Nonperturbative thermodynamics of multi-Higgs models

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Contents

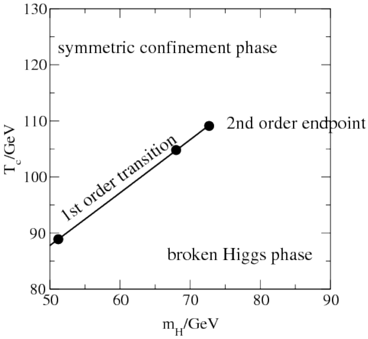
- Motivation and introduction to nonperturbative EWPT
- Application to BSM models
- More simulation techniques and outlook

First order electroweak phase transition?

NO phase transition in the SM (smooth crossover)!

Need significant modifications
to EW scale dynamics from BSM physics:

- Through radiative corrections
  to the Higgs (often $\mathcal{O}(1)$ couplings)?
- Multiple phase transitions
  before settling to the EW minimum?
- Non-renormalizable operators?



Source: hep-ph/0010275

Problems with perturbation theory

- Infrared problem: bad convergence for light bosons at high-T

$$\text{expansion parameter} \sim g^2 n_b(m) \sim \frac{g^2}{e^{m/T} - 1} \xrightarrow{m \ll T} \frac{g^2 T}{m}$$

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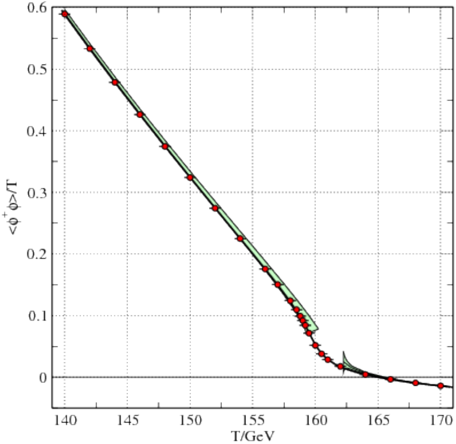
- Momentum scale $g^2 T$ is nonperturbative (Linde)

- Phase transitions are precisely due to light fields!

$$m_{T=0}^2 + g^2 T^2 \approx 0 \text{ near } T_c$$

The Standard Model crossover

Perturbation theory not reliable near T_c (predicts first order!)



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Other comparisons with nonperturbative results

Radiatively induced transition in the MSSM: Laine et al. 2012

- T_c overestimated by $\sim 7\%$ compared to lattice
- L/T_c^4 underestimated by 50%!

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Bubble nucleation in toy models: Moore et al. 2000, 2001

- Supercooling off by at least 25% at 2-loop
- Typically $> 100\%$ discrepancy in nucleation rates

How to be nonperturbative?

Nonperturbative = lattice Monte Carlo

(in practice)

Discretize the theory directly in imaginary time?

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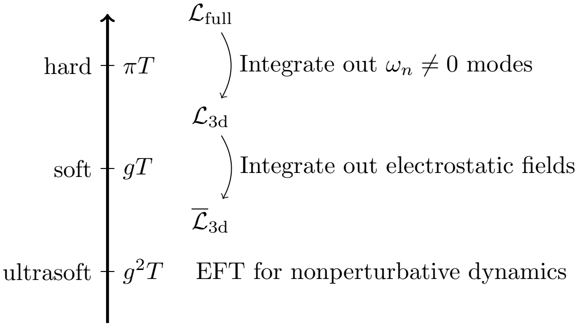
Discretize the theory directly in imaginary time? NO!

- Weak interactions (with fermions) not well understood on the lattice
- Difficult to fit all relevant scales on finite lattice $(\pi T, gT, g^2 T)$
- Controlling the continuum limit is complicated if the theory has many parameters

Effective theory approach

$$\phi(\tau,\mathbf{x}) = T\sum_n \tilde{\phi}(\mathbf{p})e^{i\omega_n\tau},\ \omega_n = \begin{cases} 2\pi nT & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$



High-T EFTs on the lattice

$$S_{3\text{d}} = \int d^3x \, \Big[\frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\phi)^\dagger(D_i\phi) + \bar{m}^2\phi^\dagger\phi + \bar{\lambda}(\phi^\dagger\phi)^2 + \mathcal{L}_{\text{BSM}} + \text{higher-order operators}\Big]$$

Fermions integrated out $\rightarrow$ discretization OK

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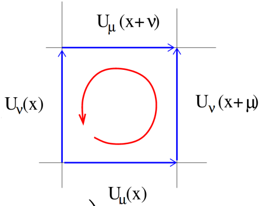

$$\begin{aligned} S_{\text{latt}} =& \beta_G \sum_{x,i<j} \left(1 - \frac{1}{2}P_{ij}(x)\right) \\ &+ \sum_{x,i} a\Big(\text{Tr}\,\Phi^\dagger(x)\Phi(x) - \text{Re}\,\text{Tr}\,\Phi^\dagger(x)U_i(x)\Phi(x+i)\Big) \\ &+ \sum_x a^3\Big(\bar{m}^2_{\text{latt}} \underbrace{\frac{1}{2}\text{Tr}\,\Phi^\dagger\Phi}_{=\phi^\dagger\phi} + \bar{\lambda}\left[\frac{1}{2}\text{Tr}\,\Phi^\dagger\Phi\right]^2 + \mathcal{L}_{\text{BSM}} + \dots\Big) \end{aligned}$$

Renormalization and continuum limit in 3d

Challenge in QFT simulations:

How to extrapolate $a \to 0$ while preserving long-distance physics?

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Challenge in QFT simulations:

How to extrapolate $a \to 0$ while preserving long-distance physics?

- Drop operators beyond dimension 4 in $S_{\text{latt}} \to$ **super-renormalizable** theory

$$\text{Lagrangian parameters:} \quad \bar{m}^2_{\text{latt}} = \bar{m}^2_{\text{cont}} + \text{counterterm}$$
$$\text{Condensates:} \quad \langle \phi^\dagger \phi \rangle_{\text{latt}} = \langle \phi^\dagger \phi \rangle_{\text{cont}} + \text{counterterm}$$

- **Exact** relations to continuum parameters once counterterms are known (2-loop calculation in 3d)

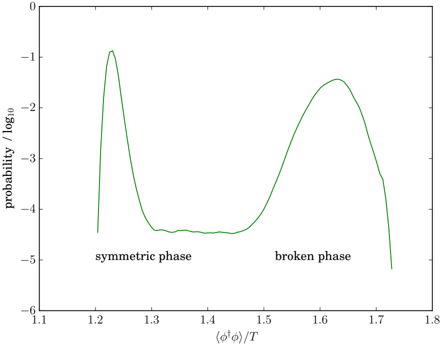
Simulating the electroweak phase transition

Simulation

I. Sample field configurations in the canonical ensemble ($\rho\{\phi\} \propto e^{-S[\phi]}$)

II. Calculate expectation values with Monte Carlo

For EWPT, we measure local condensates $\langle \phi^\dagger \phi \rangle$, $\langle (\phi^\dagger \phi)^2 \rangle \ldots$



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Application to the Two Higgs Doublet Model (2HDM)

$$V(\phi_1, \phi_2) = m_1^2 \phi_1^\dagger \phi_1 + m_2^2 \phi_2^\dagger \phi_2 + m_{12}^2 \phi_1^\dagger \phi_2 + \cdots + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \ldots$$

- Goal: compare lattice simulations to standard perturbative calculations
- Resulting EFT after dimensional reduction:

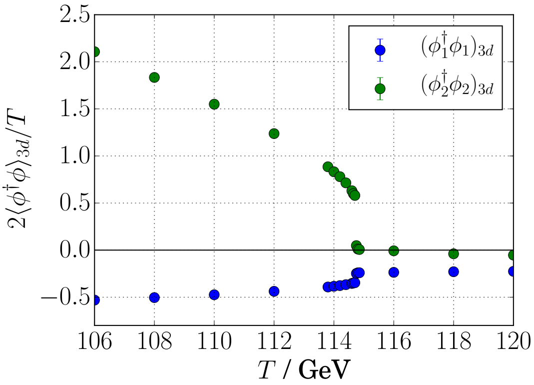
$$\begin{aligned}\bar{\mathcal{L}}_{3\text{d}} =& \frac{1}{4}(F_{rs})^2 + (D_r\phi_1)^\dagger (D_r\phi_1) + (D_r\phi_2)^\dagger (D_r\phi_2) + \overline{m}_1^2(T)\phi_1^\dagger\phi_1 \\ &+ \overline{m}_2^2(T)\phi_2^\dagger\phi_2 + \overline{m}_{12}^2(T)\phi_1^\dagger\phi_2 + \cdots + \overline{\lambda}_3(T)(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \ldots\end{aligned}$$

Application to the Two Higgs Doublet Model (2HDM)

- Suppose the $T = 0$ minimum is "mostly SM": $\tan\beta = v_2/v_1 \gg 1$

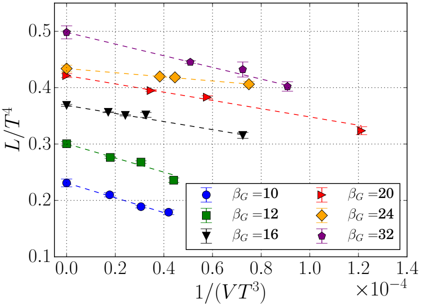
- Study radiatively-induced transition from the high-T symmetric phase to the EW minimum

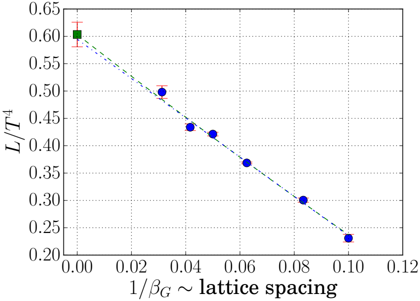
Kainulainen et al. 1904.01329



Continuum and infinite volume extrapolation

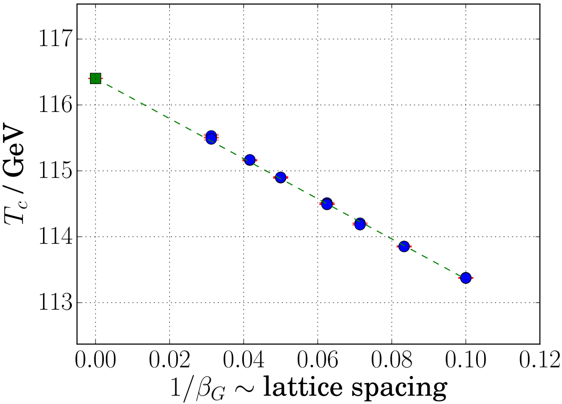
Measure latent heat from condensate discontinuity



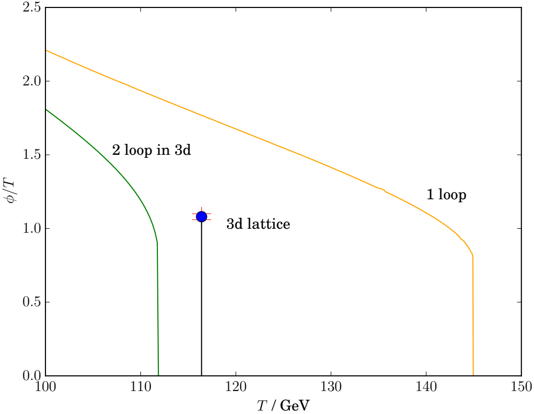


More extrapolations: critical temperature

Statistical errors from Monte Carlo are typically under control



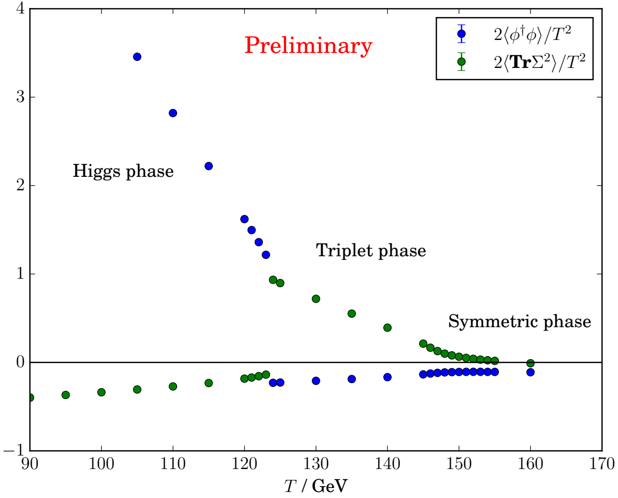
Comparison with perturbation theory (2HDM)



- Large perturbative effect from 2-loop corrections
- 2-loop is closer to the lattice result but not quite the same

Multiple phase transitions?

SM + $\mathrm{SU}(2)$ triplet scalar (finite lattice):



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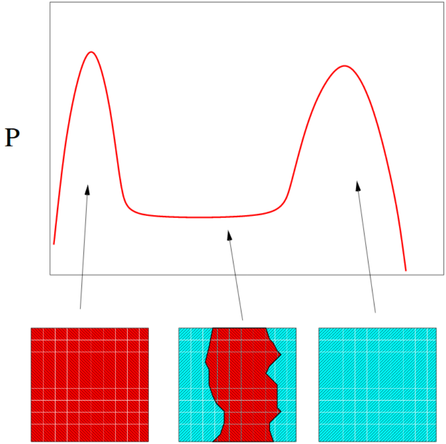
Non-extremal configurations

Field configurations outside the pure phases are exponentially rare due to extra free energy in the phase interface



Multicanonical method

Enhance probability of mixed-phase configurations by using **multicanonical ensemble**:

$$\rho\{\phi\} \propto e^{-S[\phi]+W[\phi]} \quad \text{with appropriate weight function } W$$

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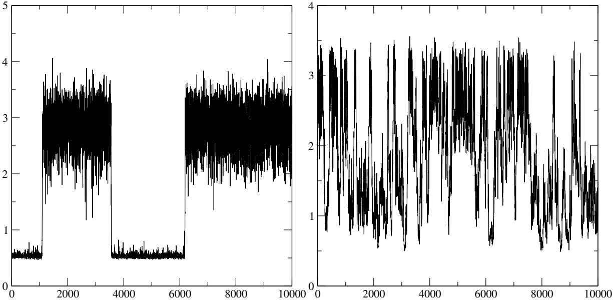
Evolution of the condensate without and with multicanonical weighting:



Multicanonical method: summary

$$\rho\{\phi\} \propto e^{-S[\phi]+W[\phi]}$$

- Absolutely necessary for studying thermal evolution from one phase to another (in strong transitions)
- Critical bubbles, interface tension, ...
- Drawback: additional CPU time spent calculating W

Outlook

The 3d approach allows for nonperturbative treatment of high-T field theories. Accuracy is still limited by that of the 4d $\rightarrow$3d mapping!

- Can be a problem in very strong transitions where the mass spectrum changes significantly
- Error in the SM is $\sim 1\%$

The EFT reproduces thermodynamics (static correlators), but can also implement "real time" with hard thermal loops

- At order $\ln(g)$, gauge field dynamics is classical Langevin

  Bödeker 1998

- Allows for nonperturbative simulations of sphaleron and nucleation rates! Moore 1998, Moore & Rummukainen 2000