A new method for a lattice QCD calculation on radiative transition

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Outline

- Motivation
- Direct method for on-shell transition factor
- A novel method for dispersion relation
- Summary and outlook

Radiative transition

- Radiative transition plays a valuable opportunity to understand the QCD and strong interaction
  - Exotic state parity: $X(3872) \to \gamma J/\psi$
  - Pseudoscalar glueball: $J/\psi \to \gamma G$
- Traditional lattice calculation: momentum extrapolation $V(q^2)$

  Feiyu Chen *et al*,arXiv:2207.04694 Xiangyu Jiang *et al*,arXiv:2206.02724

  L.-C.Gui *et al*,PRD100,054511(2019)
- New methodology for pion charge radius, no extrapolation-dependent uncertainty + high precision

  X.Feng,Y.Fu,L.-C.Jin,PRD101,051502(2020)
- Extension to radiative transition, potential application in baryon sector

An ideal test for new methodology: $J/\psi \to \gamma\eta_c$

- Important, interplay of strong and electromagnetic interaction
- Abundant, many lattice studies in the past

  J.J.Dudek *et al*,PRD73,074507(2006)
  Y.Chen *et al*(CLQCD),PRD84,034503(2011)
  L.-C.Gui *et al*,PRD100,054511(2019)
  D.Becirevic *et al*,JHEP01,028(2013)
  G.C.Donald *et al*(HPQCD),PRD86,094501(2012)

- Fundamental, intermediate contribution in three-body decay
  - $J/\psi \to \gamma\nu\bar{\nu}$
  - $J/\psi \to 3\gamma$

$J/\psi \to \gamma\eta_c$ on the lattice

- Euclidian hadroinc function

$$H_{\mu\nu}(\vec{x},t) = \langle 0|\phi_{\eta_c}(\vec{x},t)J_\nu(0)|J/\psi_\mu(p')\rangle, t>0$$

where $J_\nu = \sum_q e_q\,\bar{q}\gamma_\mu q$ ($e_q = 2/3, -1/3, -1/3, 2/3$ for $q = u, d, s, c$)

- Single-η_c dominance at long distance

$$\begin{aligned} H_{\mu\nu}(\vec{x},t) &\doteq \frac{2e_c}{m_{\eta_c}+m_{J/\psi}}\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{Z}{E}\epsilon_{\mu\nu\alpha\beta}p_\alpha p'_\beta \\ &\times V(q^2)e^{-Et+i\vec{p}\cdot\vec{x}} \end{aligned}$$

with

$$\begin{aligned} \langle 0|\phi_{\eta_c}(0)|\eta_c(\vec{p})\rangle &= Z,\ \ q^2 = (m_{J/\psi}-E)^2 - |\vec{p}|^2 \\ \langle \eta_c(\vec{p})|J_\nu(0)|J/\psi_\mu(p')\rangle &= \frac{4V(q^2)}{m_{\eta_c}+m_{J/\psi}}e_c\epsilon_{\mu\nu\alpha\beta}p_\alpha p'_\beta \end{aligned}$$

Traditional way

- Fourier transform on hadroinc function

$$\frac{(m_{\eta_c}+m_{J/\psi})Z}{2e_c\cdot Z}e^{Et}\int d^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}H_{\mu\nu}(\vec{x},t)=V(q^2)\epsilon_{\mu\nu\alpha\beta}p_\alpha p'_\beta$$

- Choosing discrete momenta $\vec{p}=2\pi/L\cdot\vec{n}$

$$\vec{n}=[0,0,1],[0,1,1],[1,1,1]\cdots$$

- Toward on-shell $V(0)$ by momentum extrapolation

$$V(q^2\neq 0),q^2=(m_{J/\psi}-E)^2-|\vec{p}|^2$$

Y.Chen *et al*(CLQCD),PRD84,034503(2011)

New method for transition factor

- Construting the scalar function $\mathcal{I}_0(t,|\vec{p}|)$

$$\begin{aligned}\mathcal{I}_0(t,|\vec{p}|) &\equiv \epsilon_{\mu\nu\alpha'\beta'}p_{\alpha'}p'_{\beta'}/(m_{J/\psi}|\vec{p}|^2)\cdot\int d^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}H_{\mu\nu}(\vec{x},t) \\ &= -\frac{4e_cZm_{J/\psi}}{m_{\eta_c}+m_{J/\psi}}V(q^2)\frac{e^{-Et}}{E}\end{aligned}$$

- $p_{\alpha'}\to i\frac{\partial}{\partial x_\alpha}$, averaging over the spatial direction for $\vec{p}$

$$\mathcal{I}_0(t,|\vec{p}|)=\int d^3\vec{x}\frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|}\epsilon_{\mu\nu\alpha 0}x_\alpha H_{\mu\nu}(\vec{x},t)$$

- For pion charge radius, $|\vec{p}|=0\Rightarrow q^2=0$ is direct
  X.Feng,Y.Fu,L.-C.Jin,PRD101,051502(2020)
  - Not accesable for radiative trasntion

New method for transition factor

$$|\vec{p}| = 0 \to q^2 = (\delta m)^2 \equiv (m_{J/\psi} - m_{\eta_c})^2$$

- Expanding $\mathcal{I}_0(t, |\vec{p}|)$ at $\vec{p} = 0$

$$\mathcal{I}_1(t, 0) \equiv -\frac{\partial \mathcal{I}_0(t, |\vec{p}|)}{\partial |\vec{p}|^2}\bigg|_{|\vec{p}|^2=0} = \frac{1}{30} \int d^3\vec{x} |\vec{x}|^2 \epsilon_{\mu\nu\alpha 0} x_\alpha H_{\mu\nu}(\vec{x}, t)$$

- Relating $\mathcal{I}_0, \mathcal{I}_1$ to $V(q^2 = 0)$

$$V(q^2) = \sum_{n=0}^{\infty} c_n \left(\frac{q^2}{m_{J/\psi}^2}\right)^n \doteq c_0 + c_1 \cdot \frac{q^2}{m_{J/\psi}^2} + \mathcal{O}(q^4/m_{J/\psi}^4)$$

- Some parameterizations

$$4 \sinh^2 \frac{E}{2} = 4 \sinh^2 \frac{m_{\eta_c}}{2} + \xi \cdot 4 \sum_i \sin \frac{\vec{p}_i}{2}$$

$$Z^2 = Z_0^2 + \eta \cdot |\vec{p}|^2$$

c_0 and c_1

We have

$$c_1 = \left[\frac{1}{\xi}\tilde{\mathcal{I}}_1(t) - \frac{\tilde{\mathcal{I}}_0(t)}{2m_{\eta_c}\sinh m_{\eta_c}}\left(1 + m_{\eta_c}t - \frac{\eta m_{\eta_c}\sinh m_{\eta_c}}{Z_0^2\xi}\right)\right] \times \frac{m_{J/\psi}^2 \sinh m_{\eta_c}}{\sinh m_{\eta_c} + \xi\delta m}$$

and

$$c_0 = \tilde{\mathcal{I}}_0(t) - c_1 \times \frac{(\delta m)^2}{m_{J/\psi}^2}$$

where $\tilde{\mathcal{I}}_n(t)$ are defined as

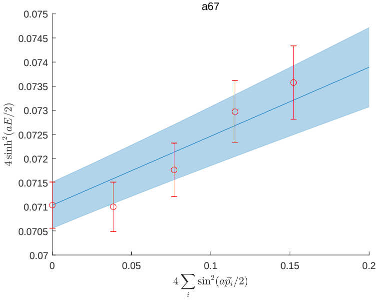
$$\tilde{\mathcal{I}}_n(t) \equiv -\frac{(m_{\eta_c} + m_{J/\psi})}{4e_c Z_0 m_{J/\psi}} m_{\eta_c} e^{m_{\eta_c}t}\mathcal{I}_n(t,0)$$

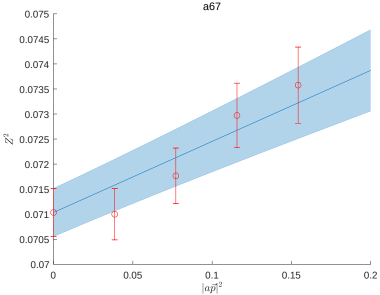
Ensembles

Ens	a(fm)	V/a^4	$a\mu_{sea}$	$N_{\rm conf} \times T_s$	m_π(MeV)
a98	0.098(3)	$24^3 \times 48$	0.0060	236×48	365
a85	0.085(2)	$24^3 \times 48$	0.0040	200×48	315
a67	0.0667(20)	$32^3 \times 64$	0.0030	200×64	300

- $N_f = 2$ twisted-mass guage configuration
- Smeared stochastic Z_4-source and point source for the propagator
- Charm quark mass is tuned by physical J/ψ mass

- Momentum extrapolation for ξ and η





- Effective energies with different momenta are needed
- Limited number of $\vec{p}$ due to the discretization

A novel approach to ξ and η

- Two point function

$$C_2(|\vec{p}|,t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} C_2(\vec{x},t) = V\frac{Z^2}{2E}\left(e^{-Et} + e^{-E(T-t)}\right)$$

- Derivative at $|\vec{p}|^2 = 0$

$$C_2'(0,t) \equiv -\left.\frac{\partial C_2(|\vec{p}|,t)}{\partial |\vec{p}|^2}\right|_{|\vec{p}|^2=0} = \frac{1}{6}\sum_{\vec{x}} |\vec{x}|^2 C_2(\vec{x},t)$$

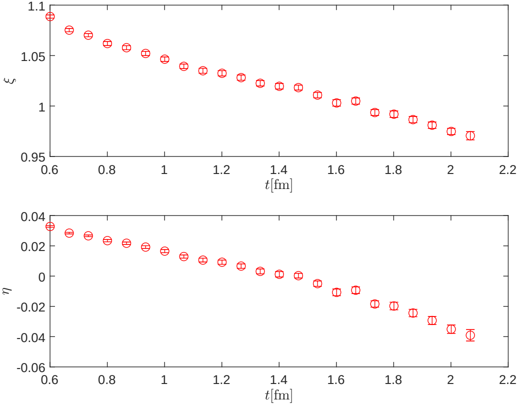
- The ratio of $C_2'(0,t)$ and $C_2(0,t)$

$$\mathcal{R}(t) = \frac{\xi}{2m\sinh m}\left[1 + mt\frac{1+e^{-m(T-2t)}(T/t-1)}{1+e^{-m(T-2t)}}\right] - \frac{\eta}{Z_0^2}$$

$$\begin{aligned} \tilde{R}(t) &\equiv \mathcal{R}(t+1) - \mathcal{R}(t) \\ &= \frac{\xi}{2\sinh m}\frac{t'\sinh m + \sinh(mt')}{\cosh m + \cosh(mt')} \end{aligned}$$

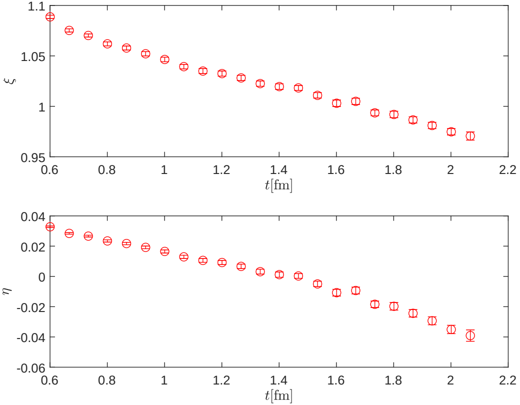
with $t' = T - 2t - 1$.

The puzzle



1.1
1.05
1
0.95
ξ
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
t[fm]
0.04
0.02
0
-0.02
-0.04
-0.06
η
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
t[fm]

The puzzle



- $\sum_{\vec{x}} |\vec{x}|^2 C_2(\vec{x}, t)$ contributes obvious finite volume effect since our small volumes $L_{max} \simeq 2.35$ fm

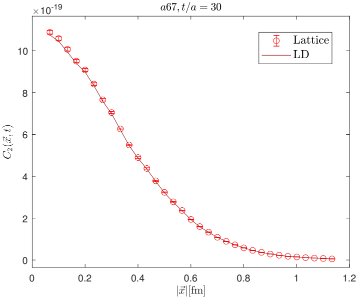
Finite volume effect

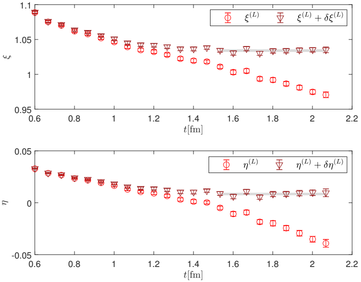
- The long-distance contribution

$$C_2^{LD}(\vec{x},t) \doteq \frac{1}{V}\sum_{\vec{p}\in\Gamma}\cos(\vec{p}\cdot\vec{x})C_2(|\vec{p}|,t)$$

- The total contribution for the quantity A, e.g. ξ, η

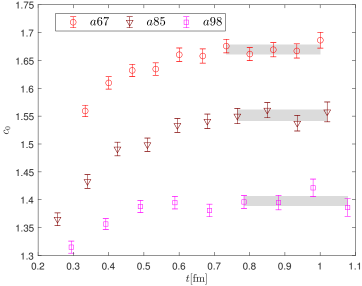
$$A = A^{(L)} + \left(A^{(\infty)} - A^{(L)}\right) \simeq A^{(L)} + \left(A_{LD}^{(\infty)} - A_{LD}^{(L)}\right)$$

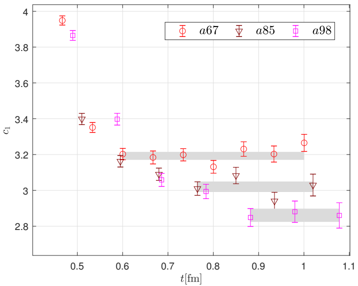




- Same strategy for c_0 and c_1

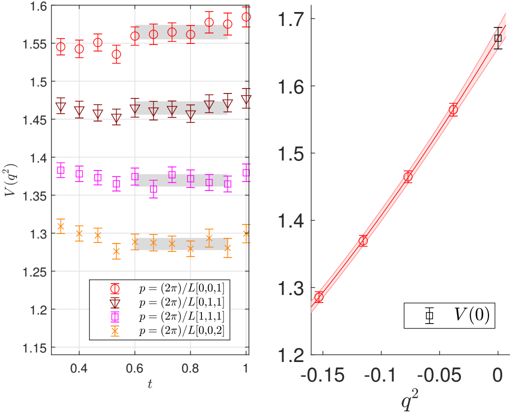
Resutls of c_0 and c_1





Traditional results of $V(0)$

- Take a67 as an example



Numerical results

- Results of ξ and η

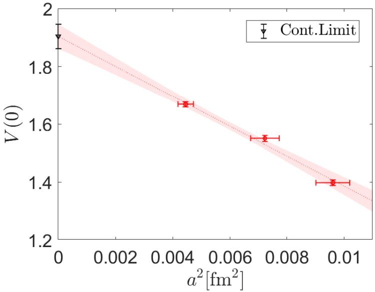
Ensemble	a67	a85	a98
$\xi_{\rm new}$	1.0341(19)	1.0071(22)	0.9202(24)
$\eta_{\rm new}$	0.0084(12)	0.0221(22)	0.0362(40)
$\xi_{\rm old}$	1.0427(38)	1.0106(35)	0.9403(51)
$\eta_{\rm old}$	0.0142(27)	0.0263(36)	0.0328(88)

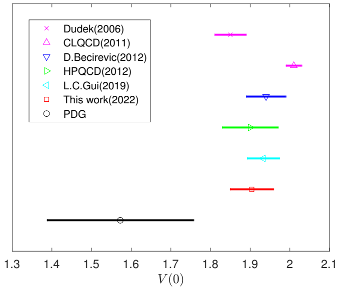
- Results of c_0 and c_1

	New		Traditional	
Ensemble	c_0	c_1	c_0	c_1
$a67$	1.669(09)	3.19(2)	1.671(16)	3.16(33)
$a85$	1.551(10)	3.02(3)	1.547(17)	2.78(37)
$a98$	1.398(09)	2.86(4)	1.366(16)	1.44(45)

The precision is improved by more than 40%

Continuous limit





- Extrapolation with lattice spacing errors included
- Using only three ensembles

Summary and outlook

- Summary
  - We present a new method to study the radiative transition, and first apply it to $J/\psi \to \gamma\eta_c$.
  - The method avoids the momentum extrapolation and improve the precision more than 40%.
  - We propose a novel method the estimate the dispersion relation without the effective energies with non-zero momenta.
- Outlook
  - The ensembles with larger L be used to further test the method.
  - Apply for other radiative transitions in baryon physics.