OperatorConversion

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1 Introduction

The general dimension-8 SMEFT Lagrangian can be written as :
$\Delta\mathcal{L}(dim-8) = \sum_j \frac{\tilde{C_j}}{\tilde{\Lambda^4}}\mathcal{O}_| = \sum_j \frac{sign(\tilde{c_j})}{\Lambda_j^4}\mathcal{O}_|$

This file will introduce the conversion method of the EFT operators from the energy scale Λ, wilson coefficient (Effective Field Theory), and vertex form factor (Effective Vertex Theory)

In this file, 3 operators will be introduced : $O_{\tilde{B}W}, O_{G+}, O_{G-}$, and their wilson coefficients : $C_{\tilde{B}W}, cg, cgm$, as well as their corresponding form factor : h_3^Z, h_3^γ, h_4. (Due to the fact the h_4^γ and h_4^Z is correlated, so here only labeled as h_4 for convenient.)

2 Conversion

In this section, some tables will be listed here for illustration.

2.1 Conversion from energy scale Λ to wilson coefficient

In this part, $O_{G_+}(cg)$ will be the example.

From the theoretical side, a table of energy cutoff scale Λ

$\sqrt{s}$	13 TeV			100 TeV		
$\mathcal{L}(ab^{-1})$	0.14	0.3	3	3	10	30
$\Lambda_{2\sigma}(TeV)$	2.1	2.4	3.3	14	16	19
$\Lambda_{5\sigma}(TeV)$	1.6	1.8	2.5	10	12	15

Table 1: Sensitivities of probes of the new physics scale Λ at $\mathcal{O}(\Lambda^{-4})$ of the nTGC pure gauge operator O_{G_+} at the 2σ and 5σ levels, observed from the reaction $pp \to Z\gamma \to l^+l^-\gamma$ at the LHC, respectively with the different luminosities

In order to convert to the limits for cg, the wilson coefficient for O_{G_+} operator, this formula is used : $cg = 1/\Lambda^4$, the unit here is TeV.

So the table for cg from the energy cutoff scale should be :

$\sqrt{s}$	13 TeV			100 TeV		
$\mathcal{L}(ab^{-1})$	0.14	0.3	3	3	10	30
$\Lambda_{2\sigma}(cg)$	0.05	0.03	8.4×10^{-3}	2.6×10^{-5}	1.5×10^{-5}	7.7×10^{-6}
$\Lambda_{5\sigma}(cg)$	0.15	0.095	0.026	1×10^{-4}	4.8×10^{-5}	2.0×10^{-5}

Table 2: Sensitivities of probes of the new physics signal strength (coupling values of the operators) at the 2σ and 5σ levels, observed from the reaction $pp \to Z\gamma \to l^+l^-\gamma$ at the LHC, respectively with the different luminosities

The other operators have same conversion method with $O_{G_+}(cg)$. From this formula mentioned above, new physics energy cutoff scale λ can be converted to the wilson coefficients for dimension-8 operators

2.2 Conversion from EFT wilson coefficient to EVT form factor

At the beginning of this part, the relationship between the operators and form factors should be well known.

The three independent form-factor parameters (h_3^Z, h_3^γ, h_4) to the cutoff scales of the corresponding dimension-8 operators $(O_{G_+}, O_{G_-}, O_{\tilde{B}W})$, as follows :

$h_4 = -\frac{C_{\tilde{G}_+}}{\Lambda^4}\frac{v^2 M_Z^2}{s_w c_w}, h_3^V = 0$, for O_{G_+}
$h_3^Z = \frac{v^2 m_Z^2}{2 s_w c_w}\frac{C_{\tilde{B}W}}{\Lambda^4}, h_3^\gamma, h_4^V = 0$, for $O_{\tilde{B}W}$
$h_3^\gamma = -\frac{c_{\tilde{G}_-}}{\Lambda^4}\frac{v^2 m_Z^2}{2 c_w}, h_3^Z, h_4^V = 0$, for O_{G_-}

Some parameters need to explain :
1. v : the vacuum expectation value in Higgs Field : 246 GeV
2. Λ : [Unit] TeV
3. m_Z : 91.1876 GeV
4. s_w : weak interaction coupling constant : $s_w^2 = 0.231, s_w = \sqrt{0.231} \approx 0.481$
5. c_w : $c_w^2 = 1 - s_w^2 = 0.769, c_w = 0.877$

Here, an example will help to understand the calculation :
This example is taken from previous $Z\nu\nu\gamma$ analysis with partial Run-2 datasets of 36.1 fb^{-1}

Parameter	Expected Limit 95% C.L. (36.1 fb^{-1})
$C_{\tilde{B}W}/\Lambda^{-4}$	(-1.3, 1.3)
h_3^Z	$(-7.7 \times 10^{-4}, 7.7 \times 10^{-4})$

Table 3: Caption

In this example, we can just replace the $C_{\tilde{B}W}/\Lambda^4$ by the value of 1.3, then use the formula :
$h_3^Z = \frac{v^2 m_Z^2}{4 s_w c_w}\frac{C_{\tilde{B}W}}{\Lambda^4} = \frac{246^2 \times 91.1876^2 [GeV^4]}{2 \times 0.481 \times 0.877}\frac{1.3}{[TeV^4]} \approx 7.7 \times 10^{-4}$

$\sqrt{s}$	13 TeV		
$\Lambda_{2\sigma}(cg)$	0.05	0.03	8.4×10^{-3}

Table 4: Illustration for the conversion from cg to h_4, this table is the limits for cg from the paper.

The results calculated from the formula is confirmed also with Dr. Ruiqing Xiao. Here I need to mention that in $Z(\nu\nu)\gamma$ analysis, their calculation from Celine's model file is not correct : https://arxiv.org/abs/1308.6323. In this paper, conjugation is not considered in operator.

The conversion of another two operators (O_{G_+}, O_{G_-} can also be calculated from the formula above.

At last, I will use the results for O_{G_+} as the example to illustrate the conversion from cg to h_4.

$\sqrt{s}$	13 TeV		
h_4	5.9×10^{-5}	3.6×10^{-5}	1×10^{-5}

Table 5: Illustration for the conversion from cg to h_4, this table is the limits for h_4, and this result can be compared with the results in paper : https://arxiv.org/abs/2206.11676v2 , table 4.

The calculated results are matched with the results listed in paper, which proves that our calculation is correct.

3 ATTENTION

The conversion method introduced in above sections is confirmed with Dr.Ruiqing Xiao.

Another thing need to mention is that when converting measured results for cg from Monte Carlo simulation or collected data. One extra step need to do :
$gO_{G+} = \tilde{B}_{\mu\nu} W^{\alpha\mu\rho}(D_\rho D_\lambda W^{\alpha\nu\lambda} + D^\nu D^\lambda W^{\alpha}_{\lambda\rho})$
$gO_{G-} = \tilde{B}_{\mu\nu} W^{\alpha\mu\rho}(D_\rho D_\lambda W^{\alpha\nu\lambda} - D^\nu D^\lambda W^{\alpha}_{\lambda\rho})$
During writing the model file, g is forget to add in the feynmann rule files, so we have to operate by hand.

So before converting cg to h_4, cg should be divided by $g_w \approx 0.65$ by hand.

Here are some explanations about g_w:
1. weak coupling constant at the Z pole : $g_w = \frac{ee}{s_w} = \sqrt{\frac{4\pi\alpha_{EW}}{s_w^2}}$
2. $\alpha_{EW} = \frac{1}{127.9}$: fine structure constant 3. $s_w^2 = 1 - (\frac{m_W}{m_Z})^2 \approx 0.231$