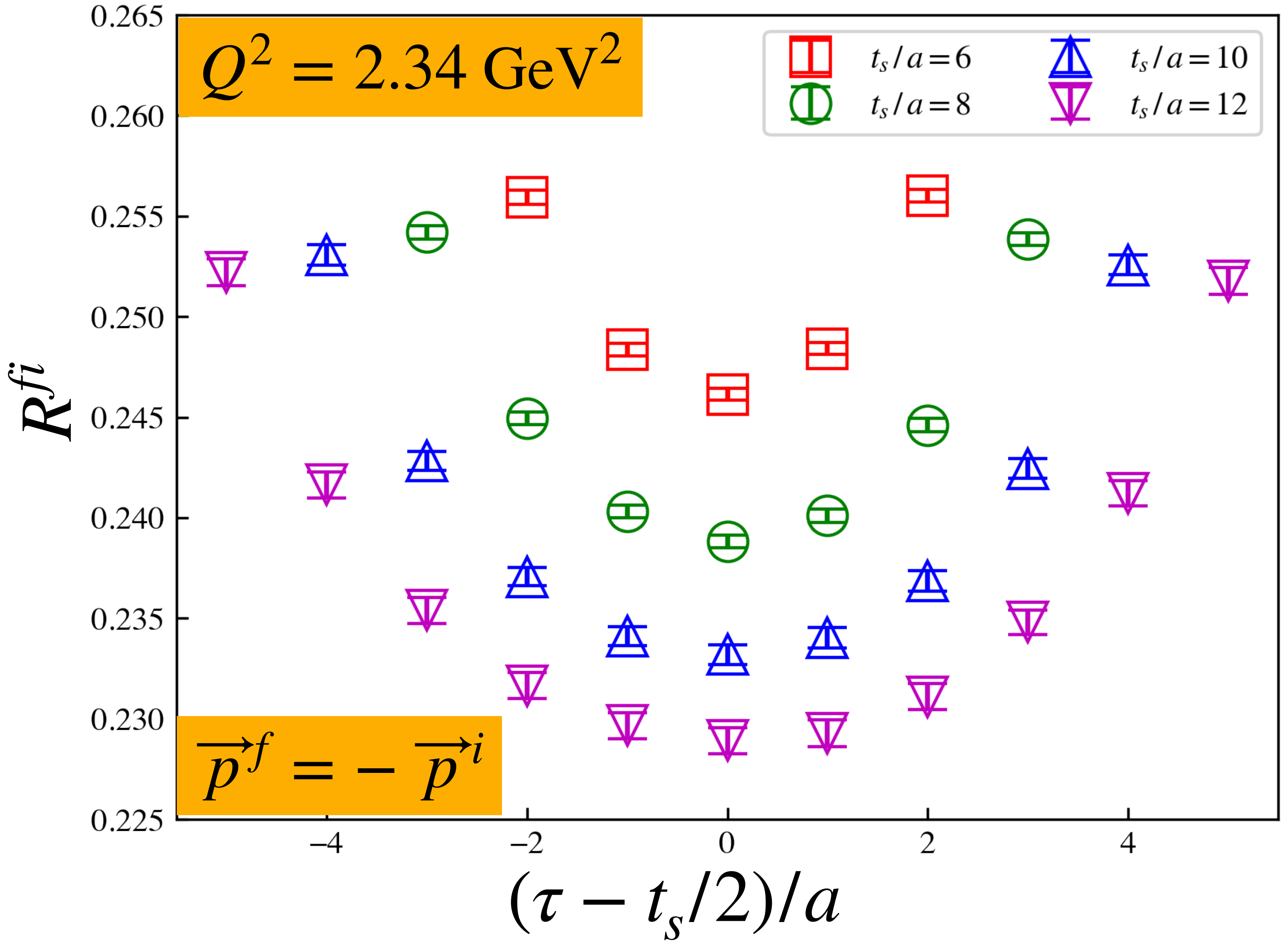
Can be suppressed by C_{2pt}

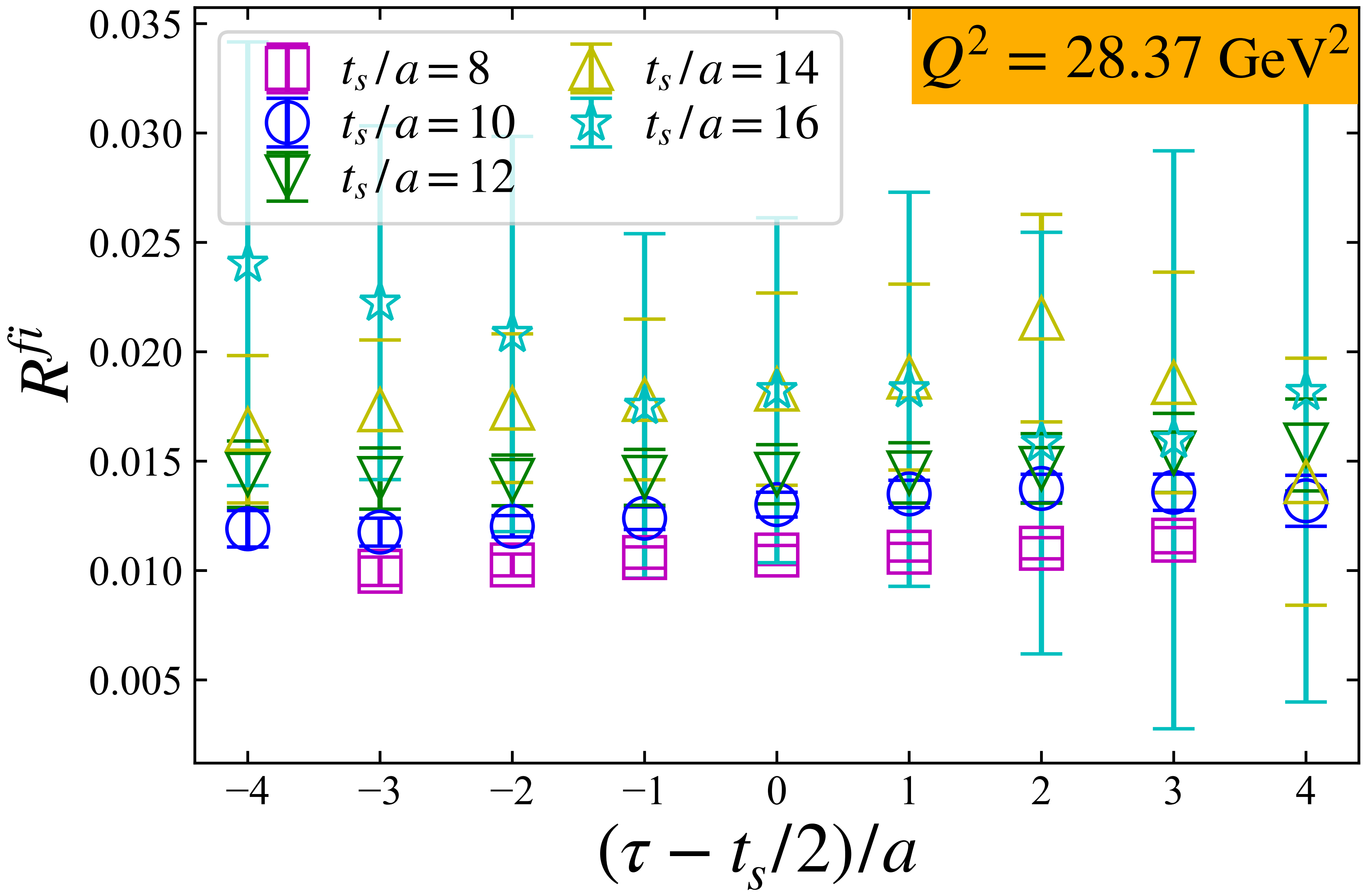
$n = k = 0$ $\downarrow$ F^B

- Take a special case $\vec{p}^{f} = -\vec{p}^{i}$ as an example 2102.06047

$$R^{fi}(\vec{p}^{f}, t_s; \vec{q}, \tau; \vec{p}^{i}, 0) \equiv \frac{C_{3pt}(\tau, t_s; \vec{p}^{i}, \vec{p}^{f})}{C_{2pt}(t_s, \vec{p}^{f})}, \quad F^B = \lim_{\tau,\, (t_s-\tau),\, t_s \to \infty} R^{fi}$$



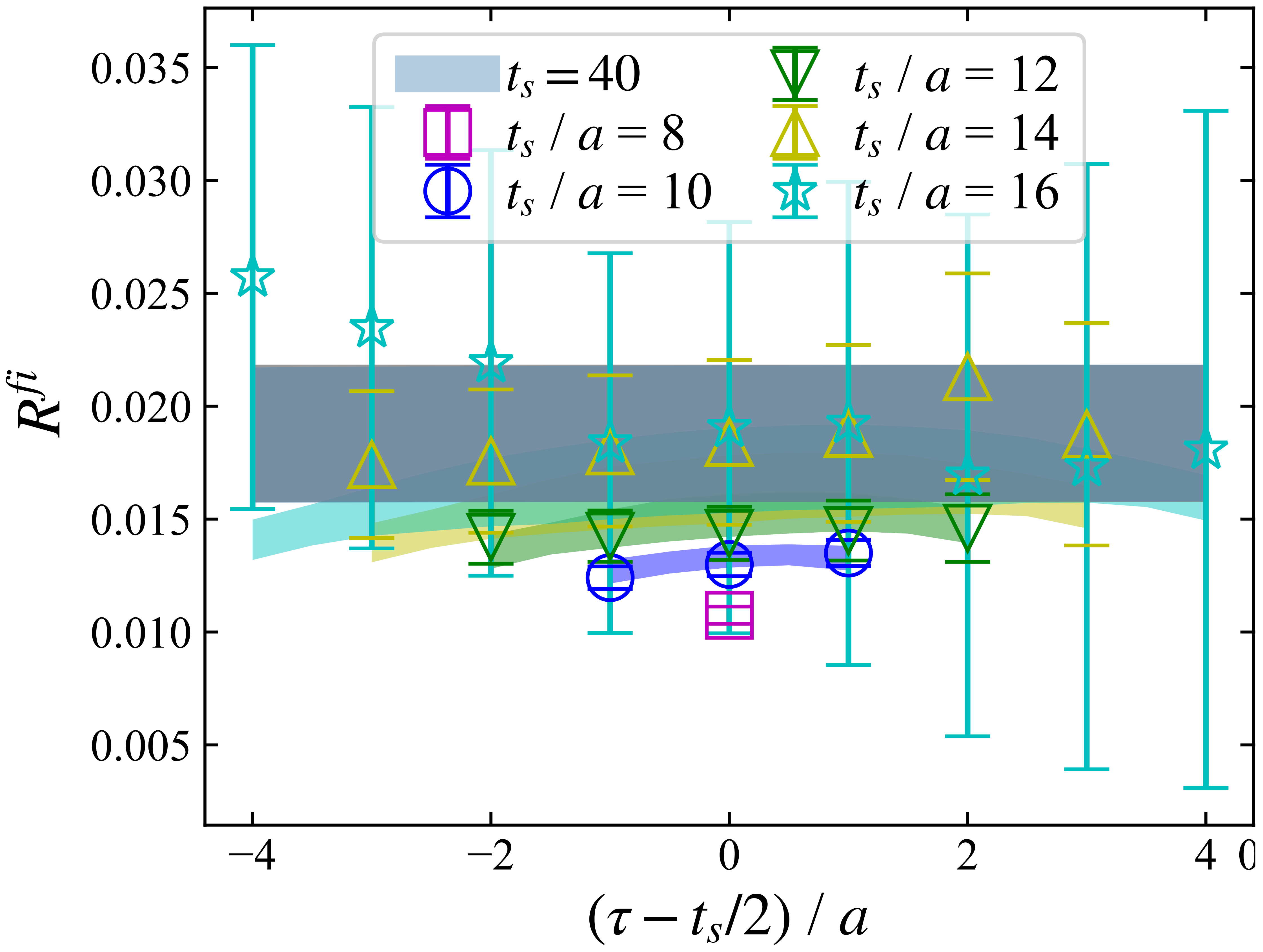




2. Extract the bare form factor

$$N_{state} = 2: R^{fi}(\tau, t_s) = \left(\underbrace{\mathcal{O}_{00}}_{F^B} + \frac{A_1}{A_0}\mathcal{O}_{11}e^{-t_s\Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{01}e^{-\tau\Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{10}e^{-(t_s-\tau)\Delta E} \right) \Big/ \left(1 + \frac{A_1}{A_0}e^{-t_s\Delta E}\right), \ \Delta E = E_1 - E_0$$

- Use the values of energy E_n and amplitude A_n extracted from C_{2pt}
- Perform a 4-parameter fit to the ratio R^{fi} to extract F^B



$Q^2 = 28.37\ \text{GeV}^2, F^B = 0.019(3)$

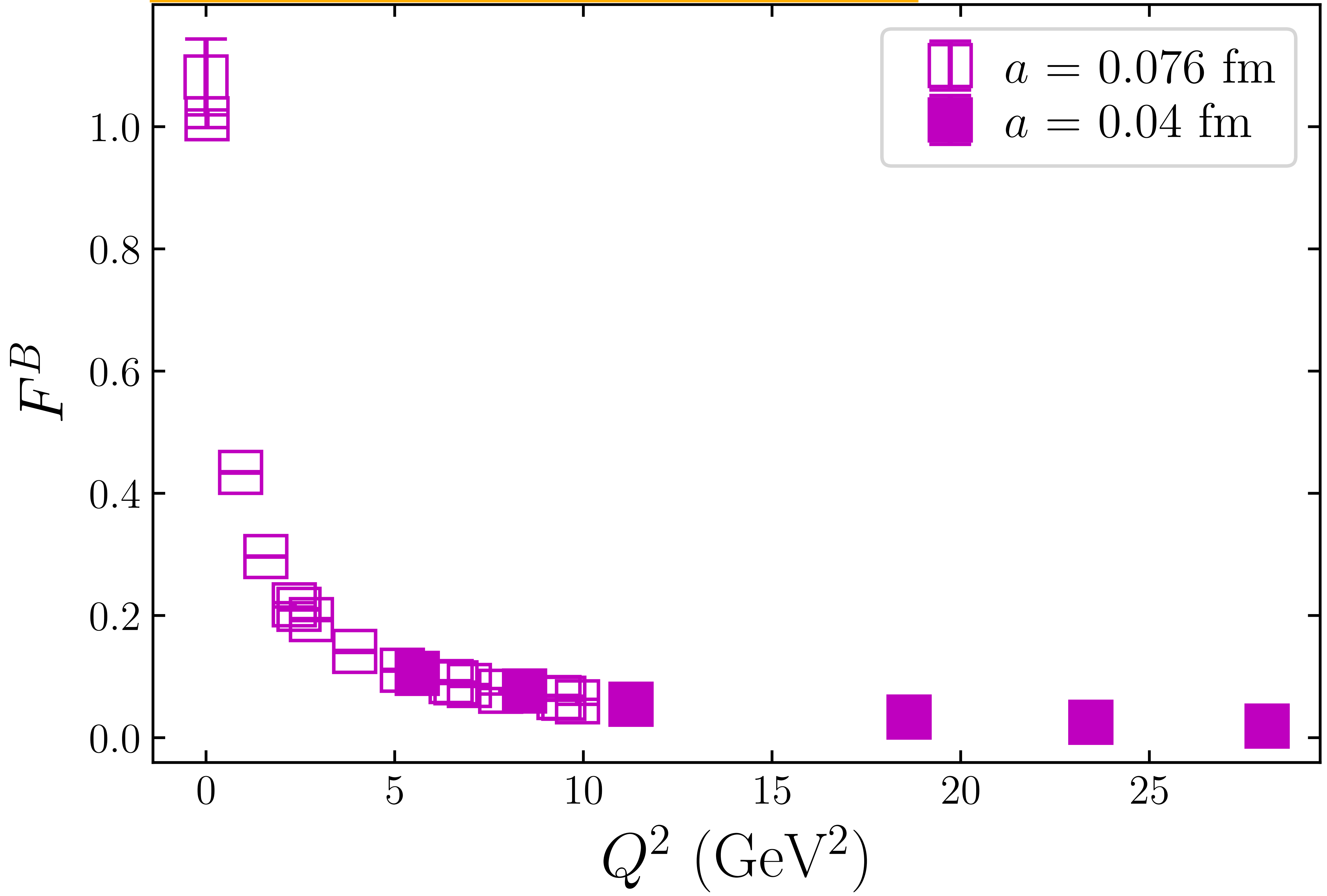
χ^2/dof: 0.73; p-value: 0.81

Extrapolation -> Grey band

$$F^B = \lim_{\tau,\ (t_s-\tau),\ t_s \to \infty} R^{fi}$$

3. Renormalization

All results of bare form factor



- F^B decreases as Q^2 increases
- Renormalization:

$$F = F^B \times Z_V$$

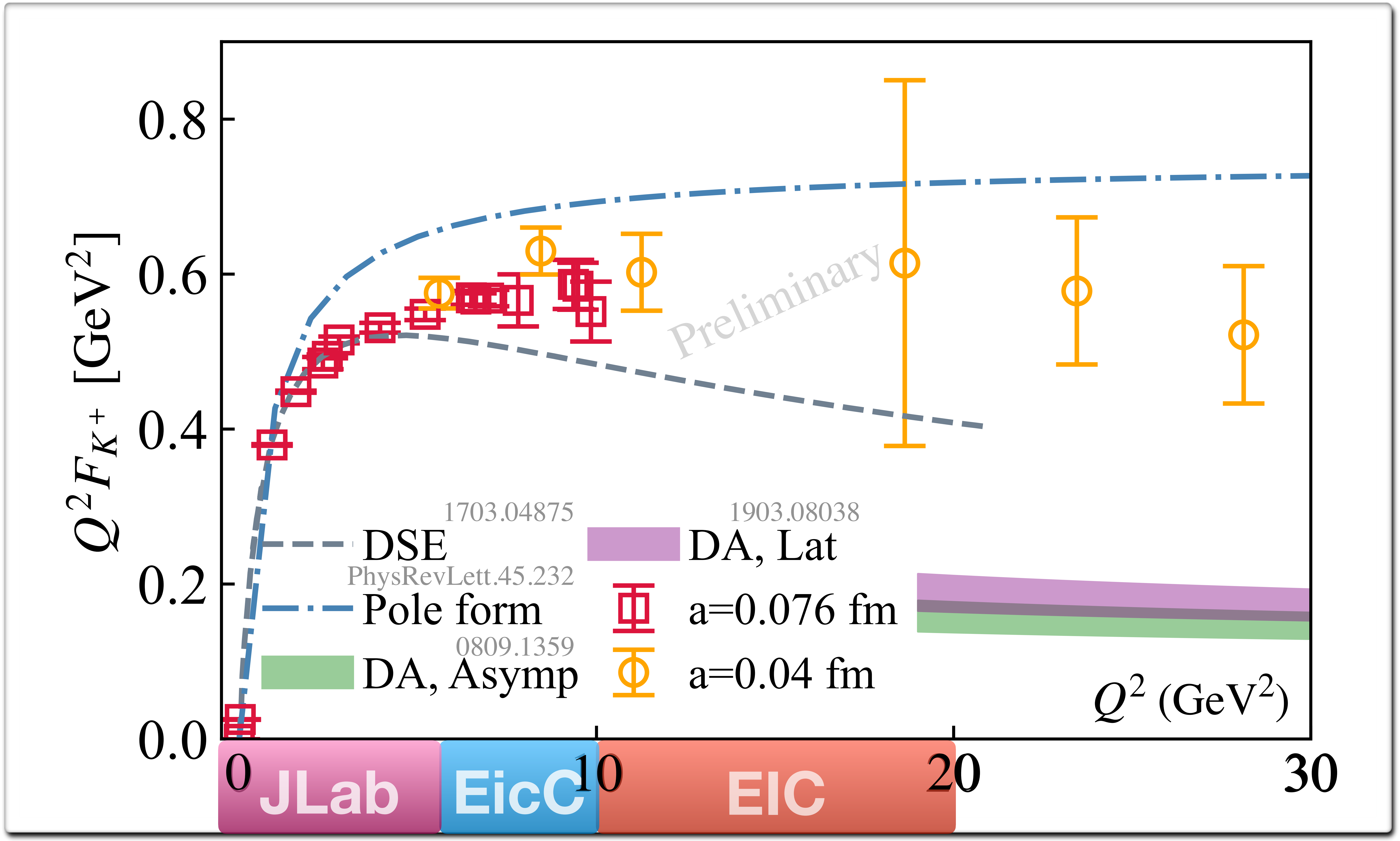
$$Z_V^{-1} = \left\langle 0 \,|\, \hat{\mathcal{O}} \,|\, 0 \right\rangle = 1.048, 1.024$$

extracted in our previous work of pion

2102.06047

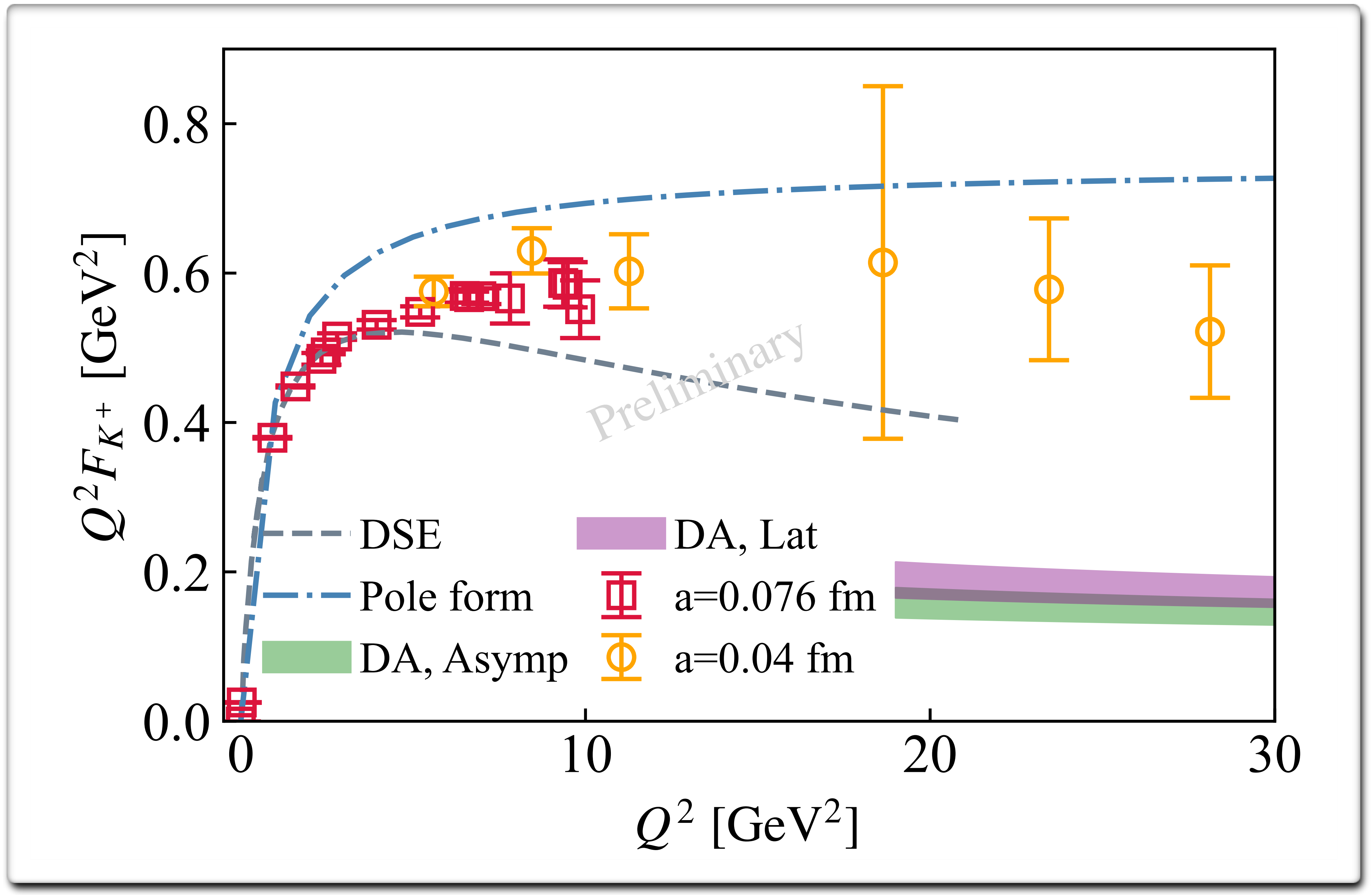
Results

Our results are the first Lattice QCD prediction of the kaon form factor at $Q^2 \geq 3$ GeV^2



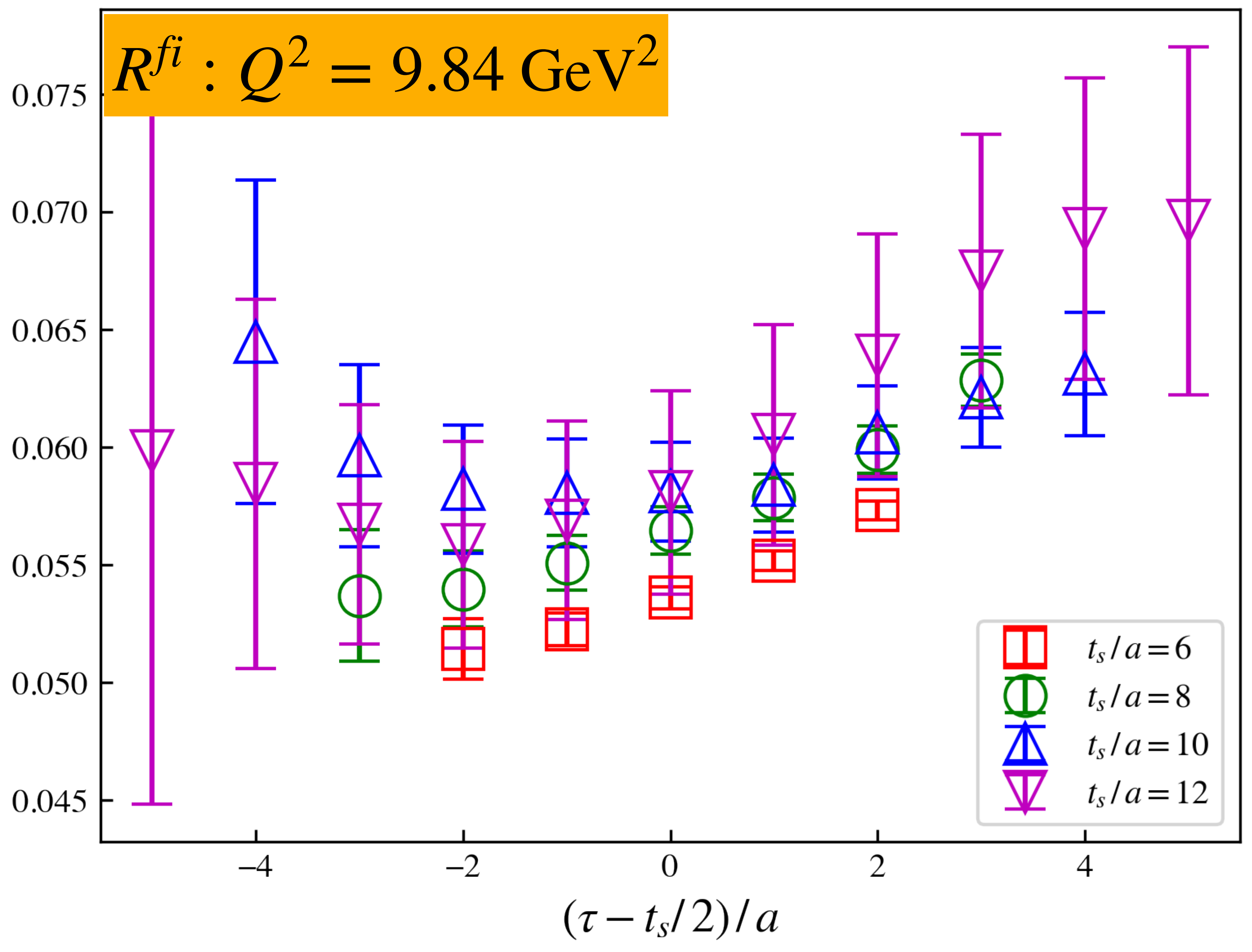
Summary

- We study the K^+ electromagnetic form factor at the physical point from the first principle using Lattice QCD
- We use improved boost smearing at the source & sink time position to achieve good signals with large Q^2 up to 28 GeV^2
- Our results of the $Q^2 F_{K^+}$ increase at lower Q^2 and then show a flat and decreased tend at higher Q^2



Thanks

Backup


R^{fi} : Q^2 = 9.84 GeV^2
0.075
0.070
0.065
0.060
0.055
0.050
0.045
−4
−2
0
2
4
$(\tau - t_s/2)/a$
$t_s/a = 6$
$t_s/a = 8$
$t_s/a = 10$
$t_s/a = 12$