

The continuum limit with various discretized fermion actions

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1. Introduction

Staggered fermion

$$S_{\text{st}}(m) = \sum_{x,y} \bar{\psi}_{\text{st}}(x) D_{\text{st}}(x, y, m) \psi_{\text{st}}(y)$$

Fermion doubling ✗
Chiral symmetry ✓

Clover fermion

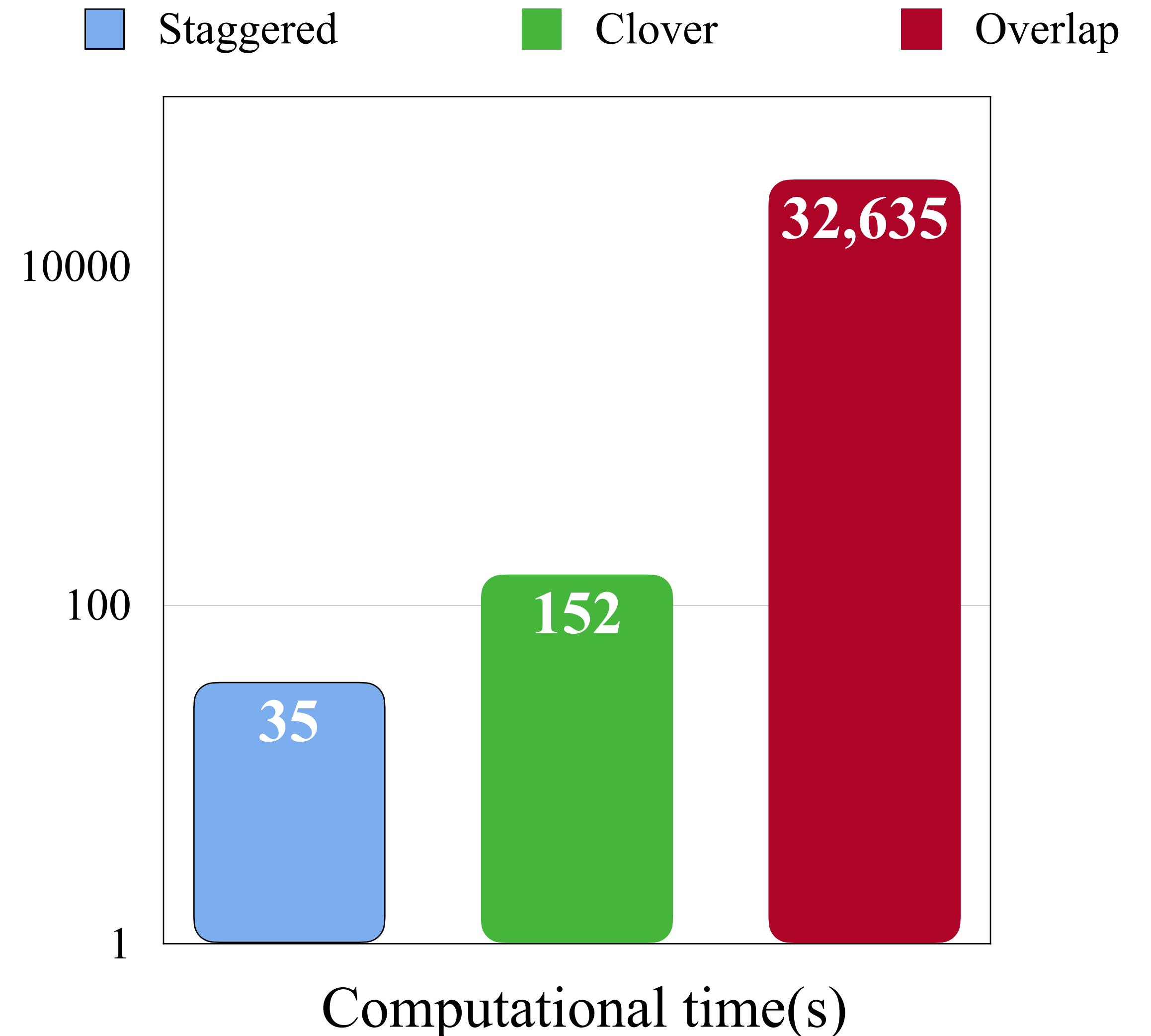
$$S_{\text{cl}}(m) = S_{\text{w}}(m) + c_{\text{sw}} \sum_x \bar{\psi}(x) \sigma_{\mu\nu} F^{\mu\nu} \psi(x)$$

Fermion doubling ✓
Chiral symmetry ✗

Overlap fermion

$$S_{\text{c}}(m) = \sum_x \bar{\psi}(x) \left[\left(1 - \frac{m}{2\rho}\right) D_{\text{ov}}(x, y, m) + \delta_{x,y} m \right] \psi(y)$$

Fermion doubling ✓
Chiral symmetry ✓



1. Introduction

Definition

MAPQ χ PT

Val_Val pion mass: $m_{\pi,VV}$

Sea_Sea pion mass: $m_{\pi,SS}$

Val_Sea pion mass: $m_{\pi,VS}$

$$\Delta_{\text{mix}}^{B/A}(m_{\pi,VV}, m_{\pi,SS}, a) \equiv m_{\pi,VS}^2 - \frac{m_{\pi,VV}^2 + m_{\pi,SS}^2}{2}$$

Innovation

1. based on the calculation at different lattice spacing
2. pion mass $m_{\pi,SS}$ is not limited to the case of $\sim 300\text{MeV}$
3. mixed action effect with kinds of the valence and sea fermion combinations

Previous works

Valence	Sea	$\delta m_{\pi} = m_{\pi,VS} - m_{\pi,SS}$ (MeV)	$a(\text{fm})$	$m_{\pi,SS}(\text{MeV})$
Overlap	Clover	153	0.09	300
DW	Staggered	30-60	0.13-0.09	310
Overlap	DW	~ 10	0.11-0.08	300-400

PoS LATTICE2007, 115 (2007)

Phys. Rev. D86, 014501 (2012)

Phys. Rev. D77, 094505 (2008)

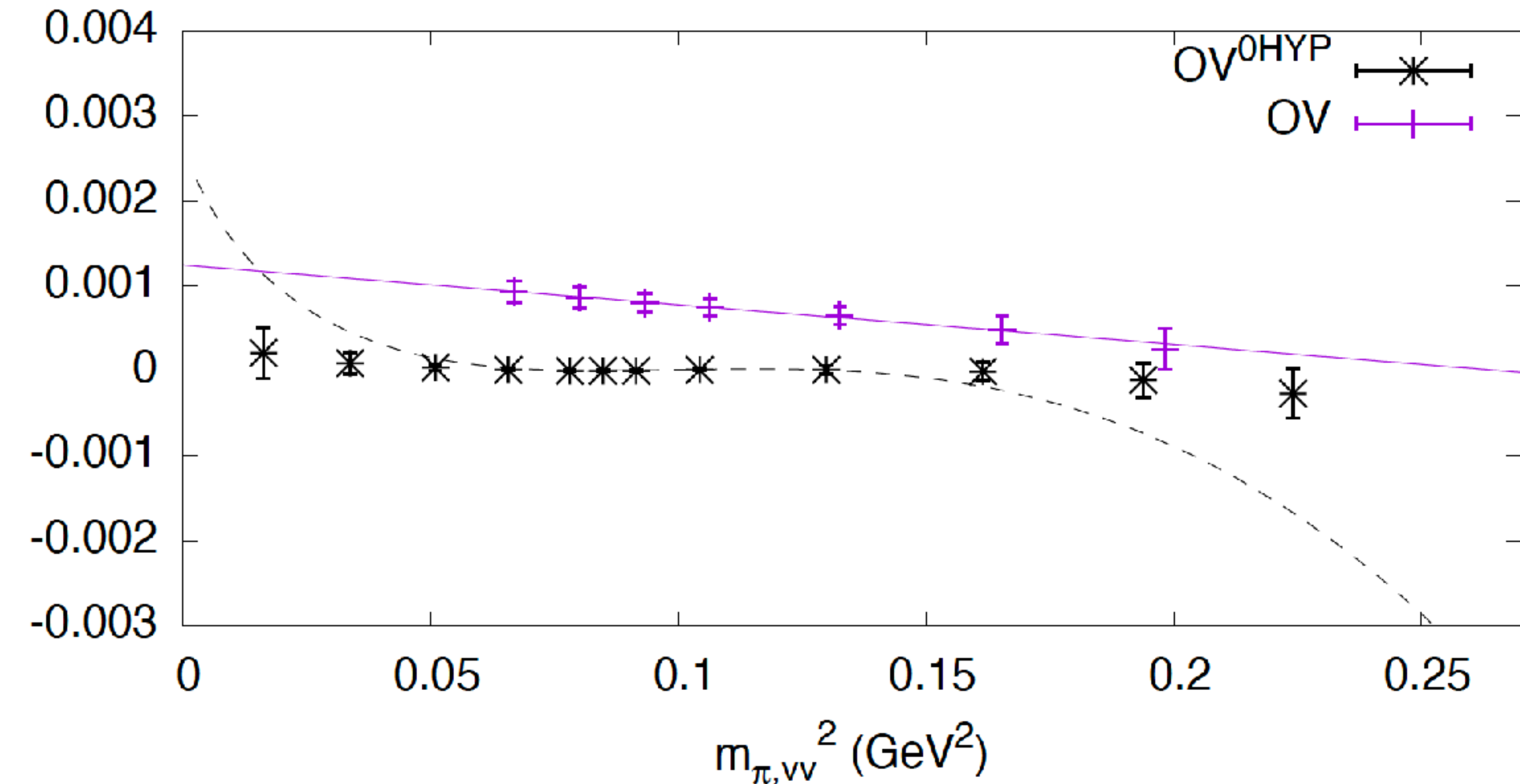
Phys. Rev. D77, 114501 (2008)

Phys. Rev. D96, 054513 (2017)

2. Methodology and setup

Contribution of the delta mix definition to the NLO at $m_{\pi, \text{VV}} \neq m_{\pi, \text{SS}}$

$\Delta_{\text{mix}}(m_{\pi, \text{VV}}, 290 \text{ MeV}, 0.11 \text{ fm}) (\text{GeV}^2)$, OV on $\text{OV}^{\text{OHYP}} + \text{IR}$



$$\Delta_{\text{mix}}(m_{\pi, \text{VV}}, m_{\pi, \text{SS}}, a) \equiv m_{\pi, \text{VS}}^2 - \frac{m_{\pi, \text{VV}}^2 + m_{\pi, \text{SS}}^2}{2}$$

$$\downarrow m_{\pi, \text{VV}} = m_{\pi, \text{SS}}$$

$$\Delta_{\text{mix, uni}}(m_{\pi}, a) \equiv \Delta_{\text{mix}}(m_{\pi}, m_{\pi}, a)$$

2. Methodology and setup

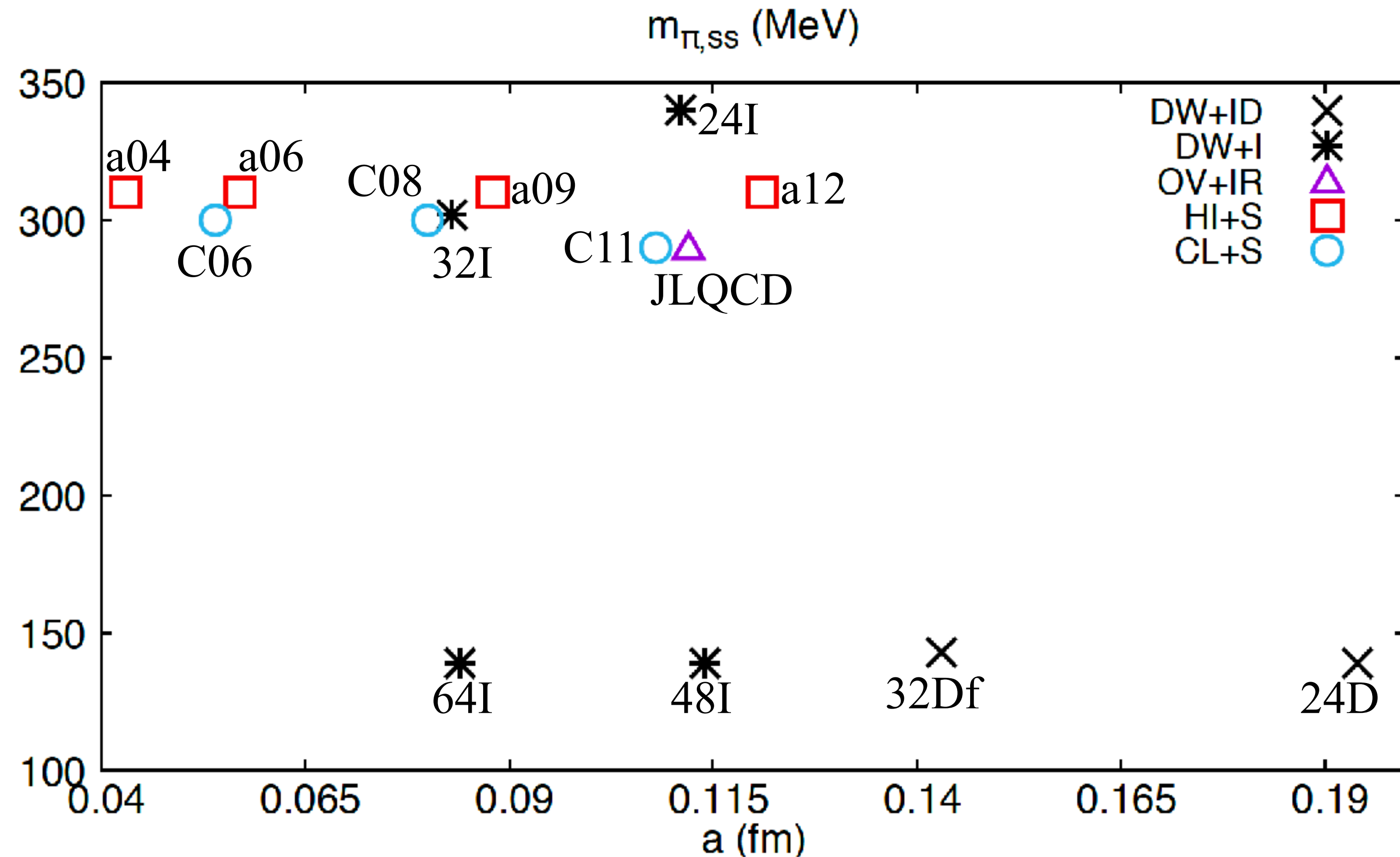
Valence fermion actions

HI: HISQ action without any additional smearing on the gauge link;

CL: Clover fermion with 1-step HYP smearing and tree level tadpole improved clover coefficient c_{sw} ;

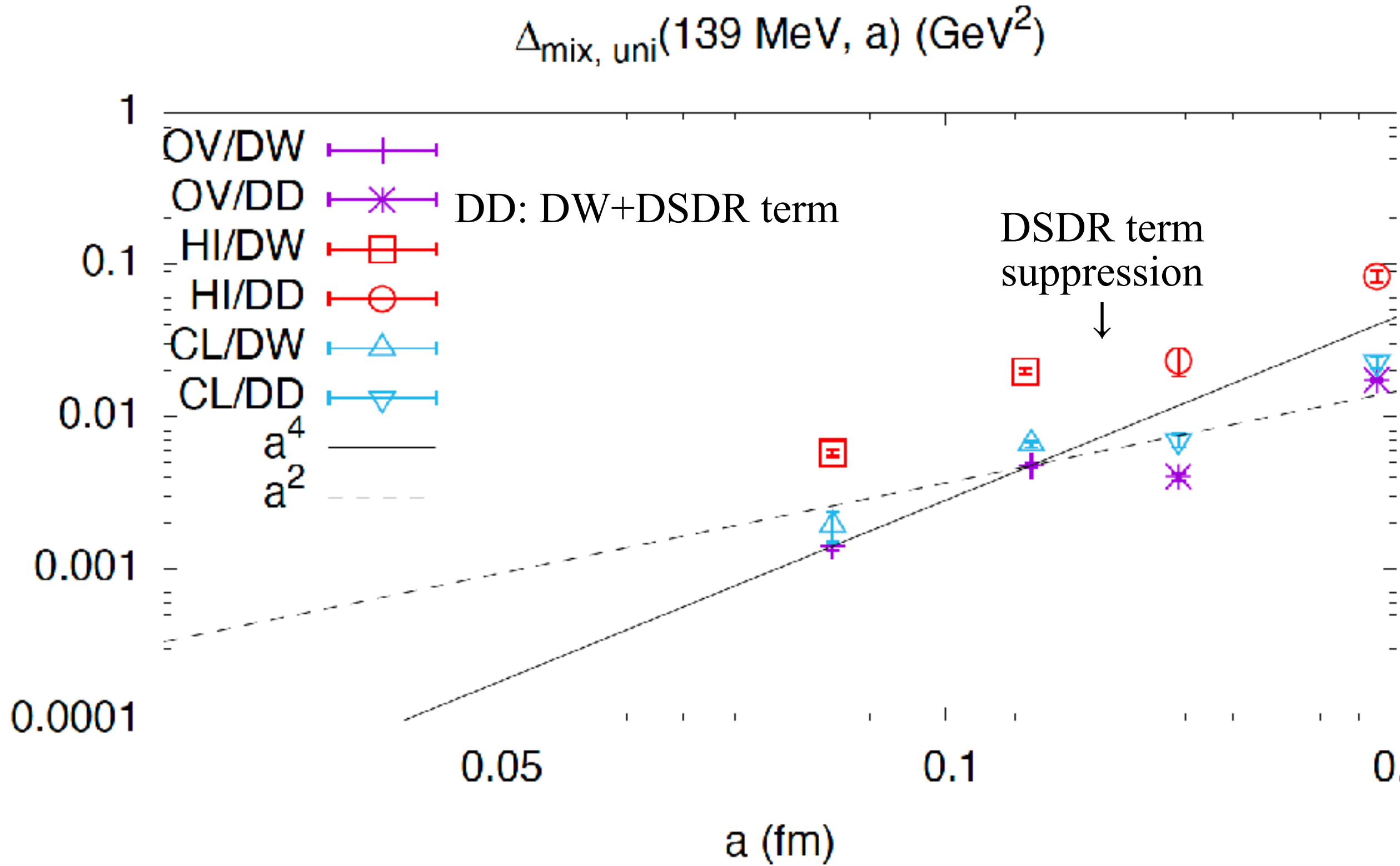
OV: Overlap fermion with 1-step HYP smearing and $\rho = 1.5$.

Sea fermion actions



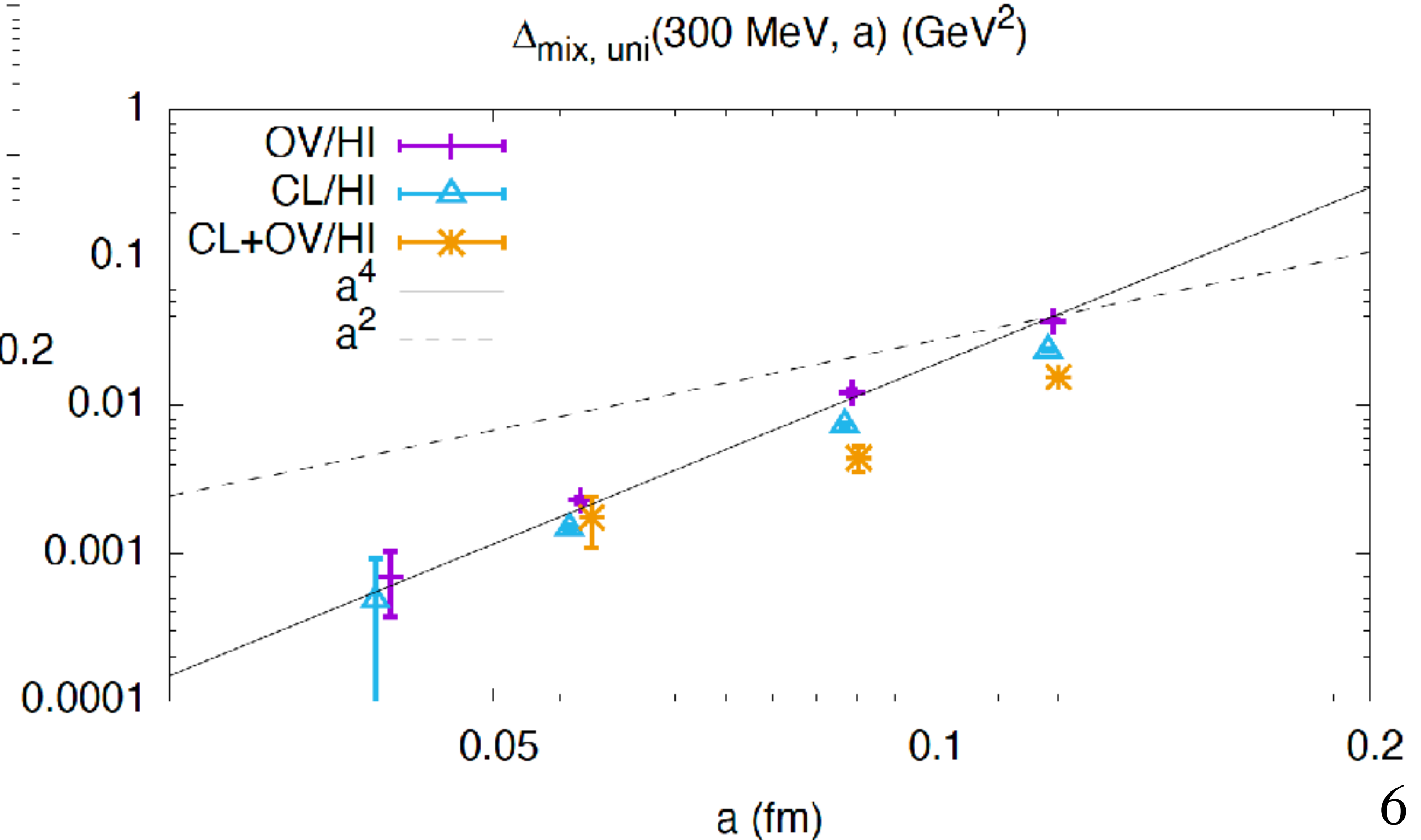
Information of the ensembles used in this calculation

3. Result



$$\bar{\Delta}_{\text{mix, uni}}^{\text{B+C/A}}(m_\pi, a) \equiv m_{\pi, \text{BC}}^2 - \frac{m_{\pi, \text{BB}}^2 + m_{\pi, \text{CC}}^2}{2} \Big|_{m_{\pi, \text{BB}}=m_{\pi, \text{CC}}=m_{\pi, \text{AA}}=m_\pi}$$

Delta mix on the HISQ sea



Effect of DSDR term on the delta mix on the DW sea

$$|\Delta_{\text{mix, uni}}^{\text{B/A}} - \Delta_{\text{mix, uni}}^{\text{C/A}}| \leq \bar{\Delta}_{\text{mix, uni}}^{\text{B+C/A}} \leq \Delta_{\text{mix, uni}}^{\text{B/A}} + \Delta_{\text{mix, uni}}^{\text{C/A}}$$

↓

$$|\Delta_{\text{mix, uni}}^{\text{OV/HI}} - \Delta_{\text{mix, uni}}^{\text{CL/HI}}| \leq \bar{\Delta}_{\text{mix, uni}}^{\text{OV+CL/HI}} \leq \Delta_{\text{mix, uni}}^{\text{OV/HI}} + \Delta_{\text{mix, uni}}^{\text{CL/HI}} \rightarrow$$

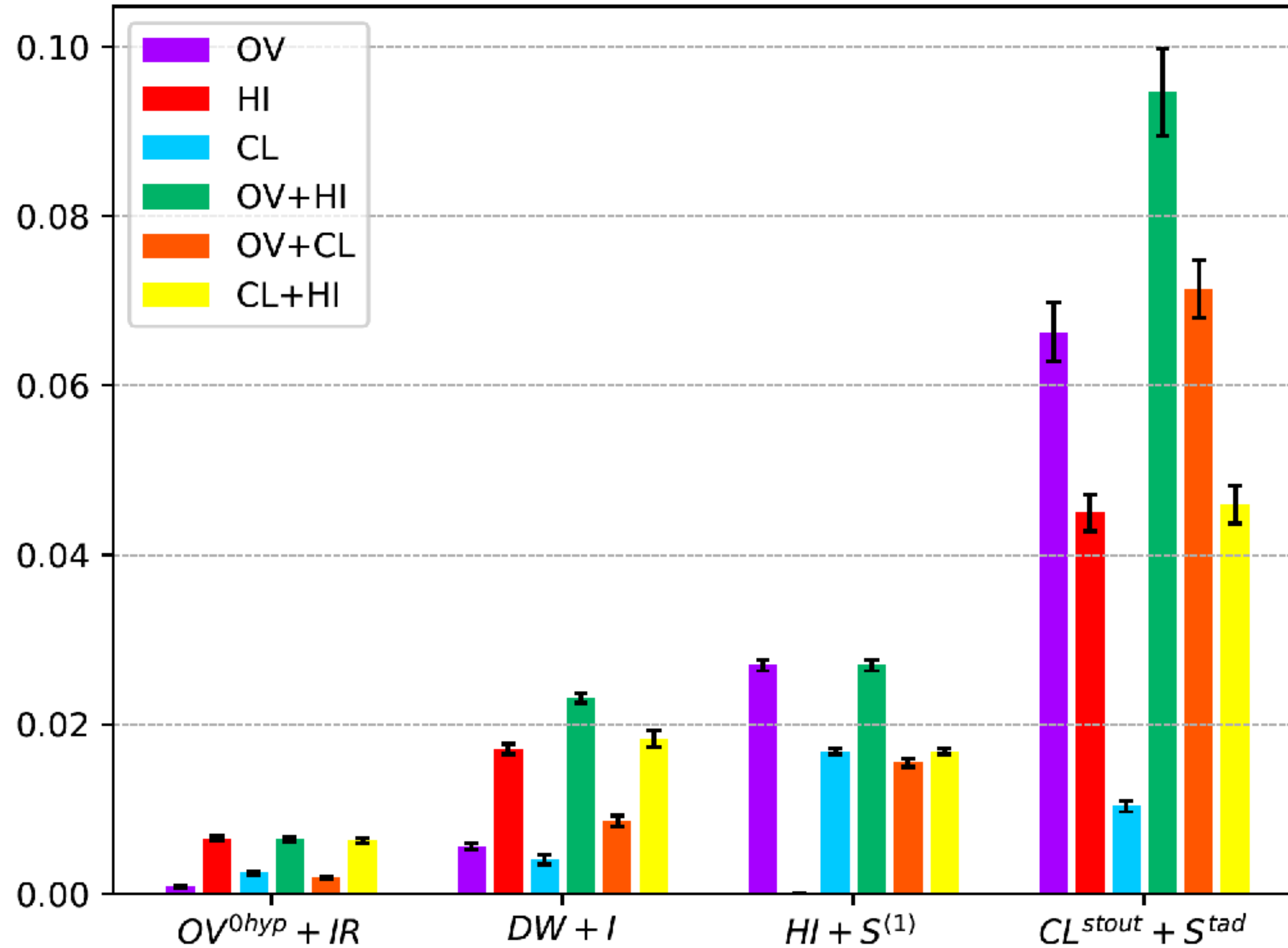
3. Result

$$\Delta_{\text{mix}}(m_{\pi,\text{VV}}, m_{\pi,\text{SS}}, a) \equiv m_{\pi,\text{VS}}^2 - \frac{m_{\pi,\text{VV}}^2 + m_{\pi,\text{SS}}^2}{2}$$

$$\Delta_{\text{mix,uni}}(m_{\pi}, a) \equiv \Delta_{\text{mix}}(m_{\pi}, m_{\pi}, a)$$

$$\bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}}(m_{\pi}, a) \equiv m_{\pi,\text{BC}}^2 - \frac{m_{\pi,\text{BB}}^2 + m_{\pi,\text{CC}}^2}{2} \Big|_{m_{\pi,\text{BB}}=m_{\pi,\text{CC}}=m_{\pi,\text{AA}}=m_{\pi}}$$

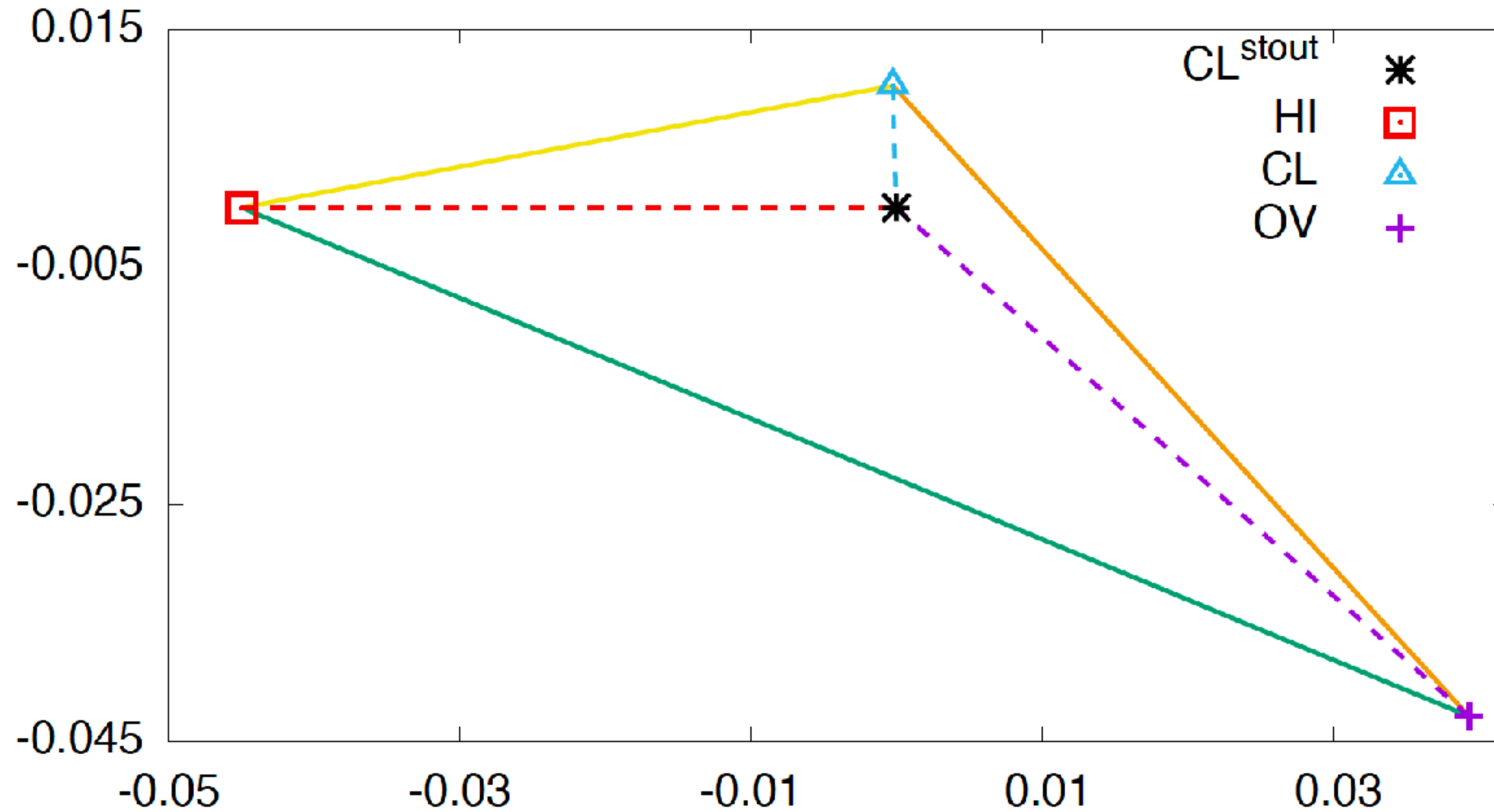
$\Delta_{\text{mix,uni}}(\text{GeV}^2)$



3. Result

$$\bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}}(m_\pi, a) \equiv m_{\pi,\text{BC}}^2 - \frac{m_{\pi,\text{BB}}^2 + m_{\pi,\text{CC}}^2}{2} \Big|_{m_{\pi,\text{BB}}=m_{\pi,\text{CC}}=m_{\pi,\text{AA}}=m_\pi}$$

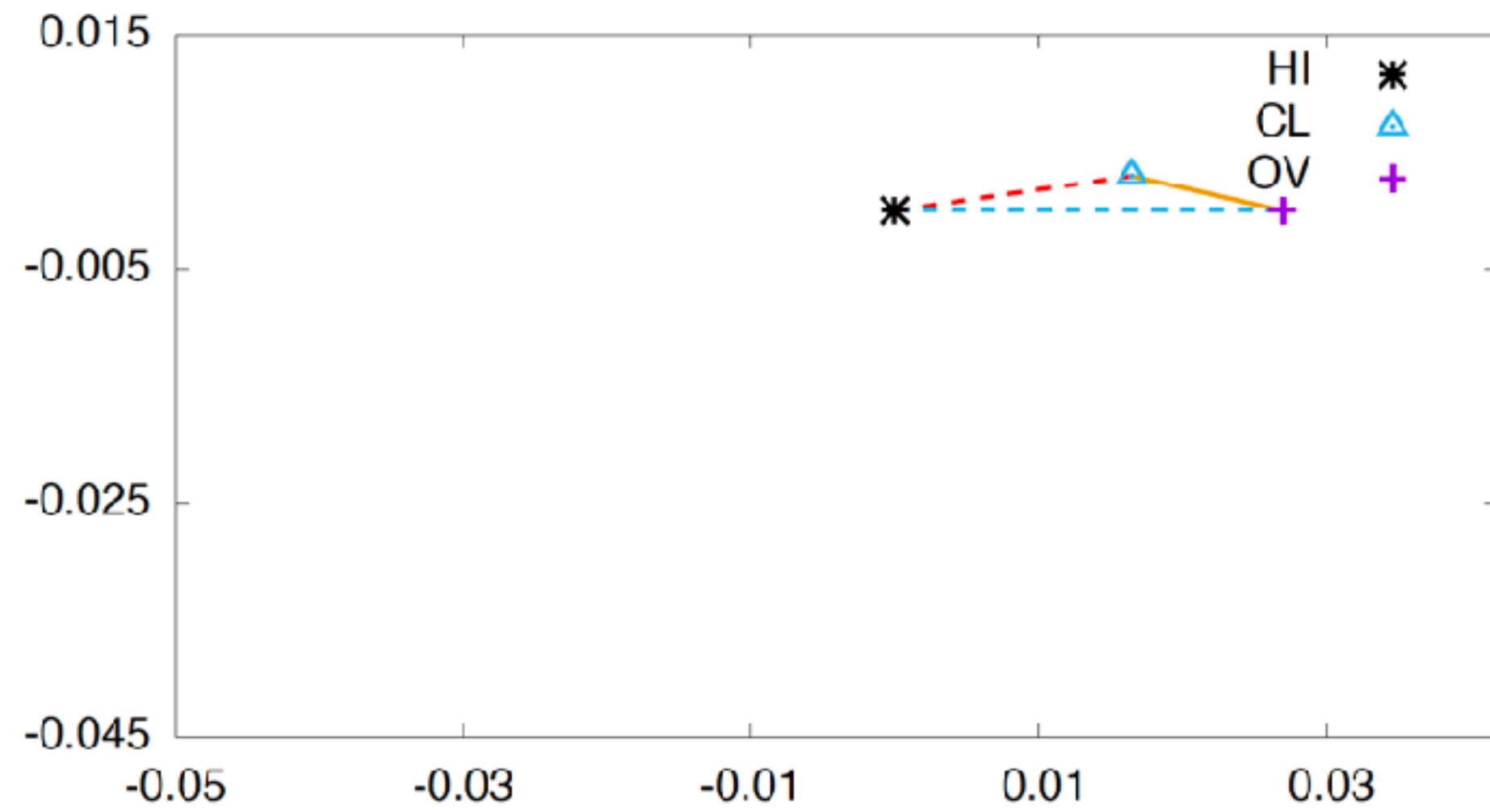
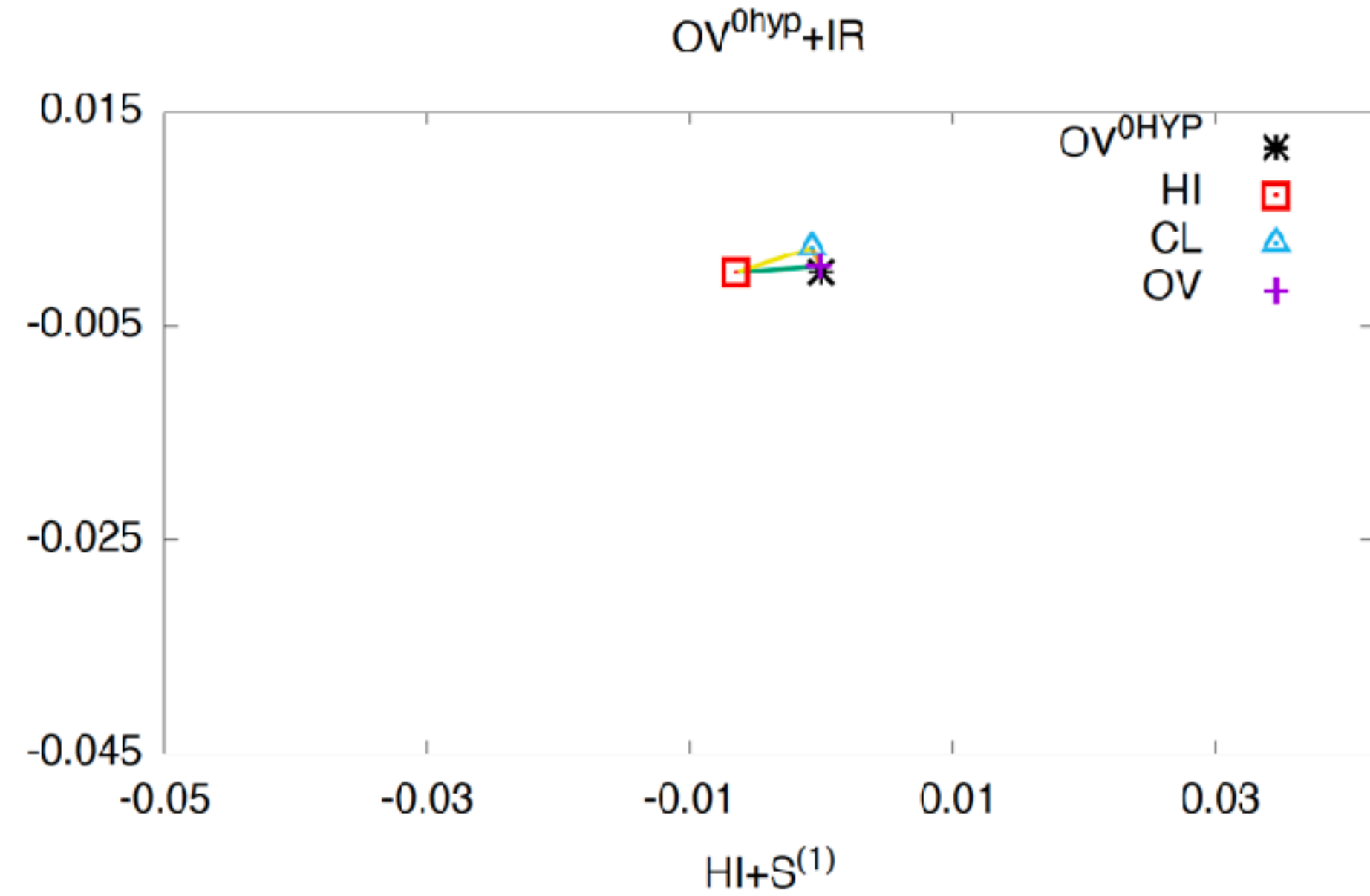
$\text{CL}^{\text{stout}} + \text{S}^{\text{stad}}$



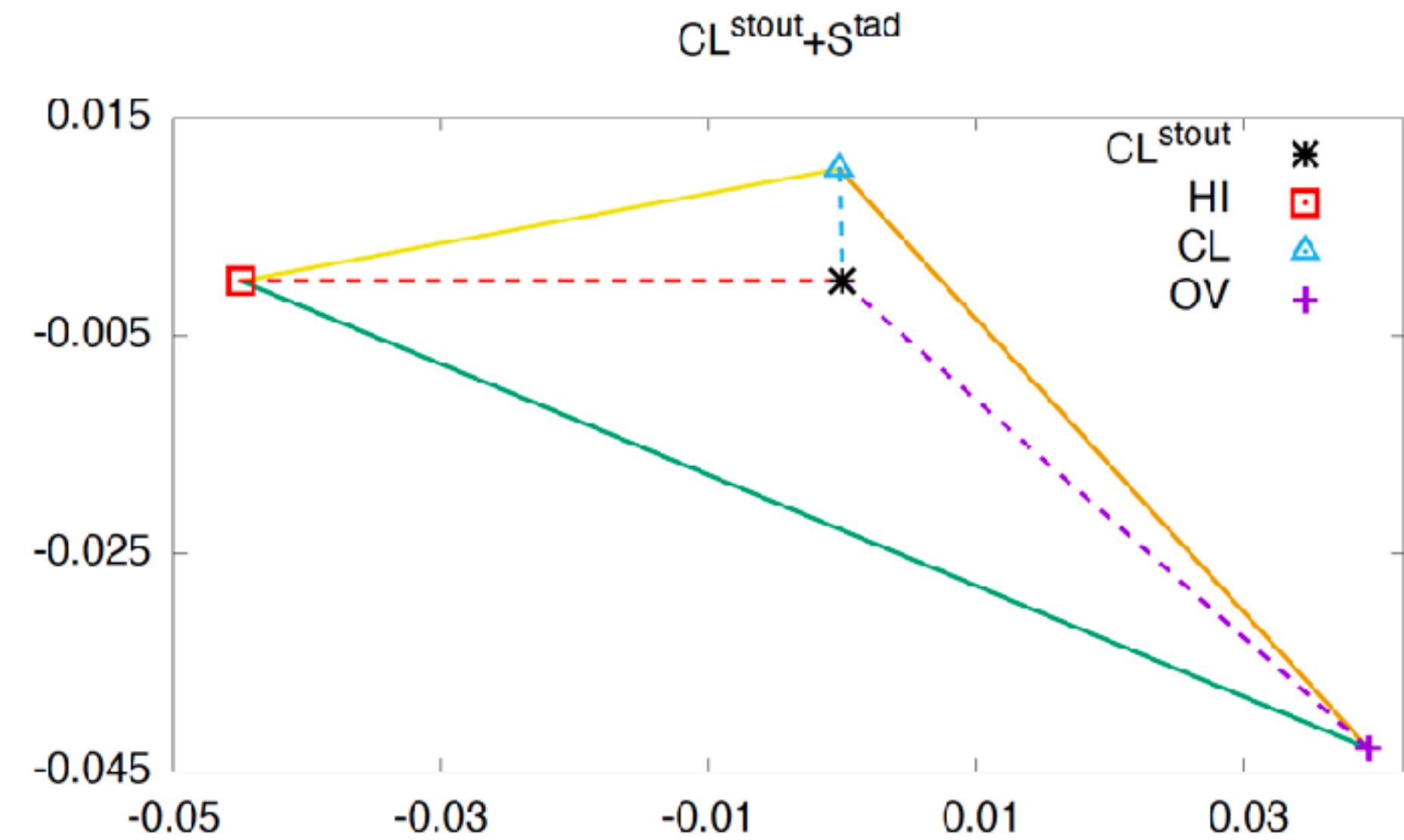
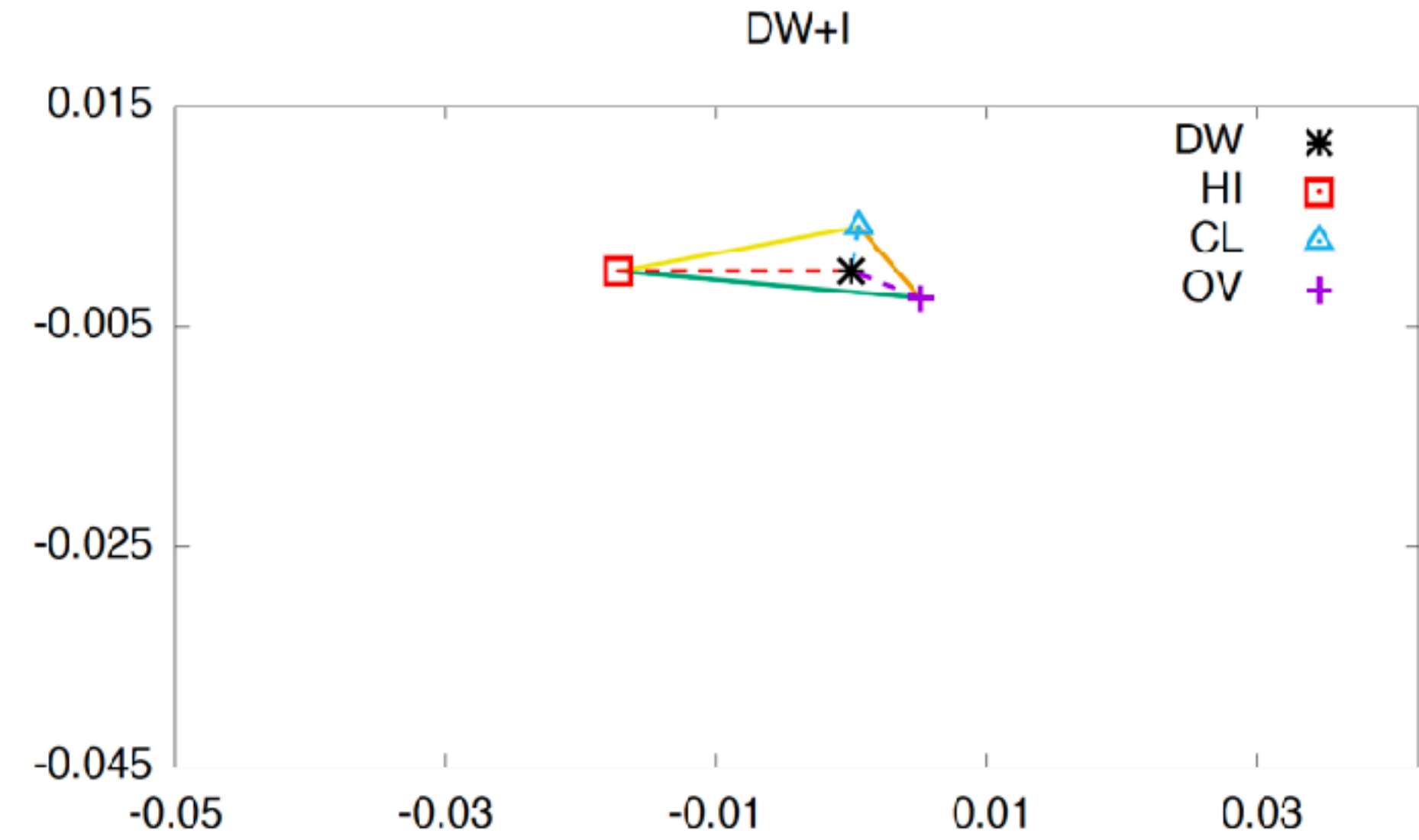
$$|\Delta_{\text{mix,uni}}^{\text{B/A}} - \Delta_{\text{mix,uni}}^{\text{C/A}}| \leq \bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}} \leq \Delta_{\text{mix,uni}}^{\text{B/A}} + \Delta_{\text{mix,uni}}^{\text{C/A}}$$

$$|\bar{\Delta}_{\text{mix,uni}}^{\text{B+D/A}} - \bar{\Delta}_{\text{mix,uni}}^{\text{C+D/A}}| \leq \bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}} \leq \bar{\Delta}_{\text{mix,uni}}^{\text{B+D/A}} + \bar{\Delta}_{\text{mix,uni}}^{\text{C+D/A}}$$

3. Result

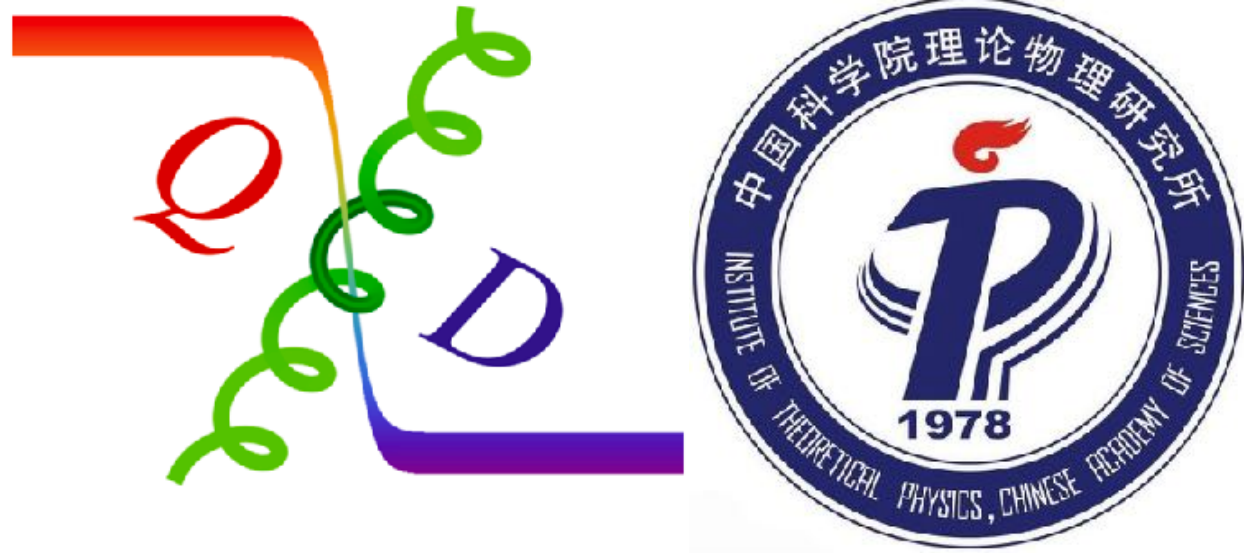


$$\bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}}(m_\pi, a) \equiv m_{\pi,\text{BC}}^2 - \frac{m_{\pi,\text{BB}}^2 + m_{\pi,\text{CC}}^2}{2} \Big|_{m_{\pi,\text{BB}}=m_{\pi,\text{CC}}=m_{\pi,\text{AA}}=m_\pi}$$



$$|\Delta_{\text{mix,uni}}^{\text{B/A}} - \Delta_{\text{mix,uni}}^{\text{C/A}}| \leq \bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}} \leq \Delta_{\text{mix,uni}}^{\text{B/A}} + \Delta_{\text{mix,uni}}^{\text{C/A}}$$

$$|\bar{\Delta}_{\text{mix,uni}}^{\text{B+D/A}} - \bar{\Delta}_{\text{mix,uni}}^{\text{C+D/A}}| \leq \bar{\Delta}_{\text{mix,uni}}^{\text{B+C/A}} \leq \bar{\Delta}_{\text{mix,uni}}^{\text{B+D/A}} + \bar{\Delta}_{\text{mix,uni}}^{\text{C+D/A}}$$



4. Summary and outlook

1. Leading mixed action effect would be a^4 , and are small when the sea fermion action has the chiral symmetry;
2. Ginsburg-Wilson relation fermion+Iwasaki gauge – smallest Δ_{mix} , $HI + S^{(1)}$ – much larger Δ_{mix} , $CL_{stout} + S_{tad}$ – largest Δ_{mix} ;
3. Mechanism of a^4 scaling of Δ_{mix} and whether the Δ_{mix} can display a^2 scaling in other observables need further studies.

THANKS FOR YOUR ATTENTION!