Type II Seesaw leptogenesis

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Sun Yat-sen university

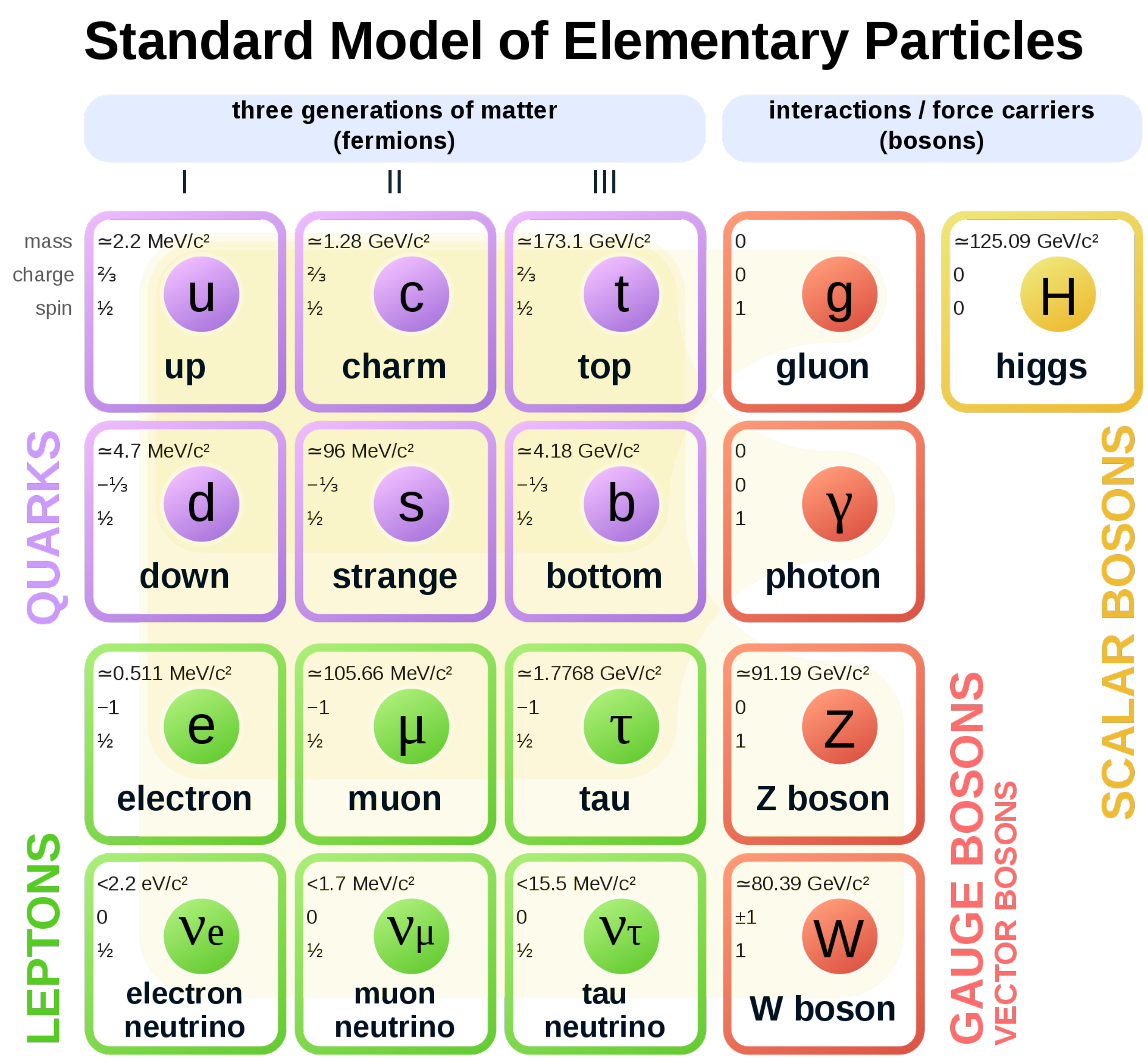
With Neil D. Barrie and Hitoshi Murayama

arXiv:2106.03381(Phys. Rev. Lett. 128, 141801) and
arXiv:2204.08202(JHEP 05 (2022) 160)

SPCS 2022

2022.11.20

Standard model



Very successful describing low energy scale physics

Observation requiring new physics

- Inflation
- Neutrino masses
- Baryon asymmetry of our universe

today's talk

- Dark matter
- Others(muon g-2?)

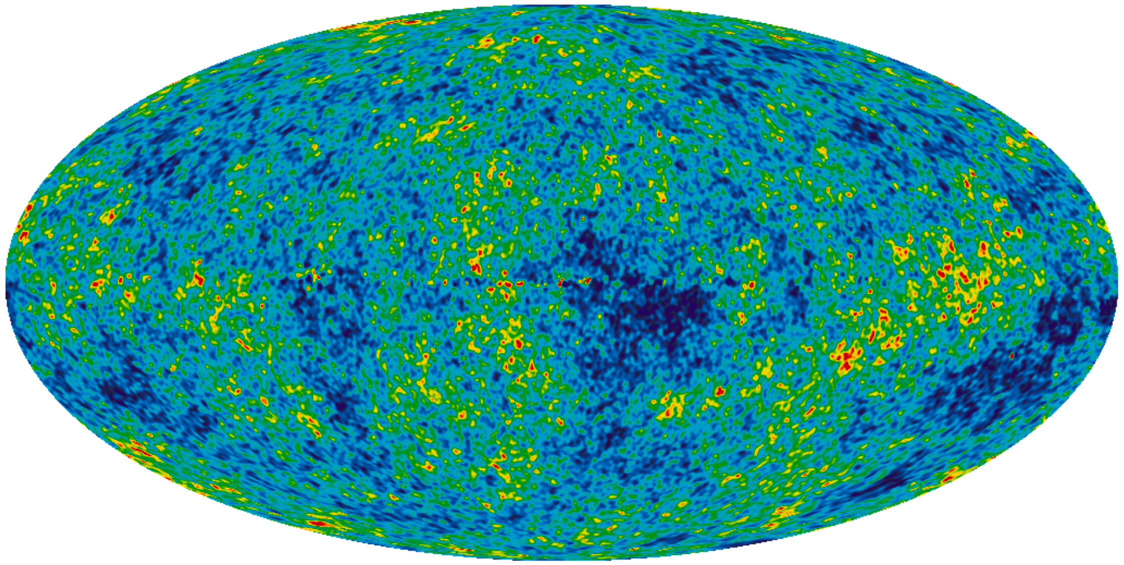
Inflation

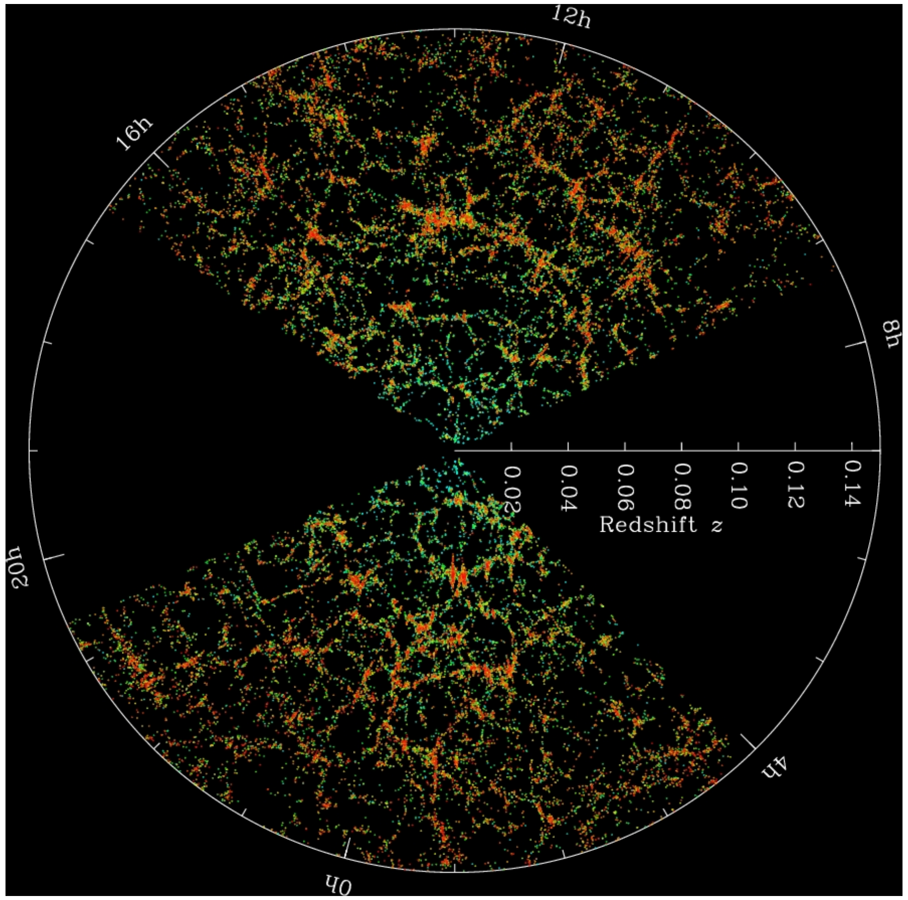
Rapid expansion of the universe in the early time

- Flatness problem
- Horizon problem
- Monopole problem?
- Seeding the primordial anisotropies in CMB

Inflation

Generating quantum fluctuations(anisotropies in CMB)





$$\frac{\delta T}{T} \sim 10^{-5}$$

Such small fluctuations finally develops the large structure of our universe

Slow-roll inflation

Assuming a scalar field, with equation of motion

$$\boxed{\begin{aligned}&\ddot{\phi}+3H\dot{\phi}+\frac{\partial V}{\partial\phi}=0\\ &H^2=\frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right)\end{aligned}}$$

Slow roll condition $\quad \dot{\phi}^2 \ll V(\phi) \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|\,,\,|V_{,\phi}|$

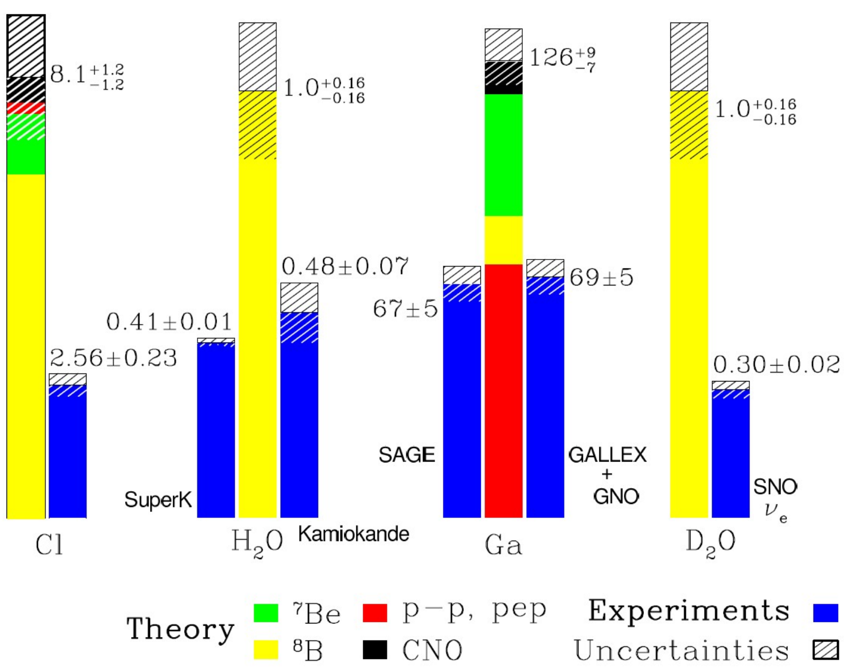
$$\epsilon_{\rm v}(\phi)\equiv\frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_{\rm v}(\phi)\equiv M_{\rm pl}^2\frac{V_{,\phi\phi}}{V} \qquad \epsilon_{\rm v},|\eta_{\rm v}|\ll 1$$

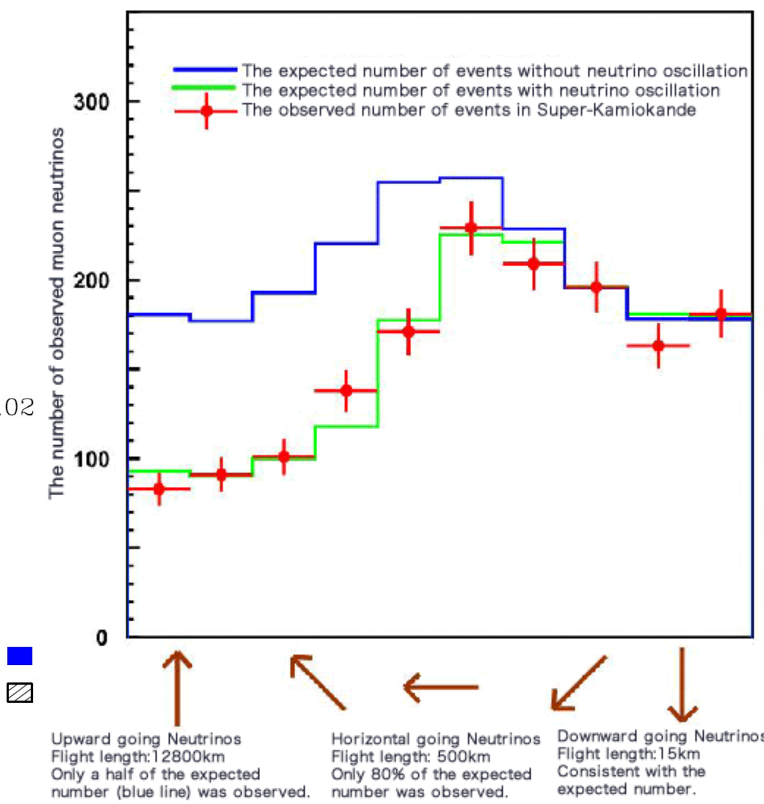
$$\begin{aligned}H^2 &\approx \frac{1}{3}V(\phi)\approx \text{const.}\\ \dot{\phi} &\approx -\frac{V_{,\phi}}{3H}\,,\end{aligned} \quad\longrightarrow\quad a(t)=a_0e^{Ht} \qquad Ht\gtrsim 60$$

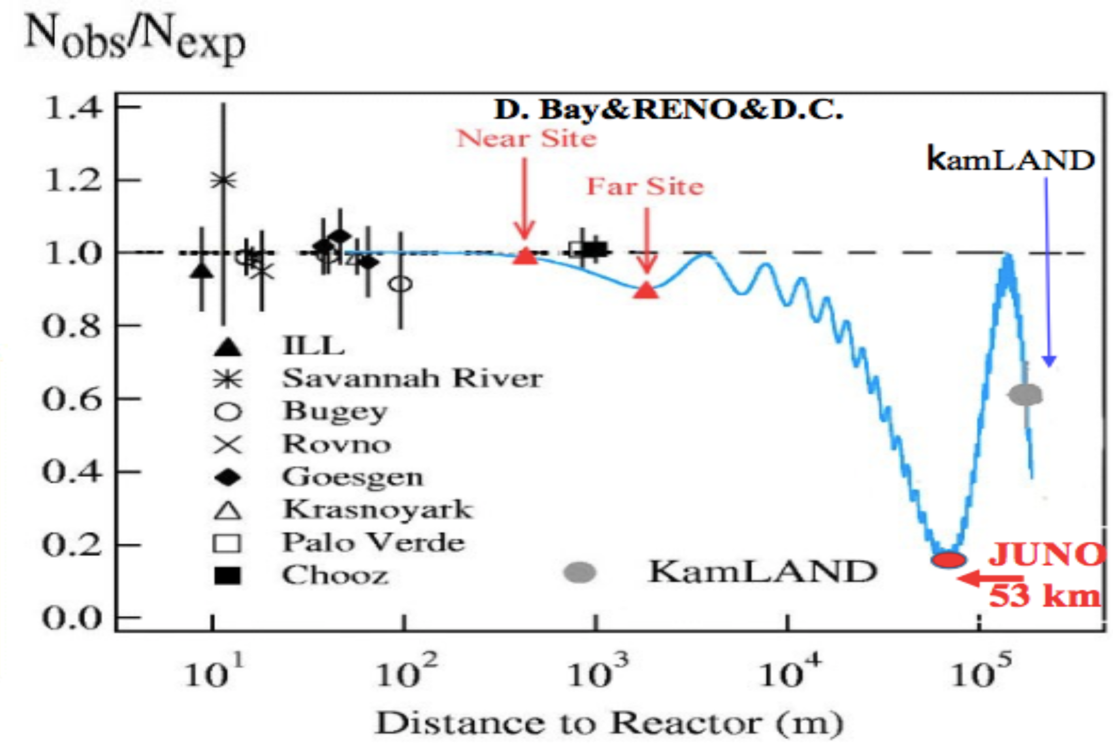
Daniel Baumann, TASI Lectures on Inflation

Neutrino masses

Neutrino oscillation requiring massive neutrinos







Solar Neutrino oscillations

$$\theta_{12}$$

Atmospheric Neutrino Oscillations

$$\theta_{23}$$

Reactor Neutrino Oscillations

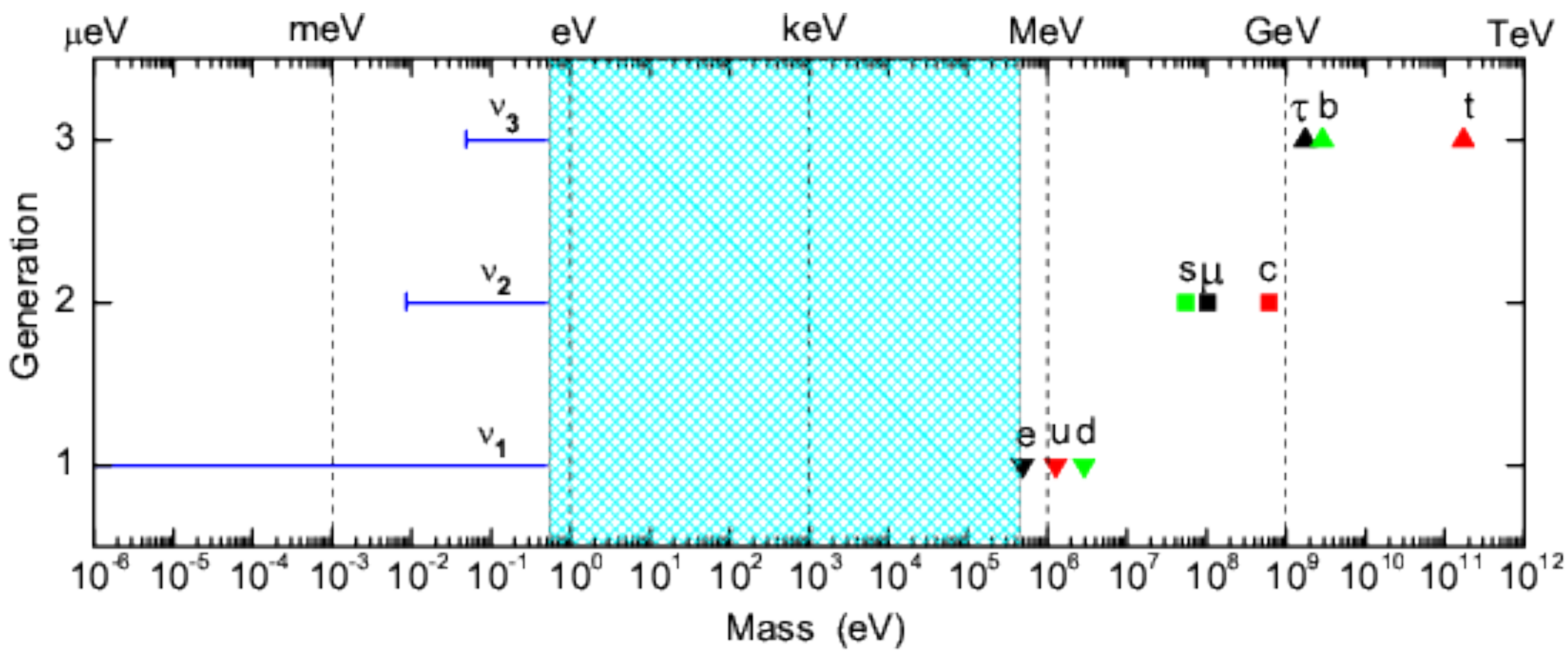
$$\theta_{13}$$

$$\Delta m^2_{21} \simeq 7.42 \times 10^{-5}\ \mathrm{eV}^2 \qquad |\Delta m^2_{13}| \approx |\Delta m^2_{23}| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$

At least a neutrino mass larger or similar to 0.05 eV

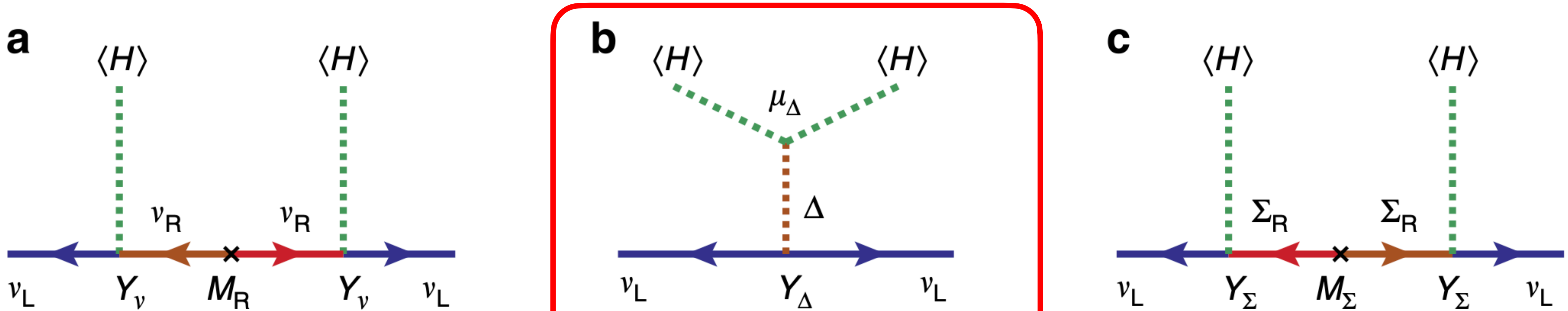
Neutrino masses vs other fermion masses

A large hierarchy comparing with other fermion masses



Three types of seesaw model(tree level)

Tommy Ohlsson, Shun Zhou, Nature Commun. 5 (2014) 5153



$M_\nu = -\langle H\rangle^2 Y_\nu M_R^{-1} Y_\nu^T$

$M_\nu = \langle H\rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$

$M_\nu = -\langle H\rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T$

SM + 3 singlets fermions

SM + 1 triplet Higgs

SM + 3 triplet fermions

Providing a minimal framework

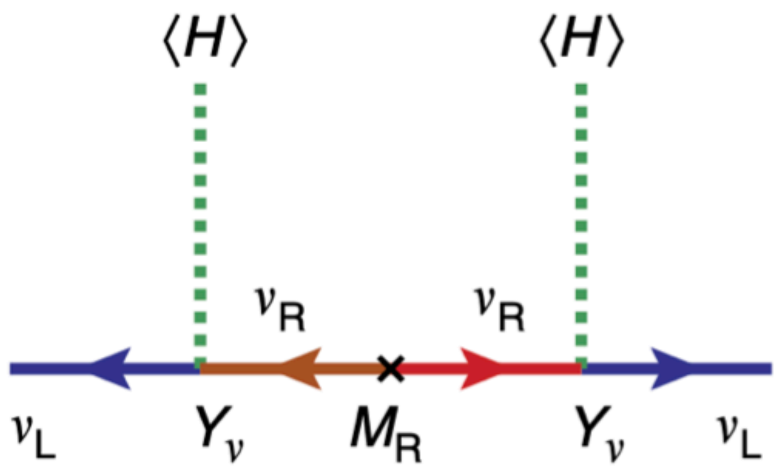
Origin of neutrino masses: type I seesaw

加入三个单态中性右手中微子 N(1, 1, 0)

$$\mathcal{L} = \mathcal{L}_{SM} + y_\nu \tilde{H}\bar{L}N - M_R \bar{N}^c N$$

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{1}{2}\frac{y_\nu^2 < H >^2}{M_R}$$



$M_\nu = -\langle H\rangle^2 Y_\nu M_R^{-1} Y_\nu^T$

Origin of neutrino masses: type II seesaw

$$H(2,1/2),\ \Delta(3,1),\ L(2,-1/2)$$

$$H = \begin{pmatrix} h^+ \\ h \end{pmatrix},\quad \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_{Yukawa} = \mathcal{L}^{\rm SM}_{\rm Yukawa} - \boxed{\frac{1}{2} y_{ij} \bar{L}^c_i \Delta L_j} + h.c.$$

$$\downarrow$$

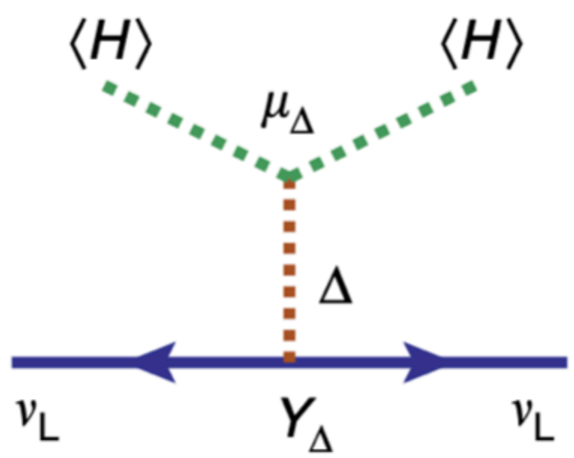
$$\frac{1}{2} y_{ij} \Delta^0 \bar{\nu}^c \nu + h.c.$$

- Giving neutrino mass matrix with vev of Delta
- at the same time Delta get a lepton number -2

$$H(2,1/2),\ \Delta(3,1),\ L(2,-1/2)$$

$$\begin{aligned} V(H,\Delta) &= -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \lambda_1 (H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) \\ &\quad + \lambda_2 (\mathrm{Tr}(\Delta^\dagger\Delta))^2 + \lambda_3 \mathrm{Tr}(\Delta^\dagger\Delta)^2 + \lambda_4 H^\dagger \Delta\Delta^\dagger H \\ &\quad \boxed{+ \left[\mu(H^T i\sigma^2 \Delta^\dagger H) + h.c.\right] + \ldots} \end{aligned}$$

U(1)L breaking term



$$M_\nu = \langle H\rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$$

$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\mathrm{EW}}^2}{2 m_\Delta^2}$$

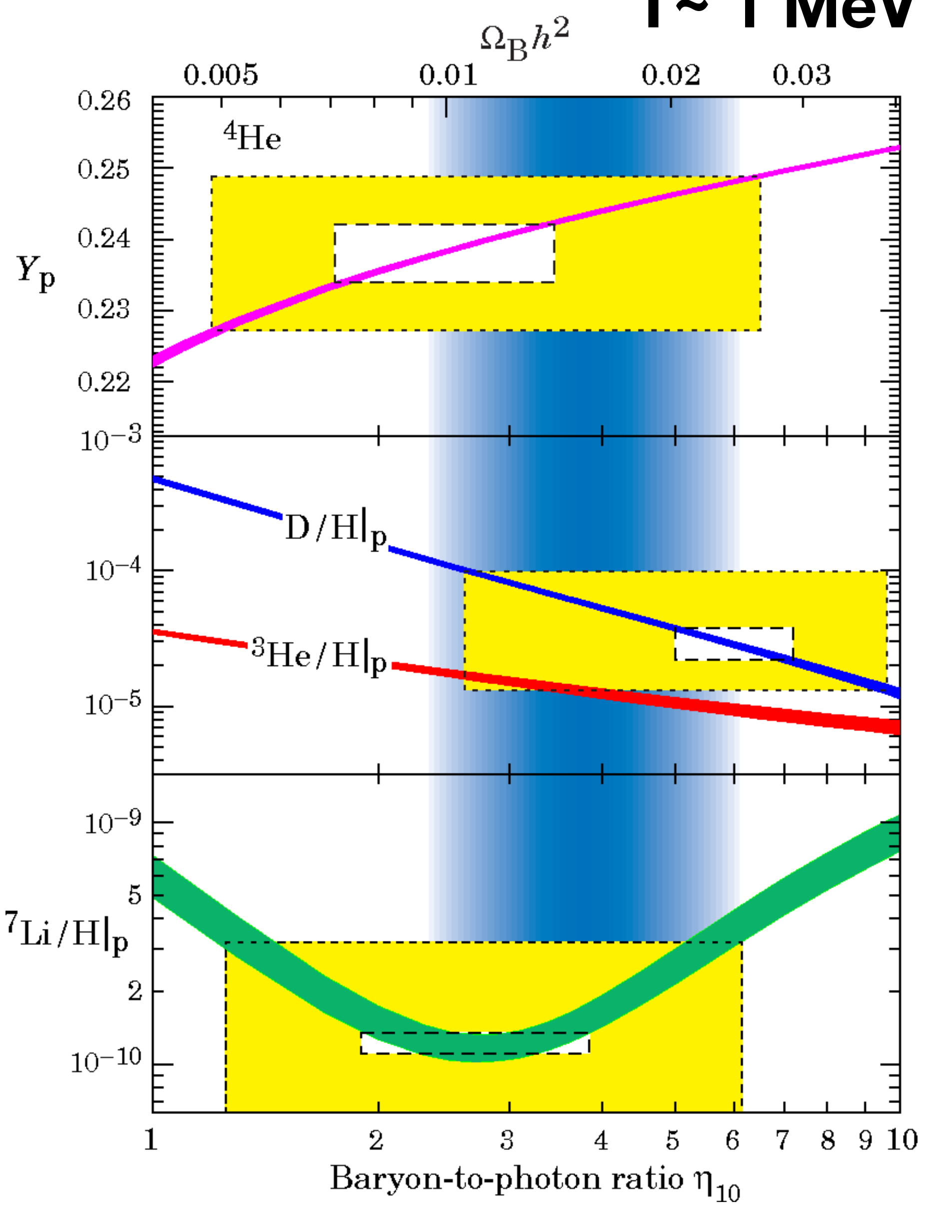
$$\mathcal{O}(1)\ \mathrm{GeV} > |\langle \Delta^0 \rangle| \gtrsim 0.05\ \mathrm{eV}$$

Neutrino masses connecting another important problem:

Baryon asymmetry of our universe

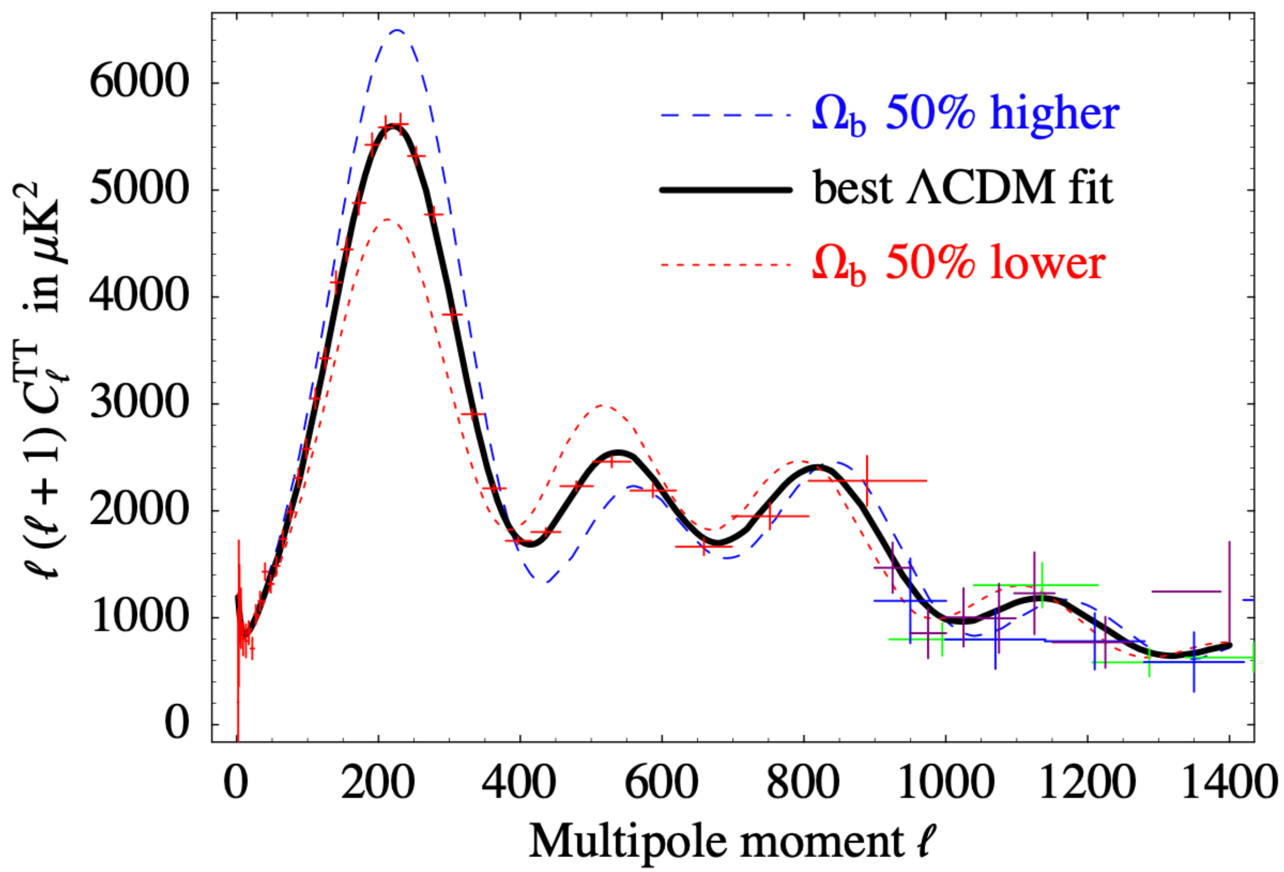
Baryon asymmetry of our universe

T~ 1 MeV



BBN

T~ 1 eV



$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

How to generate baryon asymmetry?

Assuming no baryon asymmetry in the beginning
(if any, diluted by inflation)

Sakharov conditions

1. B number violation
2. C and CP violation
3. Out of thermal equilibrium

SM has (1) (2) but not enough CP violation, (3) does not

Three popular ways to generate baryon asymmetry

- Electroweak baryogenesis

Rubakov and Shaposhnikov, 1996'
D. E. Morrissey and M. J. Ramsey-Musolf, 2012'

First order phase transition (adding scalars) + additional $\not{C}P$

- Baryogenesis via leptogenesis

$$n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$$

sphaleron process

Fukugita and Yanagida, 1986'

Type I seesaw: SM + 3 right-handed neutrinos

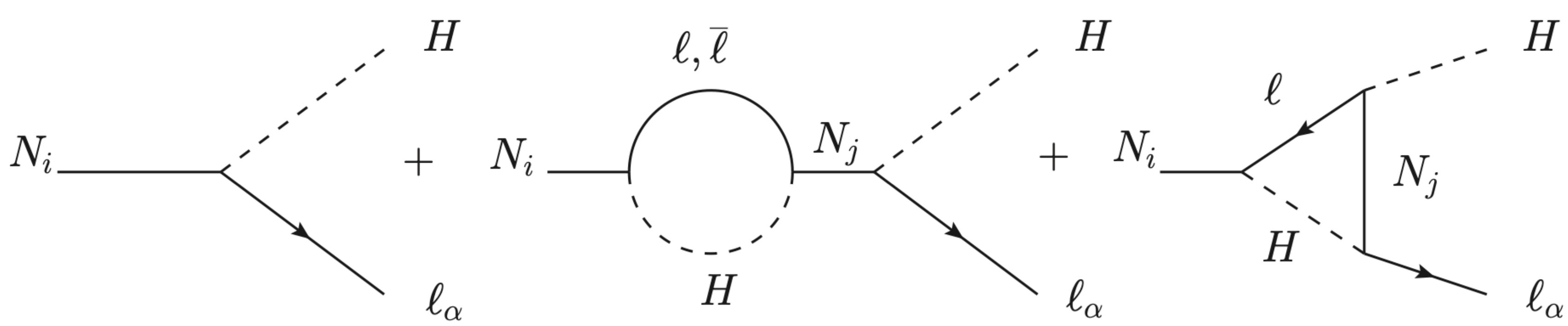
- Baryogenesis from Affleck-Dine mechanism

Affleck and Dine, 1985'

Baryogenesis via leptogenesis

Baryogenesis Without Grand Unification (**3980 citations**), Fukugita and Yanagida, 1986'

$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{R_i}}\partial\!\!\!/ N_{R_i} - \left(\frac{1}{2}M_i\overline{N^c_{R_i}}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell^a_\alpha H^b + h.c.\right)$$



$$\epsilon_{i\alpha} = \frac{\gamma\left(N_i \to \ell_\alpha H\right) - \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}{\sum_\alpha \gamma\left(N_i \to \ell_\alpha H\right) + \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}$$

$$\epsilon_i \equiv \sum_\alpha \epsilon_{i\alpha} = \frac{1}{8\pi}\frac{1}{(Y^\dagger Y)_{ii}}\sum_{j\neq i}\mathrm{Im}\left[(Y^\dagger Y)^2_{ji}\right] g\left(\frac{M_j^2}{M_i^2}\right)$$

$$g(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$$

Baryogenesis via leptogenesis

$$Y_{\Delta B} \simeq \frac{135\zeta(3)}{4\pi^4 g_*}\epsilon \times \eta \times C$$

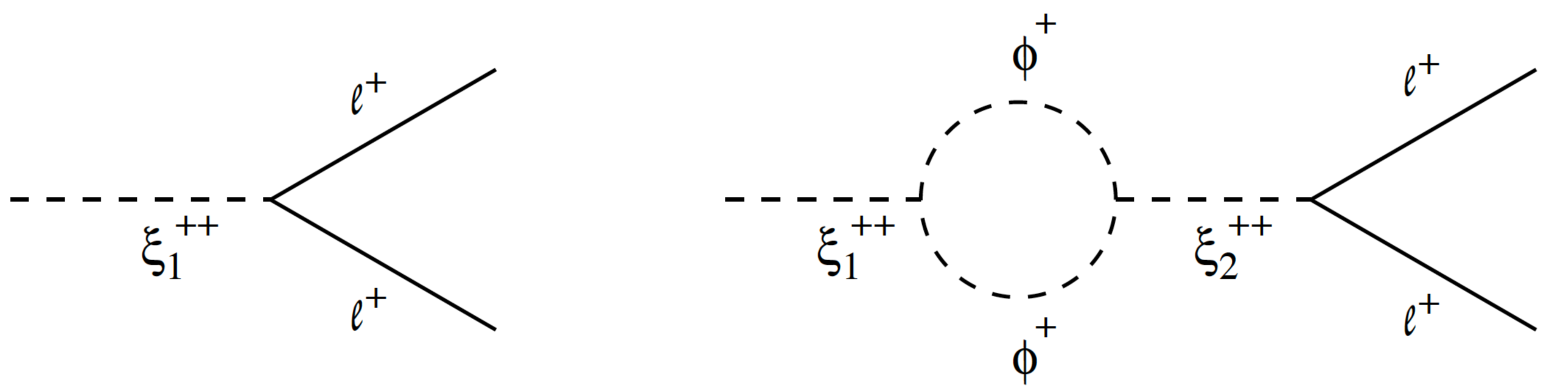
- epsilon is the efficiency of right handed neutrino decay
- eta is the wash out factor: when the N leaves thermal equilibrium
- C is the conversion rate $\quad n_B = \frac{28}{79}(\mathcal{B}-\mathcal{L})_i$
- N mass generally > 10^8 GeV, difficult to probe

Can type II seesaw realize leptogenesis?

Leptogenesis from type II seesaw?

Type II seesaw Neutrino Masses and Leptogenesis with Heavy Higgs Triplets (**533 citations**), E. Ma, U. Sarkar, Phys.Rev.Lett. 80 (1998) 5716-5719

$M \sim 10^{13}$ GeV



$$\delta_i = 2\left[B\left(\psi_i^- \to ll\right) - B\left(\psi_i^+ \to l^c l^c\right)\right]$$

$$\delta_i = \frac{Im\left[\mu_1 \mu_2^* \sum_{k,l} y_{1kl} y_{2kl}^*\right]}{8\pi^2 (M_1^2 - M_2^2)} \left[\frac{M_i}{\Gamma_i}\right]$$

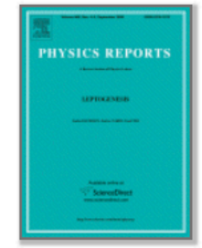
At least two triplet Higgs are needed to generate the baryon asymmetry

Leptogenesis from type II seesaw?

ELSEVIER

Physics Reports

Volume 466, Issues 4–5, September 2008, Pages 105-177



Leptogenesis (1,016 citations)

Sacha Davidson [a], Enrico Nardi [b, c], Yosef Nir [d, 1]

To calculate ϵ_T, one should use the Lagrangian terms given in eqn (2.15). While a single triplet is enough to produce three light massive neutrinos, there is a problem in leptogenesis if indeed this is the only source of neutrinos masses: The asymmetry is generated only at higher loops and in unacceptably small.

It is still possible to produce the required lepton asymmetry from a single triplet scalar decays if there are additional sources for the neutrino masses, such as type I, type III, or type II contributions from

One triplet Higgs can not generate leptogenesis!
but it is enough to give neutrino masses!

Leptogenesis from type II seesaw

PHYSICAL REVIEW LETTERS **128,** 141801 (2022)

Affleck-Dine Leptogenesis from Higgs Inflation

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We find that the triplet Higgs of the type-II seesaw mechanism can simultaneously generate the neutrino masses and observed baryon asymmetry while playing a role in inflation. We survey the allowed parameter space and determine that this is possible for triplet masses as low as a TeV, with a preference for a small vacuum expectation value for the triplet $v_\Delta < 10$ keV. This requires that the triplet Higgs must decay dominantly into the leptonic channel. Additionally, this model will be probed at the future 100 TeV collider, upcoming lepton flavor violation experiments such as Mu3e, and neutrinoless double beta decay experiments. Thus, this simple framework provides a unified solution to the three major unknowns of modern physics—inflation, the neutrino masses, and the observed baryon asymmetry—while simultaneously providing unique phenomenological predictions that will be probed terrestrially at upcoming experiments.

Affleck-Dine mechanism

Assuming phi is a complex scalar with B/L charge and potential is "flat"

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + [c_{n,m}\phi^n(\phi^*)^m + h.c] \quad m \neq n$$

$$\downarrow$$

(B/L violation)

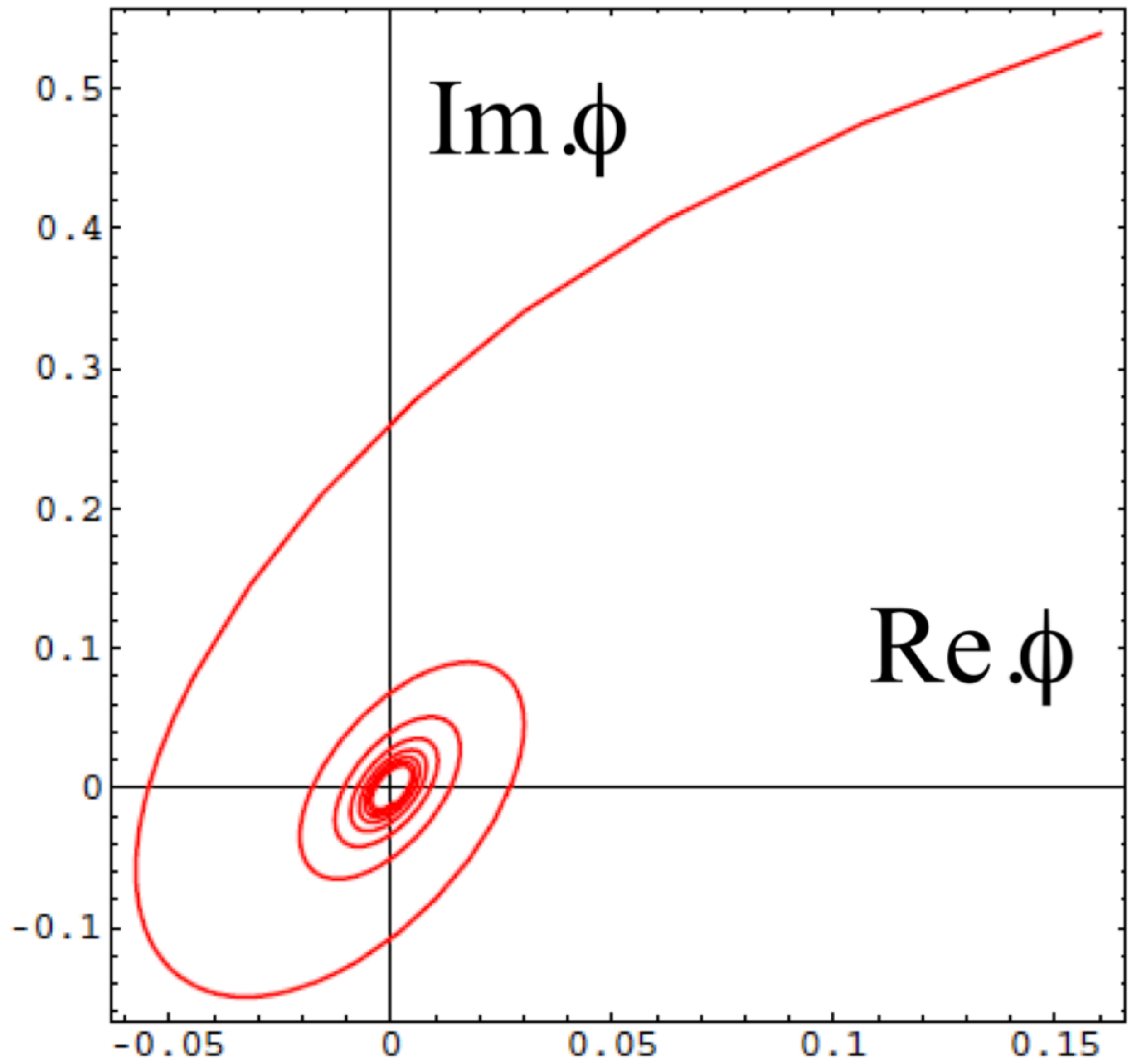
$$j_B^\mu = i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)$$

$\boxed{\phi \text{ is spatially constant}}$

$$n_B = i(\phi^*\dot{\phi} - \phi\dot{\phi}^*) = \rho_\phi^2\dot{\theta} \qquad \phi = \frac{1}{\sqrt{2}}\rho_\phi e^{i\theta}$$

A motion of theta will generate baryon number

Affleck-Dine mechanism



Affleck-Dine mechanism

Three conditions for Affleck-Dine mechanism

	Type II seesaw
● Scalar particle with initial displaced vacuum	?
● Scalar particle taking B/L charge	✓
● Small B/L violation term in the potential	✓

Combing the idea of inflation with A-D mechanism

Three conditions for Affleck-Dine mechanism

If the scalar plays the role of inflation	Type II seesaw
● Scalar particle with initial displaced vacuum	✓
● Scalar particle taking B/L charge	✓
● Small B/L violation term in the potential	✓

SM+Type II seesaw

To be consistent with inflation, we need add non-minimal couplings

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}M_P^2 R - \boxed{f(H,\Delta)R} - g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H)$$
$$-g^{\mu\nu}(D_\mu \Delta)^\dagger(D_\nu \Delta) - V(H,\Delta) + \mathcal{L}_{\text{Yukawa}}$$

$$h \equiv \frac{1}{\sqrt{2}}\rho_H e^{i\eta} \qquad \Delta^0 \equiv \frac{1}{\sqrt{2}}\rho_\Delta e^{i\theta}$$

$$F(H,\Delta) = \xi_H|h|^2 + \xi_\Delta|\Delta^0|^2 = \frac{1}{2}\xi_H\rho_H^2 + \frac{1}{2}\xi_\Delta\rho_\Delta^2$$

SM+Type II seesaw

During inflation(Oleg Lebedev and Hyun Min Lee, arXiv:1105.2284)

$$\frac{\rho_H}{\rho_\Delta} \equiv \tan\alpha = \sqrt{\frac{2\lambda_\Delta \xi_H - \lambda_{H\Delta}\xi_\Delta}{2\lambda_H \xi_\Delta - \lambda_{H\Delta}\xi_H}}$$

$$\rho_H = \varphi \sin\alpha, \;\; \rho_\Delta = \varphi \cos\alpha$$
$$\xi \equiv \xi_H \sin^2\alpha + \xi_\Delta \cos^2\alpha$$

Similar to SUSY case, but mixing with a general angle

SM+Type II seesaw

Finally the model can be simplified as

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_p^2}{2}R - \frac{\xi}{2}\varphi^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$
$$-\frac{1}{2}\varphi^2\cos^2\alpha\ g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\varphi,\theta)$$

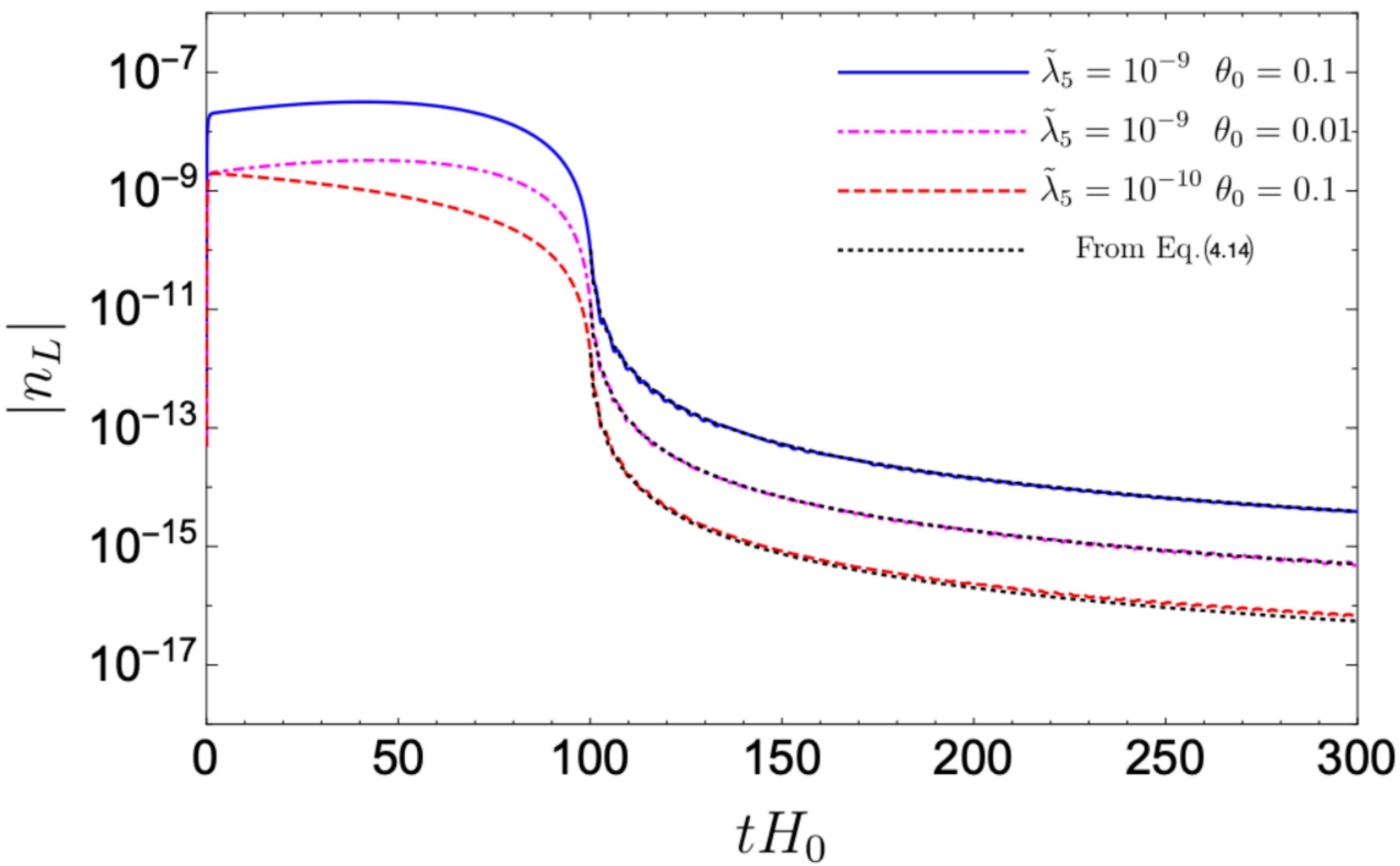
$$V(\varphi,\theta) = \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2\varphi^3\left(\tilde{\mu} + \frac{\tilde{\lambda}_5}{M_p}\varphi^2\right)\cos\theta$$

We need keep the theta term, because

$$n_L = Q_L\varphi^2\dot{\theta}\cos^2\alpha$$

Lepton number generation

$$\xi = 300,\ \lambda = 4.5 \cdot 10^{-5}$$
$$\chi_0 = 6.0 M_p,\ \dot{\chi}_0 = 0,\ \text{and}\ \theta_0 = 0$$



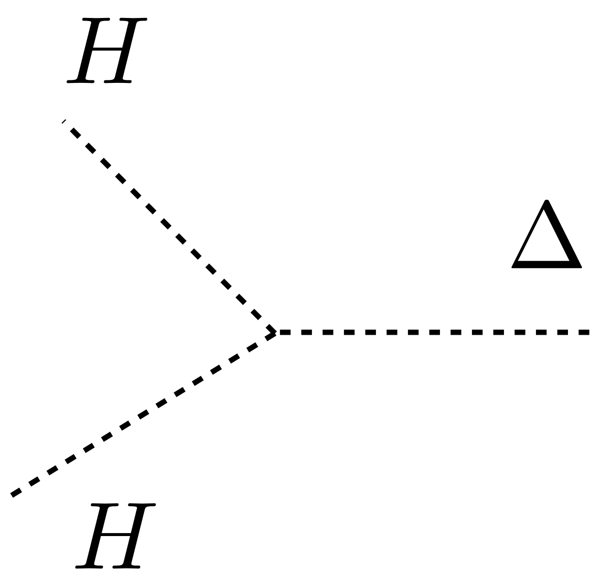
SM+Type II seesaw

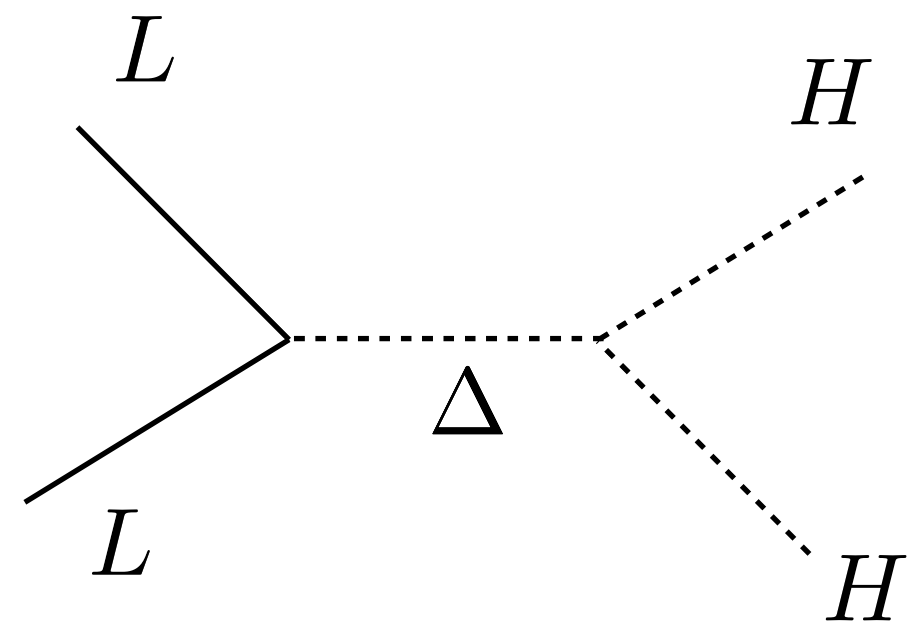
$$T_{\rm reh} \approx 2.2 \cdot 10^{14}\ \text{GeV}$$

$$\eta_B = \frac{n_B}{s}\bigg|_{\rm reh} = \eta_B^{\rm obs}\left(\frac{|n_{L_{\rm end}}|/M_p^3}{1.3\cdot 10^{-16}}\right)\left(\frac{g_*}{112.75}\right)^{-\frac{1}{4}}$$

$$\tilde{\lambda}_5 = 7\cdot 10^{-15}\ \text{for}\ \theta_0 = 0.1$$

Wash out process



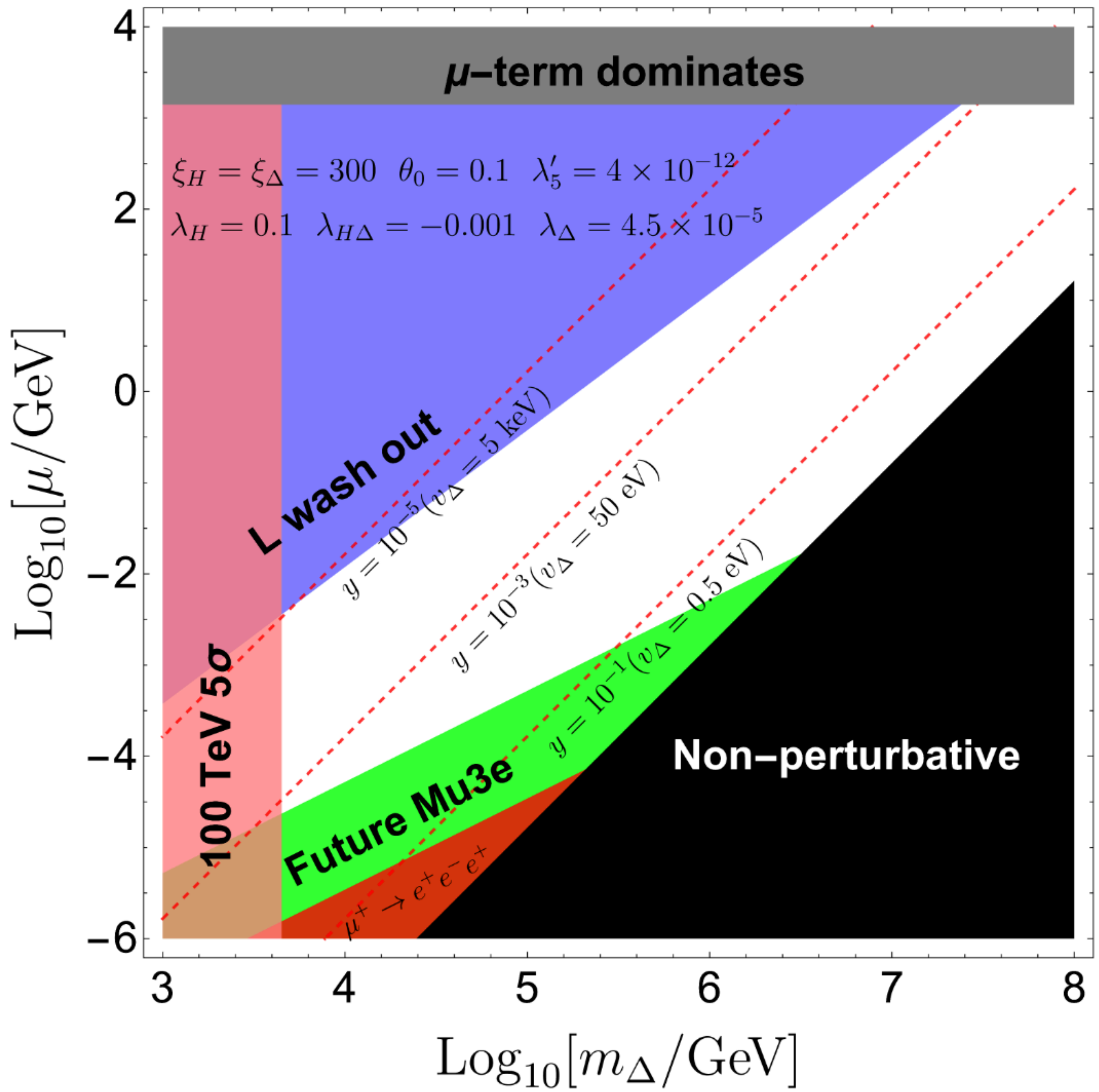


$$\frac{\mu^2}{8\pi m_\Delta} < H(m) = \frac{m_\Delta^2}{M_P}$$

$$m_\Delta < 10^{12}\ \text{GeV}$$

A small mu term is preferred

SM+Type II seesaw

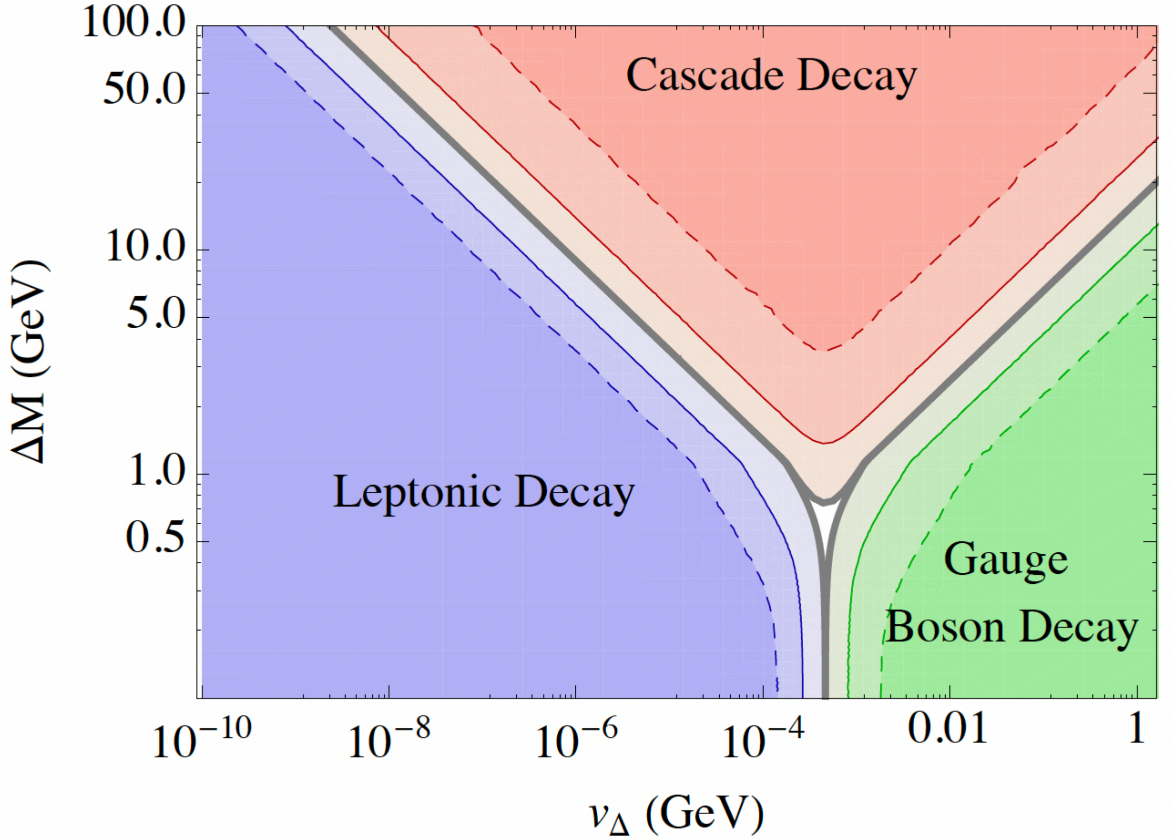


- Triplet Higgs could be as light as TeV
- Vacuum value < 10 keV, traditional type II seesaw < 1 GeV

Phenomenology implications I: collider physics

Decay of the doubly-charged Higgs

$$\Delta M = m_{\Delta^{++}} - m_{\Delta^{+}}$$



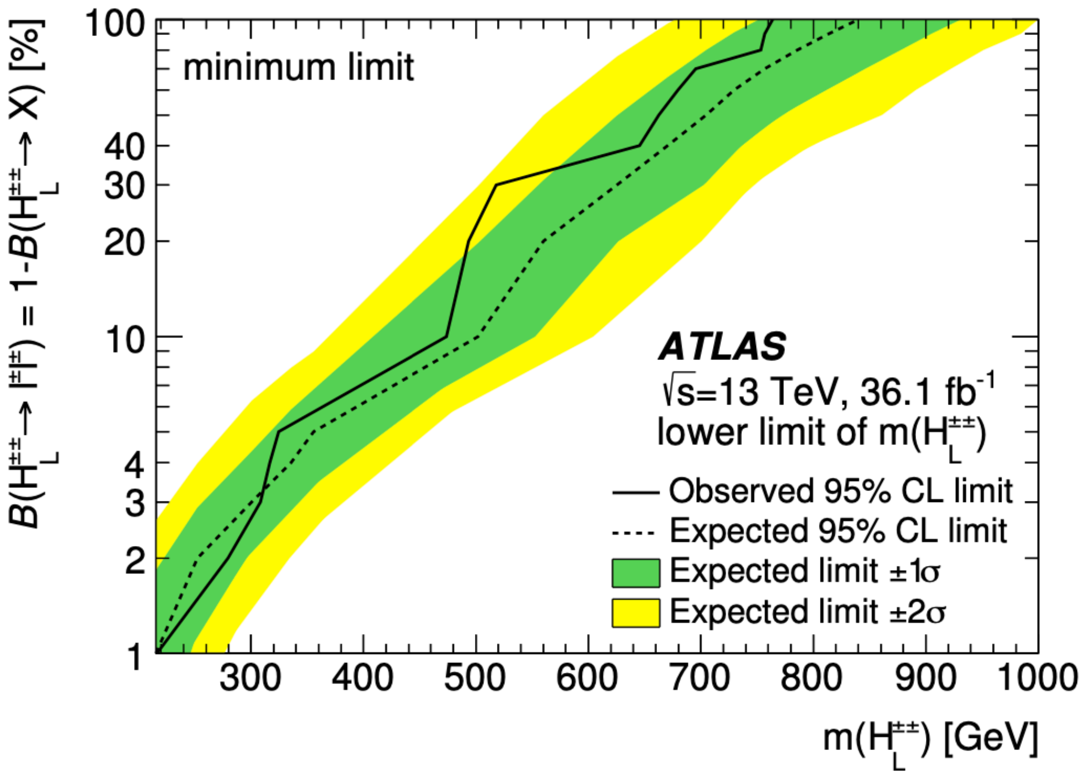
For v > 1 MeV, mainly decay gauge bosons

For v < 0.1 MeV, mainly decay leptons

Phenomenology implications I: collider physics

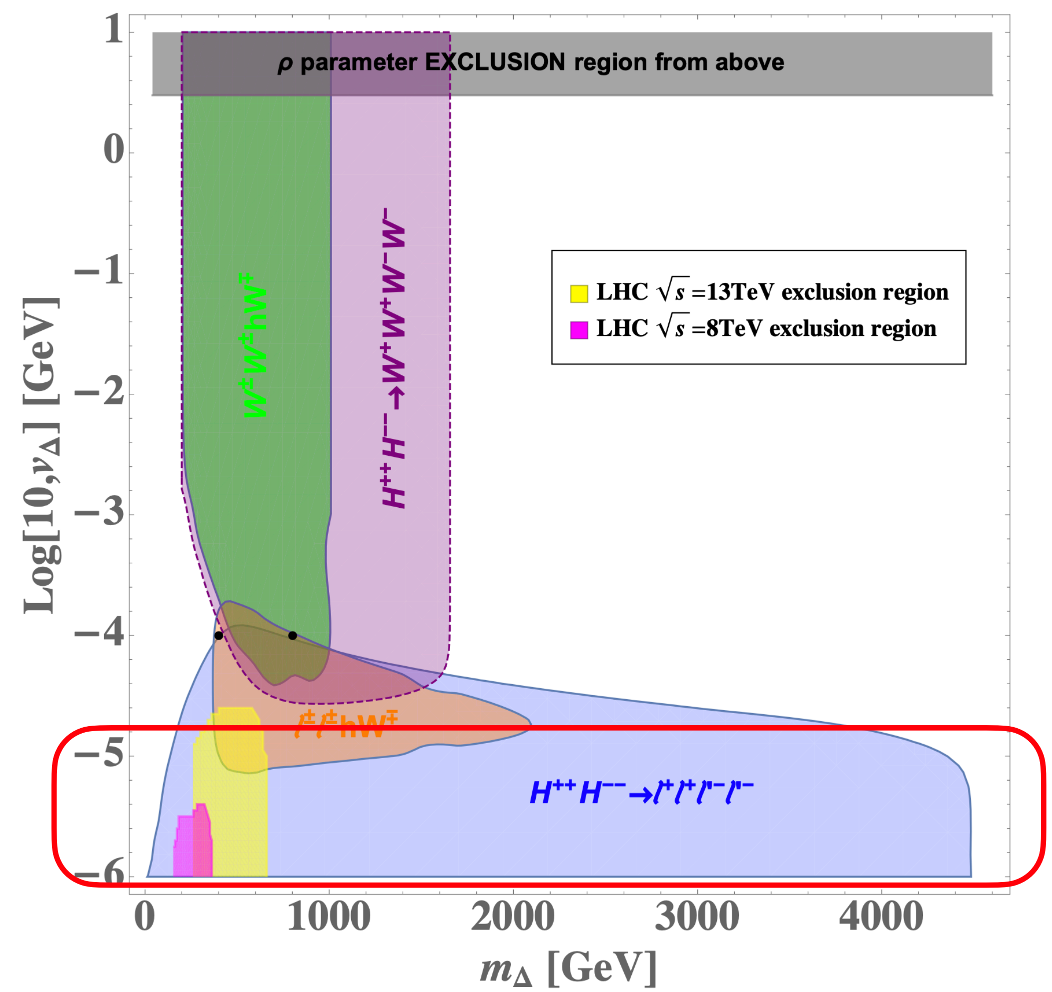
Current limit from LHC

ATLAS, Eur. Phys. J. C 78 (2018) 199



Future reach

Y. Du, A. Dunbrack, M. J. Ramsey-Musolf, J. Yu, JHEP01(2019)101

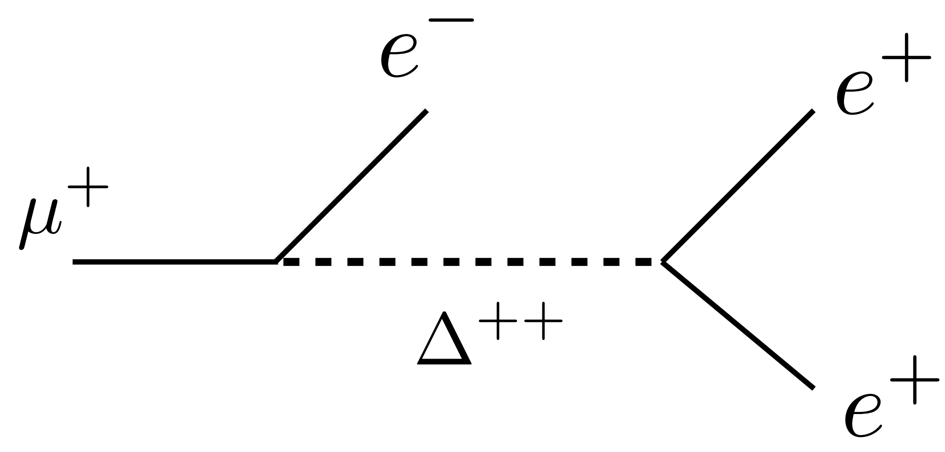


5 sigma discover region @100 TeV collider

Smoking gun: observing doubly-charged Higgs from leptonic channel

Phenomenology implications II: flavor physics

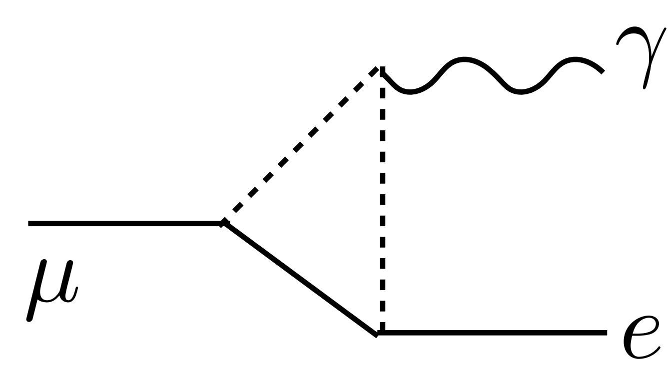
$$\mathcal{B}(\mu^+ \to e^+ e^- e^+) = \frac{|y_{\mu e} y^\dagger_{ee}|^2}{16 G_F^2 m^4_{\Delta^{++}}}$$



$$\mathcal{B}(\mu^+ \to e^+ e^- e^+) \leqslant 1.0 \times 10^{-12}$$

$$\mathcal{B}(\mu \to e\gamma) \simeq \frac{\alpha}{3072\pi} \frac{\left|(y^\dagger y)_{e\mu}\right|^2}{G_F^2} \left(\frac{1}{m^2_{\Delta^+}} + \frac{8}{m^2_{\Delta^{++}}} \right)^2$$

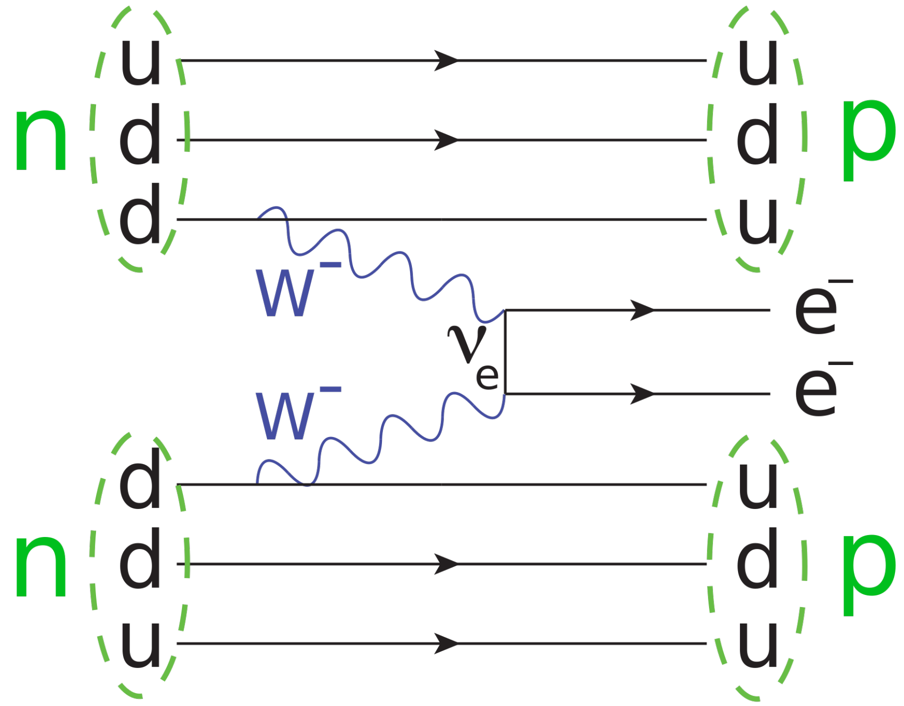
$$\mathcal{B}(\mu \to e\gamma) < 4.2 \times 10^{-13}$$

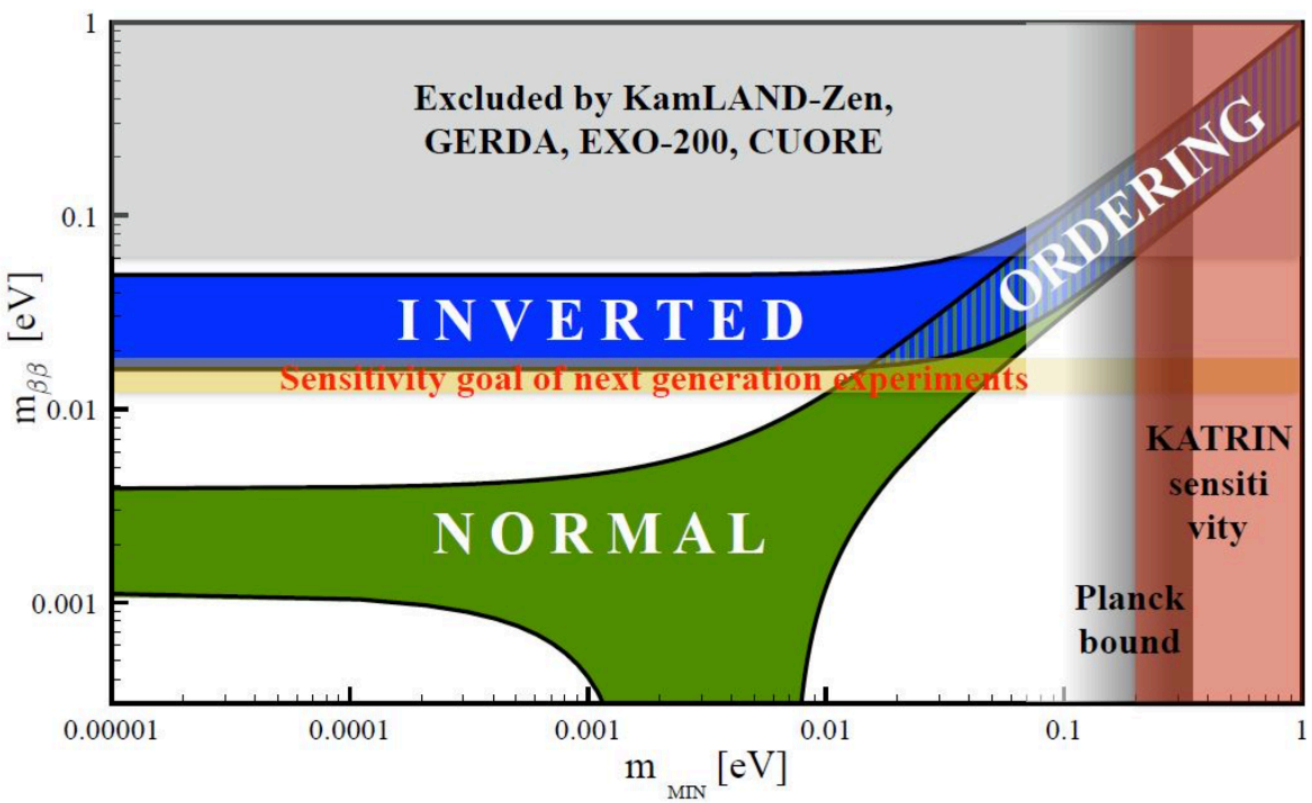


Phenomenology implications III: neutrino physics

- Neutrino must be majorana type

Neutrinoless double beta decay

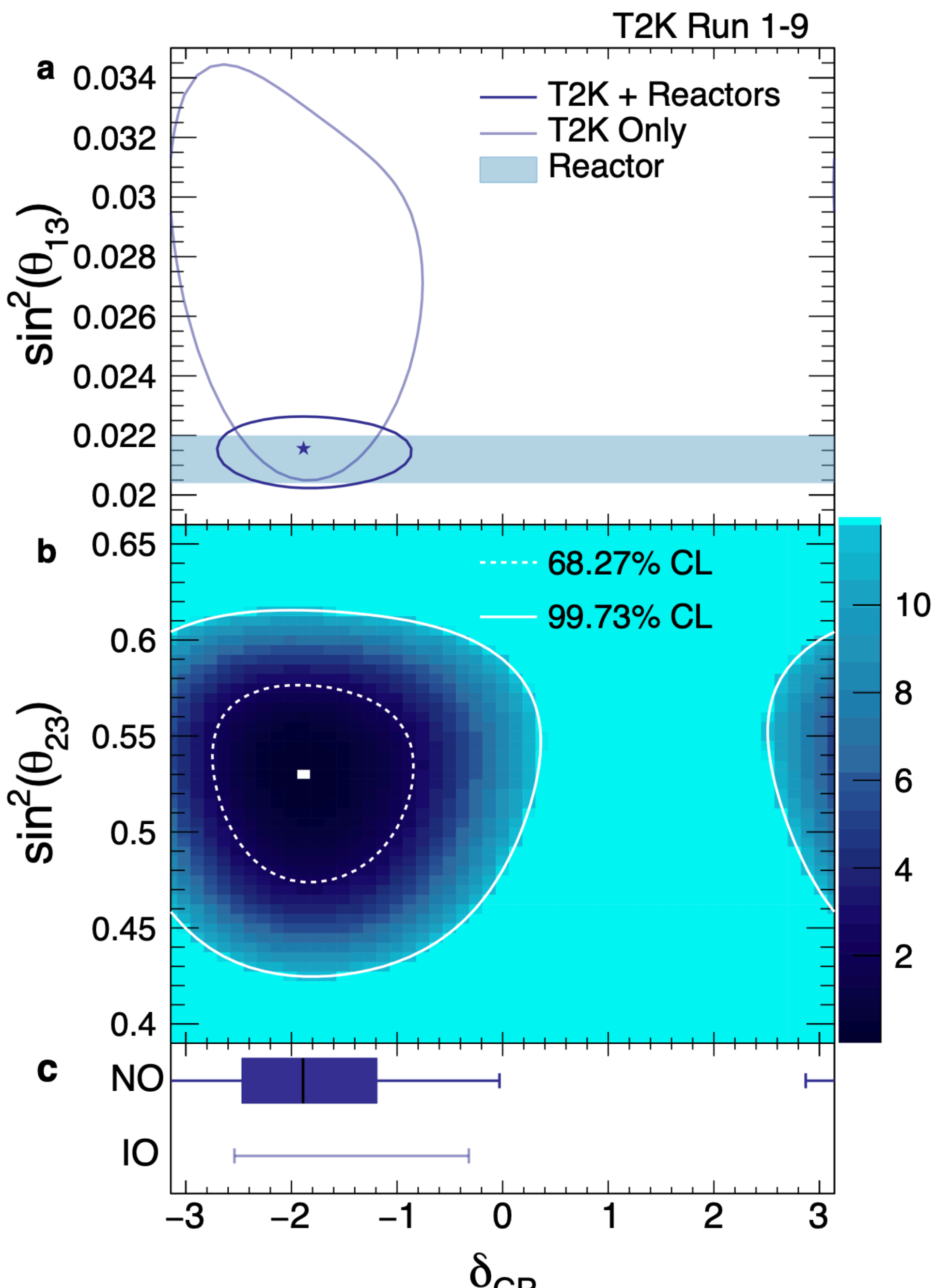




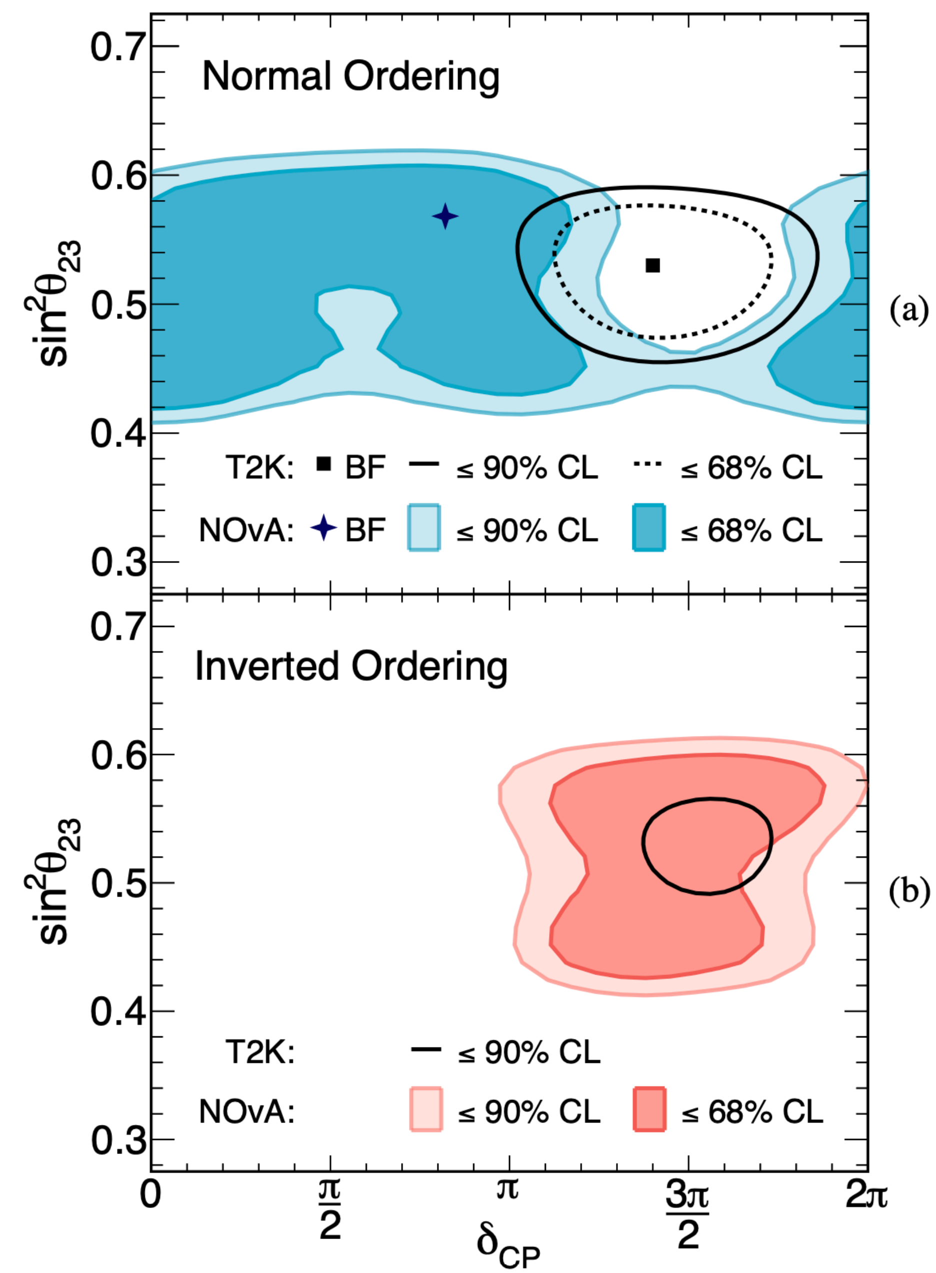
Phenomenology implications III: neutrino physics

- CP violation in neutrino sector

T2K, 19'



Nova, 21'



Leptogenesis even without CP violation

Phenomenology implications IV: cosmological signal

- Tensor to scalar ratio, within the future reach of LiteBIRD

$$0.0033 < r < 0.0048$$

- Non-Gaussian signature, model dependent

- Imprint of isocurvature signature from baryon matter

- Gravitational wave from preheating

Summary

- One simple extension of SM, three problems can be solved: inflation, baryogenesis and neutrino masses

- Unique signatures at collider, LFV violation, neutrino experiments and astronomy observations

- More studies in future

Thanks

Summary

- One simple extension of SM, three problems can be solved: inflation, baryogenesis and neutrino masses

- Unique signatures at collider, LFV violation, neutrino experiments and astronomy observations

Back up

Slow-roll inflation

Assuming a scalar field, with equation of motion

$$\boxed{\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \\ &H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \end{aligned}}$$

Slow roll condition $\dot{\phi}^2 \ll V(\phi) \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|\,,\ |V_{,\phi}|$

$$\epsilon_{\rm v}(\phi) \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_{\rm v}(\phi) \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \qquad \epsilon_{\rm v}, |\eta_{\rm v}| \ll 1$$

$$\begin{aligned} H^2 &\approx \frac{1}{3}V(\phi) \approx \text{const.} \\ \dot{\phi} &\approx -\frac{V_{,\phi}}{3H}\,, \end{aligned} \longrightarrow \quad a(t) = a_0 e^{Ht} \qquad Ht \gtrsim 60$$

Daniel Baumann, TASI Lectures on Inflation

Slow-roll inflation

标量扰动

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_{pl}^4}\frac{1}{\epsilon_V}\Big|_{k=aH}$$

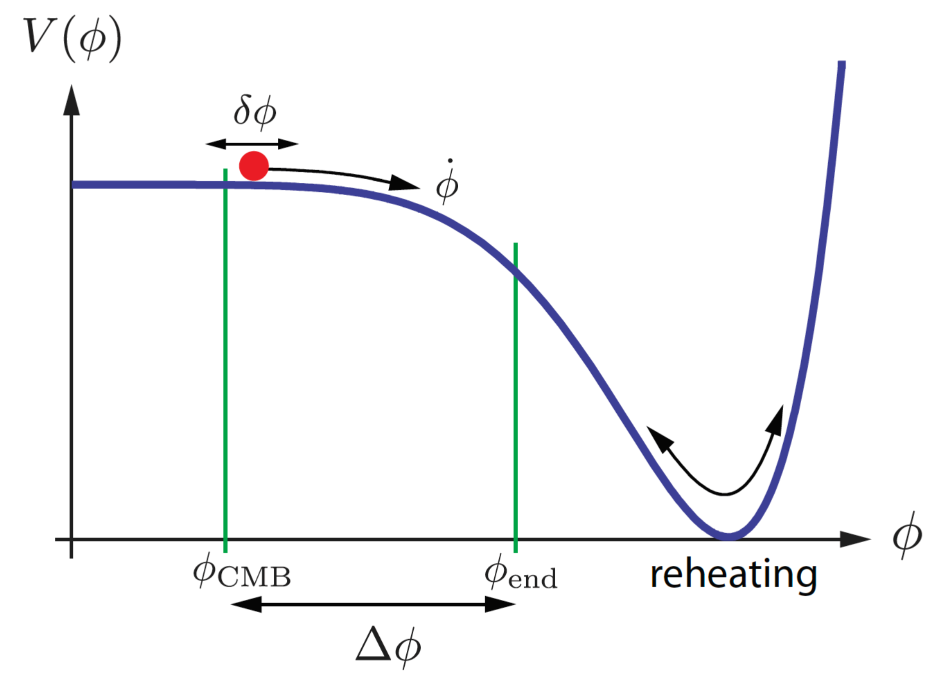
$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} = 2\eta_V - 6\epsilon_V$$

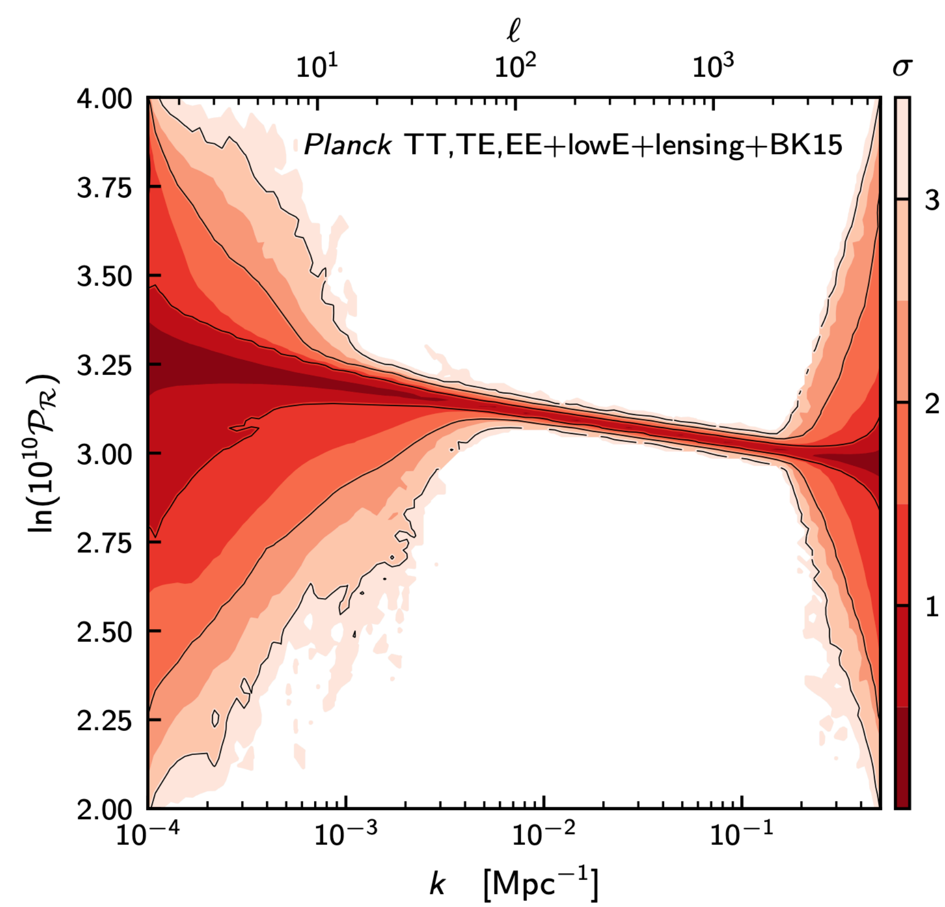
$n_s \simeq 0.965$ n=1 to be scale invariant

张量扰动

$$\Delta_t^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_{pl}^4}\Big|_{k=aH} \qquad r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

tensor-scalar ratio $r \lesssim 0.056$

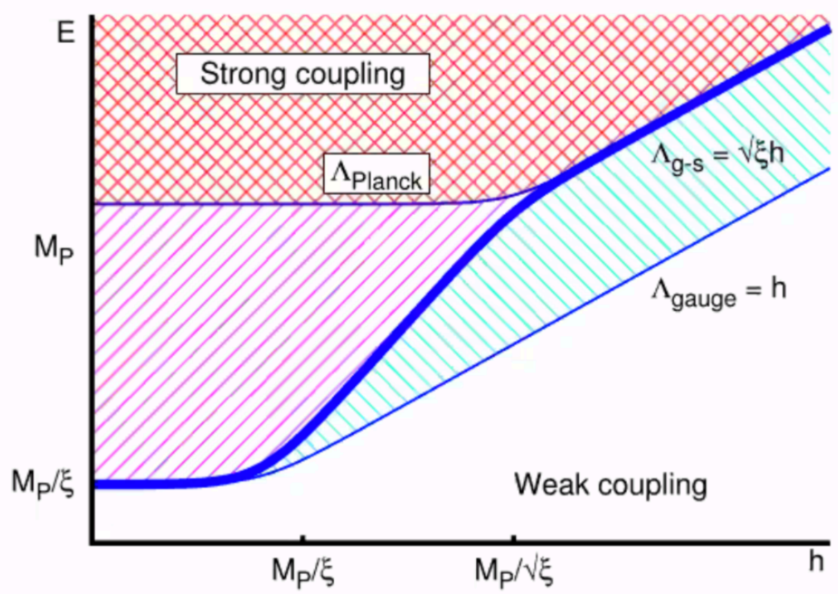




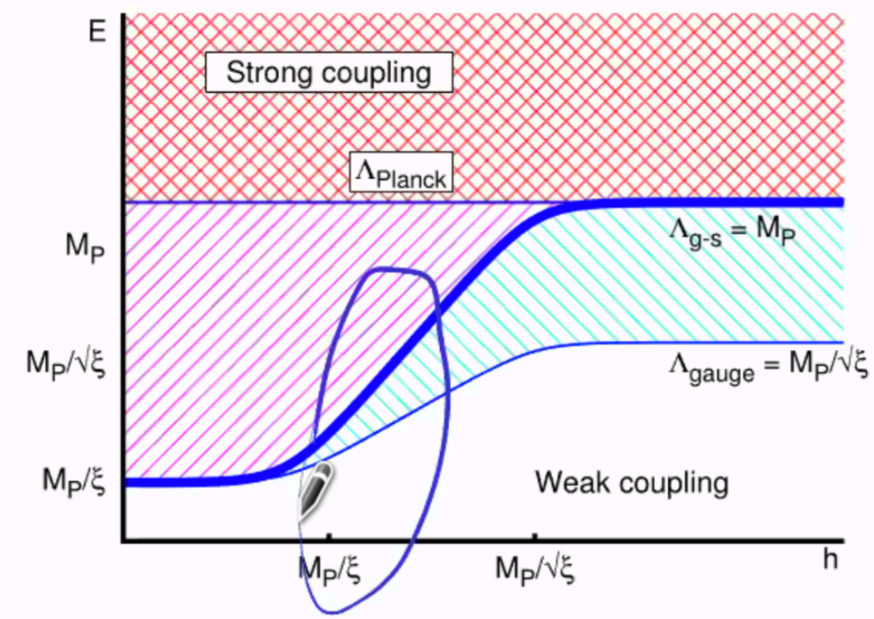
Unitary problem for Higgs inflation

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Relevant scales at inflation

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Problems during reheating

SM+Type II seesaw

$$\chi = \sqrt{\frac{3}{2}} M_p \log\left(1 + \frac{\xi_H |h|^2}{M_p^2} + \frac{\xi_\Delta (|\Delta^0|)^2}{M_p^2}\right) \quad \text{and} \quad \kappa = \frac{\rho_H}{\rho_\Delta}$$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left(1 + \frac{1}{6}\frac{\kappa^2+1}{\xi_H\kappa^2+\xi_\Delta}\right)(\partial_\mu\chi)^2 + \frac{M_p}{\sqrt{6}}\frac{(\xi_\Delta-\xi_H)\kappa}{(\xi_H\kappa^2+\xi_\Delta)^2}(\partial_\mu\chi)(\partial^\mu\kappa) + \frac{M_p^2}{2}\frac{\xi_H^2\kappa^2+\xi_\Delta^2}{(\xi_H\kappa^2+\xi_\Delta)^3}(\partial_\mu\kappa)^2 \,.$$

$\xi \equiv \xi_H + \xi_\Delta \gg 1$

$$\mathcal{L}_{\rm kin} = \frac{1}{2}(\partial_\mu\chi)^2 + \frac{M_p^2}{2}\frac{\xi_H^2\kappa^2+\xi_\Delta^2}{(\xi_H\kappa^2+\xi_\Delta)^3}(\partial_\mu\kappa)^2$$

SM+Type II seesaw

$$U = \frac{\lambda_H \kappa^4 + \lambda_{h\Delta}\kappa^2 + \lambda_\Delta}{4(\xi_H \kappa^2 + \xi_\Delta)^2} M_p^4$$

$$\begin{aligned}
&(1)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H > 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta > 0\ , \quad && \kappa = \sqrt{\frac{2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta}{2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H}} \\
&(2)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H > 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta < 0\ , \quad && \kappa = 0\ , \\
&(3)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H < 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta > 0\ , \quad && \kappa = \infty\ , \\
&(4)\ 2\lambda_H\xi_\Delta - \lambda_{h\Delta}\xi_H < 0\ ,\ 2\lambda_\Delta\xi_H - \lambda_{h\Delta}\xi_\Delta < 0\ , \quad && \kappa = 0, \infty\ .
\end{aligned}$$

Kappa is almost fixed during inflation

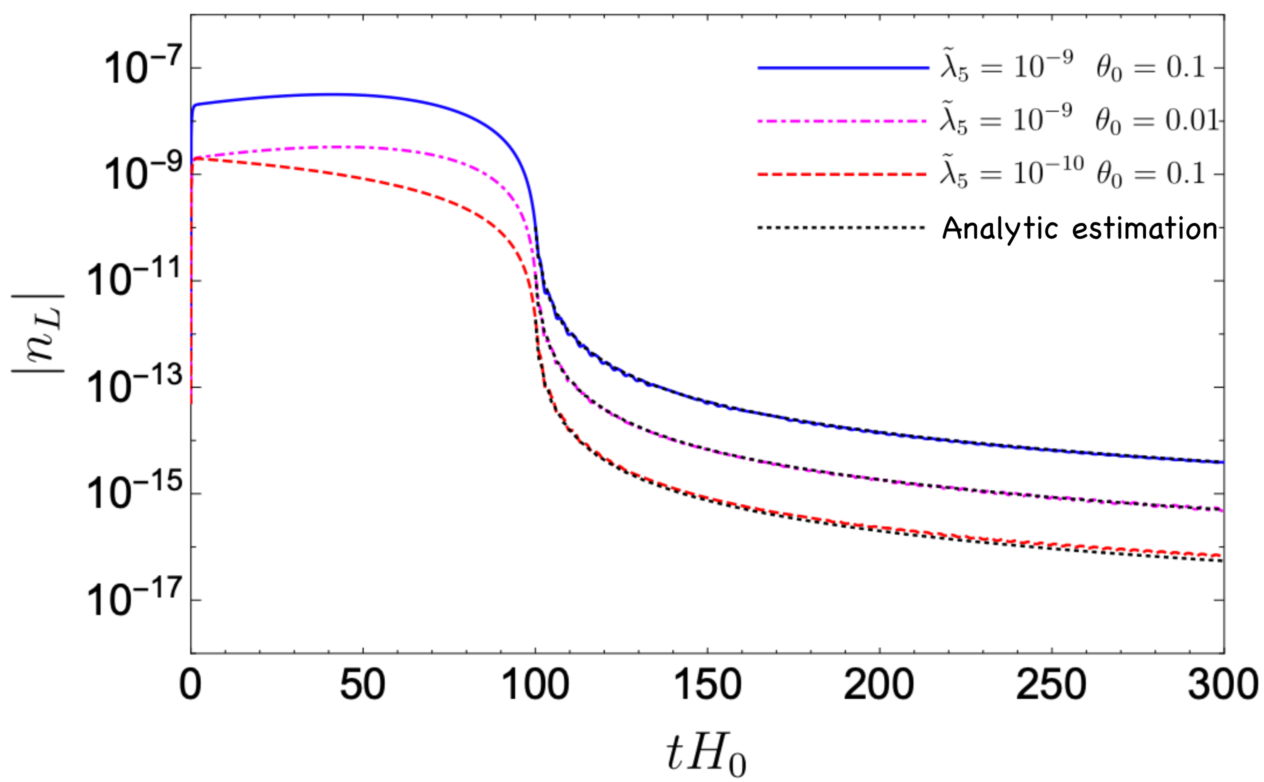
After inflation

$$\frac{d[a^3 f(\chi)\dot{\theta}]}{dt} = \frac{d[a^3 n_L/(Q_L \Omega^2)]}{dt} = a^3 U_{,\theta}$$

If U,theta red shifted faster than a^3, we can ignore the last term

$$n_L(t) = n_{L\mathrm{end}} \frac{\Omega^2(\chi)}{\Omega^2(\chi_{\mathrm{end}})} \left(\frac{a}{a_{\mathrm{end}}}\right)^{-3}$$

After inflation

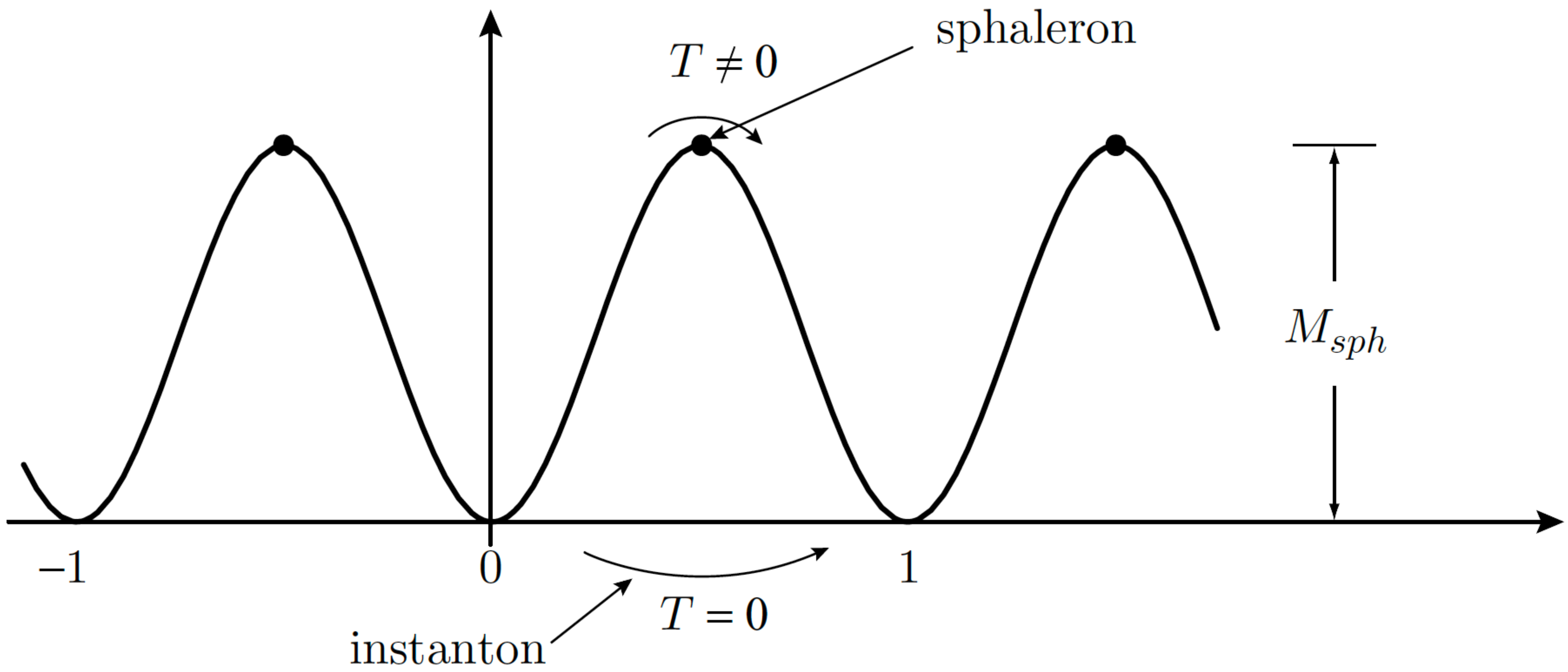


After inflation, nL is just conserved with additional factor Omega

Instanton, sphaleron process

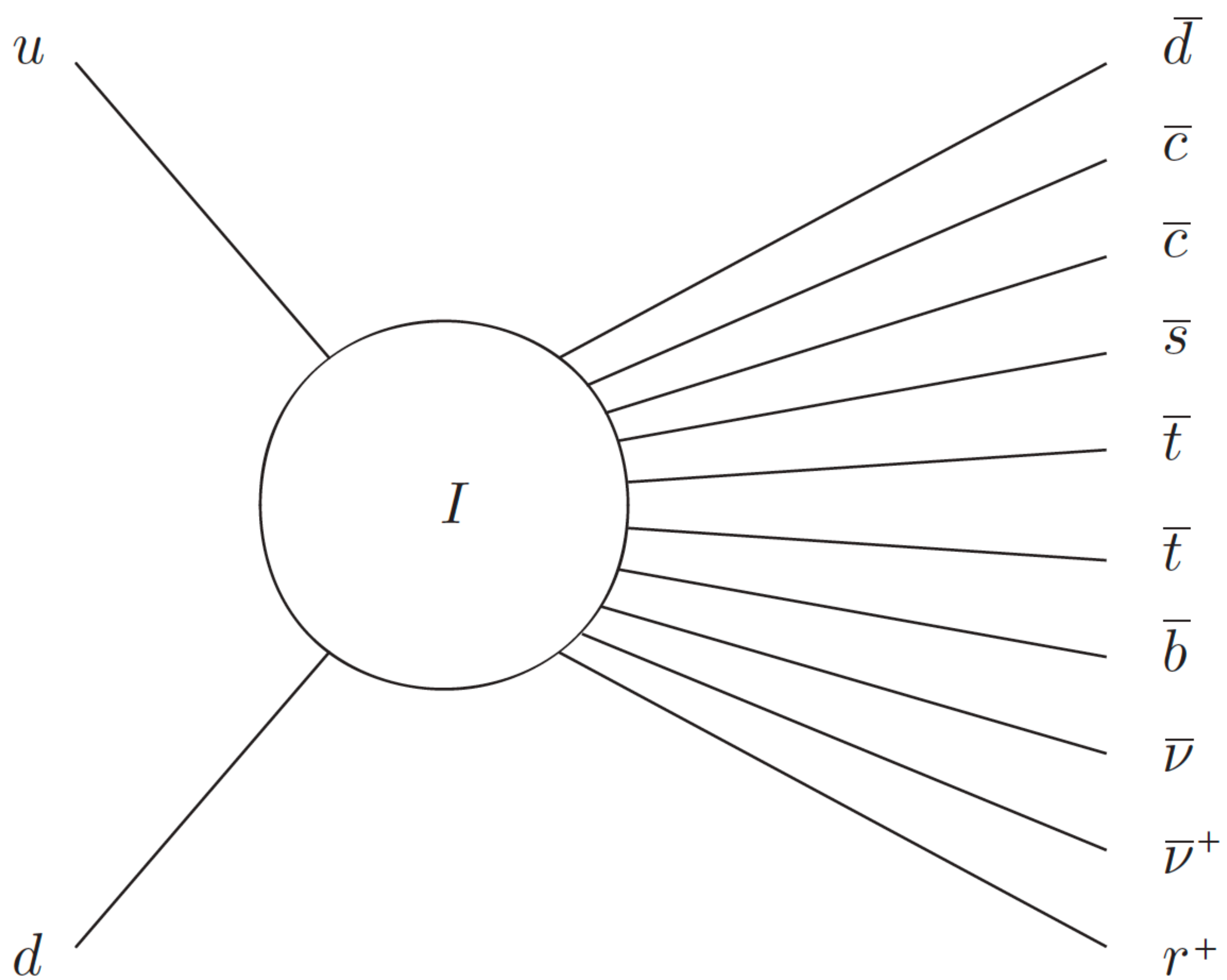
Effective for T> 100 GeV

$$\exp\left(-\frac{M_{sph}(T)}{T}\right) \sim \exp\left(-2\pi\frac{M_W(T)}{\alpha_w T}\right)$$



$$\Gamma \propto \exp\left(-\frac{4\pi}{\alpha}\right)$$

Instanton, sphaleron



Baryon asymmetry via leptogenesis

1. the sphaleron interactions themselves:

$$\boxed{\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0}$$

2. a similar relation for QCD sphalerons:

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0.$$

3. vanishing of the total hypercharge of the universe:

$$\sum_i \left(\mu_{q_i} - 2\mu_{\bar{u}_i} + \mu_{\bar{d}_i} - \mu_{\ell_i} + \mu_{\bar{e}_i}\right) + \frac{2}{N}\mu_H = 0$$

4. the quark and lepton Yukawa couplings give relations:

$$\mu_{q_i} - \mu_\phi - \mu_{d_j} = 0, \quad \mu_{q_i} - \mu_\phi - \mu_{u_j} = 0, \quad \mu_{\ell_i} - \mu_\phi - \mu_{e_j} = 0.$$

$$B = \frac{8N+4}{22N+13}(\mathcal{B} - \mathcal{L})_i$$

SM+Type II seesaw

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \xi \frac{\varphi^2}{M_p^2}$$

$$\frac{d\chi}{d\varphi} = \frac{\sqrt{(6\xi^2\varphi^2/M_p^2) + \Omega^2}}{\Omega^2}$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} f(\chi) g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - U(\chi, \theta)$$

$$f(\chi) \equiv \frac{\varphi(\chi)^2 \cos^2 \alpha}{\Omega^2(\chi)}, \quad \text{and} \quad U(\chi, \theta) \equiv \frac{V(\varphi(\chi), \theta)}{\Omega^4(\chi)}$$

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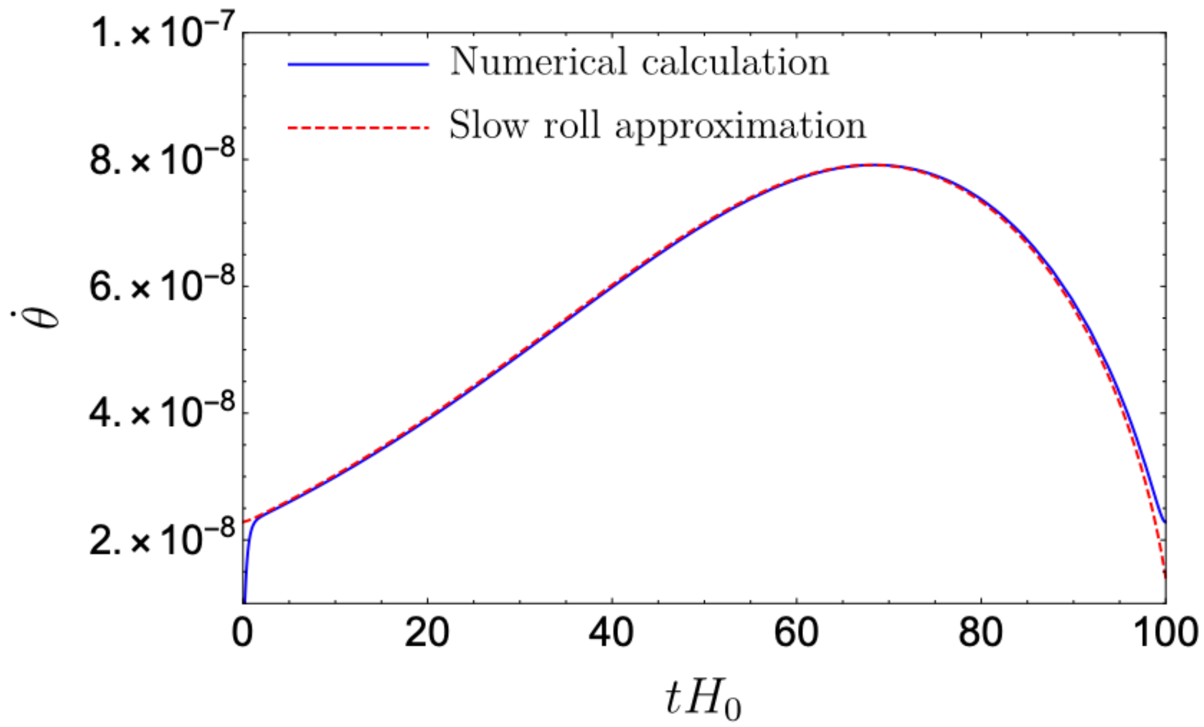
Motion during inflation

$$\ddot{\chi} - \frac{1}{2}f_{,\chi}\dot{\theta}^2 + 3H\dot{\chi} + U_{,\chi} = 0$$
$$\ddot{\theta} + \frac{f_{,\chi}}{f(\chi)}\dot{\theta}\dot{\chi} + 3H\dot{\theta} + \frac{1}{f(\chi)}U_{,\theta} = 0$$

SM+Type II seesaw

$$\xi = 300,\ \lambda = 4.5 \cdot 10^{-5},\ \ \tilde{\lambda}_5 = 10^{-9}$$
$$\chi_0 = 6.0 M_p,\ \dot{\chi}_0 = 0,\ \text{and}\ \dot{\theta}_0 = 0$$

Theta follow slow roll



$$\begin{aligned} n_{L\text{end}} &= Q_L \varphi_{\text{end}}^2 \dot{\theta}_{\text{end}} \cos^2\alpha \\ &\simeq -\mathcal{O}(1) Q_L \varphi_{\text{end}}^2 \frac{M_p U_{,\theta}}{f(\chi_{\text{end}})\sqrt{3U_{\text{end}}}} \cos^2\alpha \\ &\simeq -\mathcal{O}(1) Q_L \tilde{\lambda}_5 \varphi_{\text{end}}^3 \sin\theta_{\text{end}} / \sqrt{3\lambda} \end{aligned}$$

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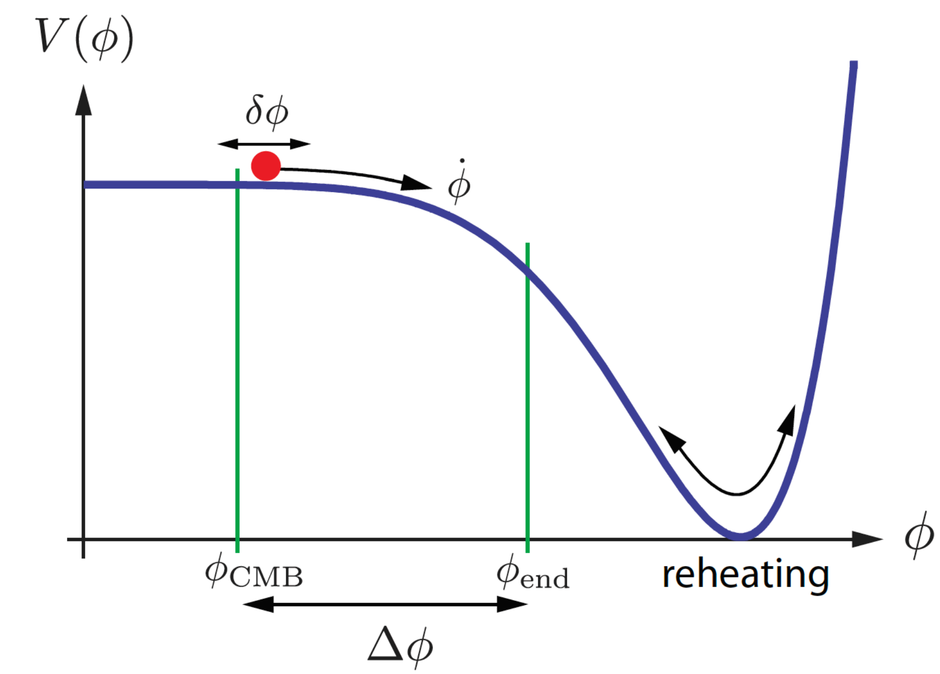
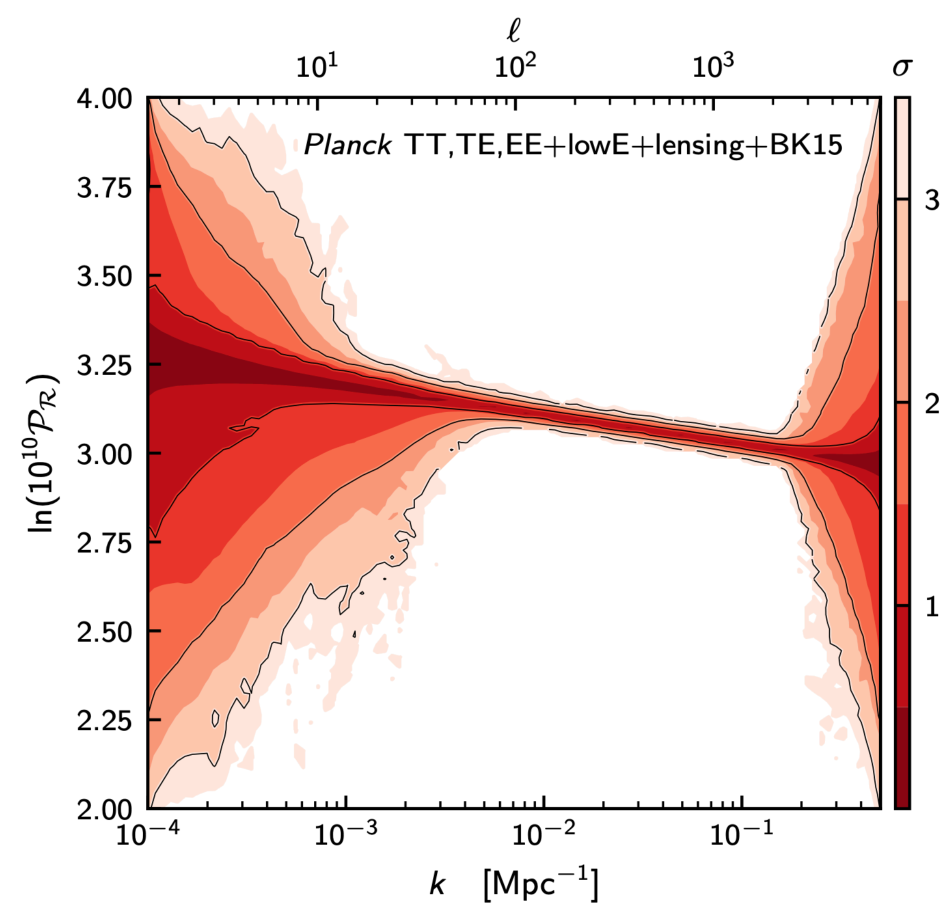
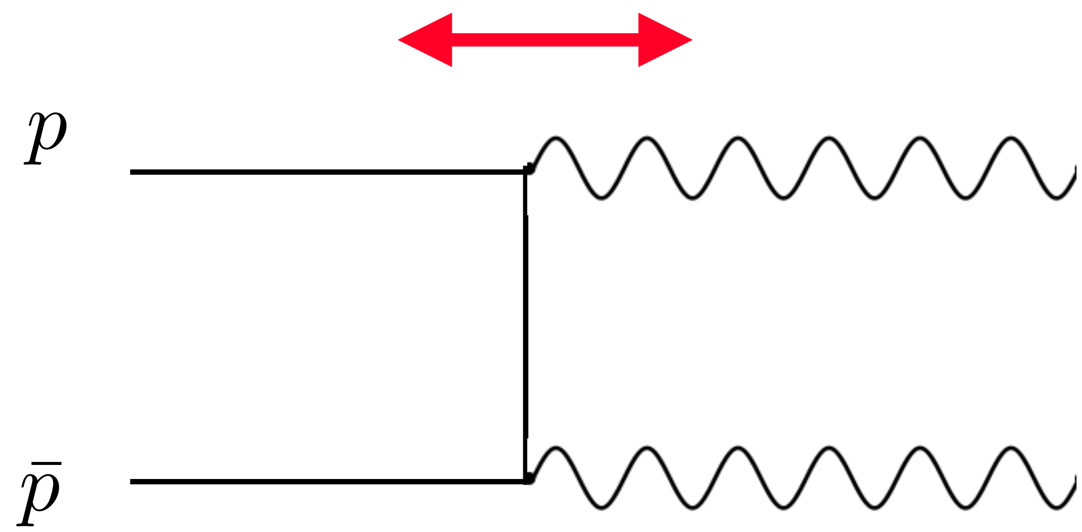




Baryon asymmetry of our universe



- T > 1 GeV, $n_b \sim n_{\bar{b}} \sim n_\gamma$
- T < 1 MeV, no baryon asymmetry $\frac{n_b}{n_\gamma} = \frac{n_{\bar{b}}}{n_\gamma} \sim 10^{-20}$
- T < 1 MeV, baryon asymmetry $\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$

two questions: why difference? why so small?