第二届中国格点量子色动力学研讨会
2022年10月8日, 上海

# Insights into Zc(3900) in a combined study of experiment and lattice data

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# Outline:

1. Introduction & Motivation

2. Covariant amplitudes of the $J/\psi$-$\pi$ and $DD^*$ systems

3. Phenomenological discussions

4. Summary

# Introduction

**Prominent features of exotic hadrons:**

**( Zc(3900), Zcs(3985), X(4020), X(6900), Pc, Pcs ...)**

**Most of them lie close to some underlying thresholds :**

| | |
|---|---|
| $Z_c(3900)$ | $\bar{D}D^*$ |
| $X(4020)$ | $\bar{D}^*D^*$ |
| $Z_{cs}(3985)$ | $D_s^- D^*$ |

❖ **Important question that follows:**

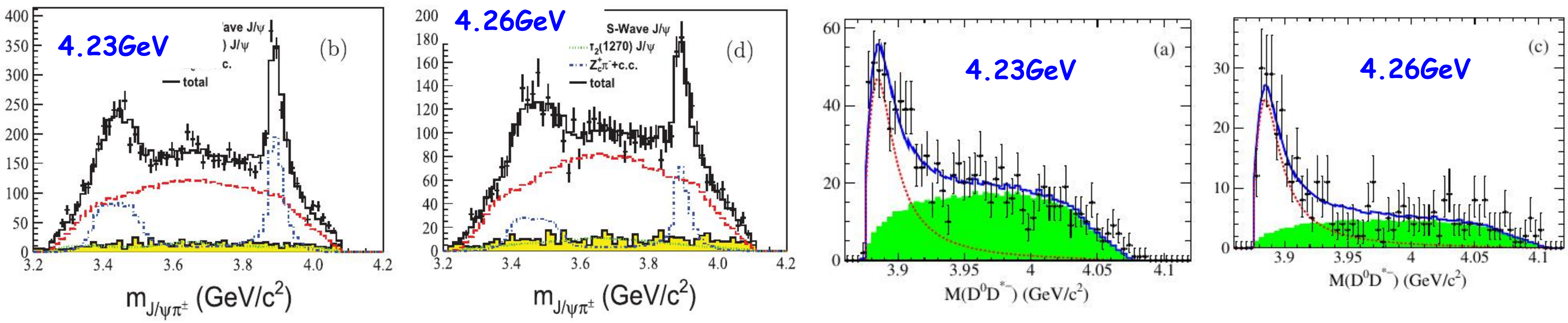
**Kinematical effects ? Or Hadron molecular ? Or Elementary/Compact state ?**

❖ **Scattering data can be useful to address these questions.**

**Too luxary for Exp to extract scattering data of D/D*.**

**LATTICE CAN provide such kinds of valuable data.**

# Zc(3900) focused in this talk



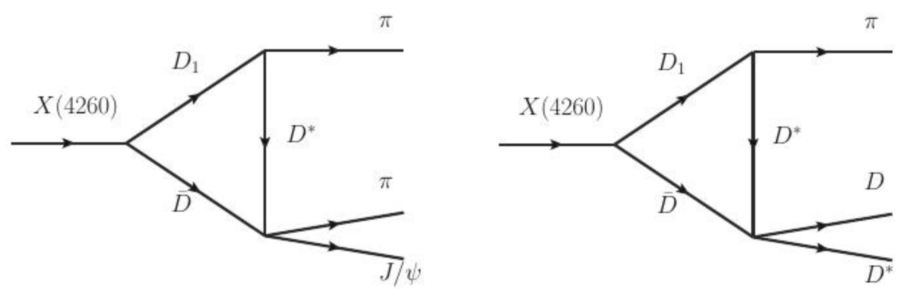

## Mechanism behind the Zc(3900) peaks:

- **Kinematical singularity**

[Wang et al., PRL'13] [Liu et al, PRD'13]

... ...

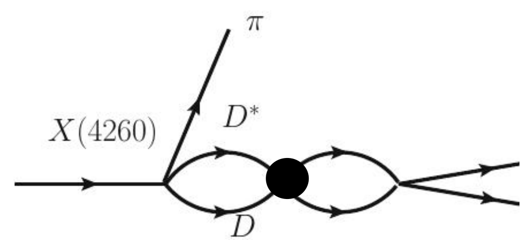
Caveat from [Gong et al., EPJC'16]




- **DD* molecular**

[Albaladejo, PLB'16] [Gong et al, PRD'16] [He et al, EPJC'18]

... ...

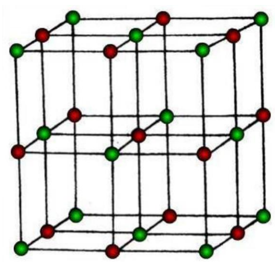


# ➢ Current status of Zc(3900) on lattice

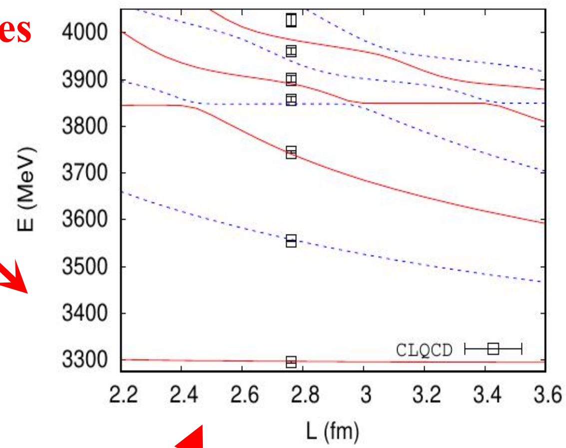
- **CLQCD: Resonace-like structure disfavored**

[Chen et al., PRD'14] [Chen et al, CPC'19]

$$\hat{p} = \frac{pL}{2\pi}$$



**Eigenenergies in finite box**




**Length of the box**

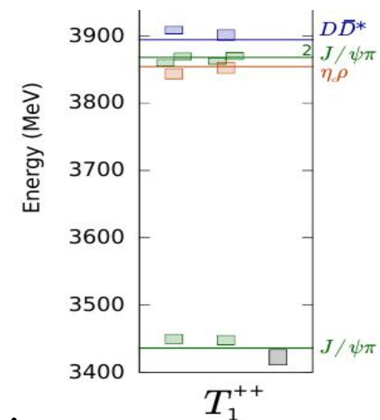
$$p \cot \delta(p) = \frac{2\pi}{L} \pi^{-3/2} \mathcal{Z}_{00}(1, \hat{p}^2)$$

Phase shifts

Luscher's Zeta function ( function of $L$, parameter free)

- **HSC: No evidence of bound state or narrow resonance**

[Cheung et al., JHEP'17]




✓ **Lattice data DO hint weak forces in the J/ψ-π and DD* system.**

✓ **Question: Do these lattice data rule out the "resonance" explanation of Zc(3900) peaks observed by Exp?**

**We try to answer this question by analyzing both data of Exp and Lattice.**

## Theoretical setups:

## Covariant amplitudes of the $J/\psi$-$\pi$ and $DD^*$ system

# Lessons from previous studies:

[Ikeda et al., PRL'16] [Albaladejo et al., PLB'16] [Ortega et al., EPJC'19]

- The off-diagnol π J/ψ - DD* interaction is important for Zc(3900).
- Constant couplings alone in the coupled J/ψ-π and DD* channels CANNOT generate any resonance.

❑ We use covariant amplitudes to study the J/ψ-π and DD* system.

$$\mathcal{L}_{D\bar{D}^*D\bar{D}^*} \sim \hat{\lambda}_1 D^\dagger D D_\mu^{*\dagger} D^{*\mu}$$

$$\begin{aligned}\mathcal{L}_{D\bar{D}^*J/\psi\pi} = \quad & \hat{\lambda}_2\psi_\mu(\nabla^\mu D^\dagger u_\nu \bar{D}^{*\nu\dagger} + \bar{D}^{*\nu}u_\nu\nabla^\mu D) + \hat{\lambda}_3\psi_\mu(\nabla^\nu D^\dagger u^\mu \bar{D}_\nu^{*\dagger} + \bar{D}_\nu^* u^\mu \nabla^\nu D) \\ & + \hat{\lambda}_4\psi_\mu(\nabla^\nu D^\dagger u_\nu \bar{D}^{*\mu\dagger} + \bar{D}^{*\mu}u_\nu\nabla^\nu D) + \hat{\lambda}_5\psi_\mu(D^\dagger\nabla^\mu u^\nu \bar{D}_\nu^{*\dagger} + \bar{D}_\nu^*\nabla^\mu u^\nu D)\end{aligned}$$

$\bar{D}^*(1)\, D(2) \to \bar{D}^*(3)\, D(4)$

$$V_{\bar{D}^*D\to\bar{D}^*D} = \lambda_1 \varepsilon_1 \cdot \varepsilon_3^*$$

$J/\psi(1)\, \pi(2) \to \bar{D}^*(3)\, D(4)$

$$V_{J/\psi\pi\to\bar{D}^*D} = \frac{\sqrt{2}}{F_\pi}\left(\lambda_2\, \varepsilon_1\cdot p_4\, \varepsilon_3^*\cdot p_2 + \lambda_3\, \varepsilon_1\cdot p_2\, \varepsilon_3^*\cdot p_4 + \lambda_4\, \varepsilon_1\cdot\varepsilon_3^*\, p_2\cdot p_4 + \lambda_5\, \varepsilon_1\cdot p_2\, \varepsilon_3^*\cdot p_2\right)$$

## Partial-wave projections in the *lS* base

$1+2 \to \bar{1}+\bar{2}$

[Gulmez et al., EPJC'19] [Du et al., EPJC'18]

$$V^J_{\ell S;\bar{\ell}\bar{S}}(s) = \frac{Y^0_{\bar{\ell}}(\hat{p}_z)}{2(2J+1)} \sum_{\sigma_1,\sigma_2,\bar{\sigma}_1,\bar{\sigma}_2,m} \int d\hat{\bar{p}}\, Y^m_{\ell}(\hat{\bar{p}})^* \, (\sigma_1\sigma_2 M|s_1 s_2 S)(m M \bar{M}|\ell S J)$$

$$(\bar{\sigma}_1\bar{\sigma}_2\bar{M}|\bar{s}_1\bar{s}_2\bar{S})(0\bar{M}\bar{M}|\bar{\ell}\bar{S}J)\, V(p_1,p_2,\bar{p}_1,\bar{p}_2,\sigma_1,\sigma_2,\bar{\sigma}_1,\bar{\sigma}_2)$$

## For the S-wave scattering of Vector + Pseudoscalar -> Vector + Pseudoscalar

$$V^J_{01;01}(s) = \frac{1}{2(2J+1)} \sum_{\sigma_1=\bar{\sigma}_1=0,\pm 1} \int d\cos\theta \; V(s,t(s,\cos\theta),\sigma_1,\bar{\sigma}_1)$$

## The polarization vectors in this convention:

$$\varepsilon(\mathbf{k},0) = \begin{pmatrix} \frac{k}{m_V}\cos\theta \\ \frac{1}{2}\left(\frac{E_k}{m_V}-1\right)\sin 2\theta \\ 0 \\ (1+\cos 2\theta)\frac{E_k}{m_V} - \cos 2\theta \end{pmatrix}, \quad \varepsilon(\mathbf{k},\pm) = \begin{pmatrix} \mp\frac{1}{\sqrt{2}}\frac{k}{m_V}\sin\theta \\ \mp\frac{1}{\sqrt{2}}\left(\frac{E_k}{m_V}\sin^2\theta + \cos^2\theta\right) \\ -\frac{i}{\sqrt{2}} \\ \mp\frac{1}{2\sqrt{2}}\left(\frac{E_k}{m_V}-1\right)\sin 2\theta \end{pmatrix}$$

- ❑ **We have verified that the helicity base can give the exactly same results with the proper definition of ploarization vectors.**

## The covariant S-wave scattering amplitudes of the J/ψ-π and DD*

$$
\begin{aligned}
V_{11}(s) &= 0\,, \\
V_{12}(s) &= \frac{\sqrt{2}}{9F_\pi M_{D^*} M_{J/\psi}}\Big\{\lambda_2\big[q_2^2(2M_{J/\psi}+E_{J/\psi})E_\pi + q_1^2(2M_{D^*}+E_{D^*})E_D\big] \\
&\quad -\lambda_4\big[q_1^2 q_2^2 + (2M_{J/\psi}+E_{J/\psi})(2M_{D^*}+E_{D^*})E_\pi E_D\big] + \lambda_5\big[q_1^2\sqrt{s}(2M_{D^*}+E_{D^*})\big]\Big\}\,, \\
V_{22}(s) &= -\frac{\lambda_1}{9M_{D^*}^2}(2M_{D^*}+E_{D^*})^2\,,
\end{aligned}
$$

## Unitarized prescription for the scattering amplitudes :

$$
T_J(s) = [1 - N_J(s)\cdot G(s)]^{-1}\cdot N_J(s)
$$

$$
N(s) = \begin{pmatrix} V_{11}(s) & V_{12}(s) \\ V_{12}(s) & V_{22}(s) \end{pmatrix} \qquad \operatorname{Im} G(s) = \frac{q(s)}{8\pi\sqrt{s}}\,, \qquad (s > s_{\rm th})\,,
$$

$$
\begin{aligned}
G(s) &= -\frac{1}{16\pi^2}\left[a_{SC}(\mu^2) + \log\frac{m_2^2}{\mu^2} - x_+\log\frac{x_+-1}{x_+} - x_-\log\frac{x_--1}{x_-}\right]\,, \\
x_\pm &= \frac{s+m_1^2-m_2^2}{2s} \pm \frac{q(s)}{\sqrt{s}}\,.
\end{aligned}
$$

## The unitarized production amplitudes

J/ψ π

$$\mathcal{P}(s) = \begin{pmatrix} P_1(s) \\ P_2(s) \end{pmatrix} = [1 - N(s) \cdot G(s)]^{-1} \cdot \alpha \,, \qquad \alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

DD*

**For the *X* -> J/ψ ππ process:**

$$M_1 = \epsilon_Y^\dagger \cdot \epsilon_{J/\psi} \big[ P_1(s) + P_1(t) \big]$$

**For the *X* -> DD*π process:**

$$M_2 = \epsilon_Y^\dagger \cdot \epsilon_{D^*} P_2(s)$$

**Finite-volume effects**

$$\Delta G(s) = \frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}|<q_{\rm max}} I(|\vec{q}|) - \int^{|\vec{q}|<q_{\rm max}} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} I(|\vec{q}|)$$

with

$$\vec{q} = \frac{2\pi}{L}\vec{n},\ (\vec{n} \in \mathbb{Z}^3)\,, \quad I(|\vec{q}|) = \frac{\omega_1 + \omega_2}{2\omega_1\omega_2\left[s - (\omega_1+\omega_2)^2\right]}\,, \quad \omega_i = \sqrt{|\vec{q}|^2 + m_i^2}\,.$$

$$\widetilde{G}(s) = G(s) + \Delta G(s)$$

**Lattice finite-volume spectra:**

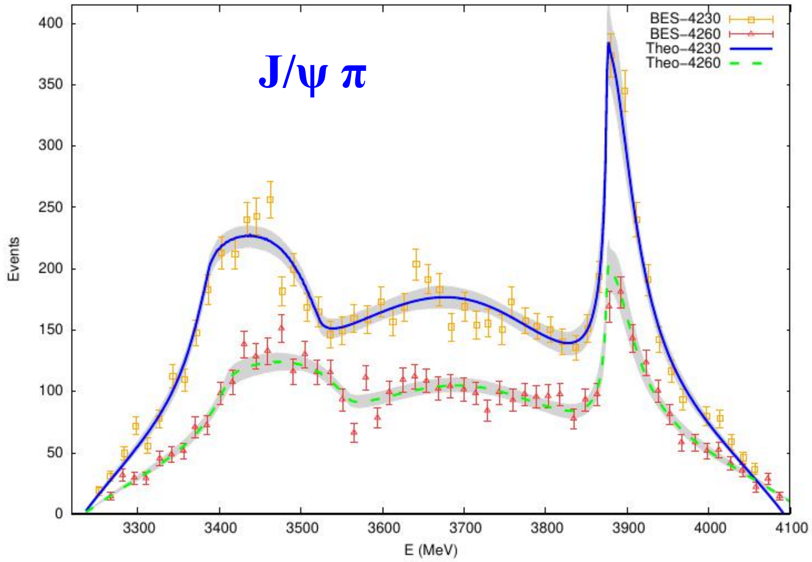
$$\det\left[1 - N_J(s)\cdot \widetilde{G}(s)\right] = 0$$

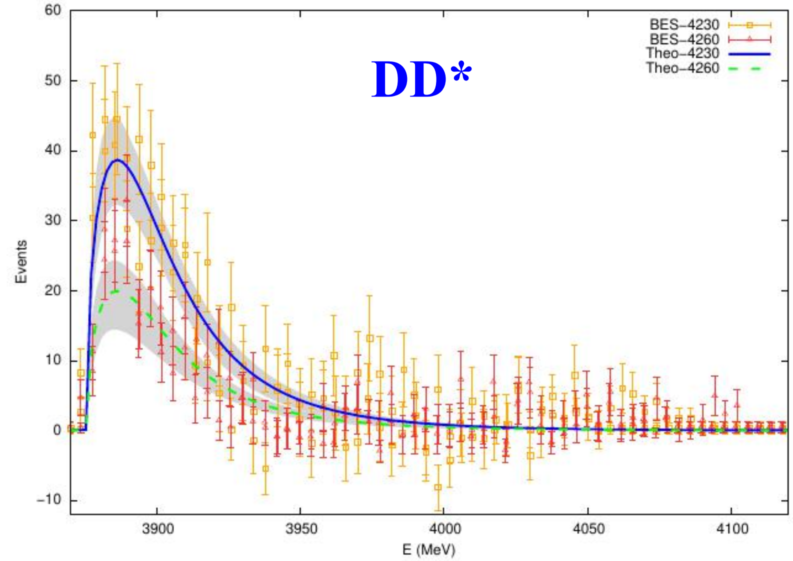
**Critical points:**

**Unitarized scattering Amp, production Amp and finite-volume Amp share the same unknown couplings in our formalism !**

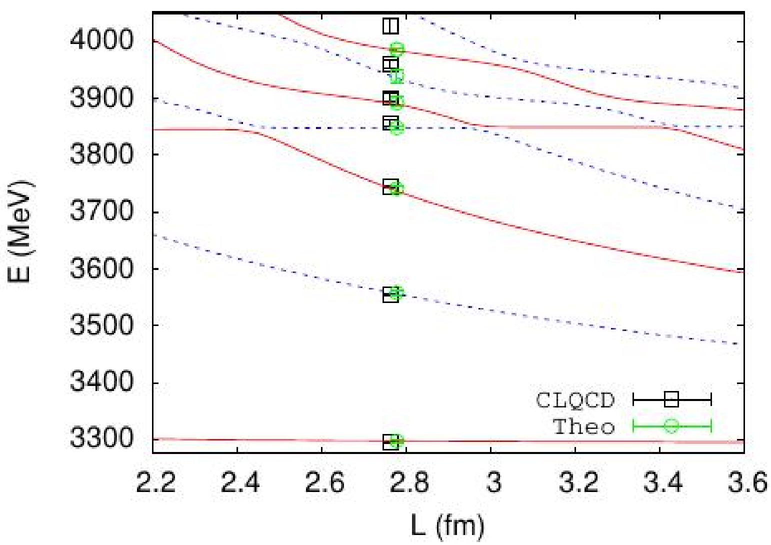
# Phenomenological discussions

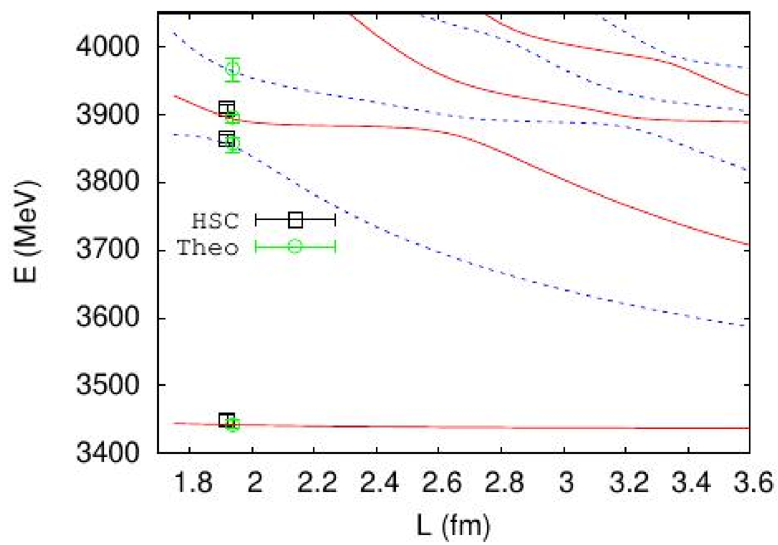
# Simultaneous fits to the data sets @ 4.23 and 4.26 GeV from BESIII




# Lattice data from CLQCD and HSC

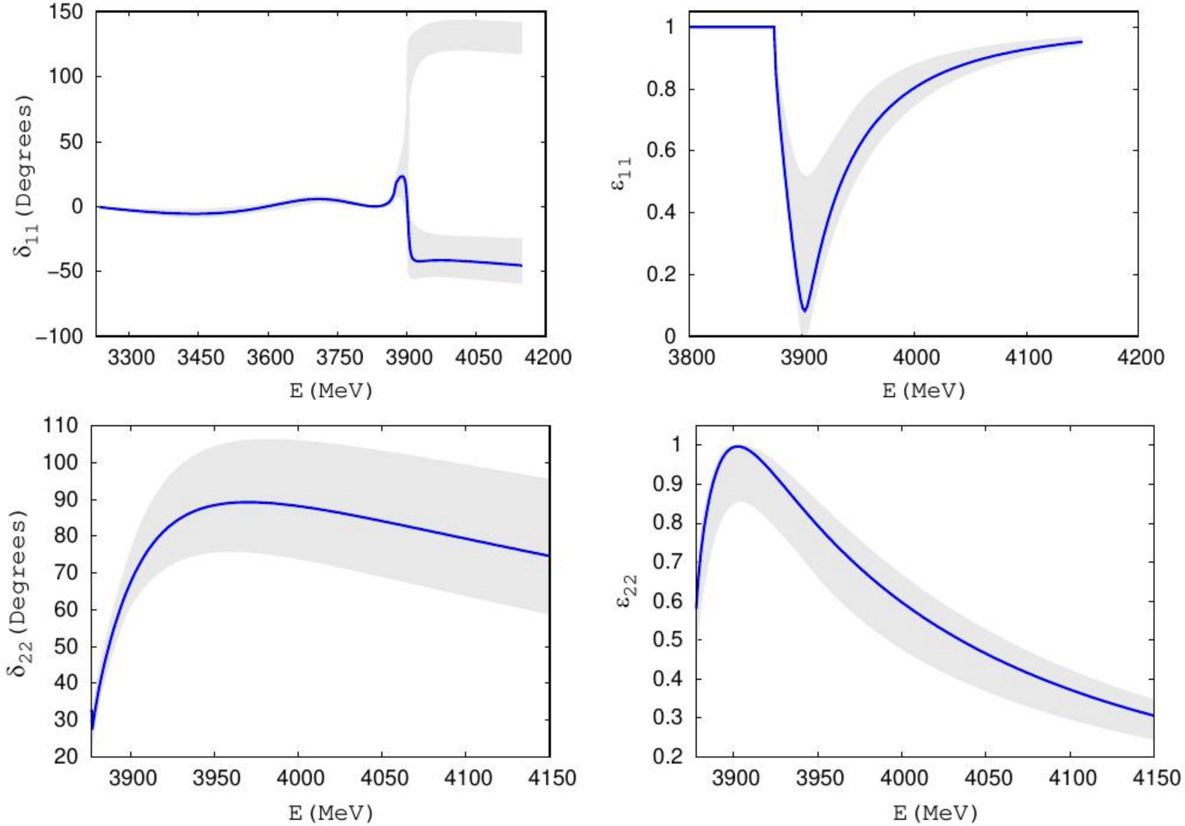


## Predictions of the phase shifts and inelasticities

**J/ψ π -> J/ψ π**

**DD* -> DD***




## Resonance pole contents in the J/ψ-π and DD* scattering amplitudes

| RS | $M_R$ (MeV) | $\Gamma_R/2$ (MeV) | $\|\gamma_1\|$ (GeV) | $\|\gamma_2\|$ (GeV) |
|---|---|---|---|---|
| III | $3874.4^{+3.7}_{-4.3}$ | $32.8^{+1.8}_{-1.6}$ | $4.3^{+0.3}_{-0.3}$ | $8.8^{+0.9}_{-0.7}$ |
| II | $3902.9^{+1.3}_{-1.3}$ | $2.5^{+2.3}_{-2.3}$ | $4.9^{+0.2}_{-0.2}$ | $8.4^{+0.3}_{-0.3}$ |
| IV | $3902.5^{+1.3}_{-2.3}$ | $3.5^{+6.4}_{-2.0}$ | $4.7^{+0.2}_{-0.4}$ | $8.7^{+0.6}_{-0.3}$ |

| RS | $M_R$ (MeV) | $\Gamma_R/2$ (MeV) | $\lvert\gamma_1\rvert$ (GeV) | $\lvert\gamma_2\rvert$ (GeV) |
|---|---|---|---|---|
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| IV | $3902.5^{+1.3}_{-2.3}$ | $3.5^{+6.4}_{-2.0}$ | $4.7^{+0.2}_{-0.4}$ | $8.7^{+0.6}_{-0.3}$ |

- **Pole counting rule:** [Morgan,NPA '91]

**Elementary particle (pair of poles in different RS) molecule (just one pole).**

**Zc(3900) sits in the "middle" of the elementary and molecule.**

- **Compositeness analysis:** [Guo, Oller, PRD '16]

$$X_k = |\gamma_k|^2 \left| \frac{dG_k^{(\mathrm{II})}(s_R)}{ds} \right|$$

$$X = X_1 + X_2 = 0.02 + 0.39 = 0.41\,, \quad (\text{Pole on RS II})\,,$$
$$X = X_1 + X_2 = 0.02 + 0.44 = 0.46\,, \quad (\text{Pole on RS III})\,,$$
$$X = X_1 + X_2 = 0.02 + 0.42 = 0.44\,, \quad (\text{Pole on RS IV})\,,$$

❑ **Similar conclusions from both methods:**

**Zc(3900) is a mixture of DD* molecule and other compact objects.**

# Summary

- **Covariant amplitudes are used to successfully analyze the Exp and lattice data to address Zc(3900).**

- **A not very standard resonance pole on fourth/second sheet is responsible for the Zc(3900), similar to the $a_0$(980) case.**

- **We demonsrtate that the current lattice data do not exclude the existence of the Zc(3900) resonance poles.**

谢谢各位的聆听！