

上海粒子物理与宇宙学专题研讨会  
中微子与暗物质物理

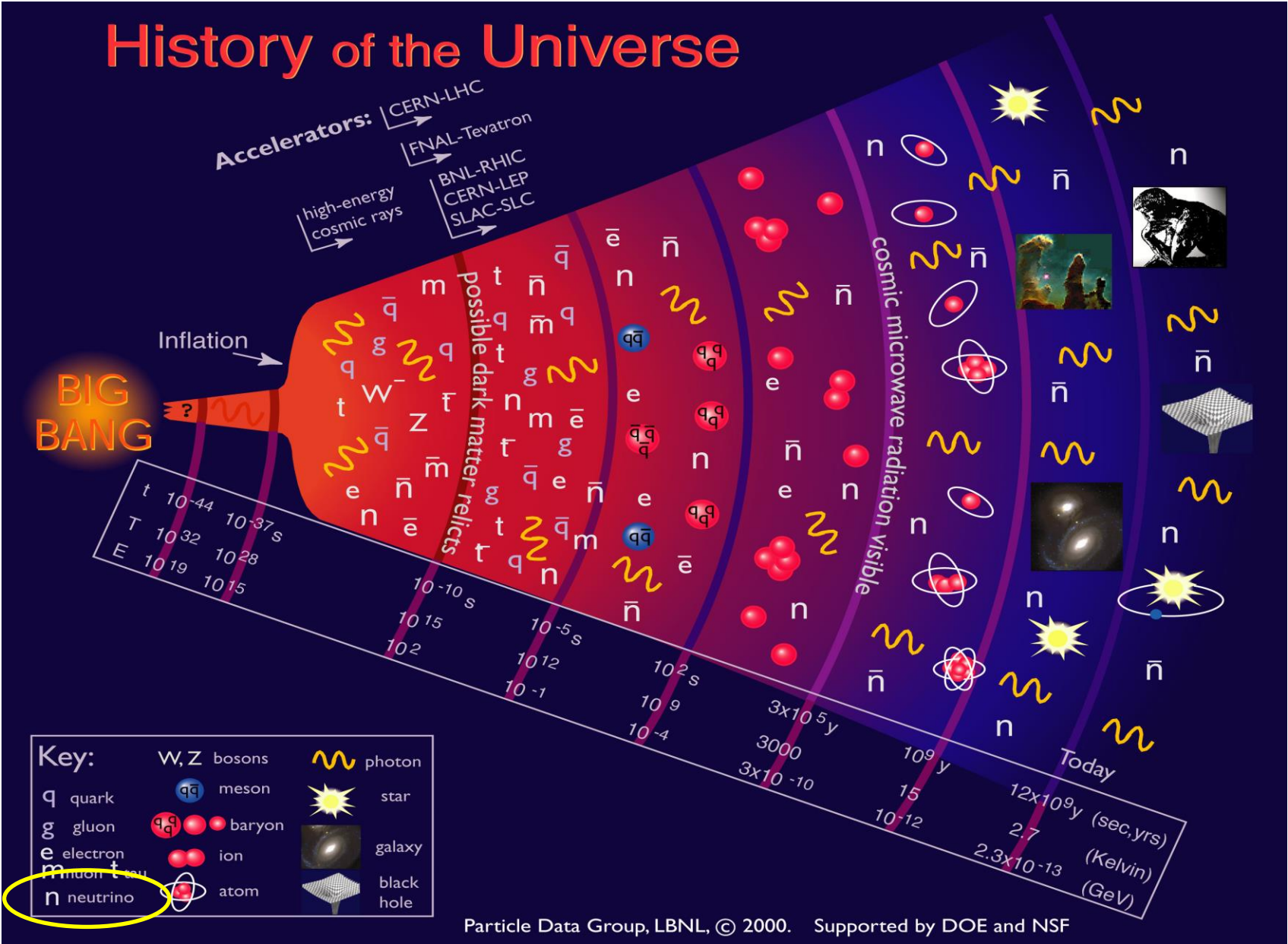


# Neutrino in Cosmology

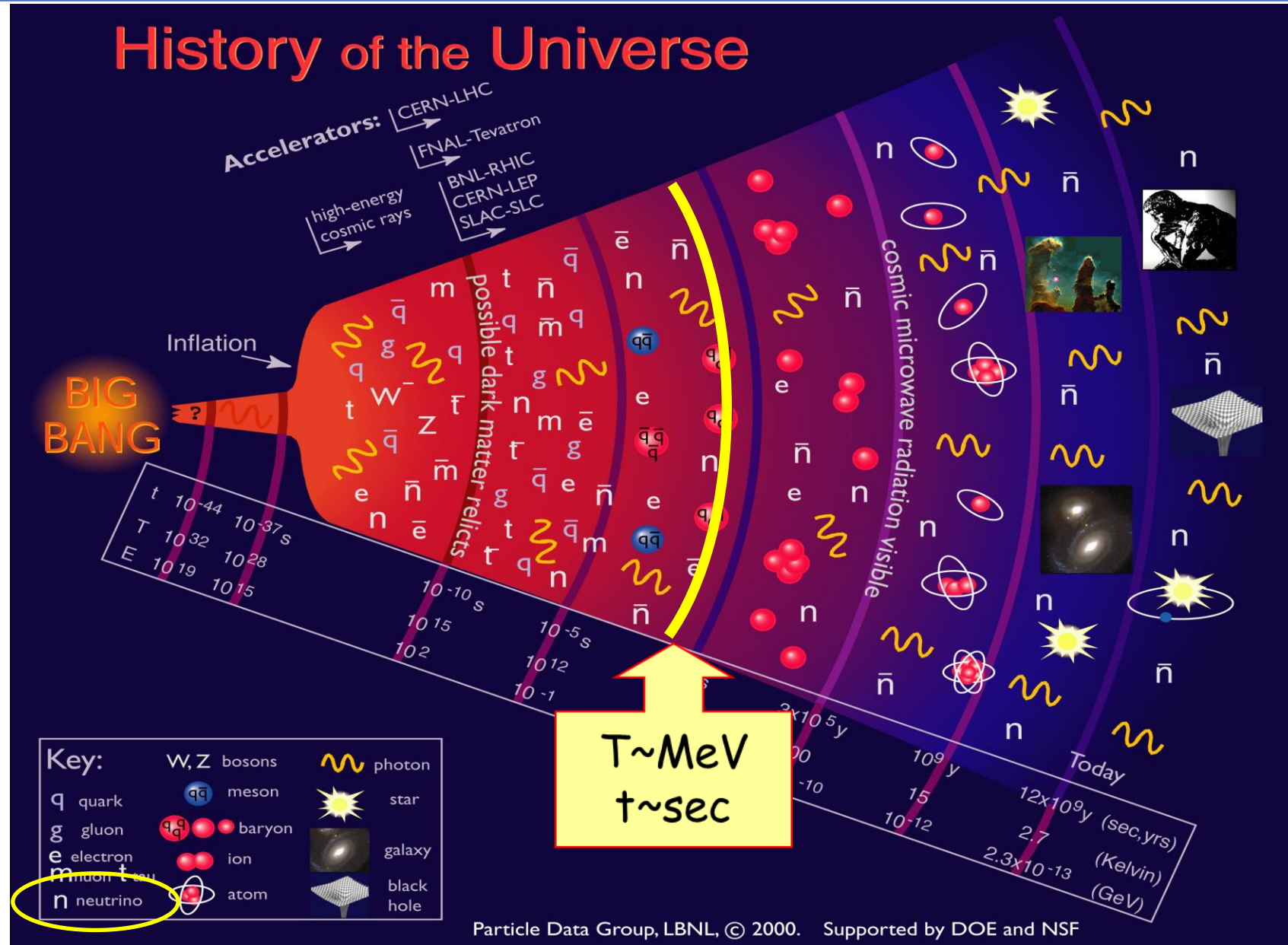
Jun-Qing Xia

Beijing Normal University  
2022-11-20

- History of Universe on Neutrino
- Effects of Neutrino on BBN, CMB & LSS
- Current Limits & Future Perspective



- $T < \text{MeV}$ , Neutrinos exit the thermal equilibrium system and begin to travel freely (CMB)

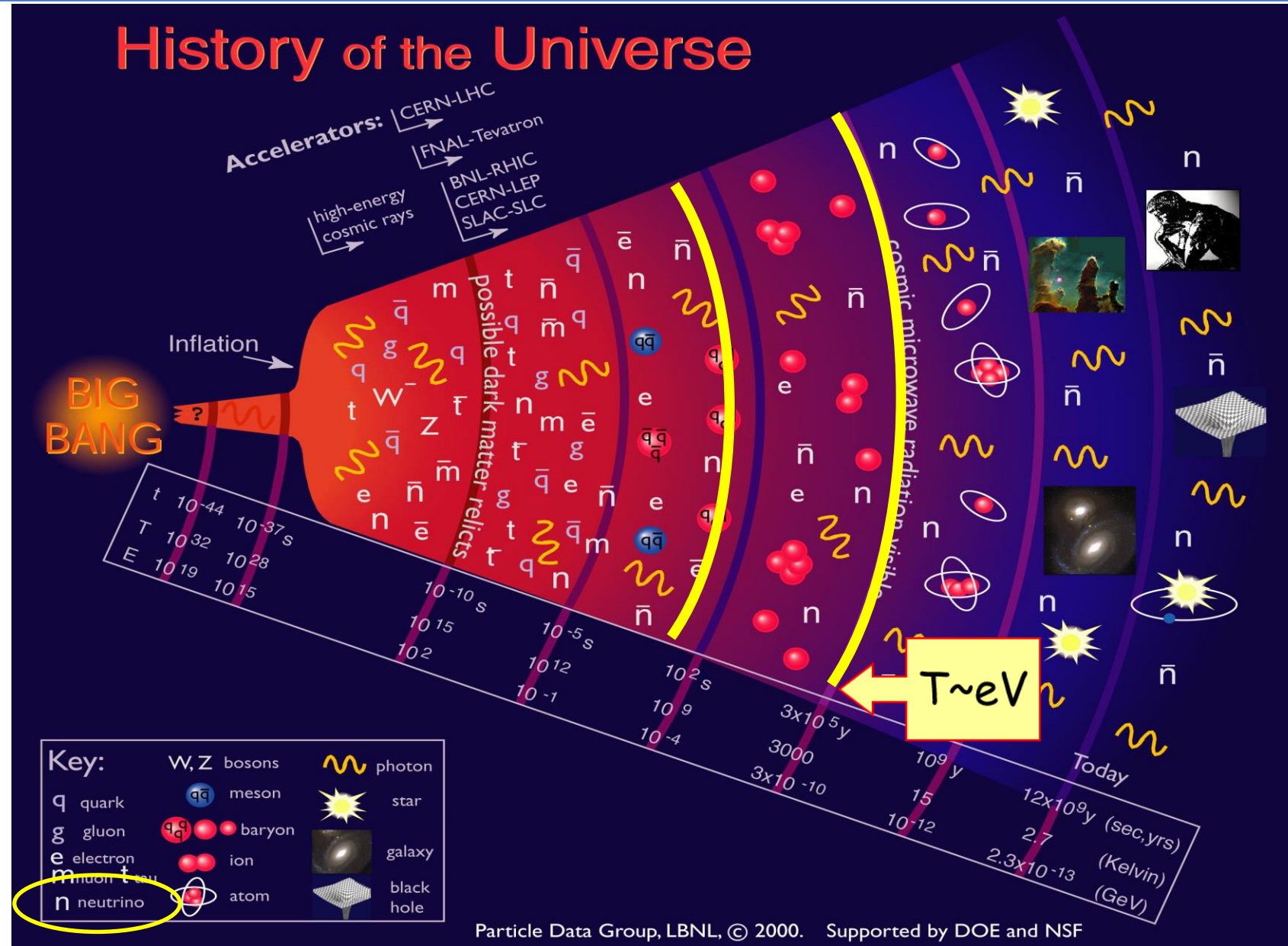


# Evolution of Universe

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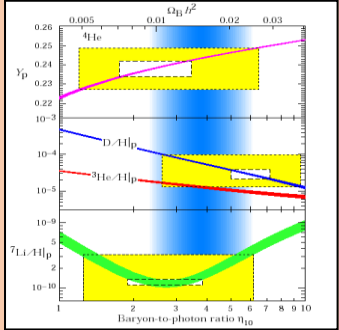
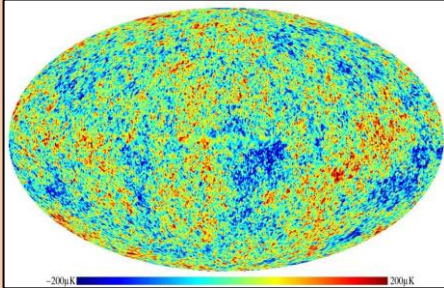
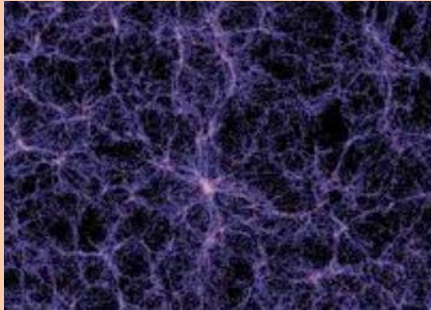
- $T > \text{eV}$ , Neutrinos are still relativistic
- $T < \text{eV}$ , Neutrinos (at least 1 flavor) start transferring to non-relativistic
- Massive Neutrinos participate the LSS formation at late time of Universe



# Effects of Neutrino

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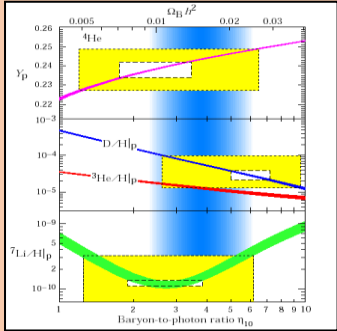
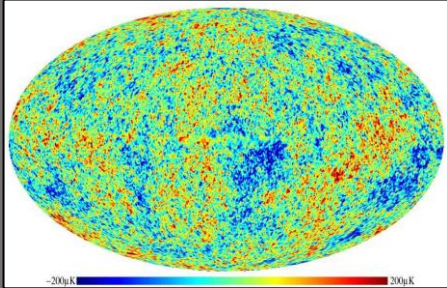
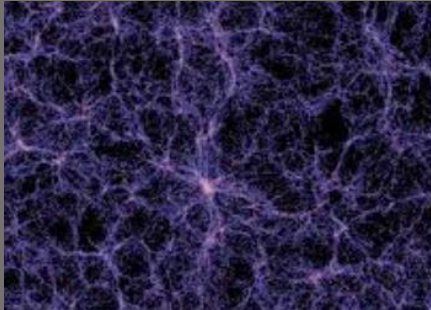
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BBN	CMB	LSS
$T \sim \text{MeV}$	$T < \text{eV}$	
$N_{\text{eff}}$	$N_{\text{eff}} \text{ \& } m_{\nu}$	

# Effects of Neutrino

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BBN	CMB	LSS
$T \sim \text{MeV}$	$T < \text{eV}$	
$N_{\text{eff}}$	$N_{\text{eff}} \text{ \& } m_{\nu}$	

- At early Universe, all particles are in thermal equilibrium and their distribution functions are:

$$f_i^{eq}(p, T) = \left[ \exp \left( \frac{E_i - \mu_i}{T} \right) \mp 1 \right]^{-1}$$

$$n = g_i \int \frac{d^2\vec{p}}{(2\pi)^3} f_i(p, T) \quad \rho = g_i \int \frac{d^2\vec{p}}{(2\pi)^3} E_i f_i(p, T)$$

VARIABLE	RELATIVISTIC		NON REL.
	BOSE	FERMI	
$n$	$\frac{\zeta(3)}{\pi^2} g T^3$	$\frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3$	$g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$
$\rho$	$\frac{\pi^2}{30} g T^4$	$\frac{7}{8} \frac{\pi^2}{30} g T^4$	$mn$

- Neutrinos decoupled at  $T \sim \text{MeV}$ , keeping a spectrum as that of a relativistic species

$$f_\nu(p, T) = \frac{1}{e^{p/T_\nu} + 1}$$

- Number density ( $\sim 112 \text{cm}^{-3}$  per flavor)

$$n_\nu = \int \frac{d^3 p}{(2\pi)^3} f_\nu(p, T_\nu) = \frac{3}{11} n_\gamma = \frac{6\zeta(3)}{11\pi^2} T_{CMB}^3$$

$$\Omega_\nu = \frac{\rho_\nu}{\rho_{\text{crit}}}$$

- Energy density

$$\rho_{\nu_i} = \int \sqrt{p^2 + m_{\nu_i}^2} \frac{d^3 p}{(2\pi)^3} f_\nu(p, T_\nu) \rightarrow \begin{cases} \frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{4/3} T_{CMB}^4 \\ m_{\nu_i} n_\nu \end{cases}$$



$$\Omega_\nu h^2 = 1.7 \times 10^{-5}$$

**massless**

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{94.1 \text{ eV}}$$

**massive**

# Neutrino Decoupled

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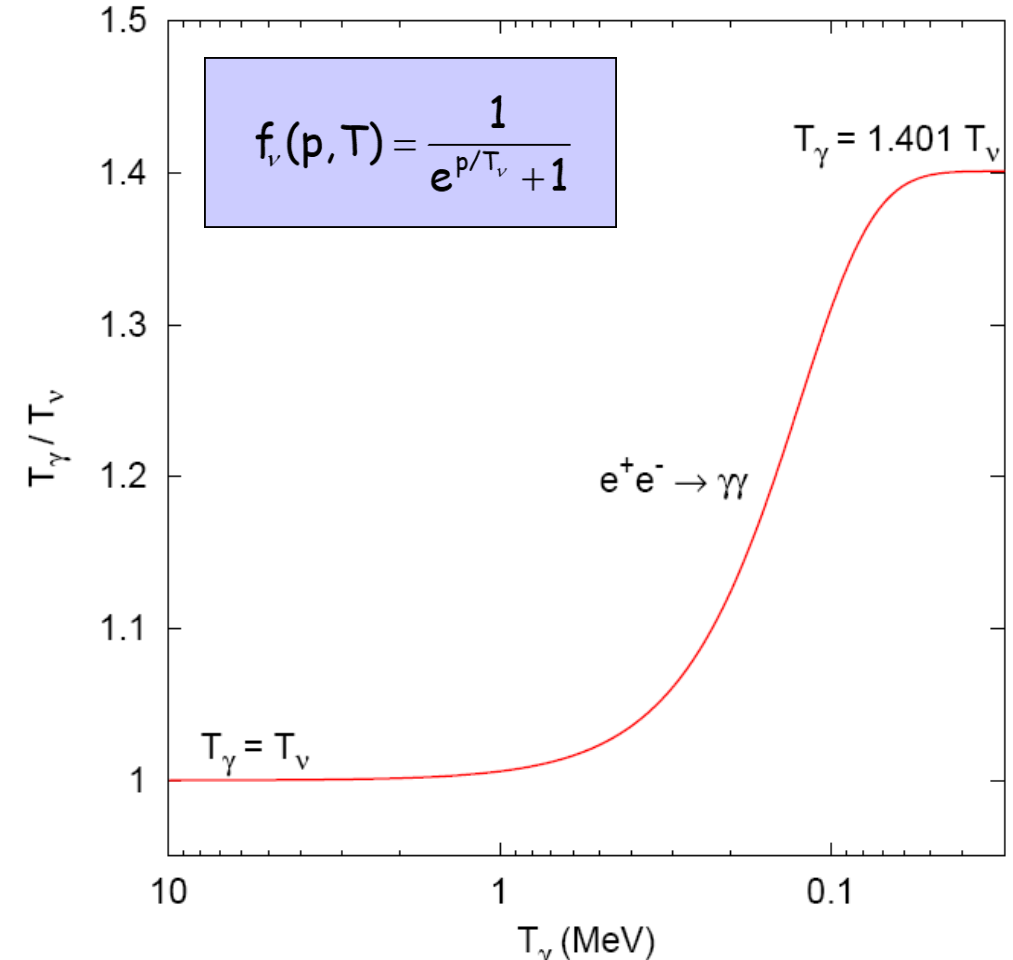
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- $T \sim \text{MeV}$ , Neutrinos start to exit the thermal equilibrium
- $T \sim m_e = 0.511 \text{ MeV}$ , electron-positron pairs annihilate to photons



heating photons, but not decoupled neutrinos.

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$



- At this stage, the radiation content of Universe contains

$$\rho_r = \rho_\gamma + \rho_\nu = \frac{\pi^2}{15} T_\gamma^4 + 3 \times \frac{7}{8} \times \frac{\pi^2}{15} T_\nu^4 = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \right] \rho_\gamma$$

$$\rho_r = \rho_\gamma + \rho_\nu = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma$$

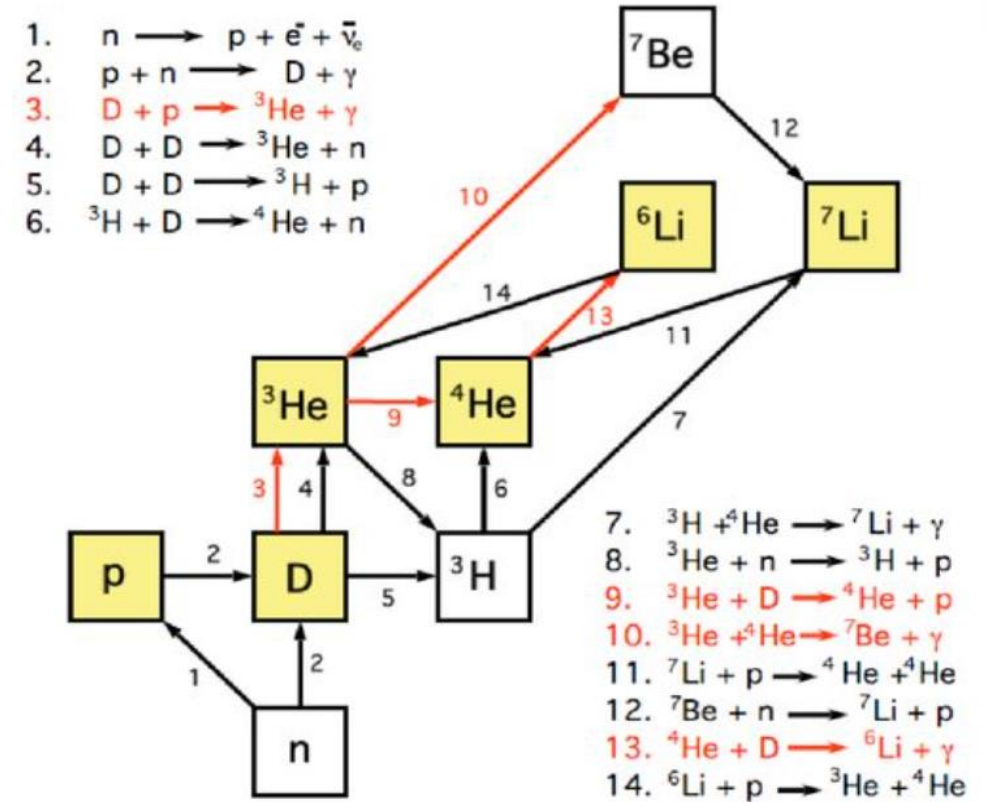
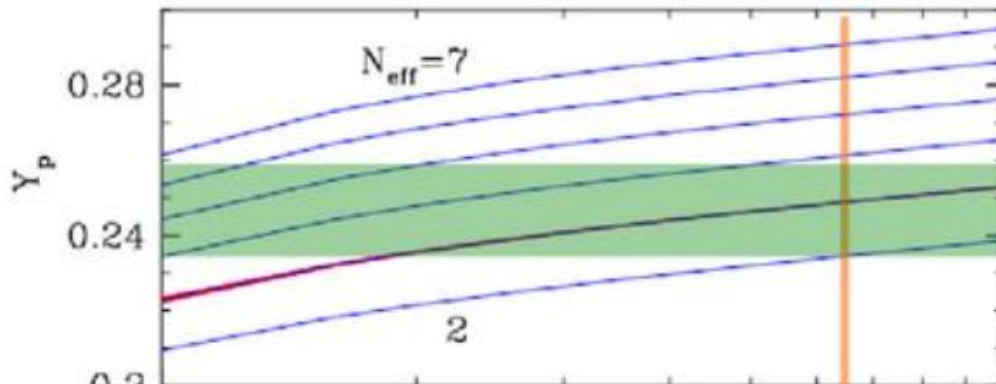
- $N_{\text{eff}}$ , effective number of relativistic neutrino-like particles
- If considering the non-instantaneous decoupling,  $N_{\text{eff}} = 3.046$
- $N_{\text{eff}}$  directly effect the expansion of Universe,  $H^2 = \frac{8\pi G\rho}{3}$

# Primordial Nucleosynthesis

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- The BBN begins with the formation of Deuterium. Nearly all the free neutrons end up bound in the most stable light element  $^4\text{He}$
- $^4\text{He}$  abundance ( $Y_p$ ) strongly depends on the expansion rate of Universe
- Its measurement can be used to constrain the  $N_{\text{eff}}$



# Recent Limits on $N_{\text{eff}}$ (2207.13133)

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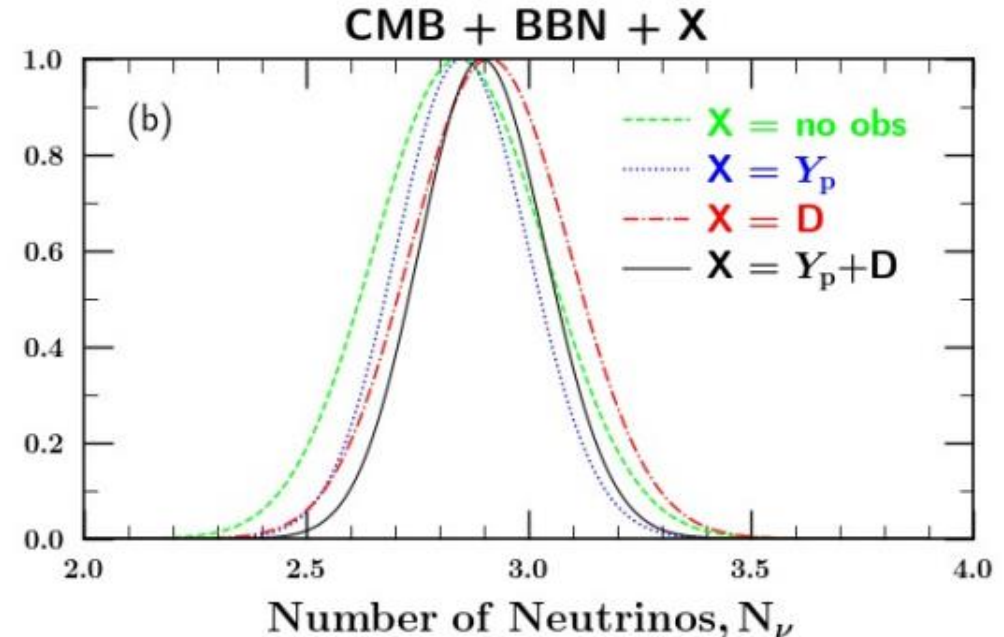
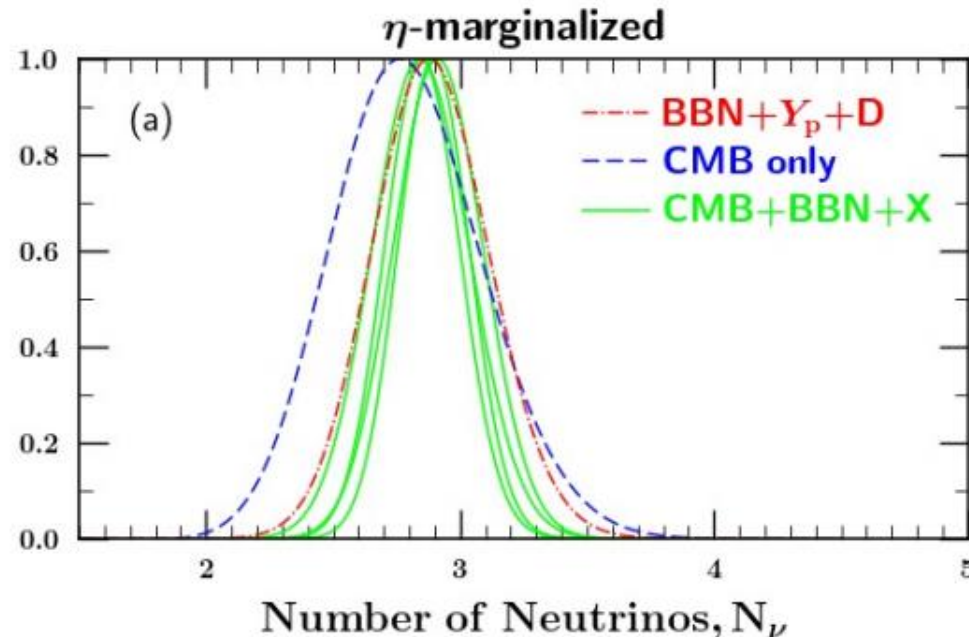
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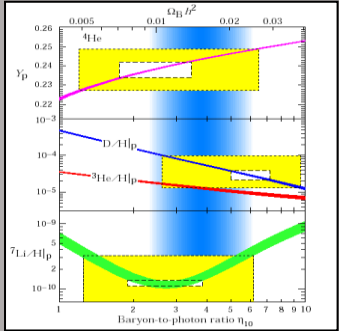
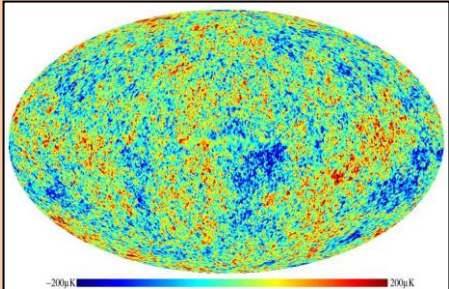
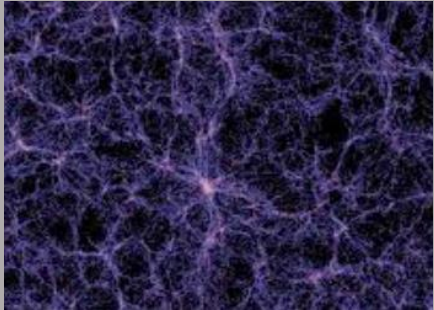
- Measurements of  $Y_p$  &  $[D/H]$  from BBN can give tight constraint

$$N_{\text{eff}} = 2.889 \pm 0.229 \ (1\sigma)$$

- If including the Planck measurement from CMB further, we have

$$N_{\text{eff}} = 2.898 \pm 0.141 \ (1\sigma)$$



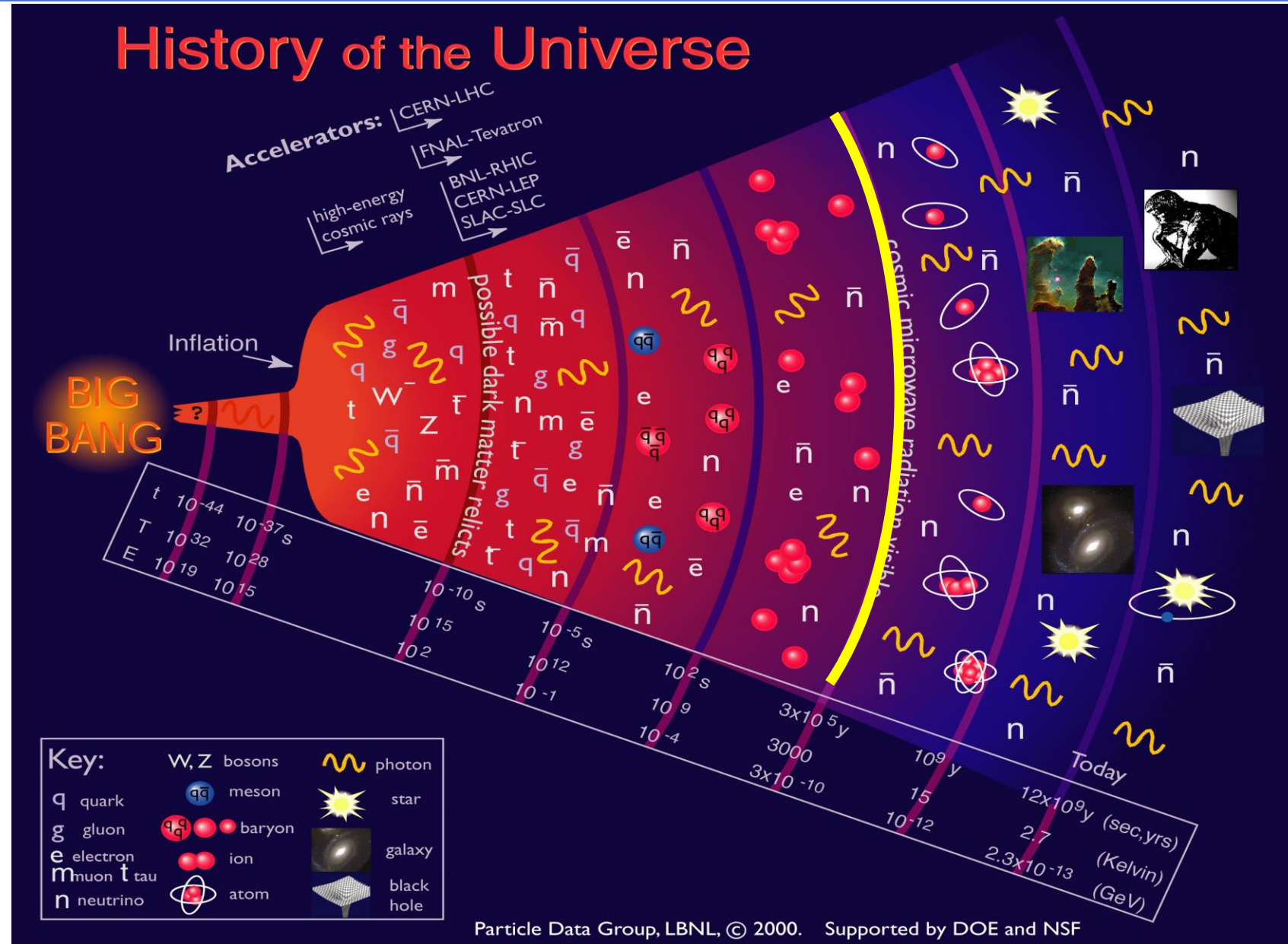
		
BBN	CMB	LSS
$T \sim \text{MeV}$	$T < \text{eV}$	
$N_{\text{eff}}$	$N_{\text{eff}} \text{ \& } m_{\nu}$	

# Cosmic Microwave Background

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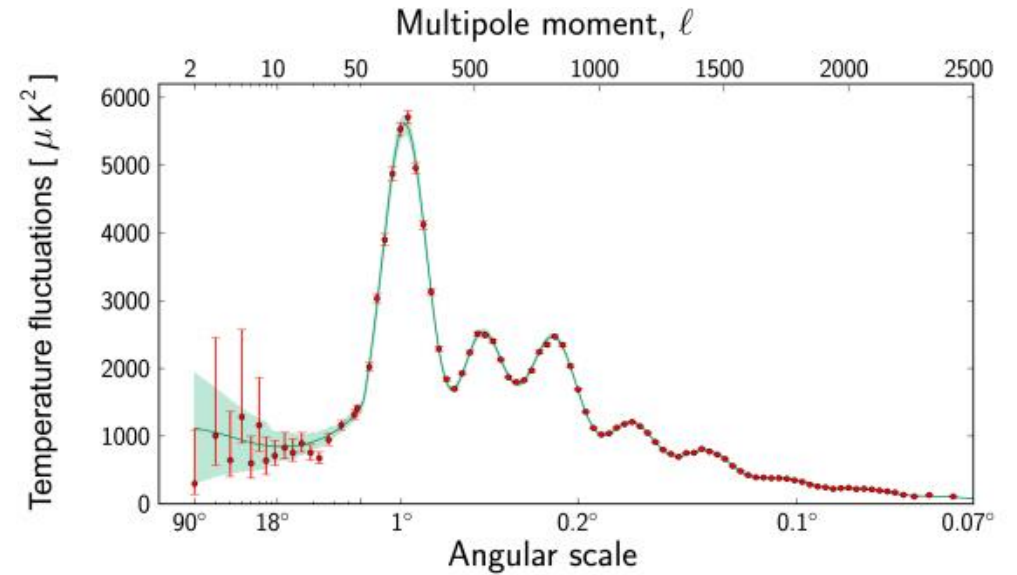
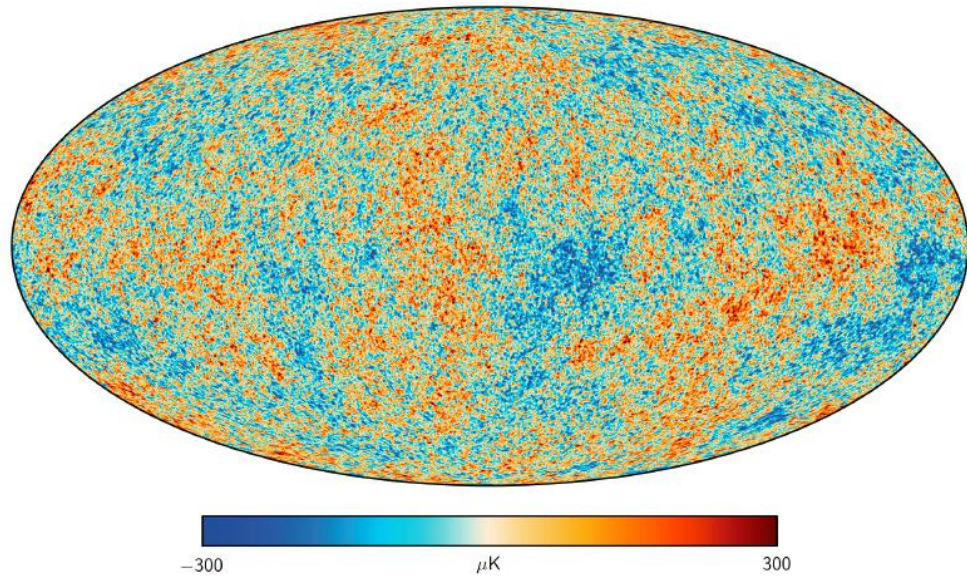
- $T \sim 0.1\text{eV}$  ( $z_{\text{cmb}} \sim 1100$ ), free electrons were captured by protons to form the neutral hydrogen
- The photon can travel freely, forming the CMB



# CMB Power Spectrum

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$$\Theta(\vec{n}) = \frac{T(\vec{n}) - \bar{T}}{\bar{T}} = \frac{\Delta T}{\bar{T}}(\vec{n})$$

$$\frac{\Delta T}{\bar{T}}(\theta, \phi) = \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$a_{\ell m} = \int d\Omega Y_{\ell m}^*(\theta, \phi) \frac{\Delta T}{\bar{T}}(\theta, \phi)$$

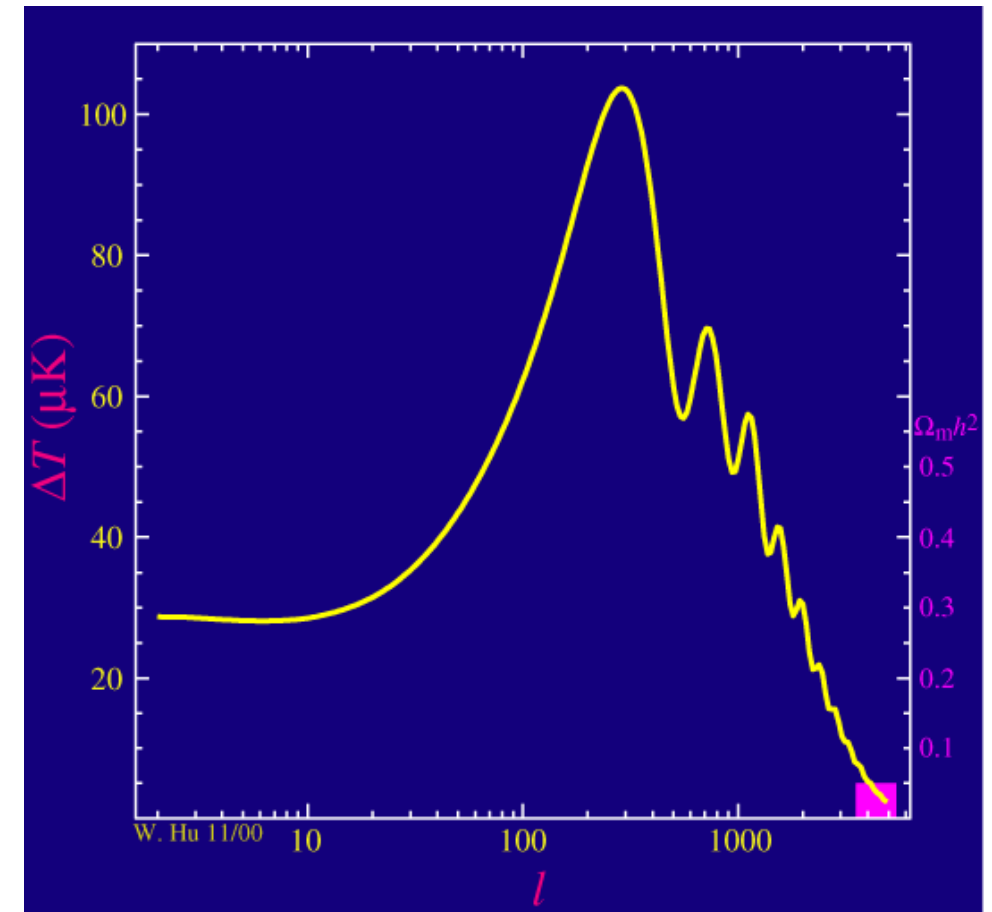
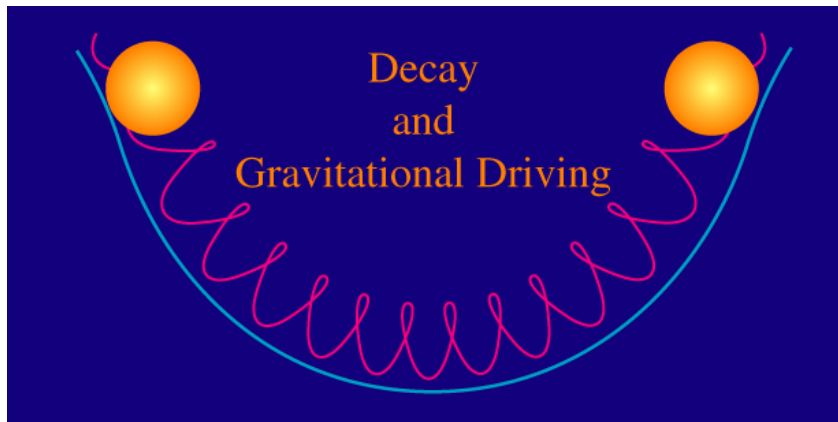
$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_{\ell}$$

# Early Integrated Sachs-Wolfe Effect

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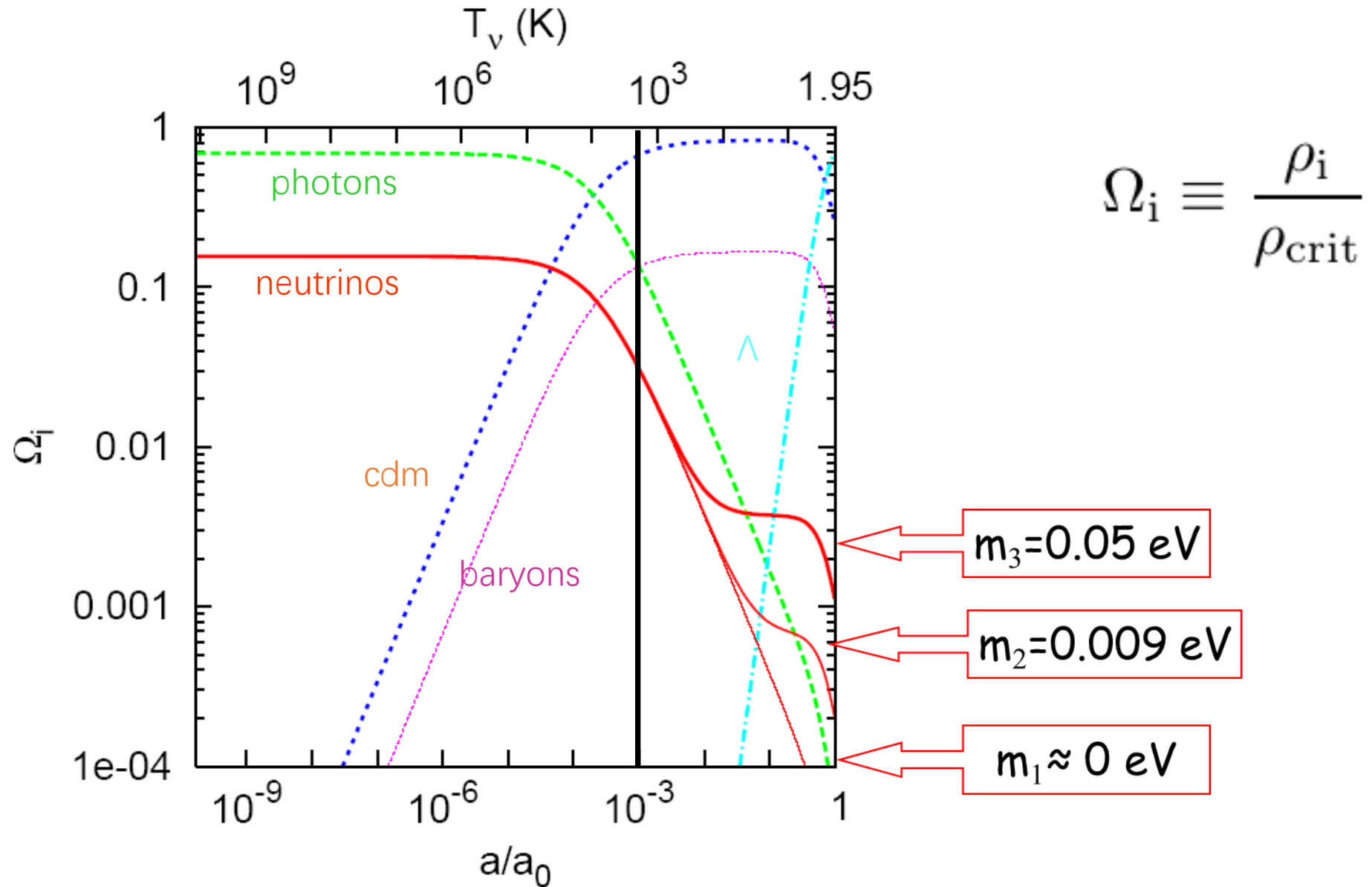
- The time-variation of gravitational potentials which occurs during the radiation-dominated, but not during the matter-dominated era, and leads to an enhancement of PS



# Evolution of Universe

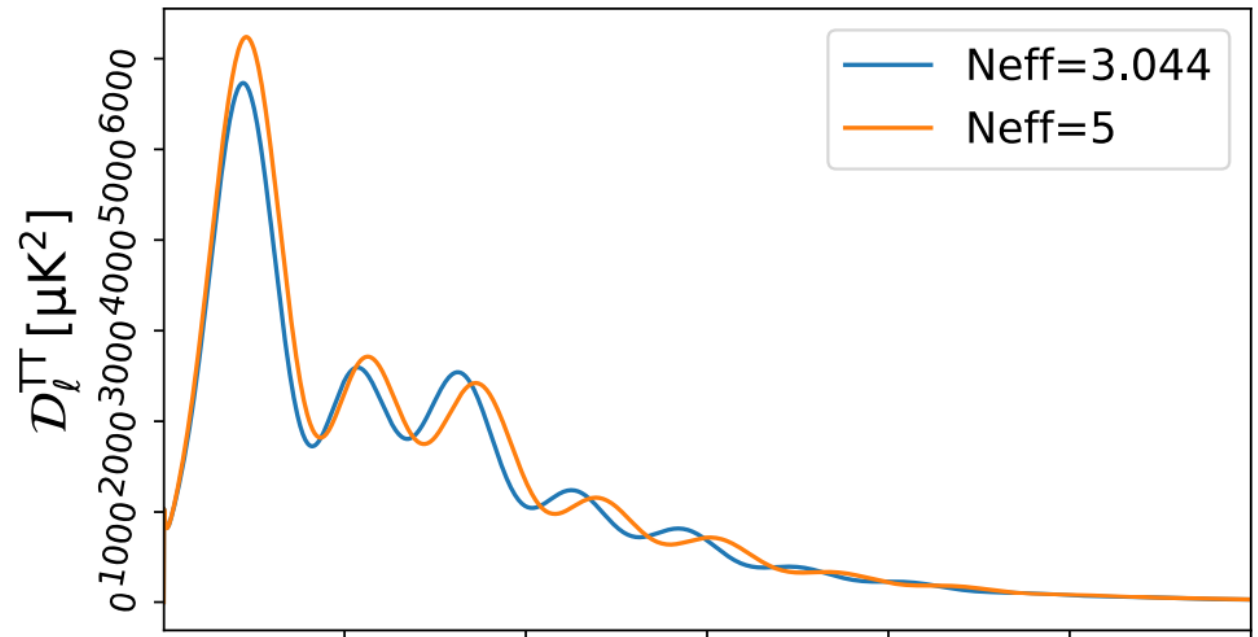
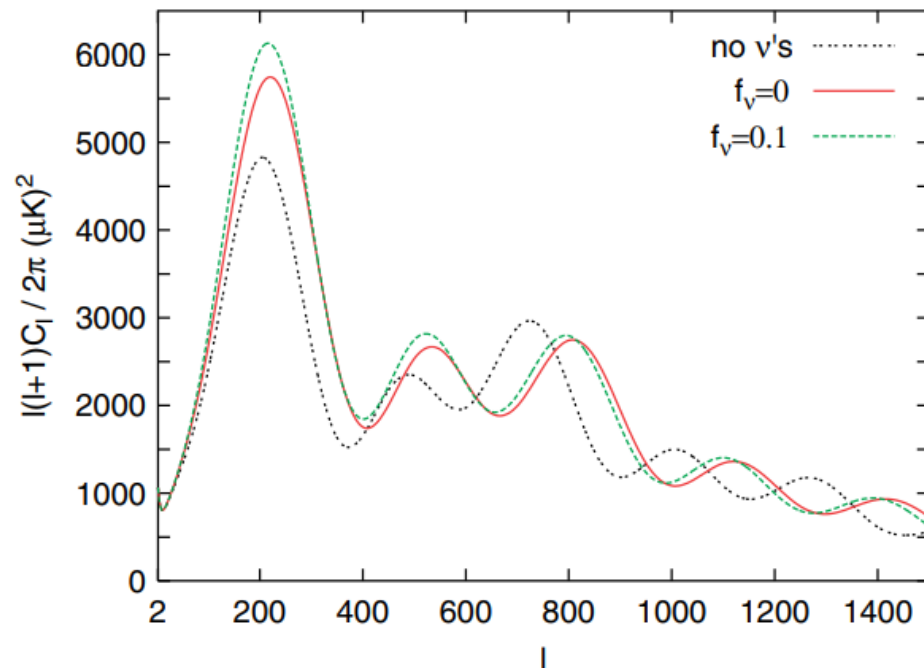
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$$\rho_r = \rho_\gamma + \rho_\nu = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma$$

- The large value of  $N_{\text{eff}}$  will increase the energy density contributions of radiation. The effect of EISW will be enhanced.



# Matter/Radiation Equality

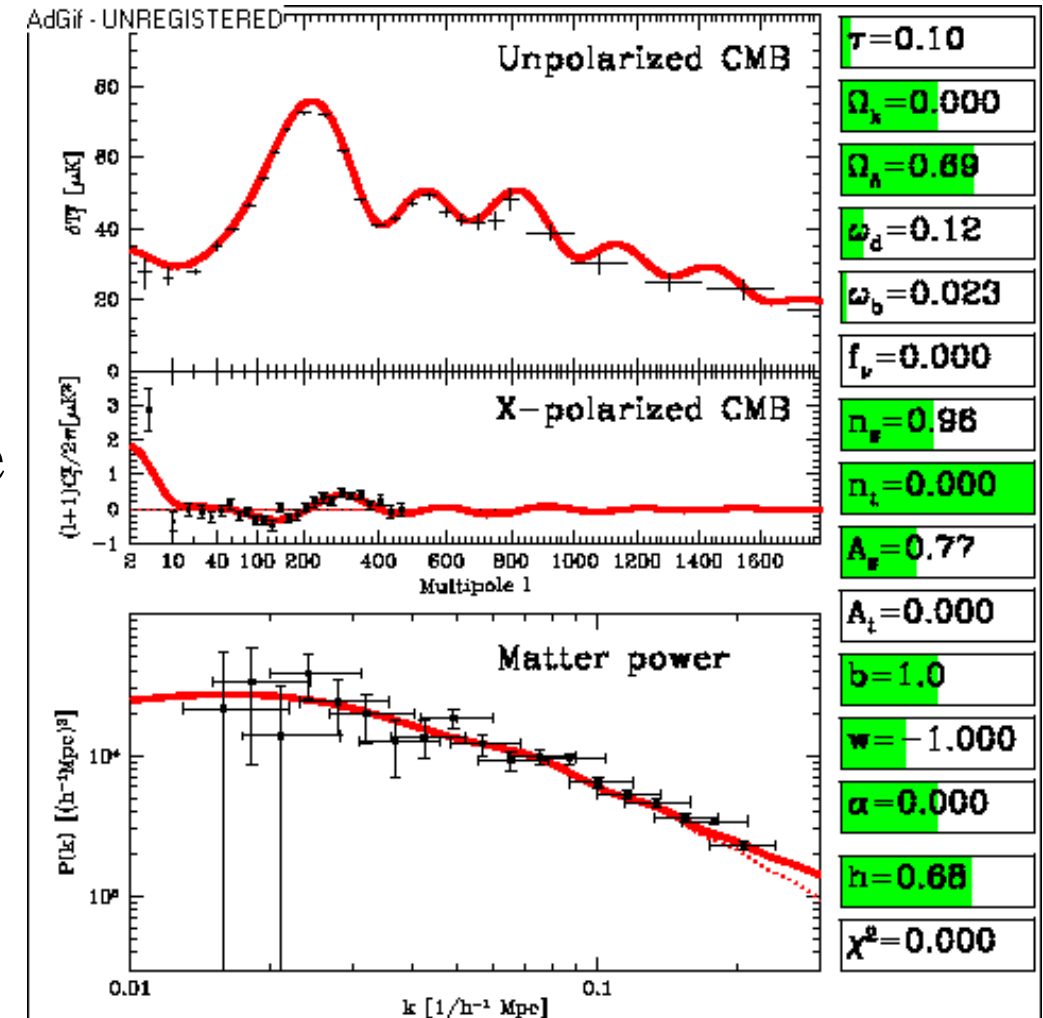
$$\rho_b + \rho_{\text{cdm}} = \rho_\gamma + \rho_\nu$$

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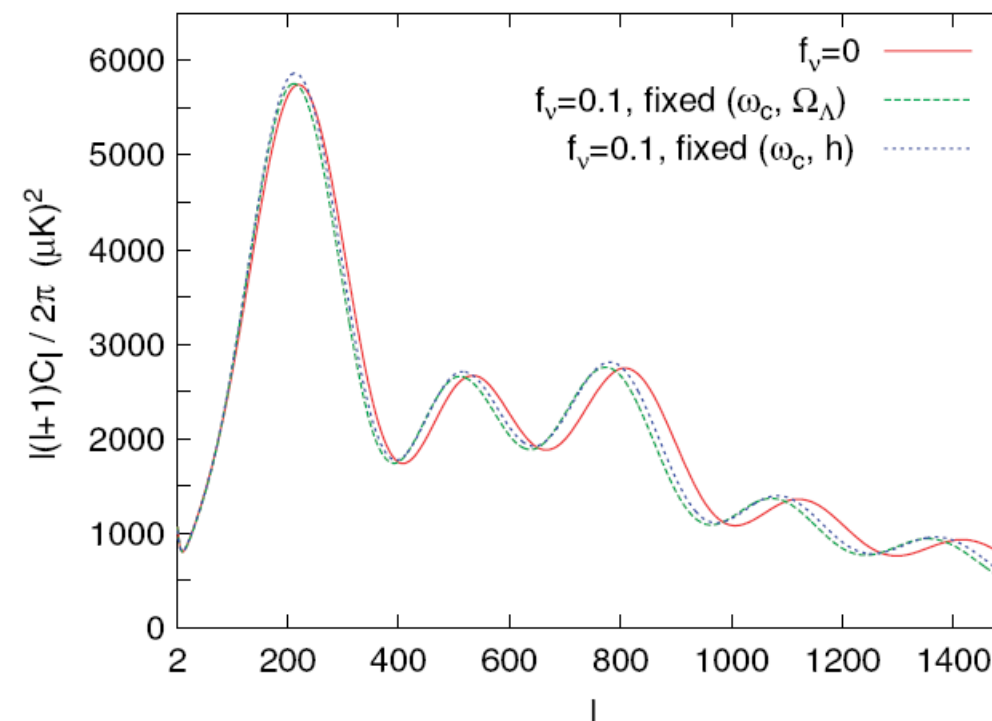
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- At  $z_{\text{eq}} \sim 3300$ , the energy density of matter and radiation are equal
- For the massive Neutrino,  $f_\nu > 0$ , it will delay the epoch of matter-radiation equality, which increases the energy density of radiation at  $z_{\text{cmb}}$ . The effect of EISW is enhanced further.

$$f_\nu \equiv \frac{\rho_\nu}{(\rho_{\text{cdm}} + \rho_b + \rho_\nu)} = \frac{\Omega_\nu}{\Omega_m}$$



- Although these effects may suggest that the CMB is exquisitely sensitive to the neutrino mass, in practice, the shape of the CMB anisotropy spectra is governed by several parameters. Changing some of parameters can mimic the effects of Neutrino.



- Latest constraints from Planck data (1807.06209):

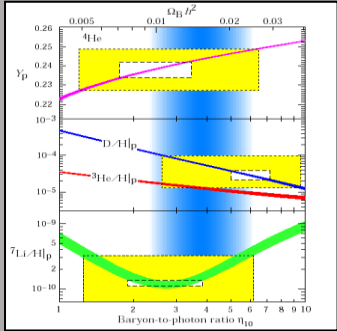
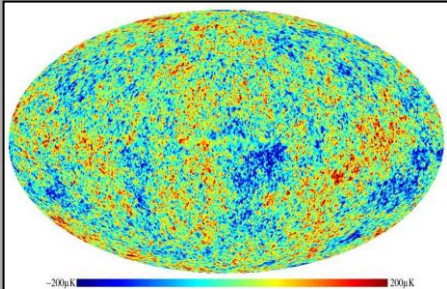
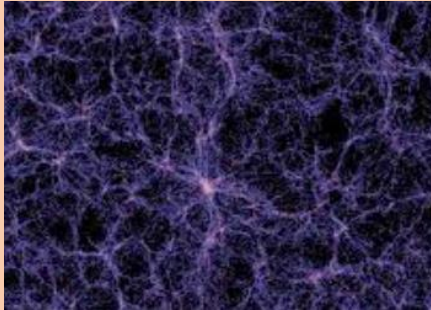
$$\sum m_\nu < 0.26 \text{ eV} \quad (95 \%, \text{Planck TT,TE,EE+lowE}).$$

$$N_{\text{eff}} = 2.92^{+0.36}_{-0.37} \quad (95 \%, \text{Planck TT,TE,EE+lowE})$$

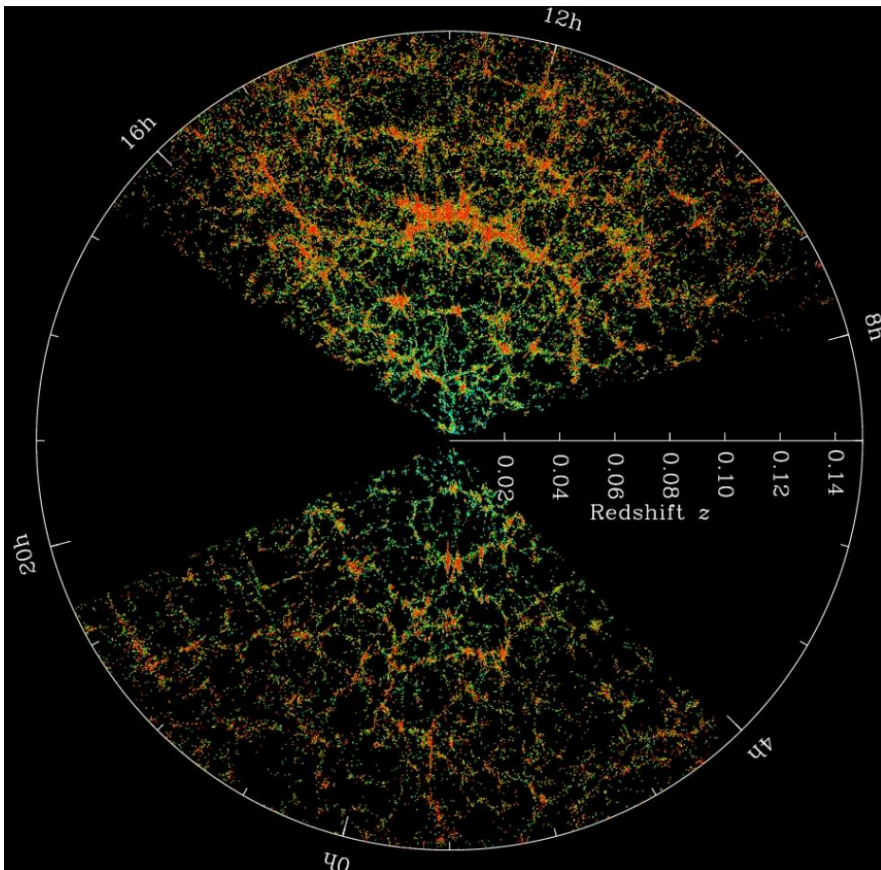
# Effects of Neutrino

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BBN	CMB	LSS
$T \sim \text{MeV}$	$T < \text{eV}$	
$N_{\text{eff}}$	$N_{\text{eff}} \text{ \& } m_{\nu}$	

- In the late time of Universe, due to the gravitational instability, the fluctuations of energy density increase exponentially and finally form the observed large scale structure.



$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\xi(\vec{r}) = \langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle$$

$$\xi(\vec{r}) = \frac{1}{(2\pi)^3} \int |\delta(\vec{k})|^2 e^{-i\vec{k} \cdot \vec{r}} d^3 k$$

$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = P(k) \delta^3(\vec{k} - \vec{k}')$$

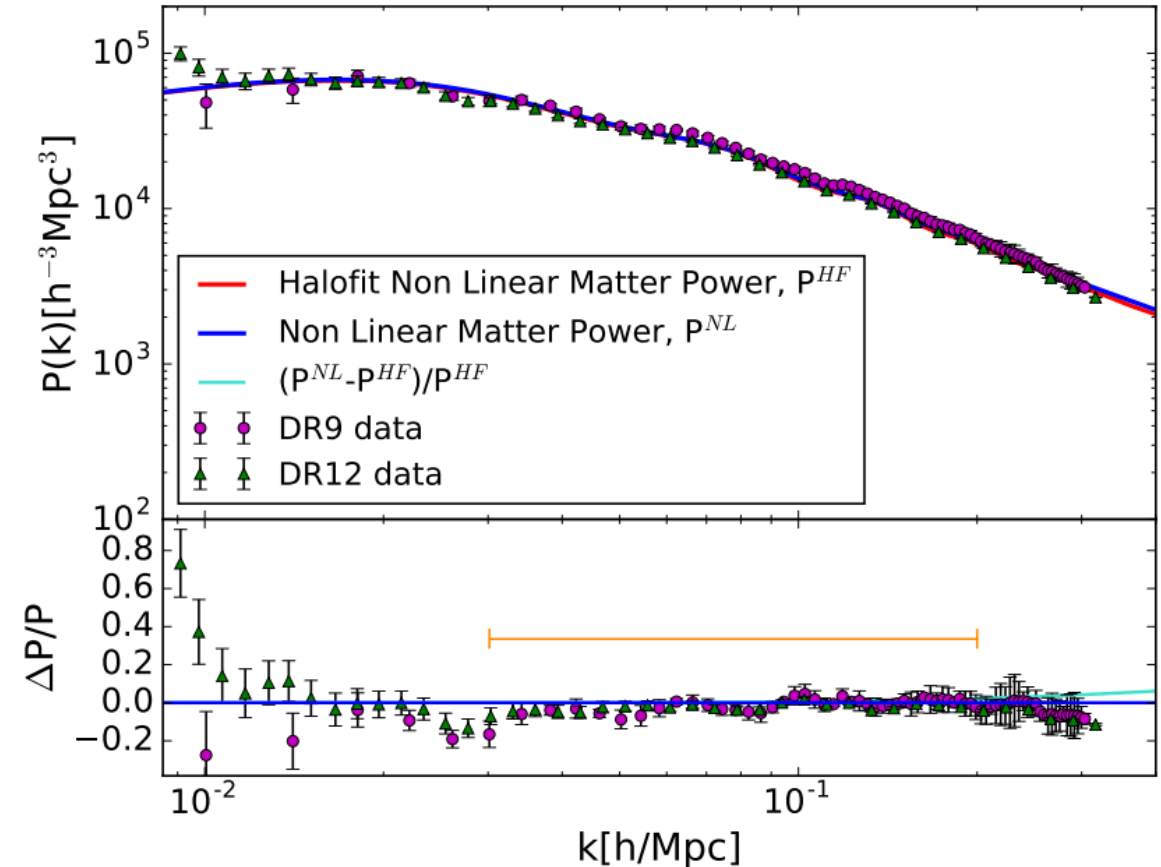
# Galaxy Power Spectrum

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- Since the linear matter power spectrum can not be observed directly, we need to analyze the clustering of galaxy
- There is a scale-independent bias factor between them

$$\text{Bias } b^2(k) = P_g(k) / P_m(k)$$

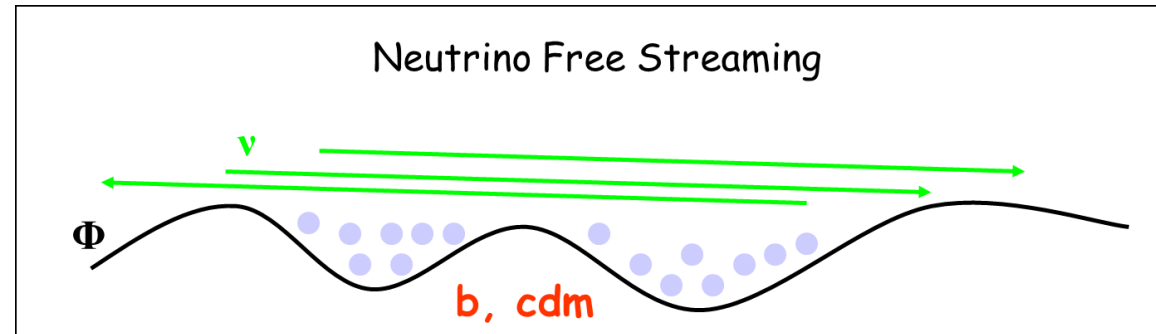


# Effect of Massive Neutrino

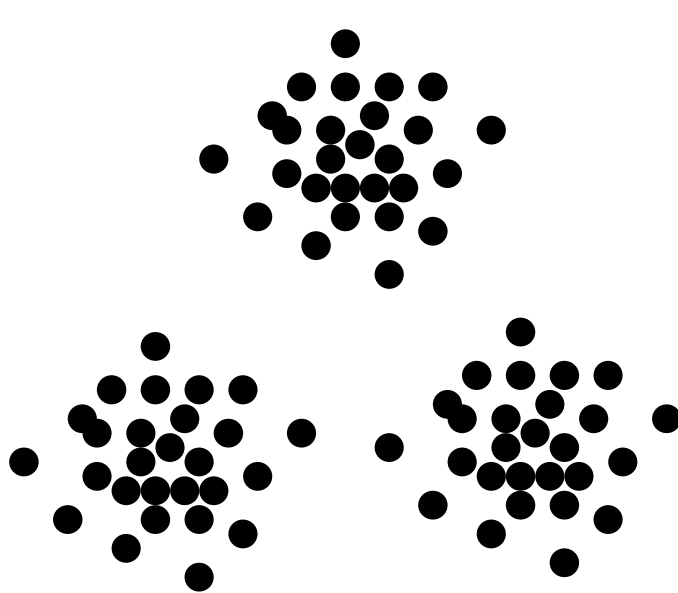
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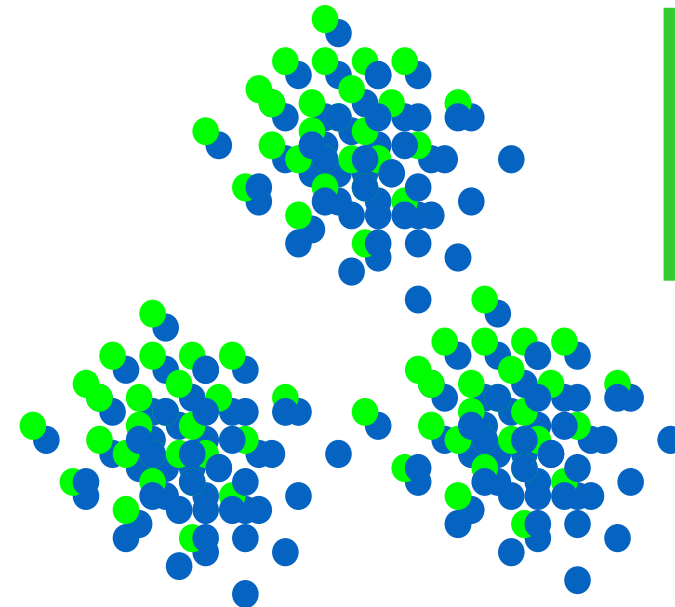
- Neutrinos cannot cluster as their thermal velocity exceeds the escape velocity of the gravitational potentials on small scales



baryons and  
CDM  
experience  
gravitational  
clustering



neutrinos  
experience  
free-streaming  
with  
 $v = c$  or  $\langle p \rangle / m$

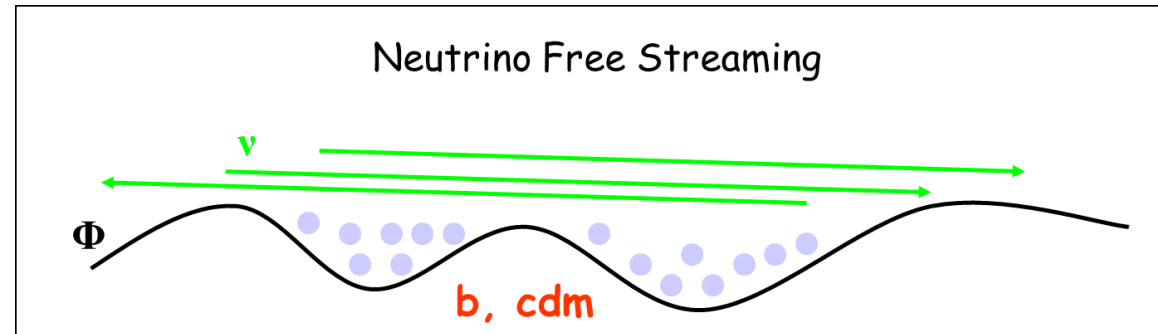


# Effect of Massive Neutrino

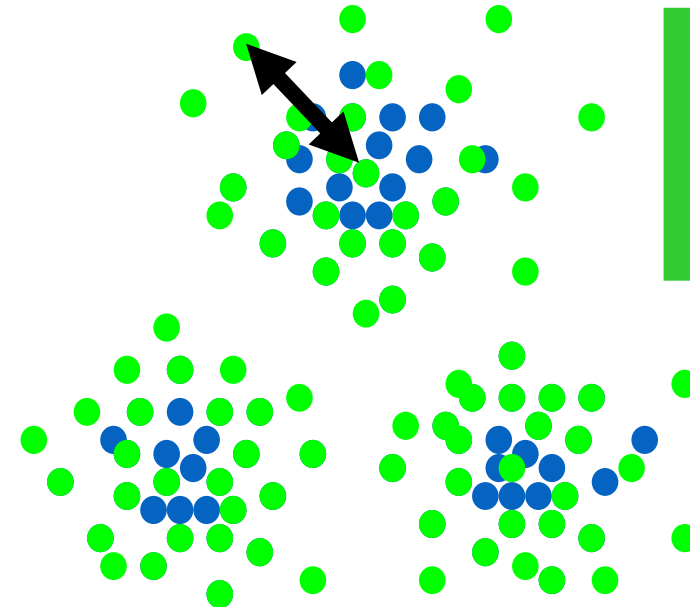
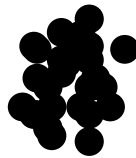
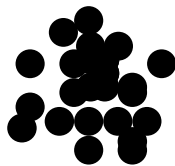
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- Neutrinos cannot cluster as their thermal velocity exceeds the escape velocity of the gravitational potentials on small scales



baryons and  
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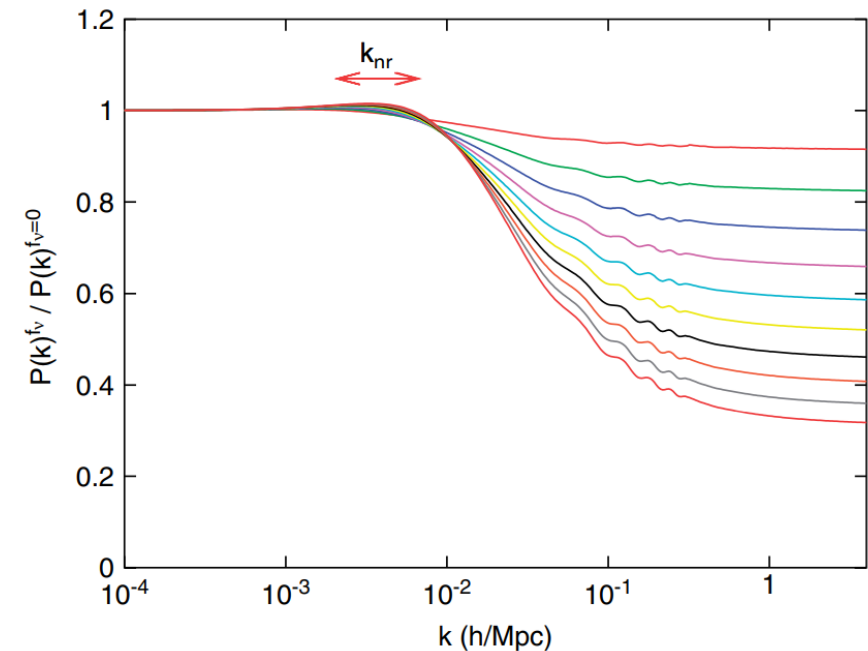
neutrinos  
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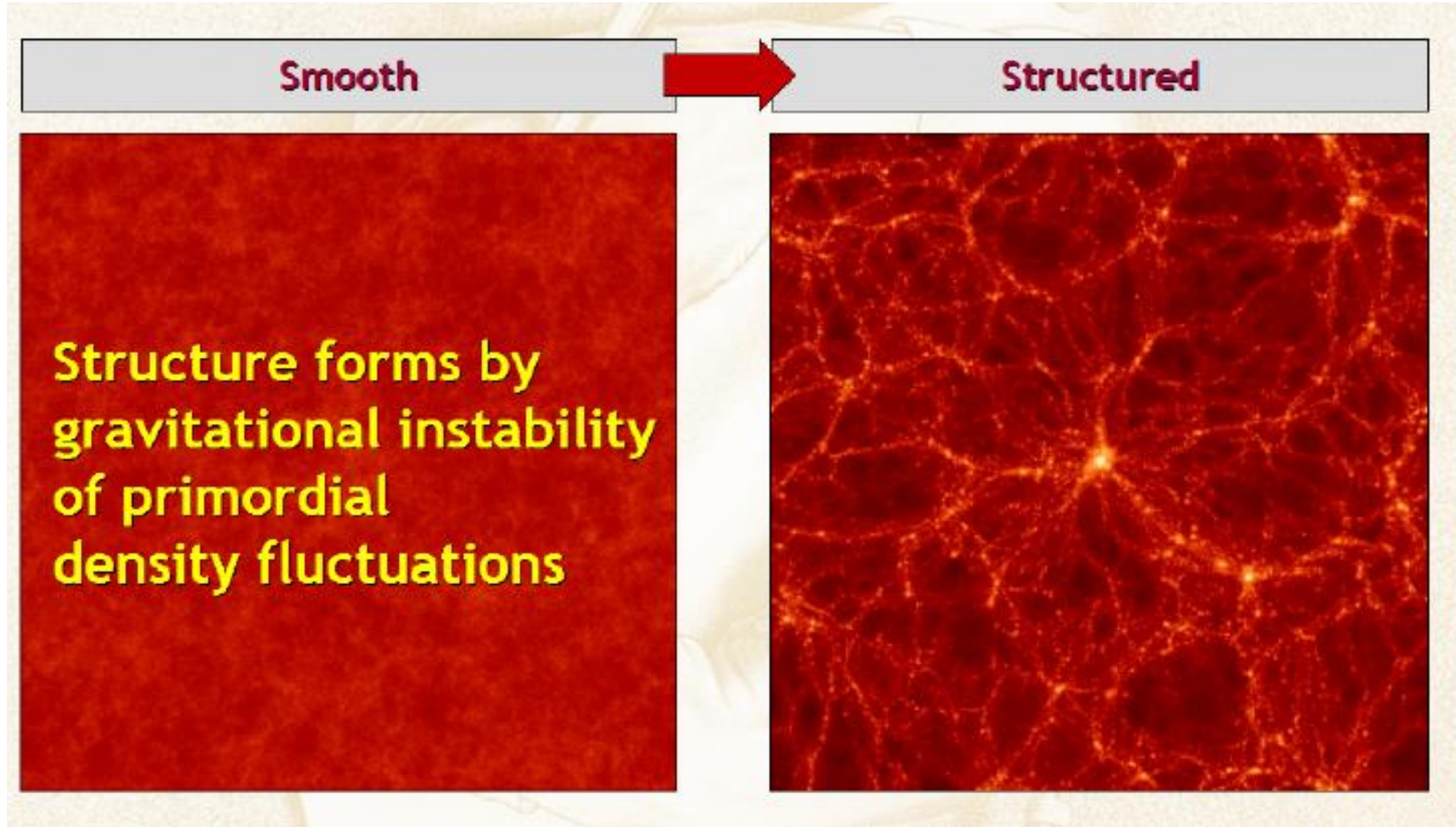
- Due to the large thermal velocities, the clustering of Neutrino can only appear on scales below the neutrino free-streaming scale

$$k_{\text{fs}} \simeq 0.018 \Omega_m^{1/2} \left( \frac{M_\nu}{1\text{eV}} \right)^{1/2} h \text{ Mpc}^{-1}$$

- For  $k > k_{\text{fs}}$ , the power spectrum is subject to a scale-independent reduction by a factor

$$\frac{P(k)^{f_\nu} - P(k)^{f_\nu=0}}{P(k)^{f_\nu=0}} \simeq -8f_\nu$$





- Latest constraints from CMB and LSS measurements (1807.06209):

$$\sum m_\nu < 0.12 \text{ eV} \quad (95 \%, \text{Planck TT,TE,EE+lowE} \\ +\text{lensing+BAO}).$$

$$N_{\text{eff}} = 2.99^{+0.34}_{-0.33} \quad (95 \%, \text{TT,TE,EE+lowE+lensing} \\ +\text{BAO}).$$

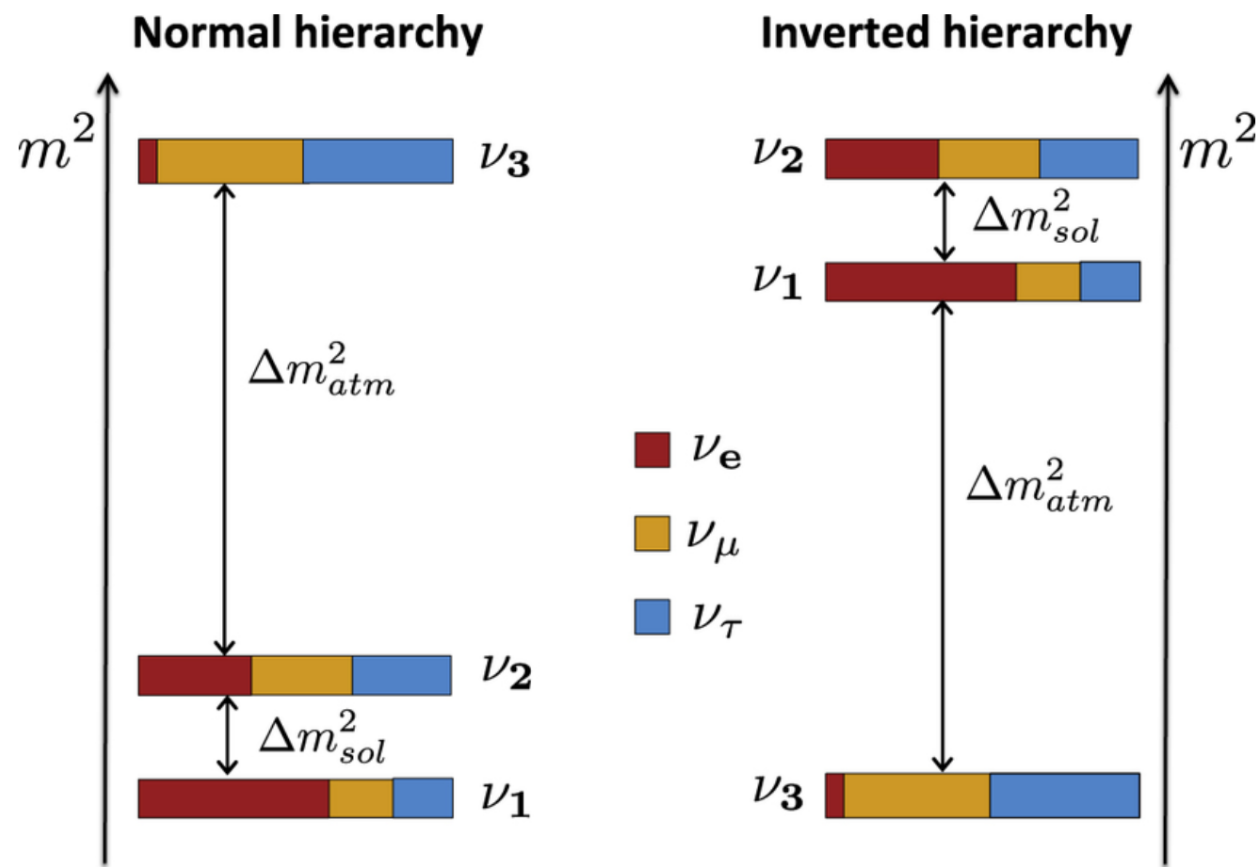
- Tightest limit on the total Neutrino mass from the combination Planck+BAO+SN Ia+RSD (2106.15267):

$$\sum m_\nu < 0.09 \text{ eV at } 95\% \text{ CL.}$$

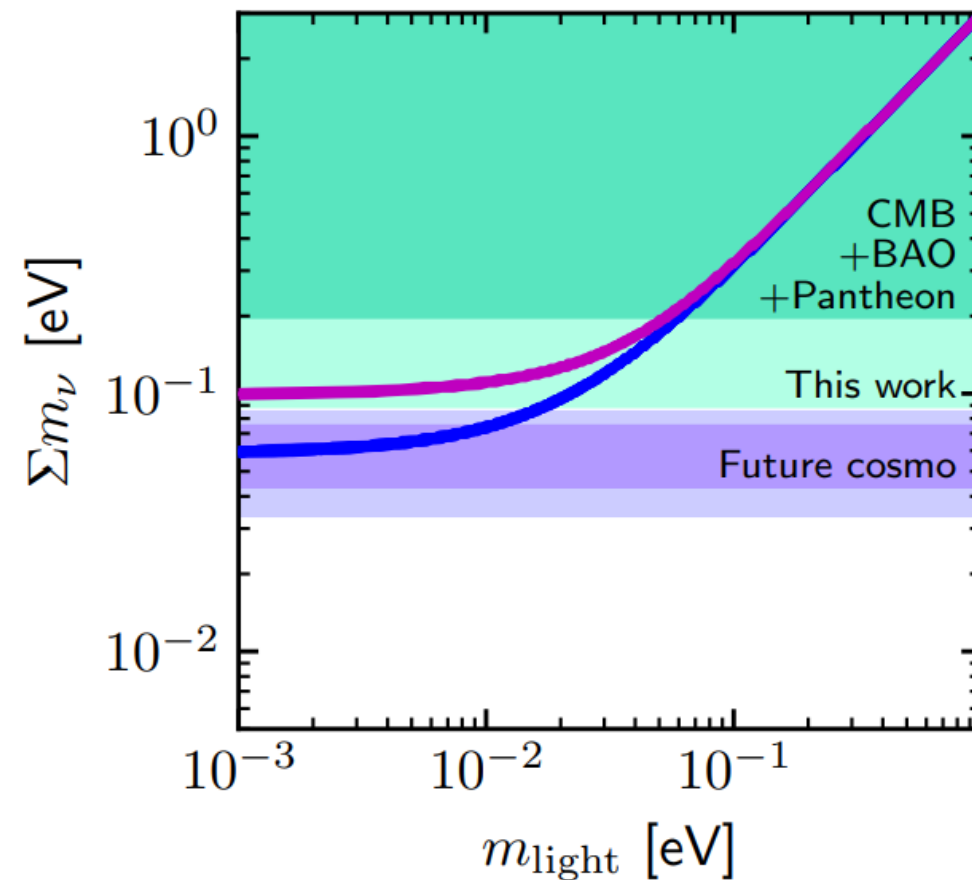
# Neutrino Mass Ordering

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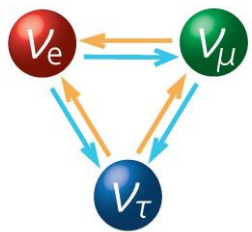
$$\sum m_\nu > 0.06 \text{ eV} \quad \sum m_\nu > 0.1 \text{ eV}$$



$$m_{\text{light}} < 0.03 \text{ eV}$$

- Besides the future CMB and LSS clustering, like CMB-S4, DESI and LSST, there are some other complimentary observables:
  - CMB Lensing, Weak Lensing, Lyman-alpha Forests
  - Cosmic Voids: the only environment where the fraction of neutrinos over CDM + baryons can be large to boost the amplitude of the effect of neutrinos (1903.05161)

Setup	$\sigma(\Sigma m_\nu)$ [meV]	$\sigma(\Sigma m_\nu)$ [meV]	$\sigma(\Omega_k)$ [ $\times 10^{-3}$ ]	$\sigma(w_0)$	$\sigma(w_a)$
S4	73	111	0.79	1.14	2.46
(+ DESI BAO)	29	76	0.48	0.13	0.41
LSST-clustering	69	91	3.33	0.42	1.22
LSST-shear	41	120	2.99	0.19	0.57
LSST-shear + clust	32	72	2.06	0.11	0.33
S4 + LSST	23	28	0.49	0.10	0.26
...	...	24	0.49	...	...



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# Thanks!