Phase Transitions in Twin Higgs Models

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Based on arXiv:1810.00574

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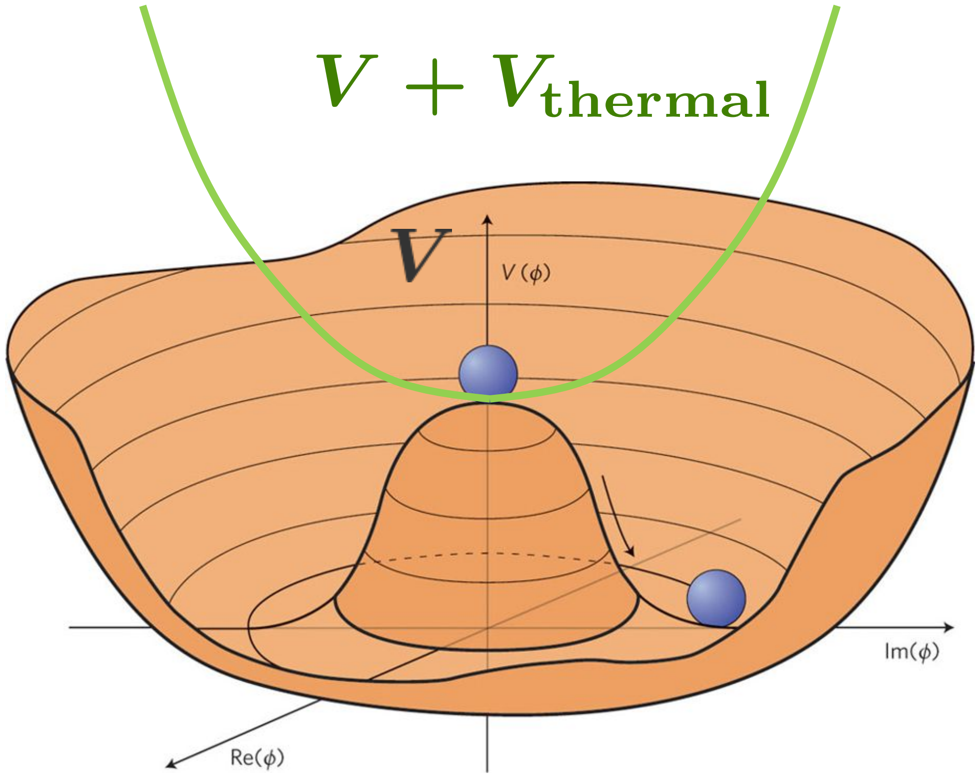
Cosmological Phase Transitions

Cosmological Phase Transitions



Spontaneous symmetry breaking

$$V = \lambda \left(|\phi|^2 - \frac{f^2}{2} \right)^2$$

Thermal fluctuation

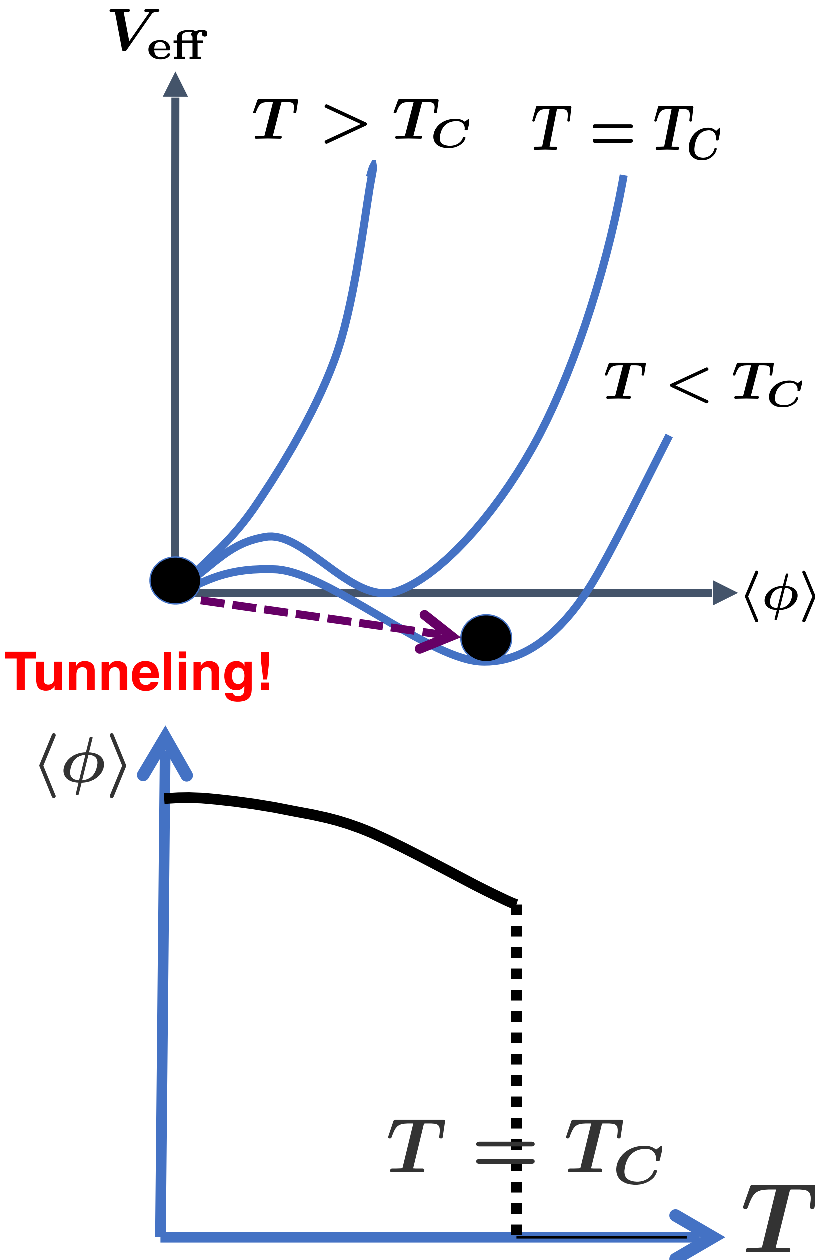
$$V_{\mathrm{thermal}} \simeq aT^2|\phi|^2, \ (a : \mathrm{const})$$

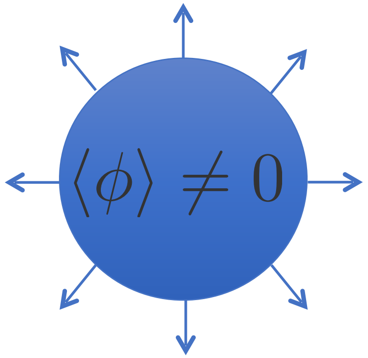
$$V + V_{\mathrm{thermal}} \simeq (aT^2 - \lambda f^2)|\phi|^2$$

$$T > T_C = \sqrt{\frac{\lambda}{a}} f \Longrightarrow \langle \phi \rangle = 0$$

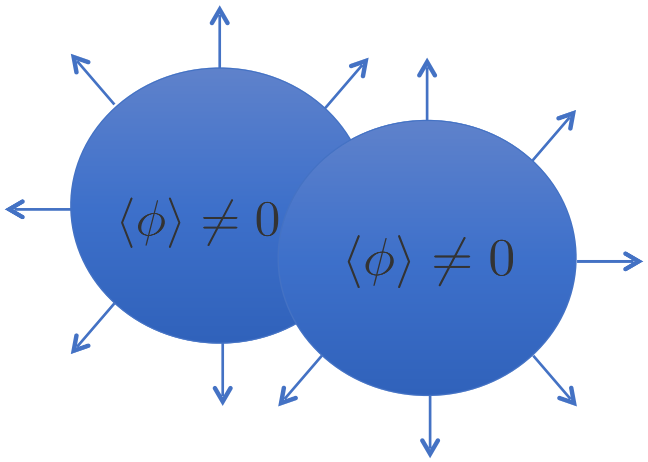
Symmetry restoration is realized in the early universe.

First-order Phase Transitions





A first-order phase transition proceeds through bubble nucleation.



There are three sources of the Gravitational Waves

Bubble collisions

[Kosowski et al. 1992]

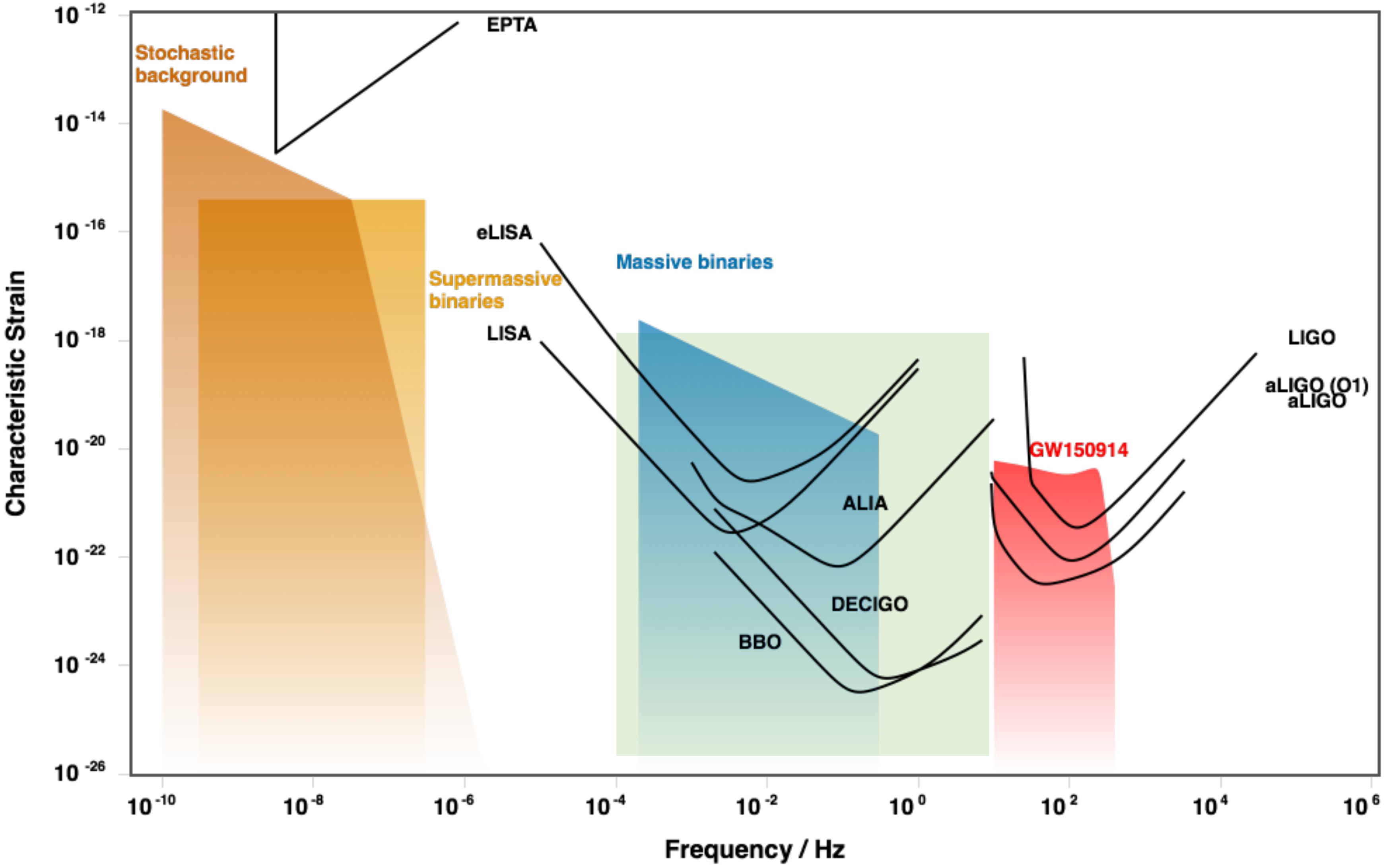
Sound Waves of the plasma

[Hindmarsh et al. 2014]

Turbulence of the plasma

[Kamionkowski et al. 1993]

Detection of Gravitational Wave (GW)



http://gwplotter.com

Motivation

Twin Higgs models

BSM physics

Constraints from Collider searches

Constraints from observation of GW

Figure

Figure

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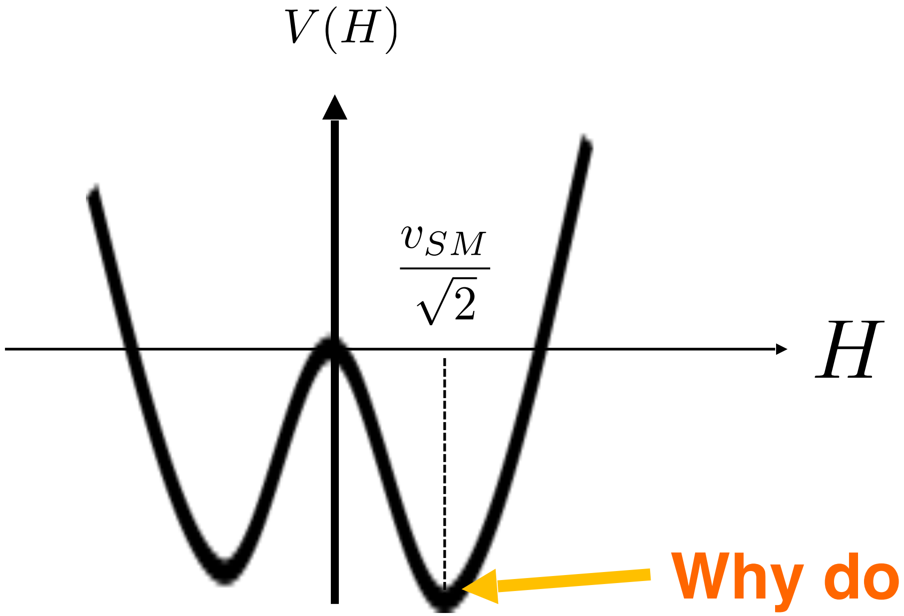
Standard Model is incomplete

SM describes phenomenology around the electroweak scale.

However, SM requires unnatural fine-tuning.



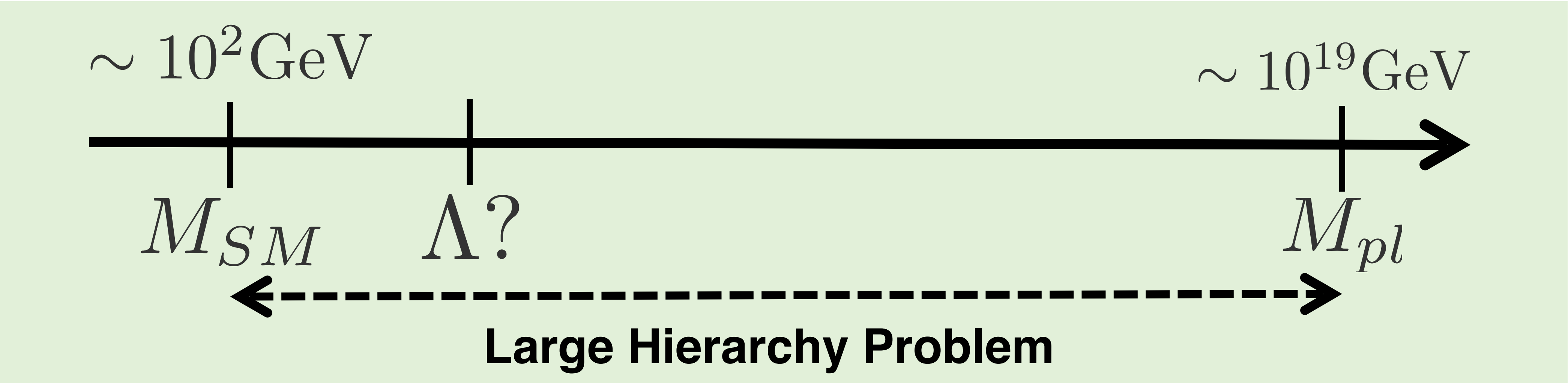
Why do we live here???

SM Higgs potential: $V(\phi) = m^2\phi^2 + \lambda\phi^4$

$$m_R^2 = m_{\text{bare}}^2 + \delta m^2$$

$\mathcal{O}(100^2)\mathrm{GeV}^2$ $\mathcal{O}(M_{\mathrm{pl}}^2)$ $\mathcal{O}(M_{\mathrm{pl}}^2)$

Where is the cut-off scale?



If SM is valid up to Planck scale: $\Lambda \sim M_{\mathrm{pl}}$

$$m_{h_R}^2 \simeq \mathcal{O}(100\ \mathrm{GeV})^2 \ll \delta m_h^2 \simeq (10^{19}\mathrm{GeV})^2$$

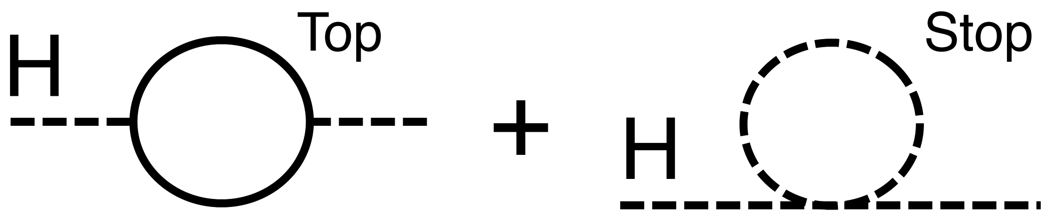
$$\Delta_{m_h} = \frac{m_{h_R}^2}{\delta m_h^2} \sim 10^{-34}$$ **(Barbieri-Giudice)**

Unnatural cancellation (fine-tuning) is needed!

Example: SUPERSYMMETRY

SUSY provides an excellent solution to the (Large) Hierarchy Problem

$$\delta m^2 =$$



$$\Lambda^2 \qquad\qquad -\Lambda^2$$

$$\simeq \frac{3y_t^2}{8\pi^2} m_{\text{stop}}^2 \log\left(\frac{\Lambda^2}{m_{\text{stop}}^2}\right)$$

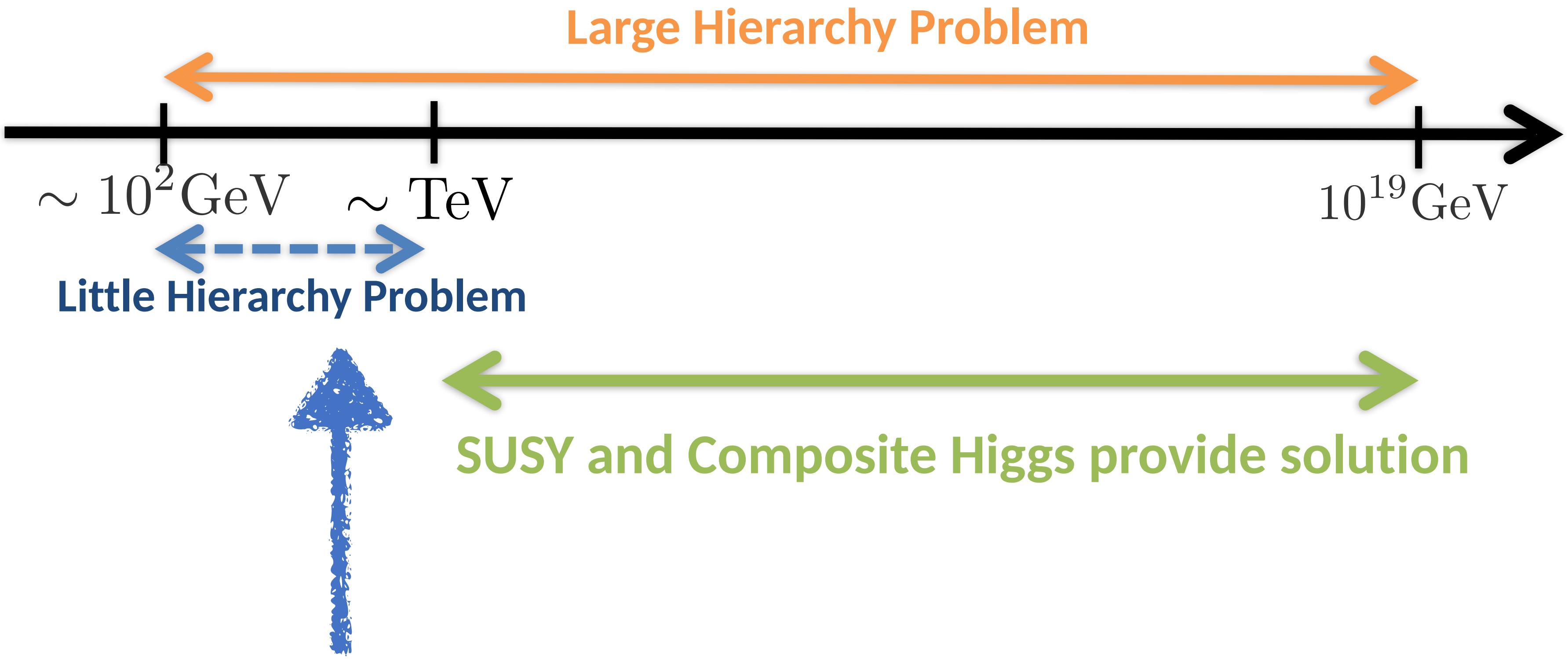
Quadratic divergence is cancelled by Top partner.
(SUSY protects quadratic divergence mass corrections.)

Soft SUSY-breaking mass is important for fine-tuning.

Scalartop is a colored state ➡ Strong bounds from Collider Searches

$$M_{\text{soft}} \gg 1\text{TeV} \quad \longrightarrow \quad \Delta_{m_h} \ll 0.01$$

Little Hierarchy Problem



Large Hierarchy Problem

$\sim 10^2\mathrm{GeV}$ $\sim \mathrm{TeV}$ $10^{19}\mathrm{GeV}$

Little Hierarchy Problem

SUSY and Composite Higgs provide solution

How to solve this problem?

Twin Higgs Models

[Chacko et al. 2005]

Twin Higgs provides solution to the (Little) Hierarchy Problem.

SM Higgs is considered as pseudo-Nambu-Goldstone Boson.

$$\mathcal{H} = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \begin{matrix} SU(2)_W \times U(1)_Y \\ SU(2)_{\widehat{W}} (\times U(1)_{\widehat{Y}}) \end{matrix} \quad V(\Phi) = \lambda \left(|\mathcal{H}|^2 - \frac{f^2}{2} \right)^2$$

$\mathcal{H}$: belongs to the (global) U(4) Fundamental Representation

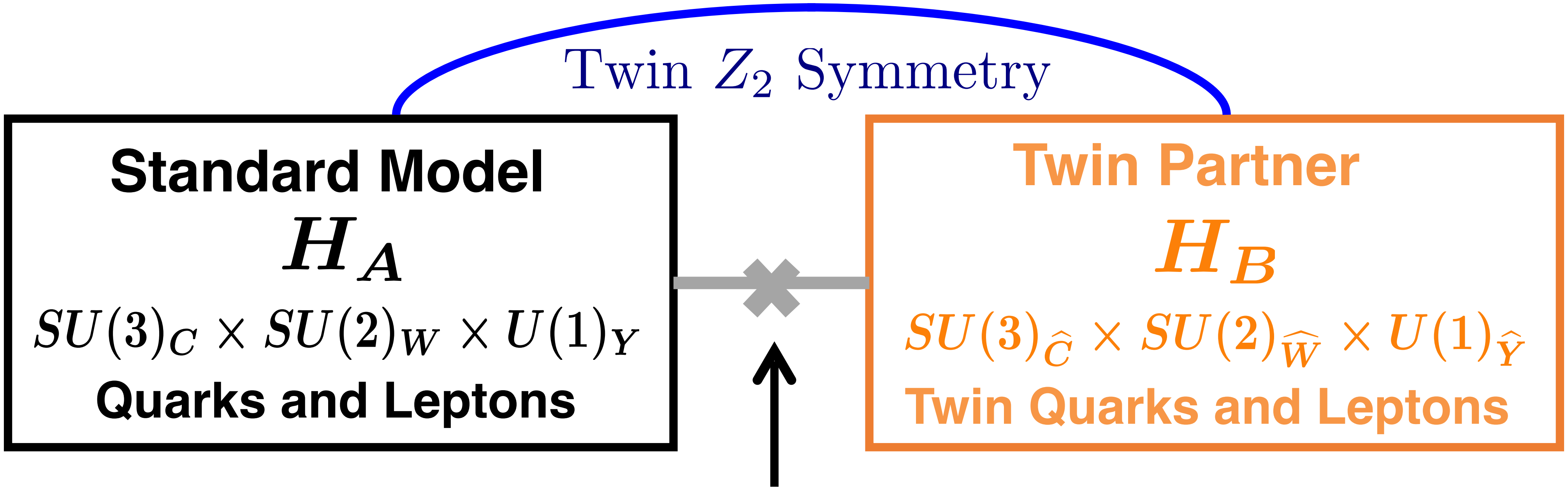
$\langle \Phi_4 \rangle = \frac{f}{\sqrt{2}}$ ➡ U(4) symmetry is spontaneously broken to U(3) symmetry.

7 Nambu-Goldstone modes arise
(4 of them are identified with SM-like Higgs)

Twin Higgs

[Chacko et al. (2016)]

copy of SM sector



Twin Z_2 Symmetry

Standard Model

H_A

$SU(3)_C \times SU(2)_W \times U(1)_Y$

Quarks and Leptons

Twin Partner

H_B

$SU(3)_{\widehat{C}} \times SU(2)_{\widehat{W}} \times U(1)_{\widehat{Y}}$

Twin Quarks and Leptons

$$\mathcal{H} = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$$

Higgs Mixing $V(\Phi) = \lambda \left(|\mathcal{H}|^2 - \frac{f^2}{2} \right)^2$

$$V_{\text{eff}} \supset \left(-\frac{3y_t^2}{8\pi^2} + \frac{9g_2^2}{64\pi^2} \right) \left(|H_A|^2 + |H_B|^2 \right) \Lambda^2$$ respects the global U(4) symmetry.

pNGB (SM-like Higgs) is insensitive to the mass correction.

Higgs potential

General Higgs potential

$$V = \lambda\left(|H_A|^2 + |H_B|^2 - \frac{f^2}{2}\right)^2 + \sigma_1 f^2 |H_A|^2 + \kappa_1\left(|H_A|^4 + |H_B|^4\right) + \rho_1 |H_A|^4$$

Spontaneous symmetry breaking $U(4) \rightarrow U(3)$

This term must be dominant compared to (explicit) U(4) breaking term.

$$\lambda \gg \sigma_1,\ \kappa_1,\ \rho_1$$

Soft twin Z_2 breaking

To satisfy constraint from Higgs coupling measurements

$$2v_A < f$$

Twin Z_2 preserving but (explicit) U(4) breaking term.

This term is naturally generated by Coleman-Weinberg (CW) potential.

Twin Z_2 and U(4) symmetries breaking term.

These quartic terms generate the SM-like Higgs mass

EWSB (Non-linear realization)

Integrating out massive mode and work in EFT for H_A

$$|H_B|^2 = \frac{f^2}{2} - |H_A|^2 \qquad (\lambda \to \infty)$$

$$V_{\rm EFT}(H_A) = -(\kappa_1 - \sigma_1) f^2 |H_A|^2 + (2\kappa_1 + \rho_1)|H_A|^4$$

This potential must be matched with SM Higgs potential.

$$2\kappa_1 + \rho_1 = \lambda_{\rm SM}, \qquad \frac{\kappa_1 - \sigma_1}{2\kappa_1 + \rho_1} = \frac{v_A^2}{f^2}$$

Tuning?

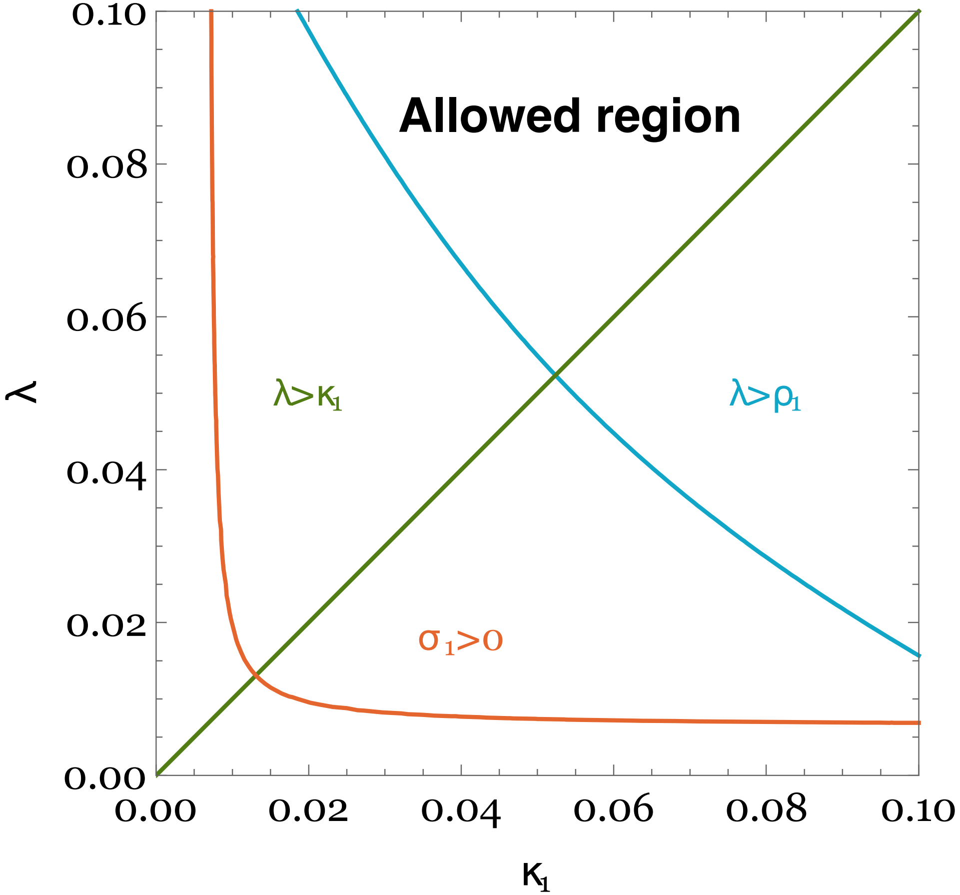
$$\Delta_\sigma \simeq 2\frac{v_{\rm SM}^2}{f^2} \qquad \Delta_\sigma > 1/10 \leftrightarrow f < 1.1\text{TeV}$$

EWSB (linear realization)

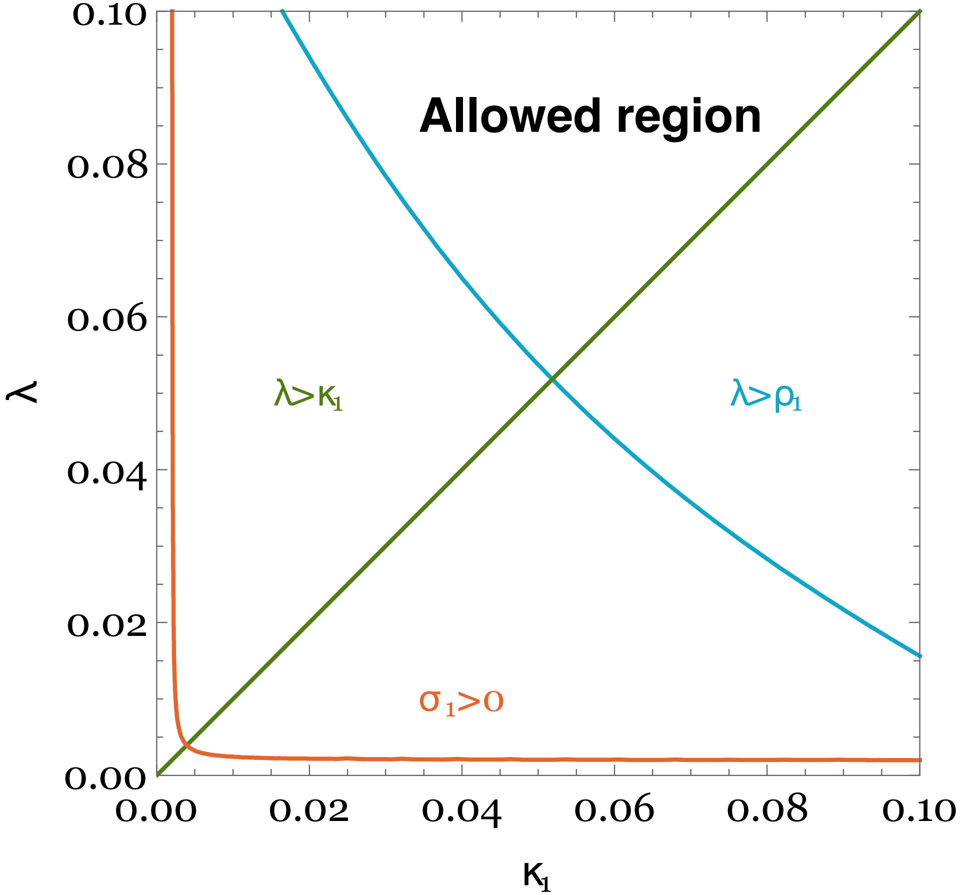
To realize correct EWSB, following conditions must be satisfied.

$$\langle H_A \rangle = v_A \simeq 246\mathrm{GeV},\ m_h \simeq 125\mathrm{GeV}$$

We solve EW vacuum and Higgs mass conditions numerically.



$v_A/f = 0.223$



$v_A/f = 0.123$

Number of NG-modes

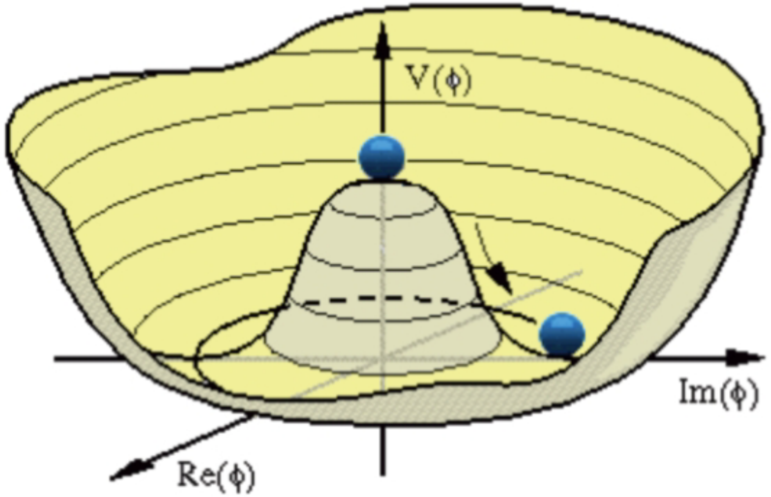
$$U(4) \supset U(2) \times U(2) \qquad \mathcal{H} = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \begin{matrix} SU(2)_W \times U(1)_Y \\ SU(2)_{\widehat{W}} (\times U(1)_{\widehat{Y}}) \end{matrix}$$

$\langle H_B \rangle \simeq f$:7 NG-modes appear and three of them eaten by $\widehat{W}$ and $\widehat{Z}$ gauge bosons.

There are 4 NG-modes

$\langle H_A \rangle = v_{\mathrm{SM}}$:three of them eaten by W and Z gauge bosons

Remaining one physical mode corresponds to SM-like Higgs!



There is an additional massive mode corresponding to the radial mode of the potential.

$$m_{\widehat{h}} \simeq \sqrt{2\lambda} f$$

Dark Higgs is singlet under U(4) symmetry.

Dark Higgs receives quadratically divergent mass correction.

Large hierarchy problem

Twin Higgs cannot solve Large Hierarchy Problem.

Dark Higgs receives quadratically divergent mass correction.

$$\Delta_{m_{\hat{h}}} = \frac{2\lambda f^2}{3y_t^2 \Lambda_{\mathrm{UV}}^2 / 8\pi^2}$$

Λ_{UV} **:cut-off scale of twin Higgs models.**

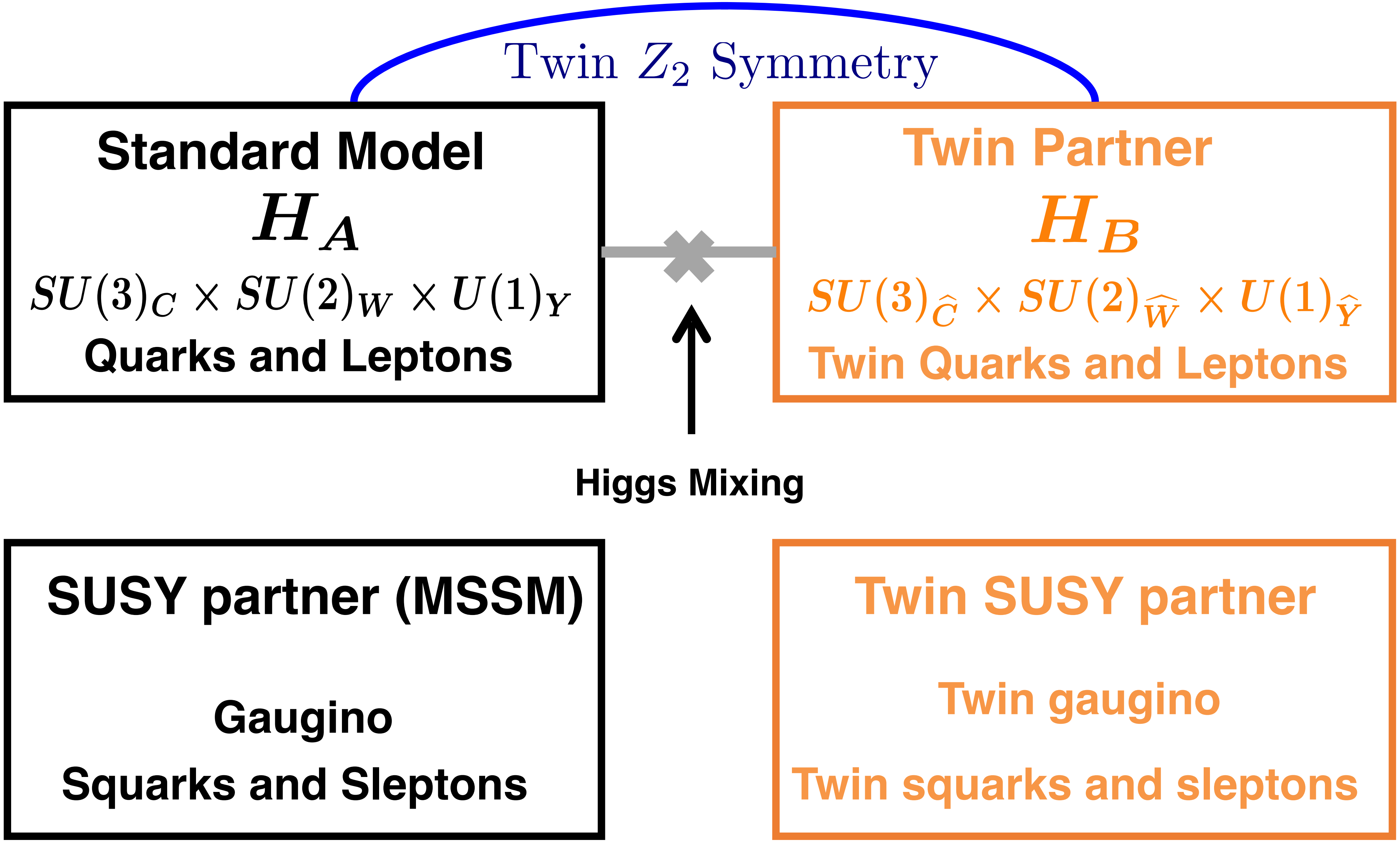
$$\Lambda_{\mathrm{UV}} = M_{\mathrm{pl}} \to \Delta_{m_{\hat{h}}} \sim 10^{-30}, \ (f \sim \mathcal{O}(\mathrm{TeV}))$$

We need UV completion to solve Large Hierarchy Problem

SUPERSYMMETRY (Weakly coupled)

Composite Higgs (Strongly coupled)

SUSY twin Higgs



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Quarks and Leptons

Twin Partner

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$SU(3)_{\widehat{C}} \times SU(2)_{\widehat{W}} \times U(1)_{\widehat{Y}}$

Twin Quarks and Leptons

Higgs Mixing

SUSY partner (MSSM)

Gaugino

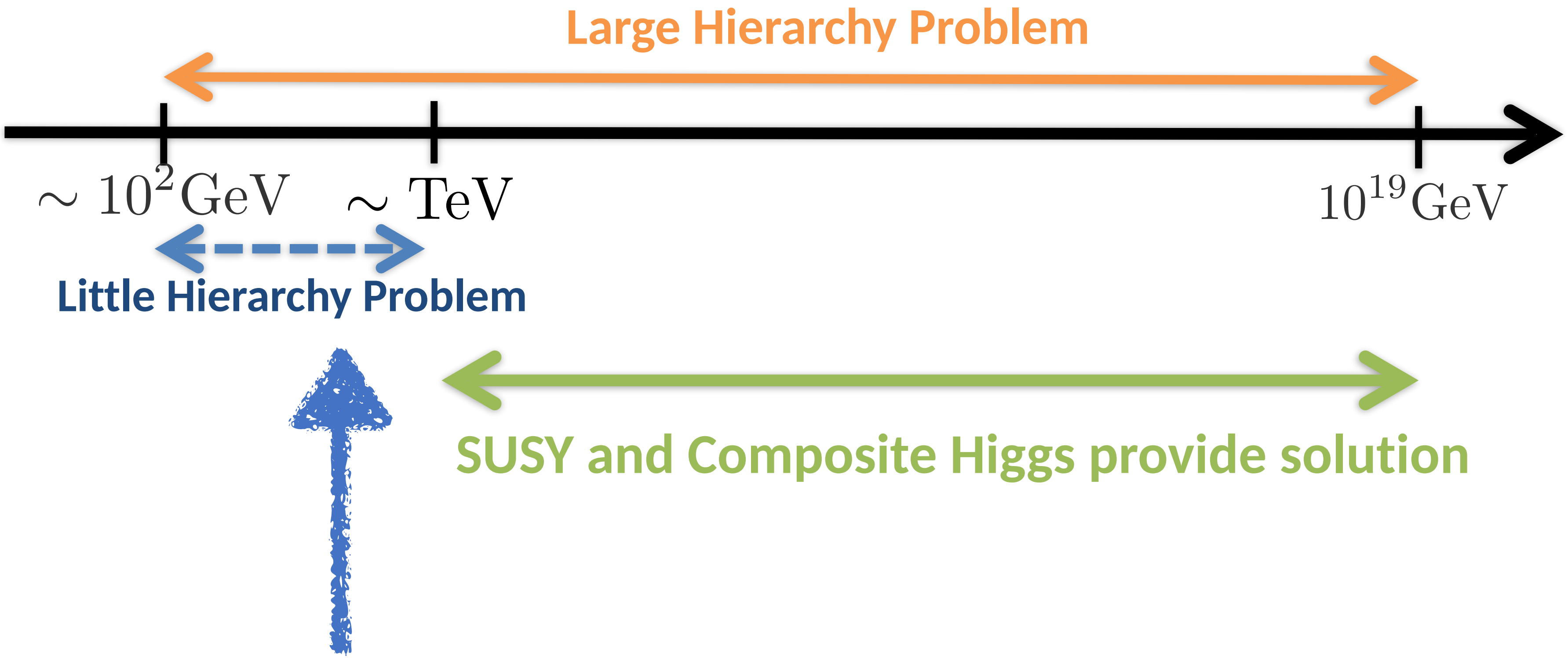
Squarks and Sleptons

Twin SUSY partner

Twin gaugino

Twin squarks and sleptons

Little Hierarchy Problem



How to solve this problem?

Supersymmetric Twin Higgs and Composite Twin Higgs give a solution to the gauge hierarchy problem.

Summary of twin Higgs models

Twin Higgs provides solution to the Little Hierarchy problem.

Twin Higgs needs UV completion to solve Large Hierarchy Problem.

Contents

Phase Transition(s) in Twin Higgs Models

There are two spontaneous symmetry breakings
(twin EW symmetry and EW symmetry)

$$(1)(0,\ 0) \Rightarrow (0,\ v_B) \Rightarrow (v_A,\ v_B)$$

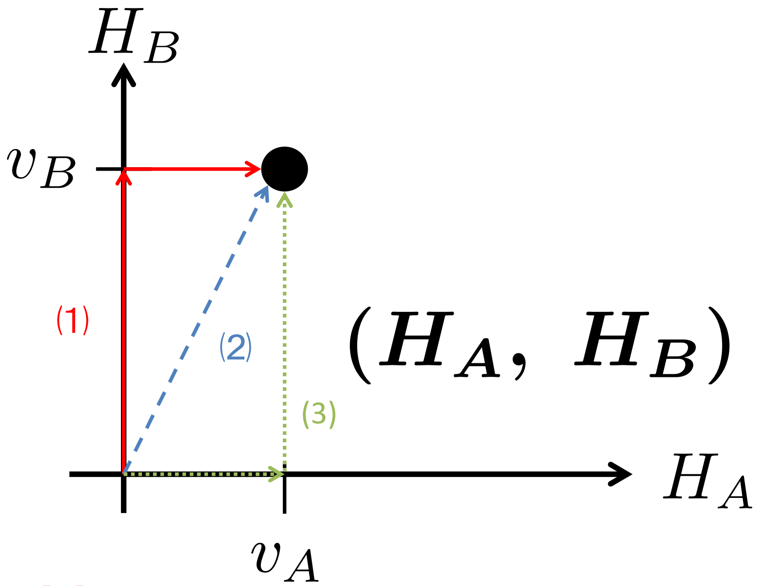
$$T_B \gg T_A$$

$$(2)(0,\ 0) \Rightarrow (v_A,\ v_B)$$

$$T_A \simeq T_B$$

$$(3)(0,\ 0) \Rightarrow (v_A,\ 0) \Rightarrow (v_A,\ v_B)$$

$$T_A \ll T_B$$



We consider the case (1) and analyze the two phase transitions.

$T_{A(B)}$: critical temperature of EW (twin EW) electroweak phase transition

Thermal mass

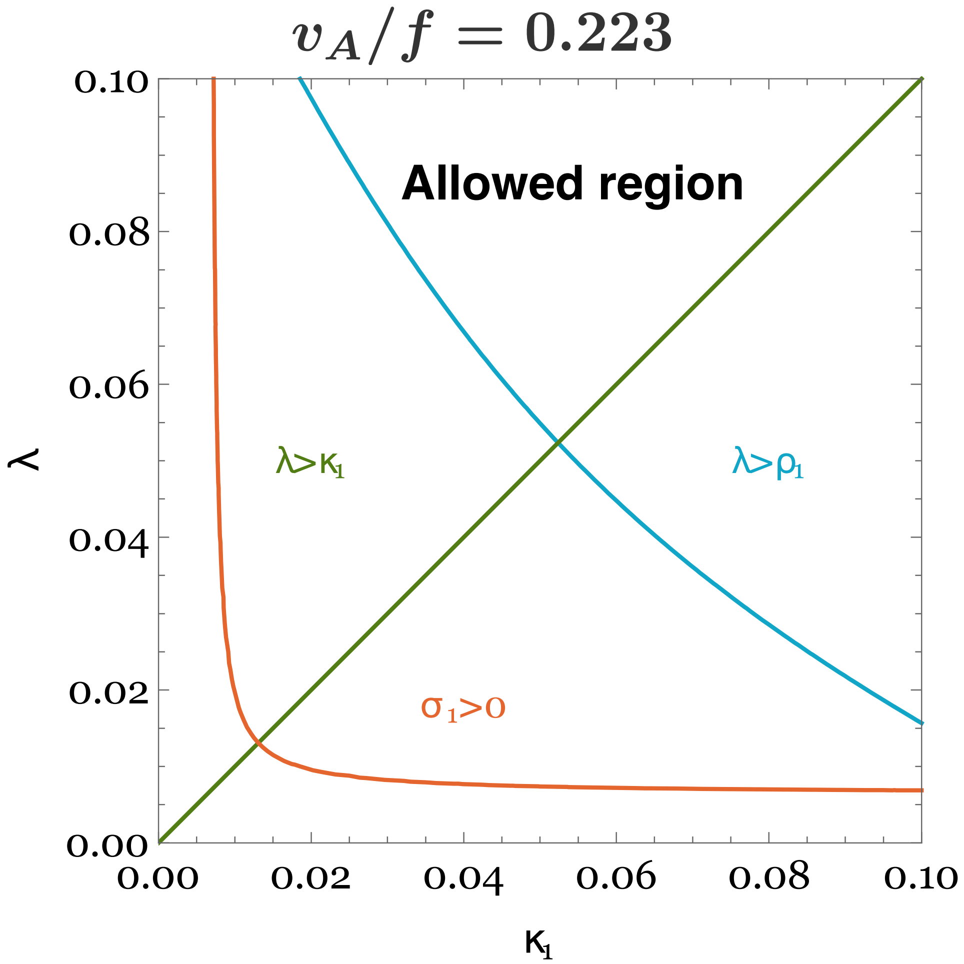
$$m_A^2(H_A,\ T) = (\zeta_A T^2 - (\lambda - \sigma_1) f^2)$$

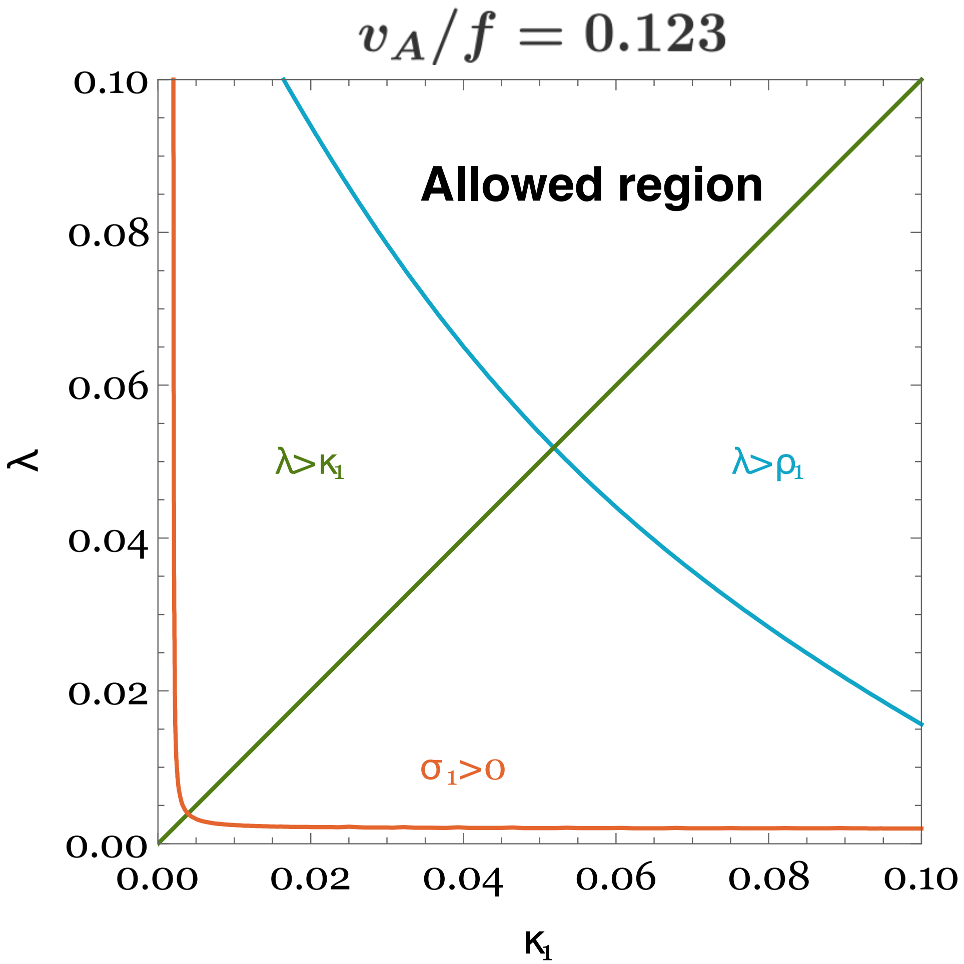
$$m_B^2(H_A,\ T) = (\zeta_B T^2 - \lambda f^2)$$

$$\frac{T_A}{T_B} = \sqrt{\frac{\zeta_B}{\zeta_A}\left(1 - \frac{\sigma_1}{\lambda}\right)}$$

$\sigma_1 > 0$ is necessary condition for two-step phase transition

Electroweak symmetry breaking(EWSB)


$v_A/f = 0.223$


$v_A/f = 0.123$

$\sigma_1 > 0$ **is necessary condition for two-step phase transition!**

Phase Transitions in Twin Higgs Models

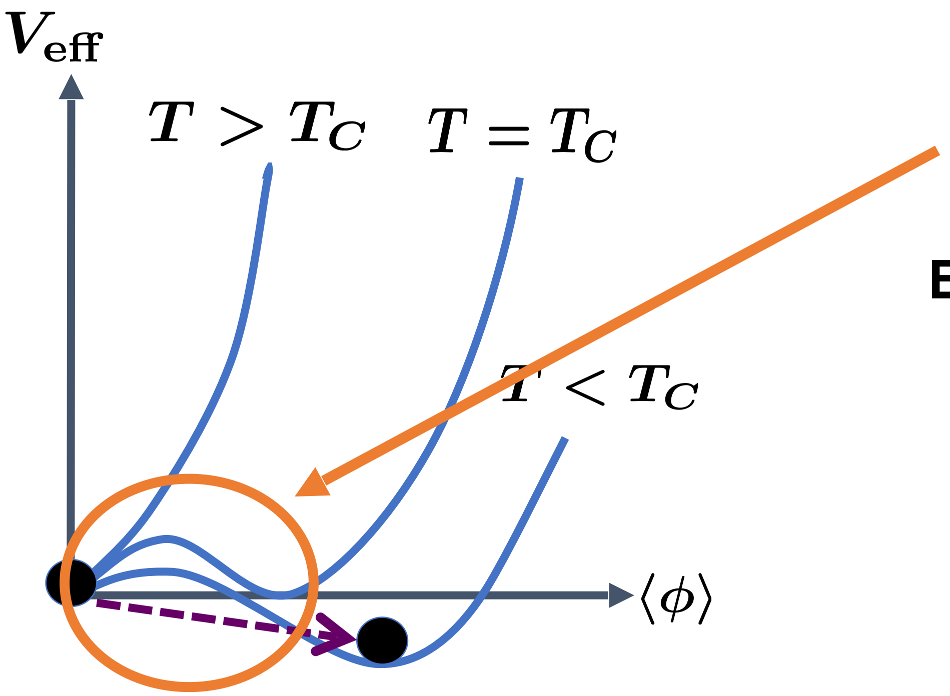
We consider following three cases:

(i) Electroweak phase transition.

(ii) U(4) breaking phase transition.

(iii) U(4) breaking phase transition with supersymmetric UV completion. (With light twin scalar top quarks.)

How to realize first-order phase transition?



Potential barrier is necessary to realize first-order phase transition.

Bosonic thermal loop generates potential barrier.

$$V_{\text{thermal}} \supset -ET\phi^3 \quad E : \text{const.}$$

Comes from Matsubara zero-mode

At zero-temperature, there is no potential barrier hence we need bosonic thermal corrections.

$$V = \lambda\left(|H_A|^2 + |H_B|^2 - \frac{f^2}{2}\right)^2 + \kappa_1\left(|H_A|^4 + |H_B|^4\right) + \sigma_1 f^2 |H_A|^2 + \rho_1 |H_A|^4 .$$

$SU(2)_{W,\widehat{W}} \times U(1)_{Y,\hat{Y}}$ **thermal corrections make potential barrier.**

Linde problem

It is difficult to analyze the phase transition with perturbative approach.

$$\begin{array}{ll} g^{2l}T^4 & \text{for } l = 1, 2 \\ g^6 T^4 \ln(T/m) & \text{for } l = 3 \\ g^6 T^4 (g^2 T/m)^{l-3} & \text{for } l > 3 \end{array}$$

expansion parameter: $\gamma = \dfrac{g^2 T}{m(\phi)} \simeq \dfrac{gT}{\phi}$

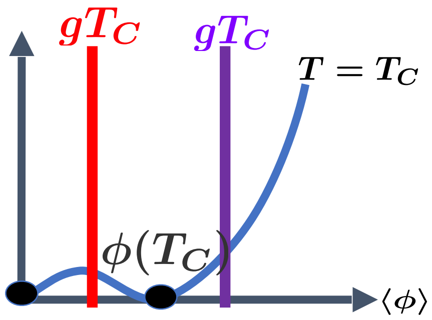
IR divergence comes from large occupation number of Bose distribution function.

$$f = \frac{1}{e^{\beta E} - 1} \quad E = \sqrt{k^2 + m^2}$$

$\gamma > 1$: perturbation theory is not reliable. [Linde.(1980)]

To clarify the order of phase transition, we need numerical study

[Peter Arnold(1994)]



$$\gamma < 1 \leftrightarrow \frac{\phi(T_C)}{T_C} > g$$

Red line does not need lattice study.
(First order phase transition)

Purple line needs lattice study.

Phase Transitions in Twin Higgs Models

We consider following three cases:

(i) Electroweak phase transition.

Order of the EWPT cannot be analyzed by perturbative method.
(At least, EWPT is “not” strong first-order.)

(ii) U(4) breaking phase transition without UV completion.

Crossover (By using lattice results)

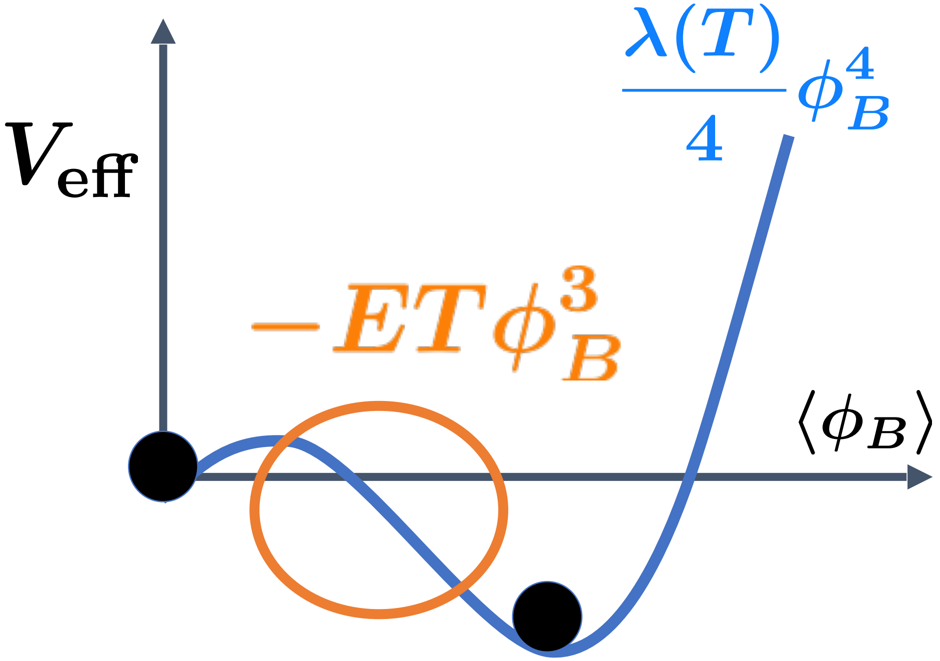
(iii) U(4) breaking phase transition with supersymmetric UV completion. (With light twin scalar top quarks.)

In this presentation, I would like to focus on this phase transition!

U(4) breaking phase transition with SUSY UV completion

Since twin stop is not colored particle, it can be light.

If it is sufficiently light, stop(scalar) loop correction makes the phase transition stronger.



$$V_{\rm eff} = \frac{M^2(T)}{2}\phi_B^2 - ET\phi_B^3 + \frac{\lambda+\kappa}{4}\phi_B^4$$

Potential barrier is generated by bosonic thermal loop.

$$E \simeq \frac{3}{32\pi}\hat{g}_2^3 + \frac{\sqrt{2}}{6\pi}\hat{y}_t^3$$

SU(2) gauge coupling: $\hat{g}_2 \simeq g_2$

top Yukawa coupling: $\hat{y}_t \simeq y_t$

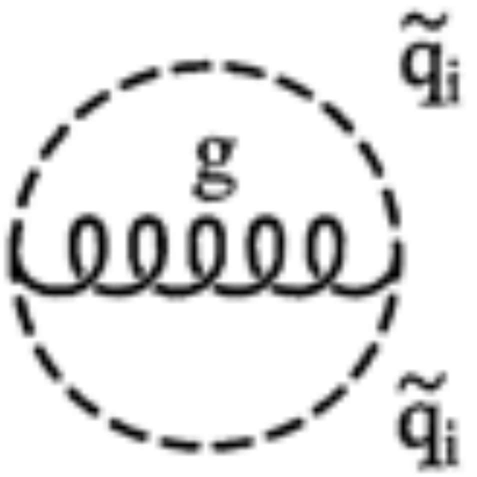
$$M^2(T) \simeq aT^2 - \lambda f^2 \qquad (a : \text{const.})$$

Twin QCD two-loop contribution

Stop-stop-gluon sunset diagram gives an additional contribution to the potential barrier.

$$V^{(2)}_{\text{thermal}} = -\frac{\widehat{g}_3^2}{2\pi^2}T^2\left((\overline{m}^2_{\tilde{t}_1^B}(\phi_B))^2\log\left(\frac{2\overline{m}^2_{\tilde{t}_1^B}(\phi_B)}{3T}\right) + (\overline{m}^2_{\tilde{t}_2^B}(\phi_B))^2\log\left(\frac{2\overline{m}^2_{\tilde{t}_2^B}(\phi_B)}{3T}\right)\right)$$

$$\overline{m}^2_{\tilde{t}^B_{1,2}} \simeq \widehat{y}_t^2\phi_B^2 + M^2_{\widehat{\text{stop}}} + aT^2 \qquad M_{\widehat{\text{stop}}}:\text{soft mass} \qquad (a:\text{constant})$$

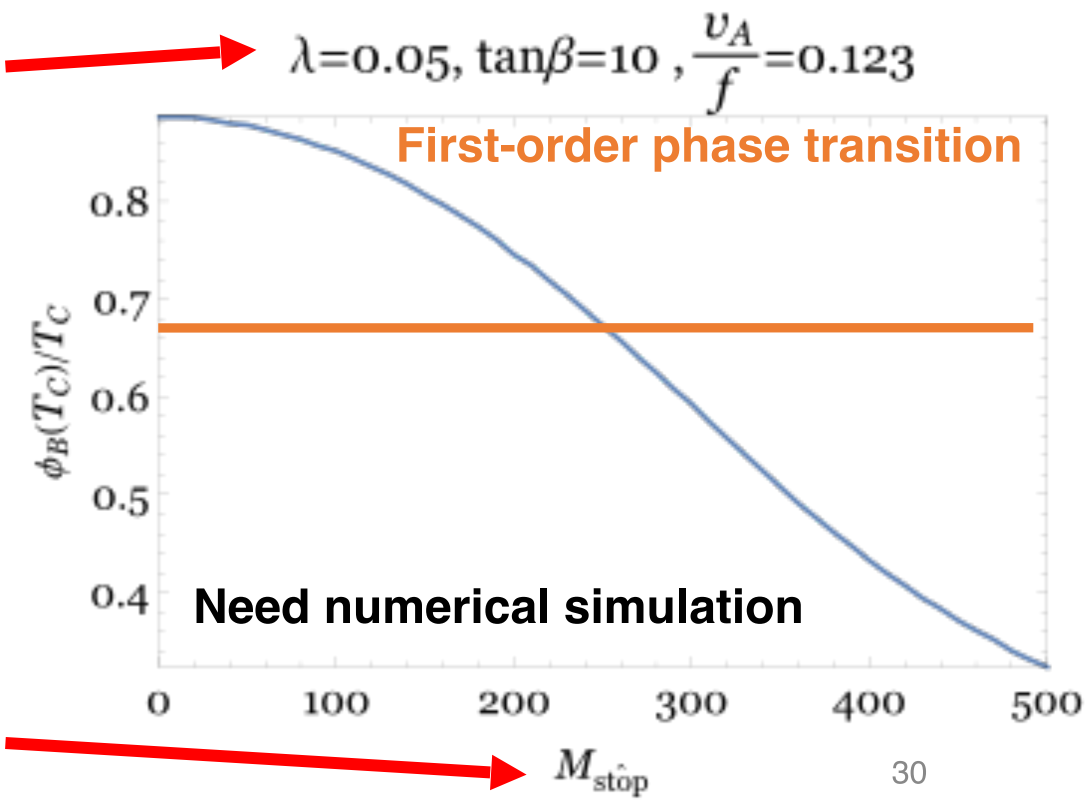


stop-stop-gluon sunset diagram

Smallest quartic coupling

Light twin stop makes phase transition first-order.

Key parameter is soft mass of twin stop.


λ=0.05, tanβ=10 , $\frac{v_A}{f}$=0.123

First-order phase transition

Need numerical simulation

Phase Transitions in Twin Higgs Models

We consider following three cases:

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Order of the EWPT cannot be analyzed by perturbative method.
(At least, EWPT is “not” strong first-order.)

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Crossover (By using lattice results)

(iii) U(4) breaking phase transition with supersymmetric UV completion. (With light twin scalar top quarks.)

First order!

U(4) breaking phase transition with light twin stop is first-order!

Can GW signals be detected by future experiments??

Overview of calculation method

Fix a model

$$\mathcal{L}(\phi,\ \psi,\ A^a_\mu, \cdots)$$

Twin Higgs model

Quantum and Thermal effects

$$V_{\rm CW}(\phi) \text{ and } V_{\rm Thermal}(\phi)$$

Solve bounce eq.

Bubble and fluid dynamics

Latent heat density: α

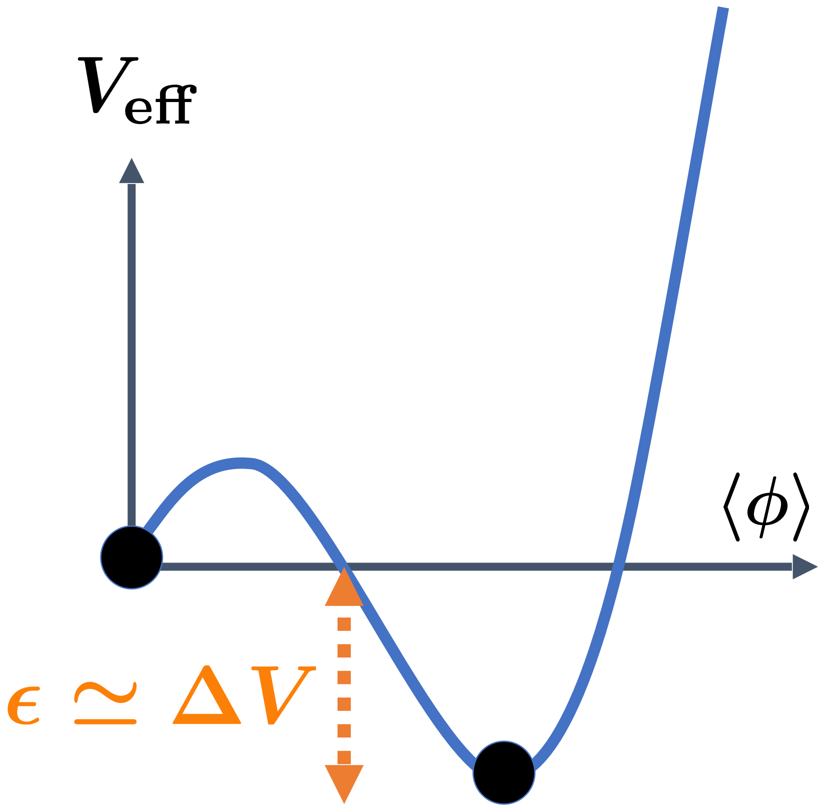
Duration of phase transition: β

etc..

GW spectrum

$$\Box h_{\mu\nu} = 16\pi G T_{\mu\nu}$$

Important parameters for GW



GW amplitude is related to two important parameters.

Latent heat density: α

Duration of phase transition: β

$$\alpha \sim \frac{\epsilon}{\rho_{\rm rad}} \qquad \Gamma \simeq \Gamma_0 e^{\beta t}$$

Γ : **Bubble nucleation rate per unit time per unit volume**

Those parameters are calculated by solving bounce equation.

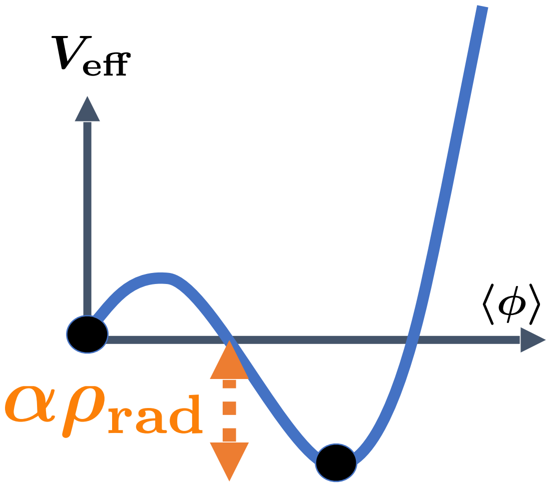
Vacuum decay rate

$$\Gamma \sim T^4 e^{-\frac{S_3}{T}}$$

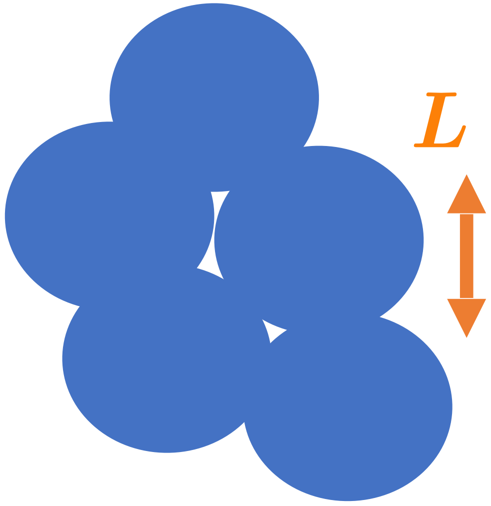
Linde.(1983)

Tunneling occurs at $\Gamma(T_*)/H^4(T_*) \sim 1$

GW amplitude



$\epsilon \sim \alpha\rho_{\rm rad}$



$L \simeq v_w \beta^{-1}$

Bubble size at collision: $L \simeq v_w \beta^{-1}$

$$\Omega_{\rm GW} \sim \frac{\rho_{\rm GW}}{\rho_{\rm crit}} \qquad \rho_{\rm crit} \simeq (1+\alpha)\rho_{\rm rad}$$

$$\rho_{\rm GW} \sim E_{\rm GW}/L^3$$ **Bubble volume** $\sim L^3$

$$E_{\rm GW} \sim \int dt P_{\rm GW} \sim P_{\rm GW}(\beta^{-1} \text{or } H^{-1})$$

Quadrupole formula: $P_{\rm GW} \sim G\dot{E}^2_{\rm kinetic}$

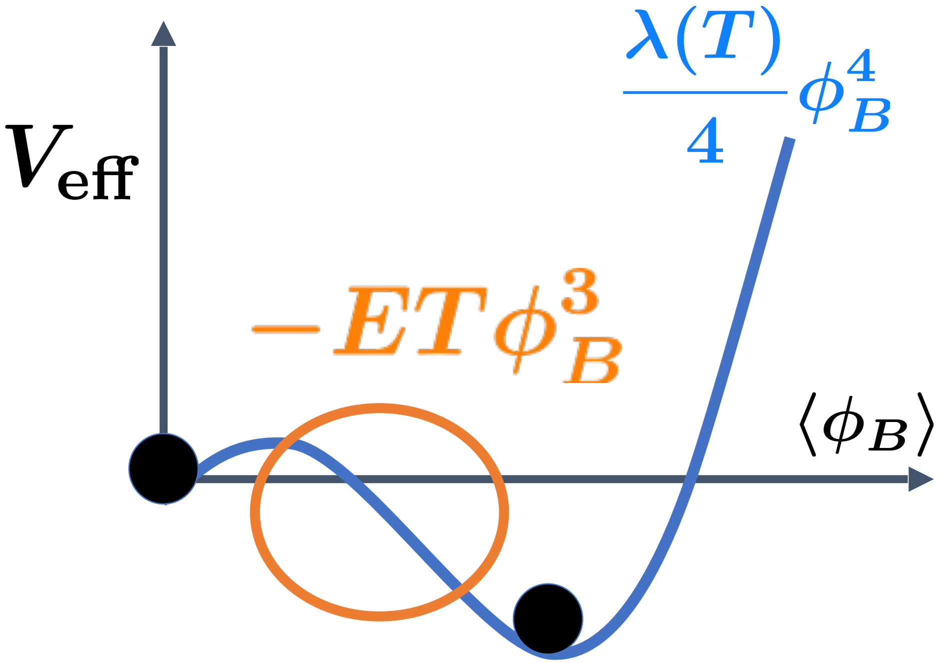
$$\dot{E}_{\rm kinetic} \sim \beta E_{\rm kinetic} \qquad E_{\rm kinetic} \sim \kappa\epsilon L^3$$

Efficiency factor: κ $\qquad G \sim \frac{H^2}{(1+\alpha)\rho_{\rm rad}}$ (Newton const.)

$$\Omega_{\rm GW} \sim \left(\frac{H}{\beta}\right)^{2 \text{ or } 1} \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 v_w^3$$

Large latent heat and long-duration enhance the GW amplitude.

Effective potential and GW



$$V_{\rm eff} = \frac{M^2(T)}{2}\phi_B^2 - ET\phi_B^3 + \frac{\lambda+\kappa}{4}\phi_B^4$$

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SU(2) gauge coupling: $\hat{g}_2 \simeq g_2$

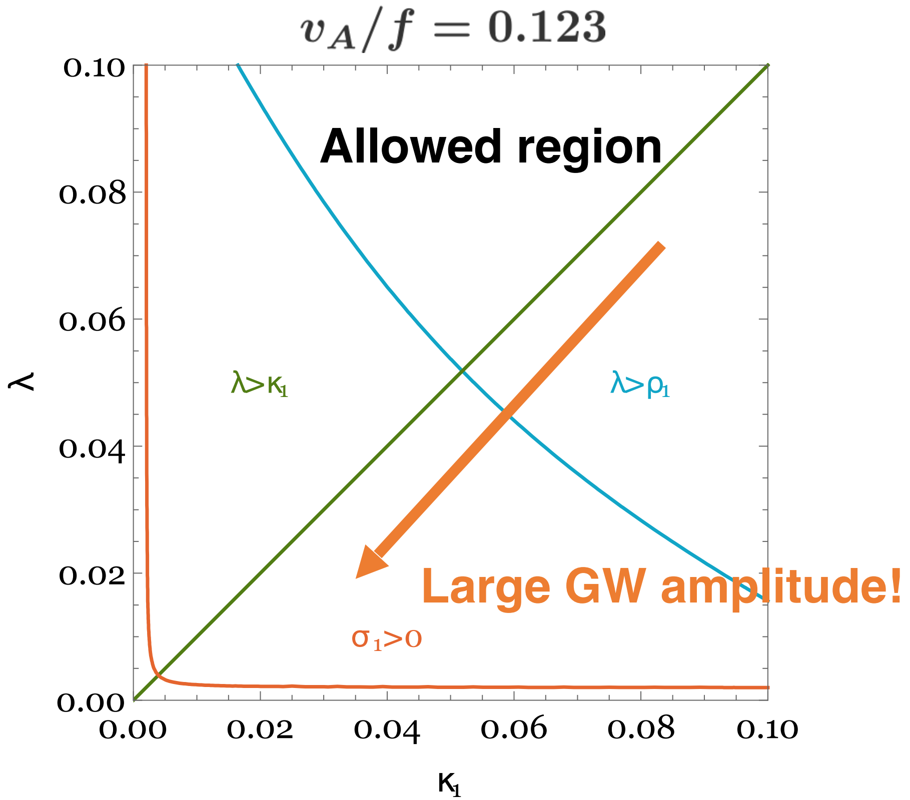
top Yukawa coupling: $\hat{y}_t \simeq y_t$

$$M^2(T) \simeq aT^2 - \lambda f^2 \qquad (a : \text{const.})$$

We (numerically) calculate the bounce eq. and found the following statement.

small	$\lambda+\kappa$	Large latent heat density and long-duration	large	$\Omega_{\rm GW}$
large	$\lambda+\kappa$	Small latent heat density and short-duration	small	$\Omega_{\rm GW}$

GW amplitude

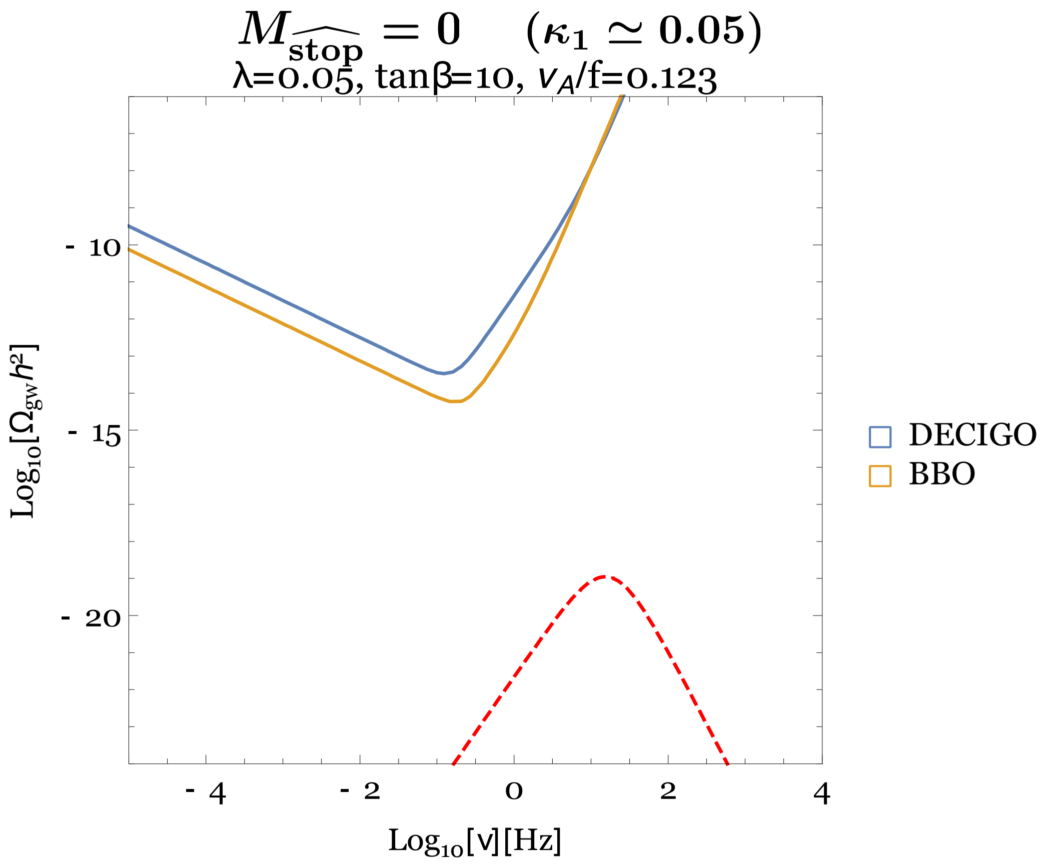

$v_A/f = 0.123$

Allowed region

Large GW amplitude!

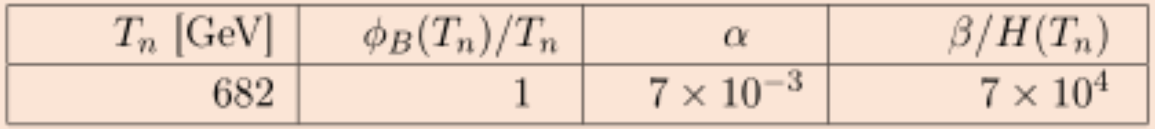
$\lambda, \ \kappa$ **bounded below to realize the SM-like Higgs mass.**

$$\lambda \gtrsim 0.05 \qquad \kappa \gtrsim 0.05$$


$M_{\widehat{\rm stop}} = 0 \qquad (\kappa_1 \simeq 0.05)$

$\lambda=0.05$, $\tan\beta=10$, $v_A/f=0.123$

Maximal GW amplitude



T_n [GeV]	$\phi_B(T_n)/T_n$	α	$\beta/H(T_n)$
682	1	7×10^{-3}	7×10^{4}

GW amplitude cannot be detected by DECIGO and BBO...

Summary

EWPT and U(4) breaking phase transition may not be first order hence there is no GW production.

U(4) breaking phase transition with light twin stop is first order. However, GW signals cannot be detected by DECIGO and BBO.

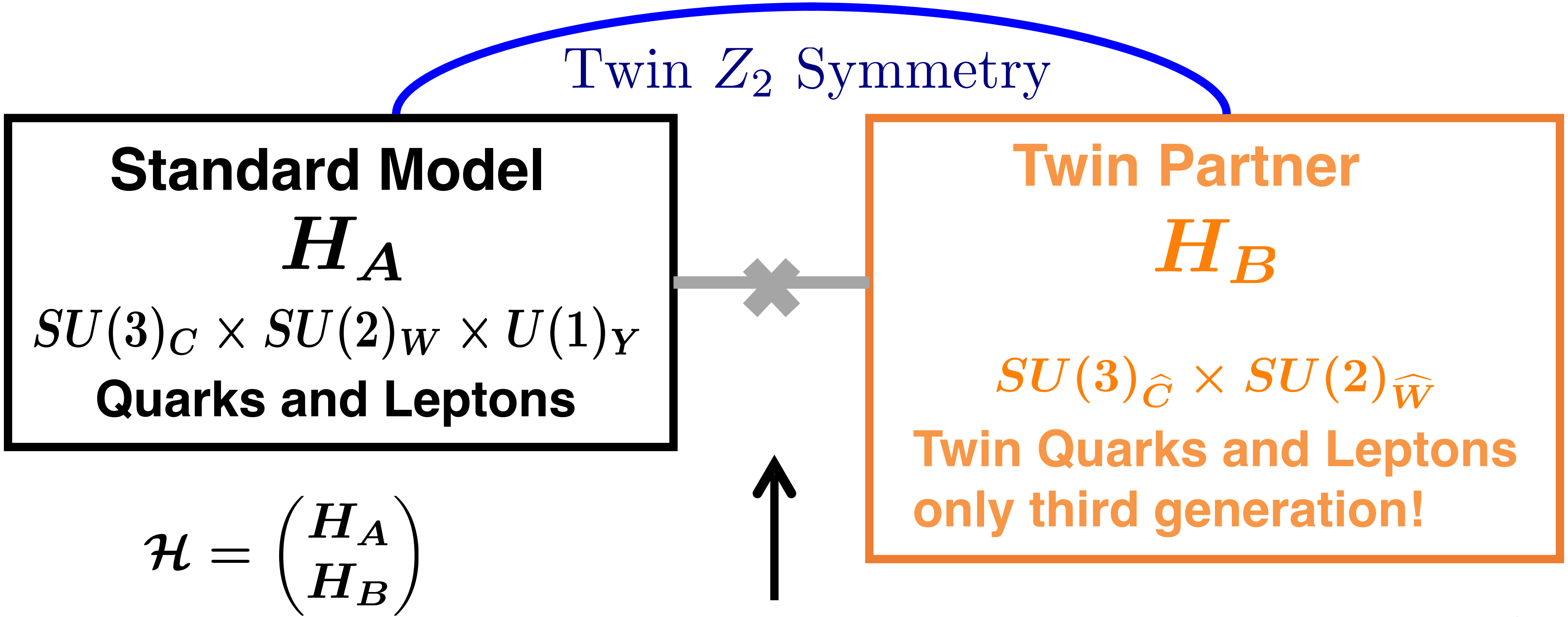
Back up

Fraternal twin Higgs model

[Craig et al.(2015)]

Exact twin Z_2 symmetry predicts many light particles which contribute to dark radiation.

(Such light particles are not necessary to solve little hierarchy problem.)



Twin Z_2 Symmetry

Standard Model

$$H_A$$

$$SU(3)_C \times SU(2)_W \times U(1)_Y$$

Quarks and Leptons

Twin Partner

$$H_B$$

$$SU(3)_{\widehat{C}} \times SU(2)_{\widehat{W}}$$

Twin Quarks and Leptons only third generation!

$$\mathcal{H} = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$$

Higgs Mixing

$$V(\Phi) = \lambda \left(|\mathcal{H}|^2 - \frac{f^2}{2} \right)^2$$

In Fraternal twin Higgs models, there is no dark radiation.

Phase Transitions in Twin Higgs Models

We consider following three cases:

(i) Electroweak phase transition.

(ii) U(4) breaking phase transition without UV completion.

(iii) U(4) breaking phase transition with supersymmetric UV completion. (With light twin scalar top quarks.)

Electroweak Phase Transition

Consider both of Higgs fields: $H_A = \begin{pmatrix} 0 \\ \frac{\phi_A}{\sqrt{2}} \end{pmatrix}, \ H_B = \begin{pmatrix} 0 \\ \frac{\phi_B}{\sqrt{2}} \end{pmatrix}$

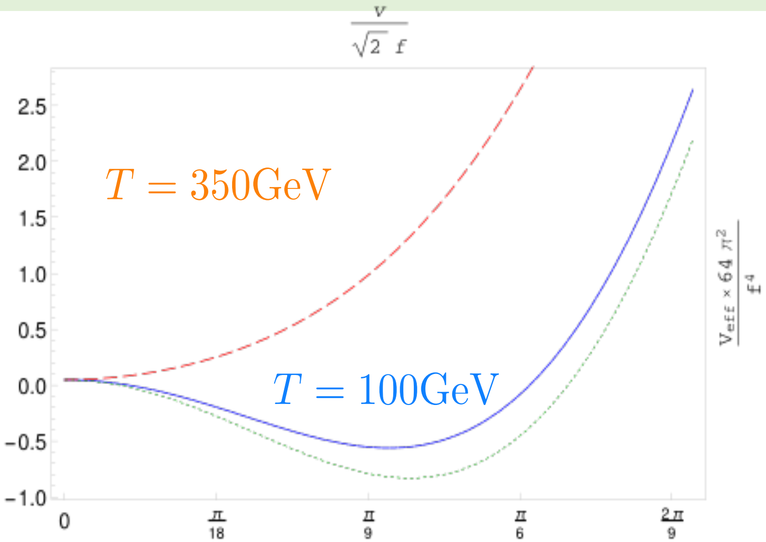
$$V(\phi_A,\ \phi_B,\ T) = V_0 + V_{\rm CW} + V_{\rm thermal} + V_{\rm ring}$$

We take into account $SU(2)_W,\ SU(2)_{\widehat{W}},\ U(1)_Y$, top and twin top quark effect.

To analyze EWPT, we integrate out massive mode ϕ_B and obtain low-energy EFT.

$$\phi_B^2 = f^2 - \phi_A^2$$

$$V(\phi_A,\ \phi_B,\ T) \to V(\phi_A,\ T)$$

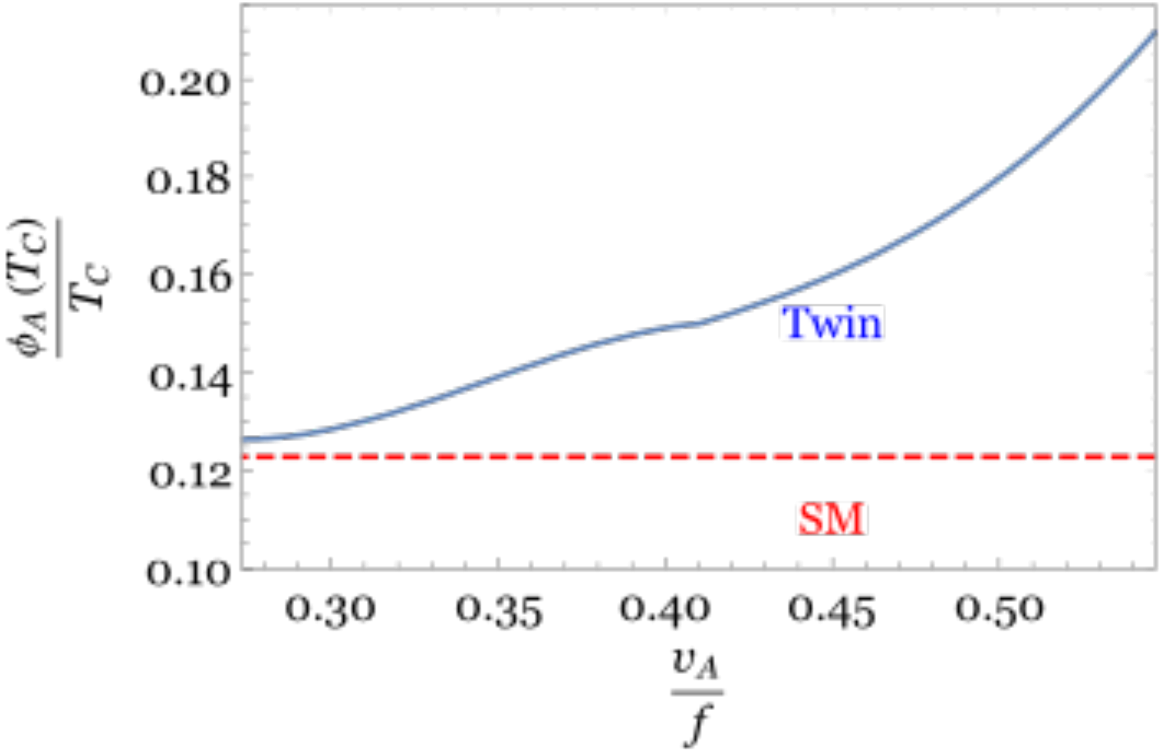


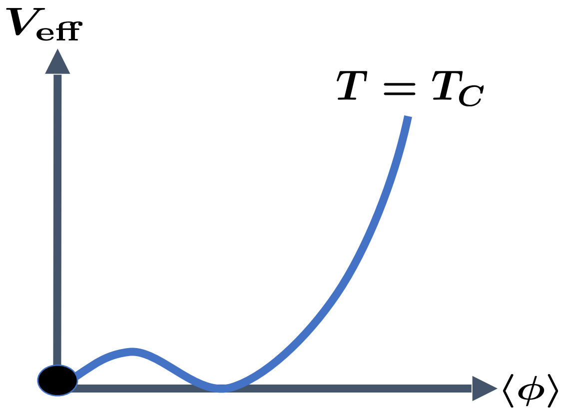
It was shown that the EW symmetry is restored at high temperature.

[Kilic and Swaminathan (2015)]

We analyze the order of the EW phase transition.

The order of electroweak phase transition in Twin Higgs





$$\gamma \simeq g_2 \frac{T_C}{\phi_A(T_C)}$$

$$\gamma < 1 \leftrightarrow \frac{\phi_A(T_C)}{T_C} > 0.65$$

Large breaking scale leads to be heavier masses of twin particles

Order of the EWPT cannot be analyzed by perturbative method.

Phase Transitions in Twin Higgs Models

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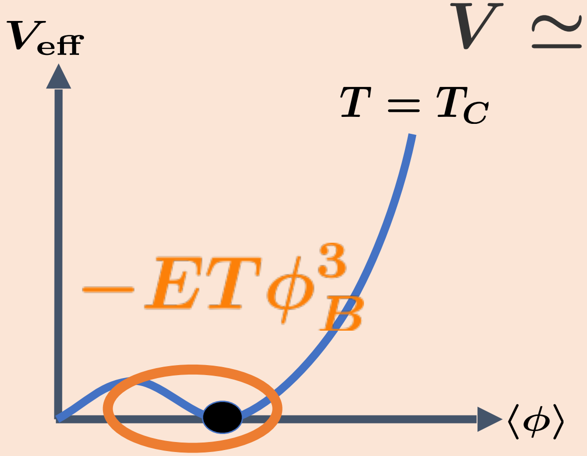
U(4) breaking phase transition without UV completion

Order parameter: $H_B = \begin{pmatrix} 0 \\ \frac{\phi_B}{\sqrt{2}} \end{pmatrix}$

One-loop resummed effective potential (with high-temperature expansion)



$$V \simeq \frac{M^2(T)}{2}\phi_B^2 - ET\phi_B^3 + \frac{\lambda + \kappa_1(T)}{4}\phi_B^4$$

$$E \simeq \frac{\widehat{g}_2^3}{16\pi} \qquad M^2(T) = -\lambda f^2 + \frac{\widehat{y}_t^2}{4}T^2 + \frac{3\widehat{g}_2^2}{16}T^2$$

$$\kappa_1(T) = \kappa - \frac{3\widehat{y}_t^4}{16\pi^2}\left(\log\left(\frac{a_f T^2}{\mu^2}\right) - \frac{3}{2}\right) + \frac{9\widehat{g}_2^4}{256\pi^2}\log\left(\left(\frac{a_b T^2}{\mu^2}\right) - \frac{3}{2}\right)$$

Due to the twin Z_2 symmetry, we have $y_t \simeq \widehat{y}_t,\ g_2 \simeq \widehat{g}_2$

U(4) breaking phase transition ⟷ **EWPT in SM (Well known result)**

Situation is similar to electroweak phase transition in SM

U(4) breaking phase transition	**Different point**	**EWPT in SM**
f	Breaking scale	v_{SM}
$\lambda + \kappa_1$	Quartic coupling	λ_{SM}

Lattice simulation shows that the order of phase transition depends only on $\lambda_{\mathrm{SM}}/g_2^2$

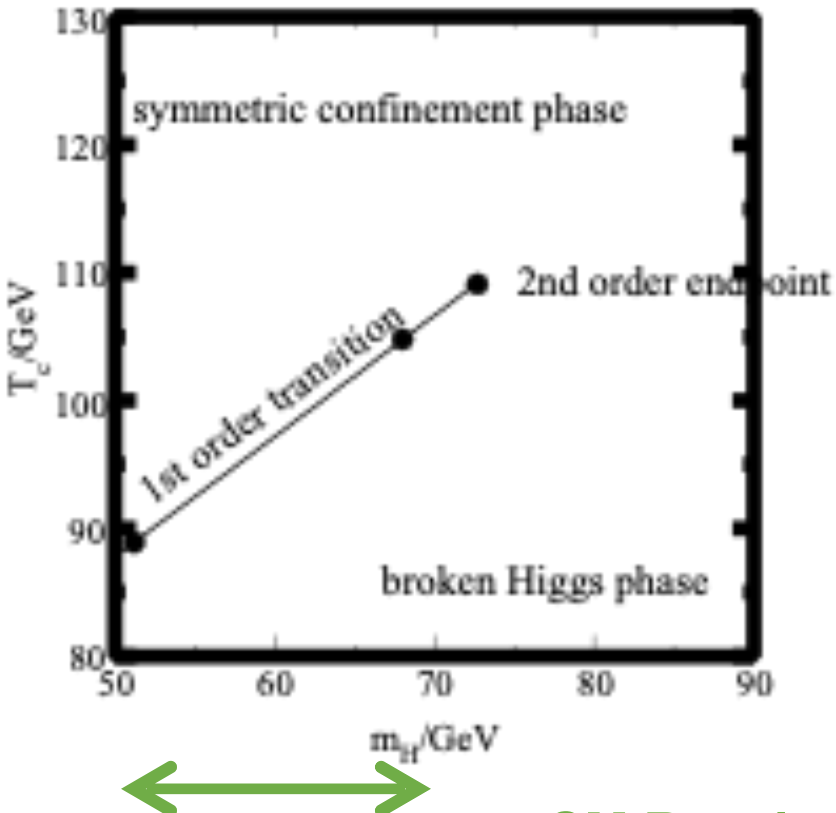
[K. Rummukainena et. al.] (1998)

We can use lattice simulation to clarify the order of U(4) breaking phase transition.

The order of U(4) breaking phase transition depends on $(\lambda + \kappa_1)/\widehat{g}_2^2$

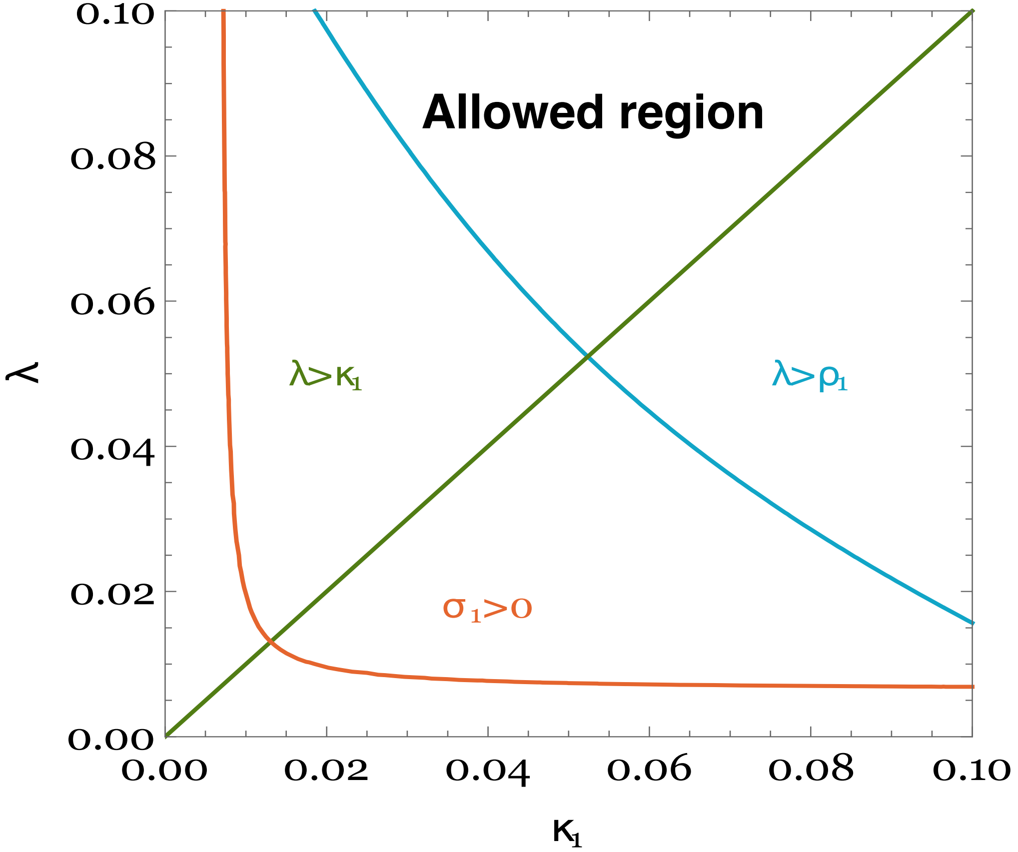
The order of U(4) breaking phase transition

M. LAINE (2000)



$\lambda_{\mathrm{SM}} < 0.05$ **SM Result**

$v_A/f = 0.223$



U(4) breaking phase transition is first order when $\lambda + \kappa_1 < 0.05$

However, smallest value allowed by theoretical bound is $\lambda + \kappa_1 > 0.1$

U(4) breaking phase transition (without UV completion) is not first order.

Phase Transitions in Twin Higgs Models

We consider following three cases:

(i) Electroweak phase transition.

Order of the EWPT cannot be analyzed by perturbative method.

(ii) U(4) breaking phase transition without UV completion.

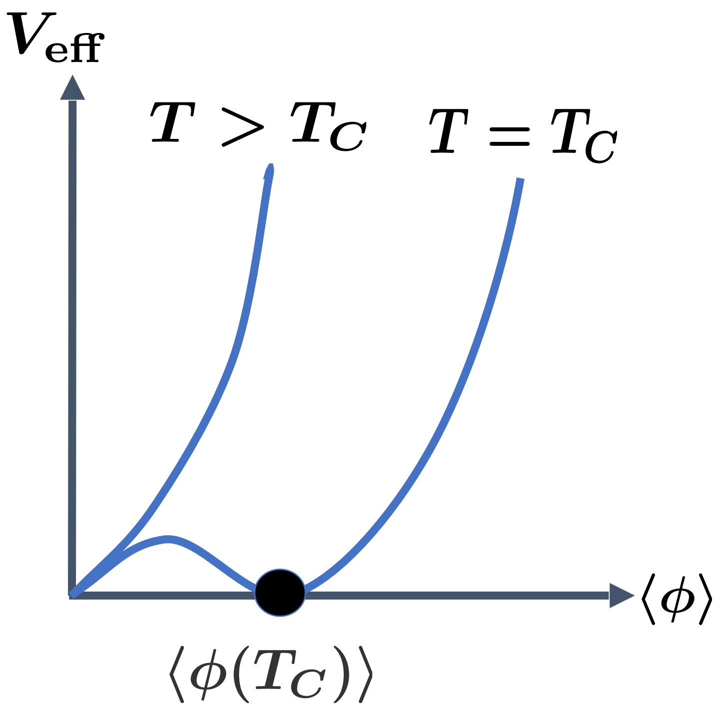
U(4) breaking phase transition (without UV completion) is not first order.

(iii) U(4) breaking phase transition with supersymmetric UV completion. (With light twin scalar top quarks.)

What have we learned from two phase transitions?

Unfortunately, two phase transitions may not be first order.

Consider simple example: $V = \frac{1}{2}M^2(T)\phi^2 - ET\phi^3 + \frac{\xi(T)}{4}\phi^4$



$$V(\phi,\, T_C) = \frac{\xi(T_C)}{4}\phi^2(\phi^2 - \langle\phi(T_C)\rangle)^2$$

$$\frac{\langle\phi(T_C)\rangle}{T_C} = \frac{2E}{\xi(T_C)} \simeq \frac{2E}{\xi}$$

Perturbative expansion requires: $\frac{\langle\phi(T_C)\rangle}{T_C} > g$

Large E and small quartic coupling are needed!

(E comes from bosonic thermal loop.)

Small Higgs mass (small quartic coupling) and a scalar field (strongly) coupled with Higgs are needed to realize first-order phase transition!!!

Reconsider our set up

$$V = \frac{1}{2}M^2(T)\phi^2 - ET\phi^3 + \frac{\xi(T)}{4}\phi^4$$

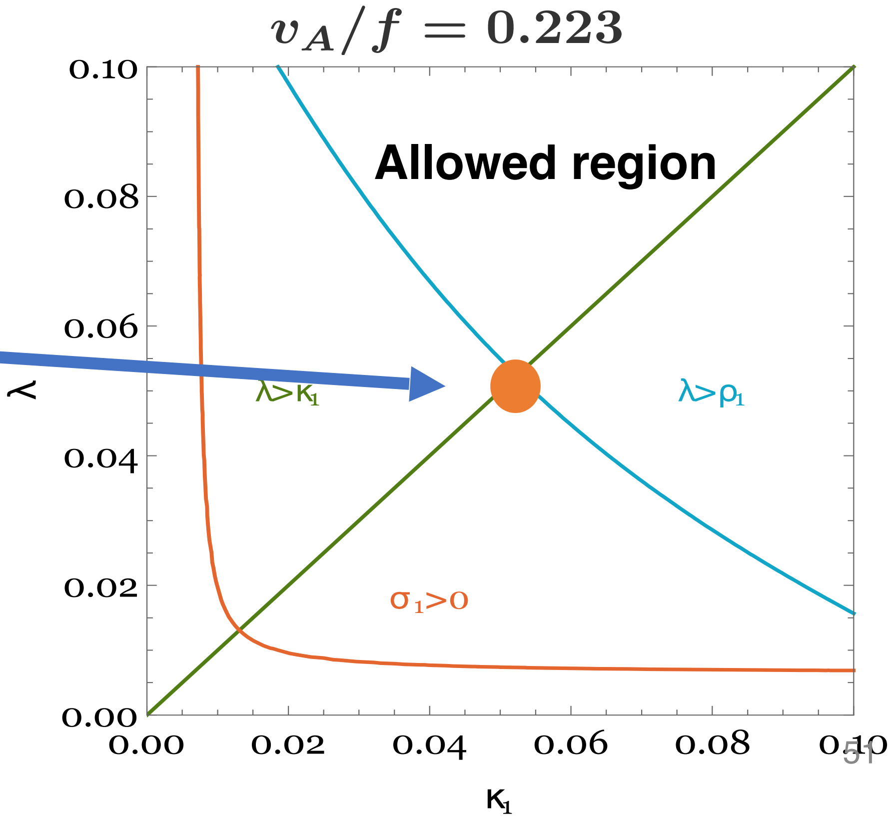
Both of the phase transition, gauge fields are only the source of potential barrier.

$$E \simeq \frac{\widehat{g}_2^3}{16\pi} \sim 10^{-3}$$

Small quartic coupling is required to realize first-order phase transition.

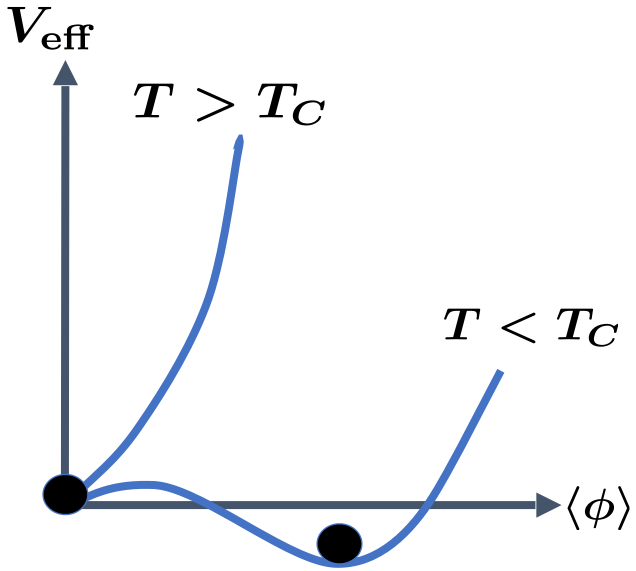
$$\frac{\langle\phi(T_C)\rangle}{T_C} \simeq 1 \leftrightarrow \lambda + \kappa_1 \simeq 0.05$$

$$\lambda + \kappa_1 \simeq 0.1$$



However, small quartic coupling cannot realize observed SM-like Higgs mass.

IR problem in Finite-temperature



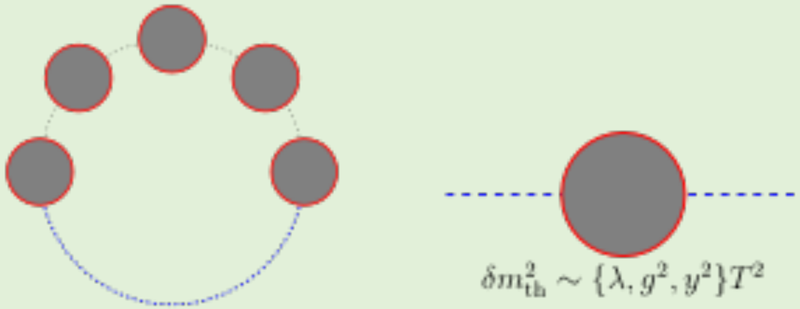
$$V = -\mu^2\phi^2 + \lambda\phi^4,\ \mu^2 > 0$$

Symmetry restoration implies

$$\mu^2 < aT^2\ (a : \text{numerical constant})$$

Dominant contribution compared to tree value

Can we believe perturbation??



$\alpha = a_B \dfrac{T^2}{m(\phi)^2}$ **:expansion parameter**

$a_B \sim \mathcal{O}(1),\ m$ **:mass inside the propagator**

Higher-order (Ring diagram) correction is non-negligible and perturbative expansion does not valid near $T = T_C$

$$\alpha \sim \frac{T_C}{m(\phi(T_C))} \sim \mathcal{O}(1)$$

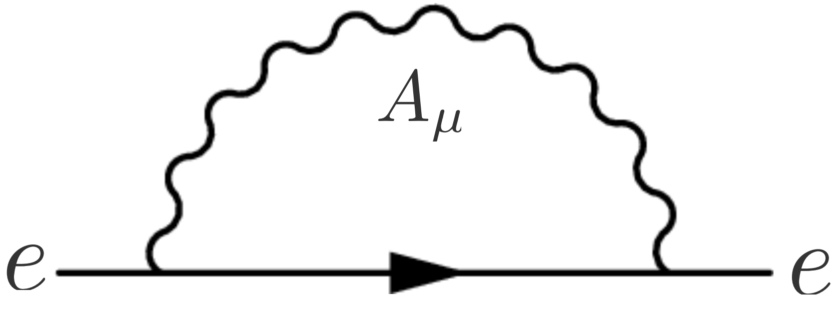
Resummation relaxes this problem

$$V_{\rm ring} = -\frac{n_B}{12\pi}\left((m^2 + \delta m^2_{\rm th})^{\frac{3}{2}} - (m^2)^{\frac{3}{2}}\right)$$

Symmetry protection

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}_L\bar{\sigma}^\mu D_\mu e_L + \bar{e}_R\sigma^\mu D_\mu e_R - m_e(\bar{e}_L e_R + \bar{e}_R e_L),$$

$$D_\mu \equiv \partial_\mu - ieA_\mu$$



	e_L	e_R
$U(1)_V$	$+1$	$+1$
$U(1)_A$	$+1$	-1

Naive dimensional estimate: $\delta m_e \sim \Lambda$

$$\Delta_{m_e} \sim 10^{-19},\ (\Lambda \sim M_{\text{pl}})$$

However $\delta m_e = m_e \frac{3\alpha}{2\pi} \log\left(\frac{\Lambda}{m_e}\right)$

Natural due to the logarithmic dependence

Why log sensitivity?

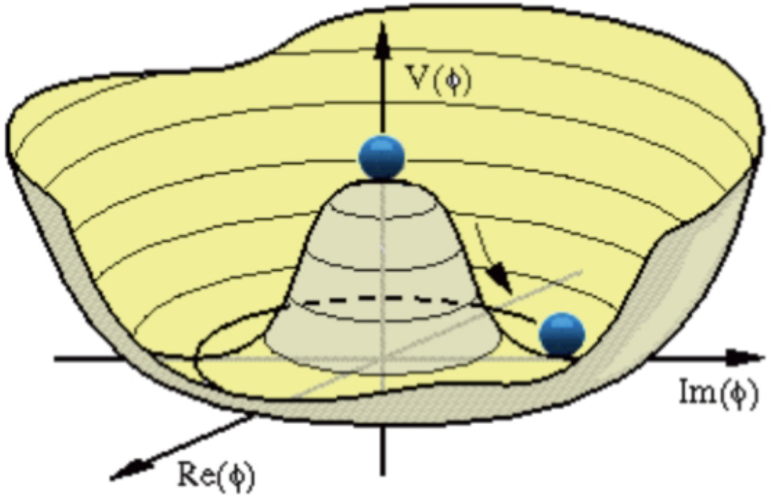
Quantum correction respects symmetry

Massless limit $m_e \to 0$, $U(1)_A$ **symmetry is restored.**

$\delta m_e \sim \Lambda$ **is forbidden by the symmetry.**

Generally, scalar field does not have symmetry ➡ **Quadratically divergent mass correction**

U(1) Toy Model (Symmetry Protection)



$$V(\phi) = \lambda \left(|\phi|^2 - \frac{f^2}{2} \right)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}} (f + \sigma(x)) e^{i\frac{a(x)}{f}}$$

$\sigma(x)$:**Massive Mode** $m_\sigma = \sqrt{2\lambda} f$

$a(x)$:**Massless NG Mode**

$$\mathcal{L}(\sigma\ , a) = -\frac{1}{2} (\partial a)^2 + \mathcal{L}_\sigma$$

NG Boson has shift-symmetry: $a(x) \rightarrow a(x) + \mathrm{const}$

Add explicit breaking source: $\mathcal{L}_{U(1)breaking} = -\rho f^3 (\phi + \phi^*)$

NG Boson acquires mass: $m_a = \sqrt{2\rho} f$

$\rho \rightarrow 0$ **U(1) symmetry is restored** ➡ $\delta m_a^2 \sim m_a^2 \log \left(\frac{\Lambda}{m} \right)$ **No fine-tuning**