Effective field theory approach to $0\nu\beta\beta$ decay with light sterile neutrinos

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GL, Michael J. Ramsey-Musolf, Juan Carlos Vasquez, 2009.01257 (PRL)

Jordy de Vries, **GL**, Michael J. Ramsey-Musolf, Juan Carlos Vasquez, 2209.03031 (JHEP)

The 2022 Shanghai Particle Physics and Cosmology Symposium:

Neutrino and Dark Matter Physics (SPCS 2022)
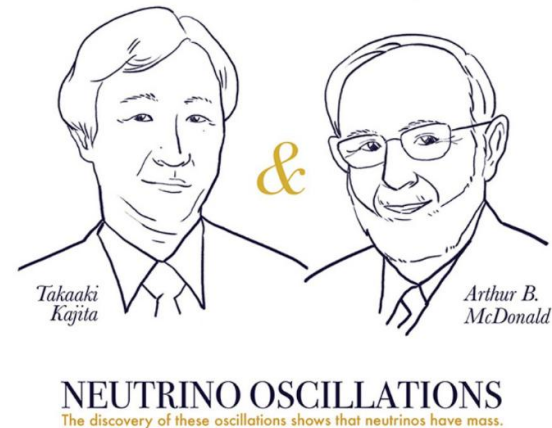
2022年11月19日

Neutrinos: what we know

- Neutrinos in the SM are **massless**

$$L_i \to \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix} \qquad m_\nu = 0$$

- Neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$
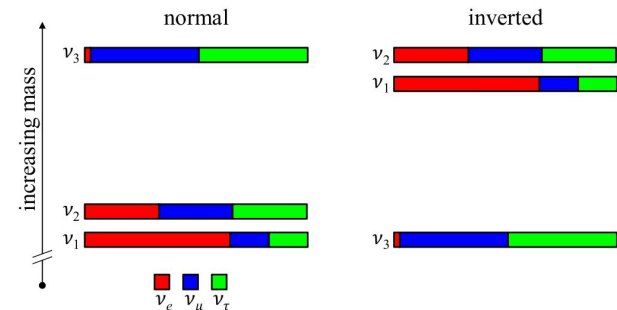

Takaaki Kajita & Arthur B. McDonald

NEUTRINO OSCILLATIONS
The discovery of these oscillations shows that neutrinos have mass.

- Neutrino oscillations require **massive** neutrinos

$$P(\nu_i \to \nu_j) \propto \Delta m_{ij}^2$$

$$\Delta m_{21}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2$$

$$|\Delta m_{31}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2$$
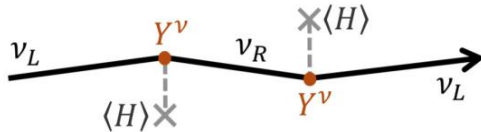

- Normal vs inverted hierarchy

Neutrinos: what we do not know

- Mass origin and Majorana nature:

  - How do neutrinos get their masses?
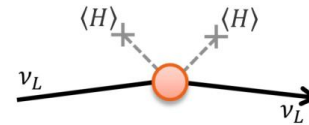 - Are they Dirac or Majorana fermions?

Dirac mass:

Majorana mass:

"Weinberg operator"

$$\mathcal{L}_D = -(Y^\nu \bar{L} H \nu_R + \text{h.c.})$$

$$\mathcal{L}_M = \frac{C_5}{\Lambda}(\bar{L}^c \tilde{H}^*)(\tilde{H}^\dagger L) + \text{h.c.}$$

S. Weinberg 1979

very small coupling

(very) large scale

a la eg. type-I, II, III seesaw

Neutrinos and lepton number violation

- How can we test if neutrinos are Dirac or Majorana fermions?

Dirac mass:

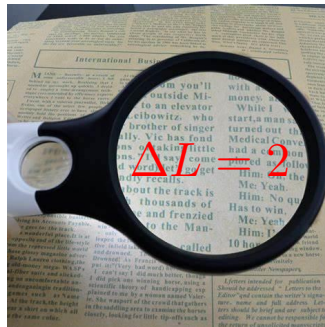$$\mathcal{L}_D = -(Y^\nu \bar{L} H \nu_R + \text{h.c.})$$

$$-1 \quad +1$$

Majorana mass:

$$\mathcal{L}_M = \frac{C_5}{\Lambda}(\bar{L}^c \tilde{H}^*)(\tilde{H}^\dagger L) + \text{h.c.}$$

$$+1 \qquad +1$$

Lepton number is violated by two units $\Delta L = 2$ if there exists Majorana neutrino mass
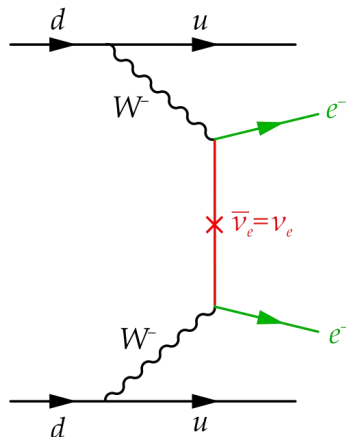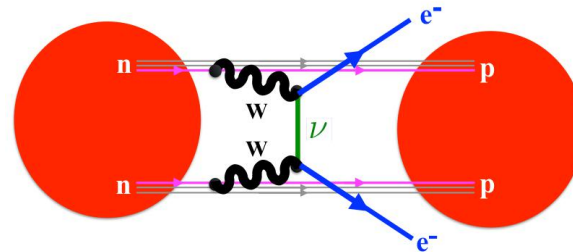
Neutrinoless double beta decay

- Why search for $0\nu\beta\beta$ decay?

  If neutrino is Majorana fermion, $0\nu\beta\beta$ decay process is induced
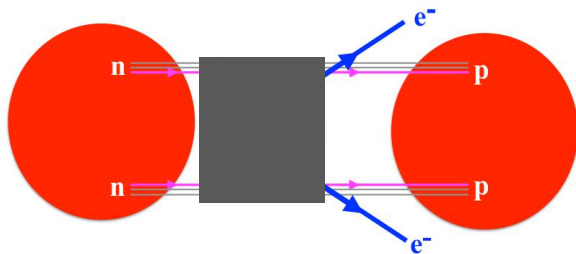
Majorana mass:

$0\nu\beta\beta$ decay:

Furry, Phys. Rev. 56 (1939) 1184

Neutrinoless double beta decay

- Why search for $0\nu\beta\beta$ decay?

  An observation of $0\nu\beta\beta$ decay implies LNV $\Delta L = 2$
  and Majorana neutrino mass

$0\nu\beta\beta$ decay:

Majorana mass:

"Black box theorem"
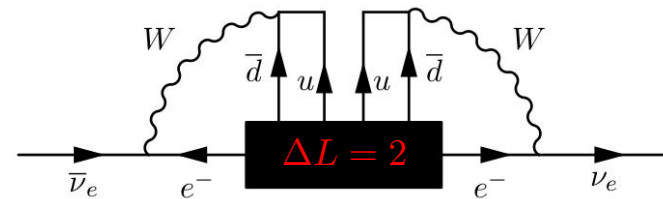
Schechter, Valle, Phys.Rev.
D25 (1982) 774

Neutrinoless double beta decay

Experimental searches for $0\nu\beta\beta$ decay in nuclei ^{136}Xe, ^{76}Ge, et al,

$$(A, Z) \to (A, Z + 2) + e^- + e^-$$

$^{A}_{Z}X$ A: mass number, # of p, n
Z: atomic number, # of p

summed energy of electrons

$Q_{\beta\beta} \sim 2 \text{ MeV}$

Neutrinoless double beta decay

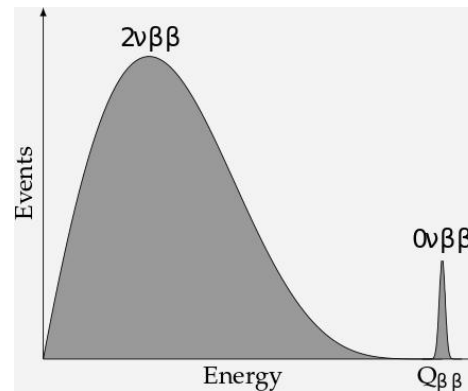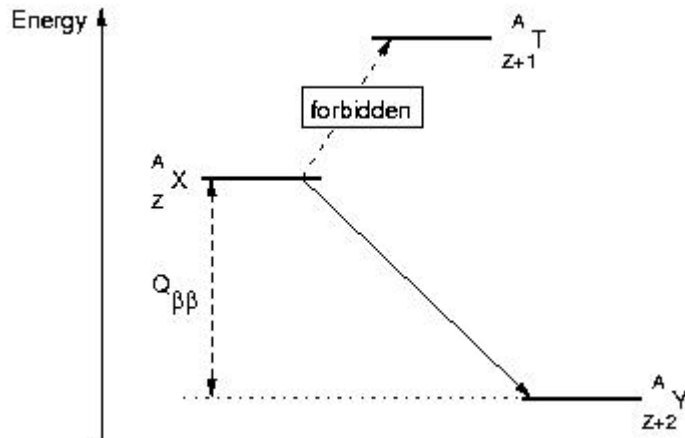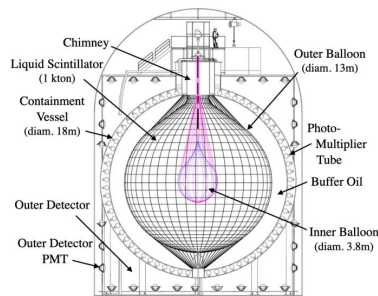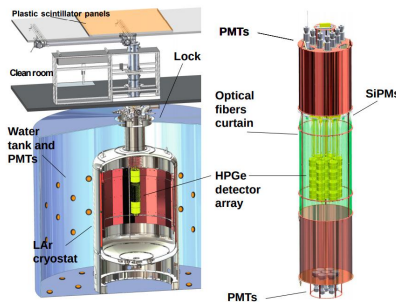## Status of experiments

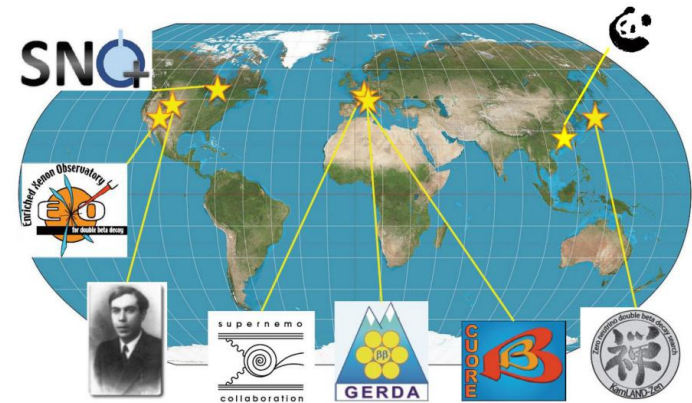KamLAND-Zen: $^{136}\text{Xe} \rightarrow {}^{136}\text{Ba} + e^- + e^-$

$$T_{1/2}^{0\nu}(\text{Xe}) > 1.07 \times 10^{26} \text{ year}$$

GERDA: $^{76}\text{Ge} \rightarrow {}^{76}\text{Se} + e^- + e^-$

$$T_{1/2}^{0\nu}(\text{Ge}) > 1.8 \times 10^{26} \text{ year}$$

PandaX, CDEX, JUNO, ...

SJM Peeters

Experiment	Isotope	Mass	Technique	Present Status	Location
CANDLES-III	^{48}Ca	300 kg	CaF$_2$ scint. crystals	Prototype	Kamioka
GERDA	^{76}Ge	≈35 kg	^{enr}Ge semicond. det.	Operating	LNGS
MAJORANA	^{76}Ge	26 kg	^{enr}Ge semicond. det.	Operating	SURF
CDEX-1T	^{76}Ge	1 ton	^{enr}Ge semicond. det.	Prototype	CJPL
LEGEND-200	^{76}Ge	200 kg	^{enr}Ge semicond. det.	Construction	LNGS
LEGEND-1000	^{76}Ge	ton	^{enr}Ge semicond. det.	Proposal	
CUPID-0	^{82}Se	5 kg	Zn^{enr}Se scintillating bolometers	Prototype	LNGS
SuperNEMO-Dem	^{82}Se	7 kg	^{enr}Se foils/tracking	Construction - 2019	Modane
SuperNEMO	^{82}Se	100 kg	^{enr}Se foils/tracking	Proposal	Modane
CMOS Imaging	^{82}Se		^{enr}Se, CMOS	Development	
					YangYang
					YangYang
					Yemi
					Canfrance
					LNGS, LSM
					LNGS
					LNGS
Tin.Tin	^{124}Sn	1 kg	Tin bolometers	Development	INO
CALDER	^{130}Te		TeO$_2$ bolometers with Cerenkov Light	Development	LNGS
CUORE	^{130}Te	1 ton	TeO$_2$ bolometers	Operating	LNGS
SNO+	^{130}Te	1.3 t	0.5% ^{enr}Te loaded liq. scint.	Construction - 2020	SNOLab
nEXO	^{136}Xe	5 t	Liq. ^{enr}Xe TPC/scint.	Proposal	
NEXT-100	^{136}Xe	100 kg	gas TPC	Prototype	Canfranc
AXEL	^{136}Xe		gas TPC	Prototype	
KamLAND-Zen	^{136}Xe	800 kg	^{enr}Xe disolved in liq. scint.	Operating	Kamioka
LZ	^{136}Xe		Dual phase Xe TPC	Construction - 2020	SURF
PANDAX-III	^{136}Xe	1 ton	Dual phase Xe TPC	Construction - 2019	CJPL
XENON1T	^{136}Xe	1 ton	Dual phase Xe TPC	Operating	LNGS
DARWIN	^{136}Xe	50 ton	Dual phase Xe TPC	Proposal	LNGS
NuDot	Various		Cherenkov and scint. detection in liq. scint.	Development	
FLARES	Various		Scint. crystals with Si photodetectors	Development	

May 28, 2020   Elliott, BB Theory Workshop

tonne-scale experiments $T_{1/2}^{0\nu} \gtrsim 10^{28}$ year
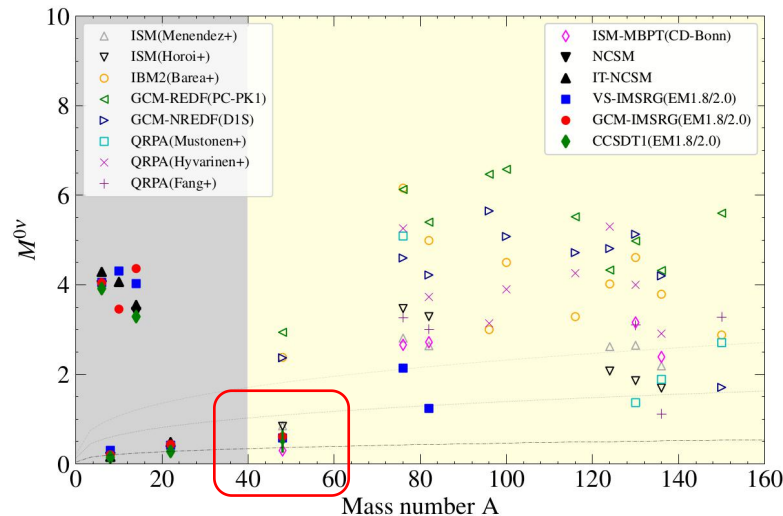
Neutrinoless double beta decay

The theoretical inverse **half-life** is expressed as

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G_{0\nu} {M_{0\nu}}^2 \langle m_{\beta\beta}\rangle^2$$

$G_{0\nu}$:   phase space factor (atomic physics)
$M_{0\nu}$:   nuclear matrix element (nuclear physics)
$\langle m_{\beta\beta}\rangle$: effective Majorana mass (particle physics)
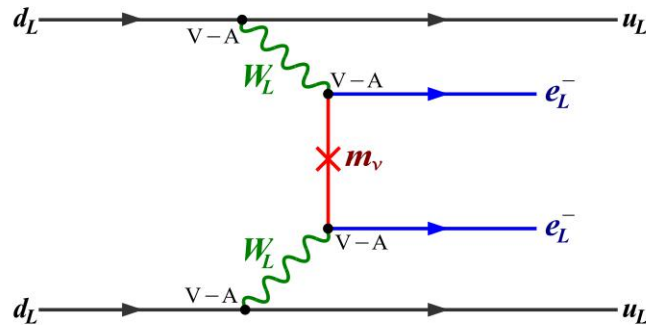

- Uncertainty in $G_{0\nu}$ is about 10%
- Accurate calculation of $M_{0\nu}$ is promising

How about $\langle m_{\beta\beta}\rangle$?

J.M. Yao 2111.15543 (PPNP);
R. Wirth, J.M. Yao, H. Hergert, 2105.05415 (PRL)

Standard mechanism

$0\nu\beta\beta$ decay is induced by the exchange of light Majorana neutrinos
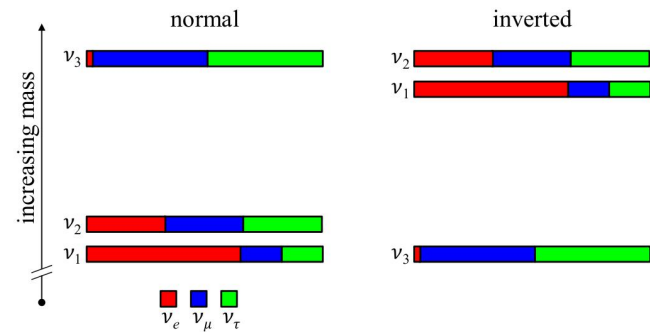

$$\langle m_{\beta\beta} \rangle = |\sum_i m_i U_{ei}^2|$$

PMNS matrix

absolute neutrino masses
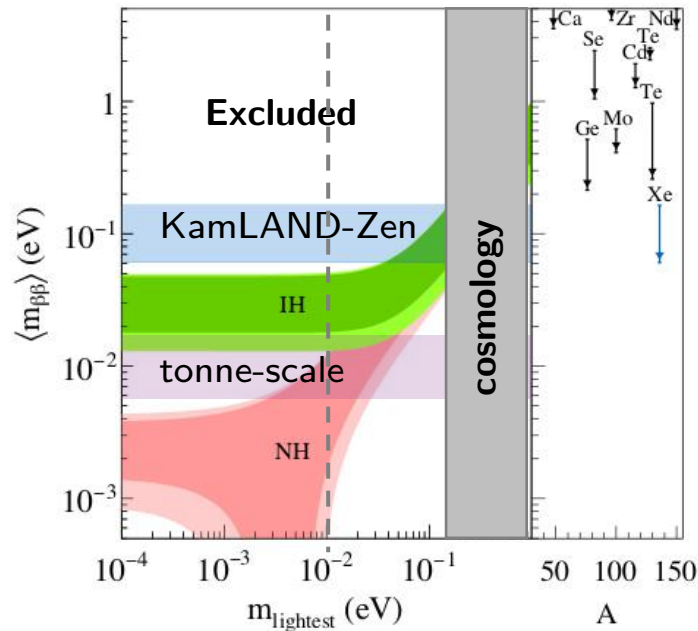
From neutrino oscillation
$\Delta m_{21}^2, |\Delta m_{31}^2|, \theta_{ij}, \delta$

The **lightest** neutrino mass, mass **hierarchy**, and **Majorana phases** are unknown

Standard mechanism

$\langle m_{\beta\beta} \rangle$ as a function of $m_{lightest}$ for NH and IH
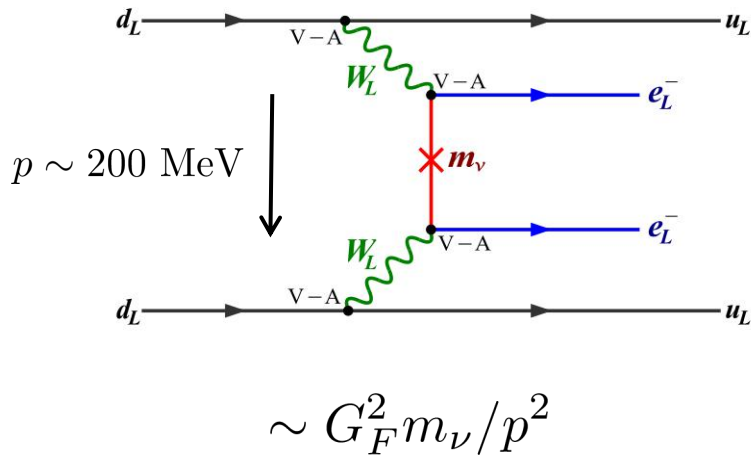

Opportunities:

- establish the mass hierarchy in $0\nu\beta\beta$ decay experiments

Challenges:

- nightmare region for a positive signal confronted with future cosmological surveys

Non-standard mechanisms

Standard mechanism:

$$\sim G_F^2 m_\nu / p^2$$

Non-standard mechanisms:

$$\sim c/\Lambda^5$$
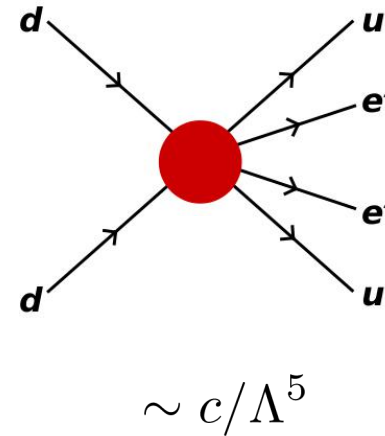
$$\frac{c/\Lambda^5}{G_F^2 m_\nu^{ee}/p^2} = c\left(\frac{3.3 \text{ TeV}}{\Lambda}\right)^5 \frac{0.1 \text{ eV}}{m_\nu^{ee}}$$

c: new coupling
Λ: heavy particle mass

It is interesting to investigate it in more details in well-motivated neutrino mass models

Minimal left-right symmetric model

Gauge group: $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Doublets:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$L_L = \begin{pmatrix} \nu \\ l \end{pmatrix}_L \qquad L_R = \begin{pmatrix} N \\ l \end{pmatrix}_R$$

Mohapatra and Senjanovic, Phys.Rev.Lett. 44 (1980) 912, Phys.Rev.D 23 (1981) 165

Bidoublet:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \implies \langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix} \qquad \boxed{\tan\beta = \frac{v_2}{v_1}}$$
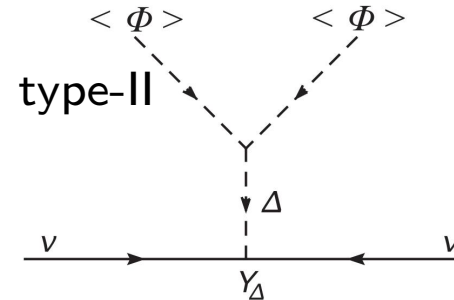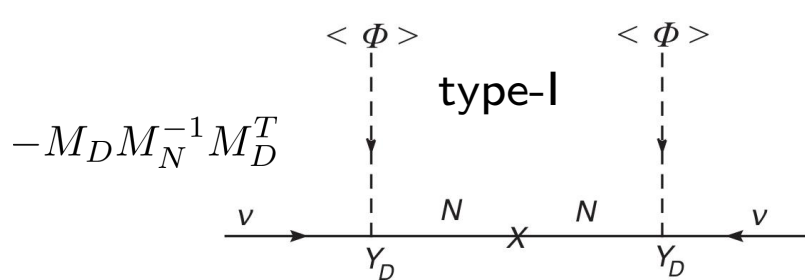
Triplets:

$$\Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix}$$

$$\implies \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \qquad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix}$$

Minimal left-right symmetric model

It provides natural origin of neutrino masses

$$-M_D M_N^{-1} M_D^T$$

type-I

type-II

$$M_L = Y_\Delta \frac{\mu v^2}{M_\Delta^2}$$
$$= \frac{v_L}{v_R} M_N$$

It is the most studied BSM model for $0\nu\beta\beta$ decay

$$L \Longleftrightarrow R$$
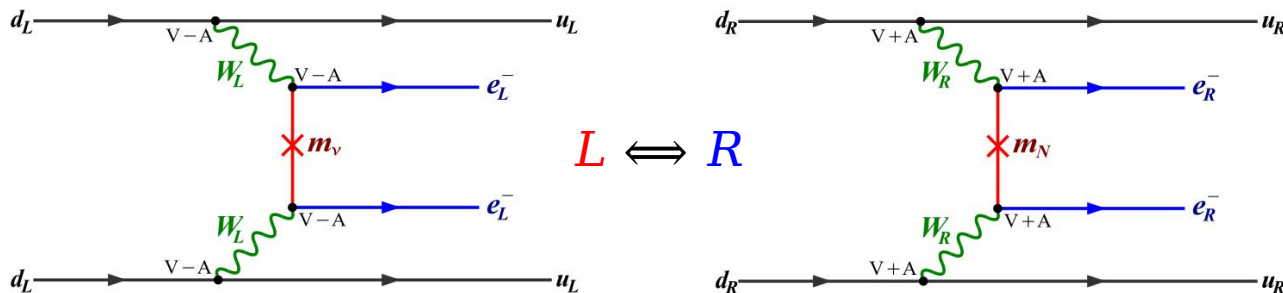
Mohapatra and Senjanovic, Phys.Rev.Lett. 44 (1980) 912, Phys.Rev.D 23 (1981) 165
Doi et al., Prog.Theor.Phys. 66 (1981) 1739
Tello et al., Phys.Rev.Lett. 106 (2011) 151801; S.-F. Ge, M. Lindner, S. Patra, 1508.07286 (JHEP);
Bhupal Dev, Goswami, Mitra Phys.Rev.D 91 (2015) 113004
and many others

Minimal left-right symmetric model

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$$-M_D M_N^{-1} M_D^T$$

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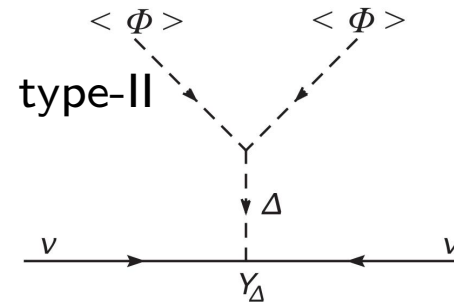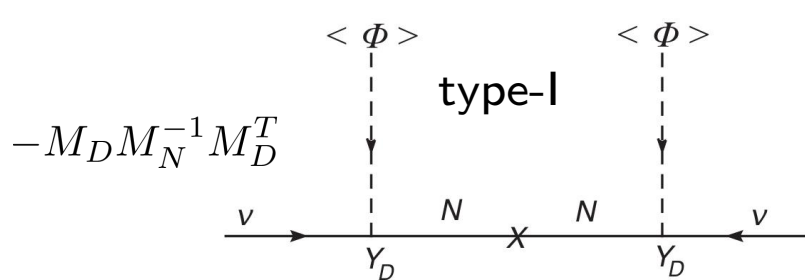
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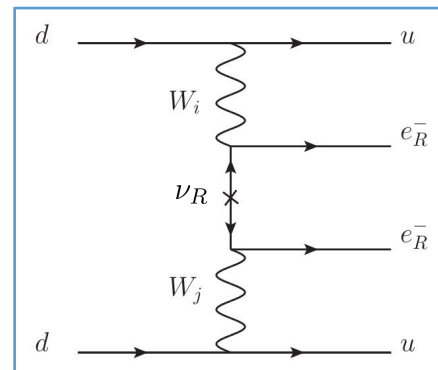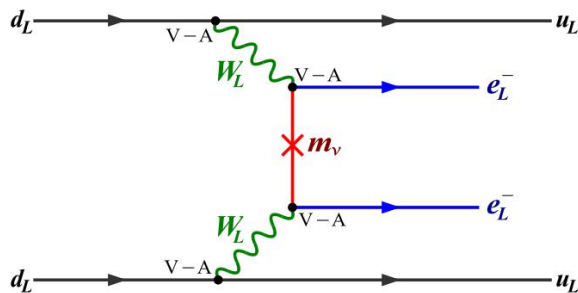
left-right mixing

$$\tan\zeta = \frac{M_W^2}{M_{W_R}^2} \sin(2\beta)$$

GL, M. J. Ramsey-Musolf, J. C. Vasquez, 2009.01257 (PRL)

New leading contribution

Chiral **enhancement** comes from the left-right mixing

$$\tan\beta = \frac{v_2}{v_1}$$
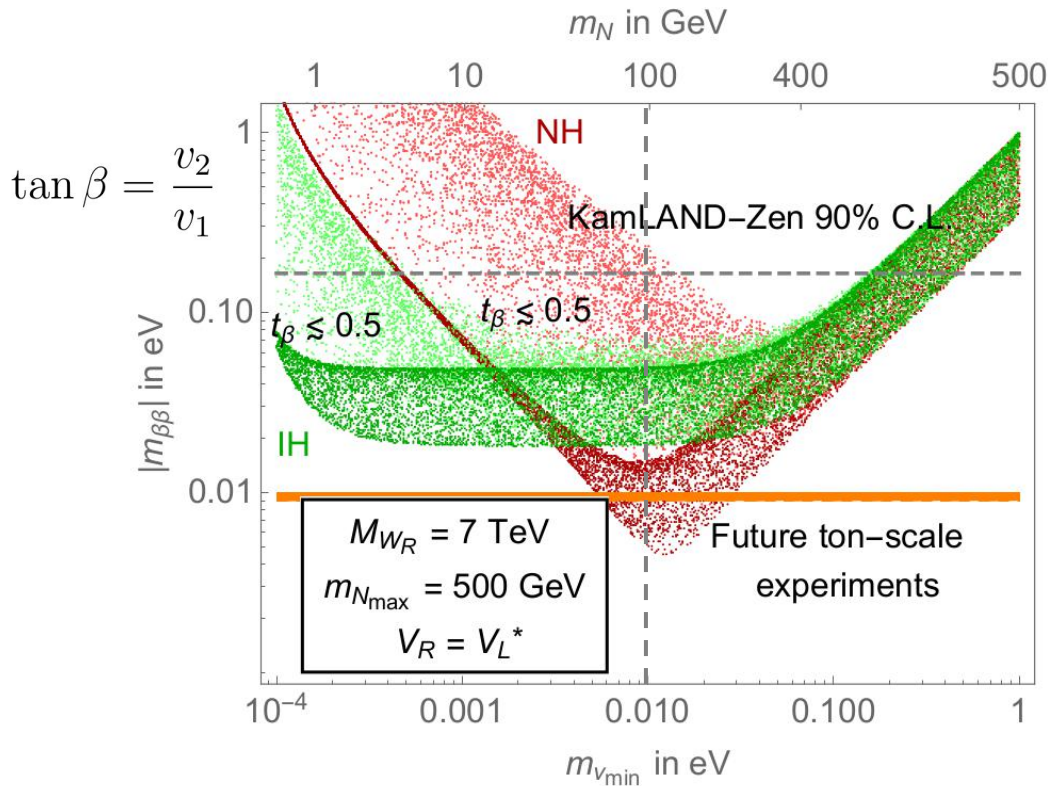
mass correlation:

$$m_N = \frac{m_1}{m_3} m_{N_{\max}} \quad \text{(NH)}$$

$$m_N \simeq 100 \text{ GeV} \cdot \frac{m_1}{0.01 \text{ eV}} \cdot \frac{m_{N_{\max}}}{500 \text{ GeV}}$$

turning point at $m_{\nu_{\min}} \sim 0.01$ eV

- **R**: standard mechanism

- **L**: non-std. mechanism

Right-handed neutrinos $m_N \lesssim 100 \text{ GeV}$

are also motivated by solving *strong CP problem*

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Strong CP problem

Brief review of strong CP problem in the SM:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - m_q e^{i\theta_q\gamma^5})q$$

- The physical parameter

  Weinberg, 1975; 't Hooft, 1976

$$\bar{\theta} = \theta + \theta_q$$

- With multiple flavors of quarks

$$\bar{\theta} = \theta + \arg\det M_Q$$

- Severe constraint from neutron EDM measurements

$$\bar{\theta} < 10^{-10}$$

It is <u>unnaturally</u> small since CP violation in weak interactions $\sim \mathcal{O}(1)$

Strong CP problem

- Several solutions to address the strong CP problem, eg.

    - Peccei-Quinn symmetry and the axion: promote $\bar{\theta}$ to be a dynamic field

      Peccei, Quinn, 1977

    - Parity solution: the strong CP (P) problem can be solved in P-symmetric theories

      Mohapatra, Senjanovic, 1978

- In the *minimal left-right symmetric model*,

$$Q_L \leftrightarrow Q_R \qquad \Phi \leftrightarrow \Phi^\dagger$$

The Yukawa interaction $\bar{Q}_L Y_Q \Phi Q_R$, so that

$$Y_Q \leftrightarrow Y_Q^\dagger$$

But, the quark mass matrix

$$M_Q = Y_Q \langle \Phi \rangle$$

credit by K. S. Babu

It is challenge to make the VEVs of Φ real

Strong CP problem

- The scalar potential

$$V \supset \alpha_2 \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\Delta_L \Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\Delta_R \Delta_R^\dagger) \right] + \mathrm{h.c.}$$

P. Duka, J. Gluza, M. Zralek, Annals Phys. 280 (2000) 336

α_2 is generally complex $\Rightarrow$ $\langle \Phi \rangle$ is complex $\qquad \langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$

$$\Rightarrow \alpha \text{ is non-zero}$$

At tree level: $\quad \bar{\theta}_{\mathrm{tree}} = \dfrac{m_t}{2m_b} \tan(2\beta) \sin\alpha$

But even if $\alpha = 0$, α_2 becomes complex due to loop corrections

R. Kuchimanchi, 1408.6382 (PRD)

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Strong CP problem

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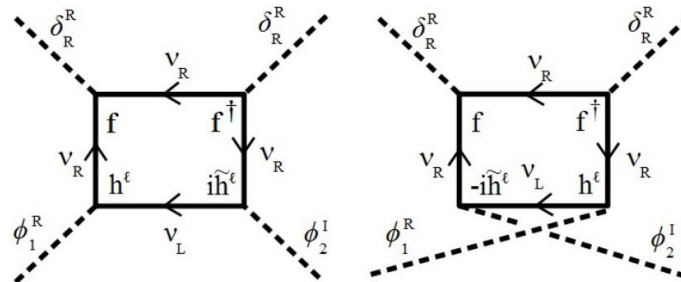
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But even if $\alpha = 0$, α_2 becomes complex due to loop corrections

$$\bar{\theta}_{\text{loop}} \propto \sqrt{\frac{m_\nu}{M_W}} \times \sqrt{\frac{m_N}{M_W}} \times \left(\frac{m_N}{M_W}\right)^2$$

For $M_{W_R} = 5 \text{ TeV}$,

$$\Longrightarrow \quad m_N \lesssim 100 \text{ GeV}$$

$$\boxed{(m_\nu/M_W)^{1/2} \lesssim 10^{-6}}$$

(neutrino mixing matrices are not included)

Sterile neutrinos

From the flavor basis to the mass basis

$$N_m = \begin{pmatrix} \nu'_L \\ \nu'^c_R \end{pmatrix} = U^\dagger \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} \qquad U = \begin{pmatrix} U_{\mathrm{PMNS}} & S \\ T & U_R \end{pmatrix}$$

Majorana states: $\quad \nu = (\nu_1, \cdots, \nu_6)^T \equiv N_m + N_m^c$

active: $\quad \nu_a \equiv \nu'_L + \nu'^c_L = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$ $\qquad$ sterile: $\quad \nu_s \equiv \nu'_R + \nu'^c_R = \begin{pmatrix} \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix}$

ν_s are sterile since they interact with W boson feebly, proportional to

- S_{ei} $(i = 1,2,3)$ for ν_4, ν_5, ν_6, respectively $\quad \boxed{S = RU_R}$ $\quad \boxed{R = M_D M_R^{-1}}$

- the left-right mixing parameter $\tan\zeta = \dfrac{M_W^2}{M_{W_R}^2}\sin(2\beta)$

Sterile neutrinos

How does the $0\nu\beta\beta$ decay half-life or $m_{\beta\beta}$ depend on the sterile neutrino mass m_i $(i = 4,5,6)$?

$0\nu\beta\beta$ decay amplitude

$$-q^2 \sim (200 \text{ MeV})^2$$
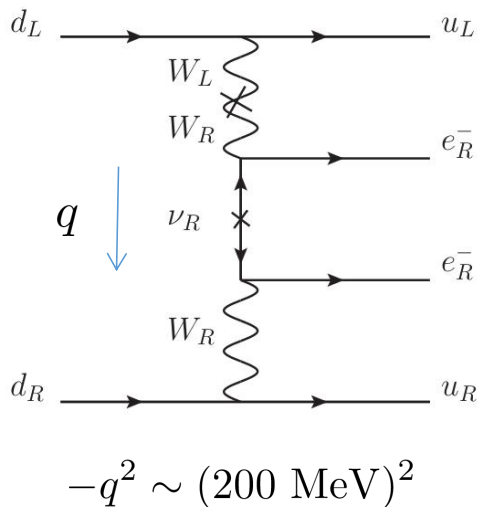
the mass dependence:

$$P_R \frac{\slashed{q} + m_i}{q^2 - m_i^2} P_R = P_R \frac{m_i}{q^2 - m_i^2} P_R$$

$$m_i^2 \ll -q^2 \qquad\qquad m_i^2 \gg -q^2$$
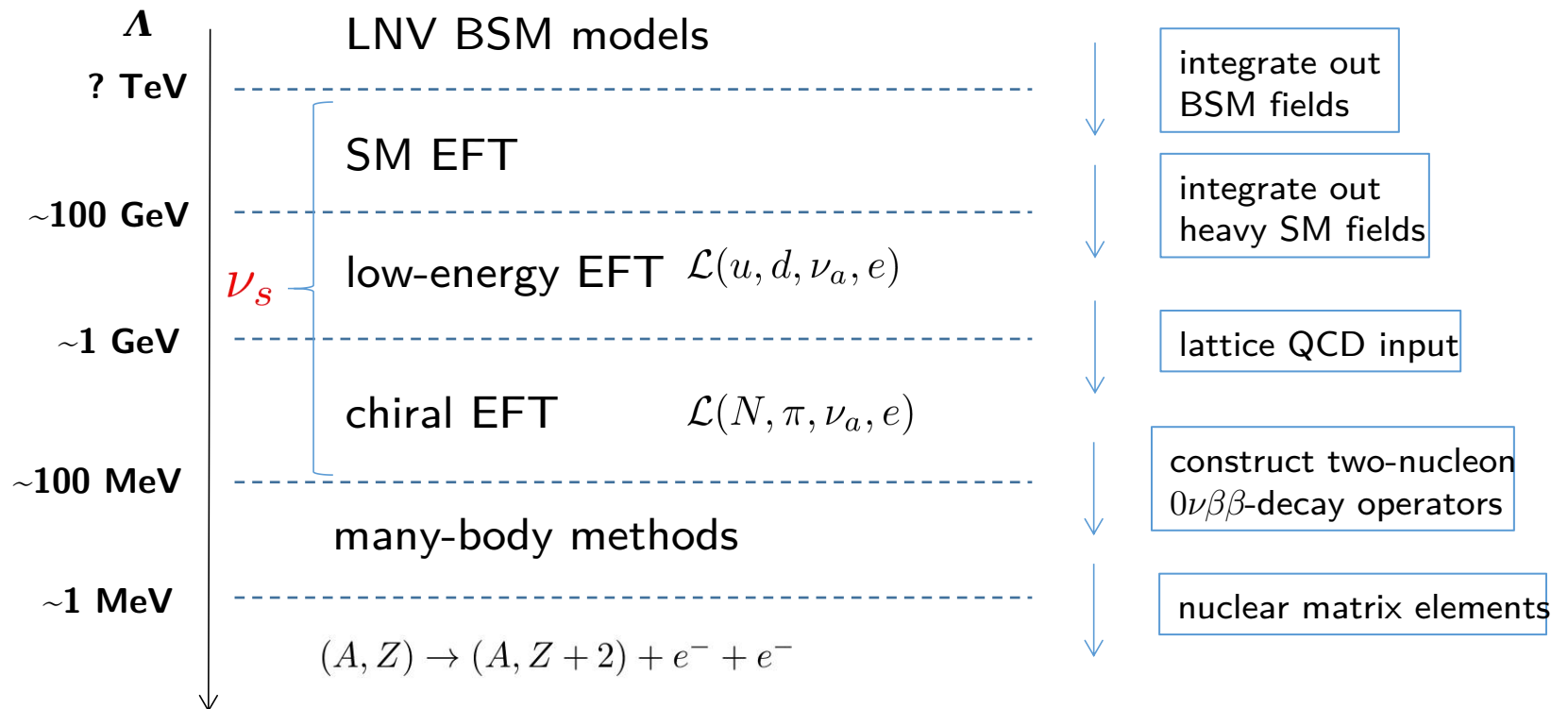
$$P_R \frac{m_i}{q^2} P_R \qquad\qquad -P_R \frac{1}{m_i} P_R$$

It is more involved for $m_i^2 \sim -q^2$, difficulties come from

- low-energy constants (LECs): hadronic level

- nuclear matrix elements (NMEs): nuclear level
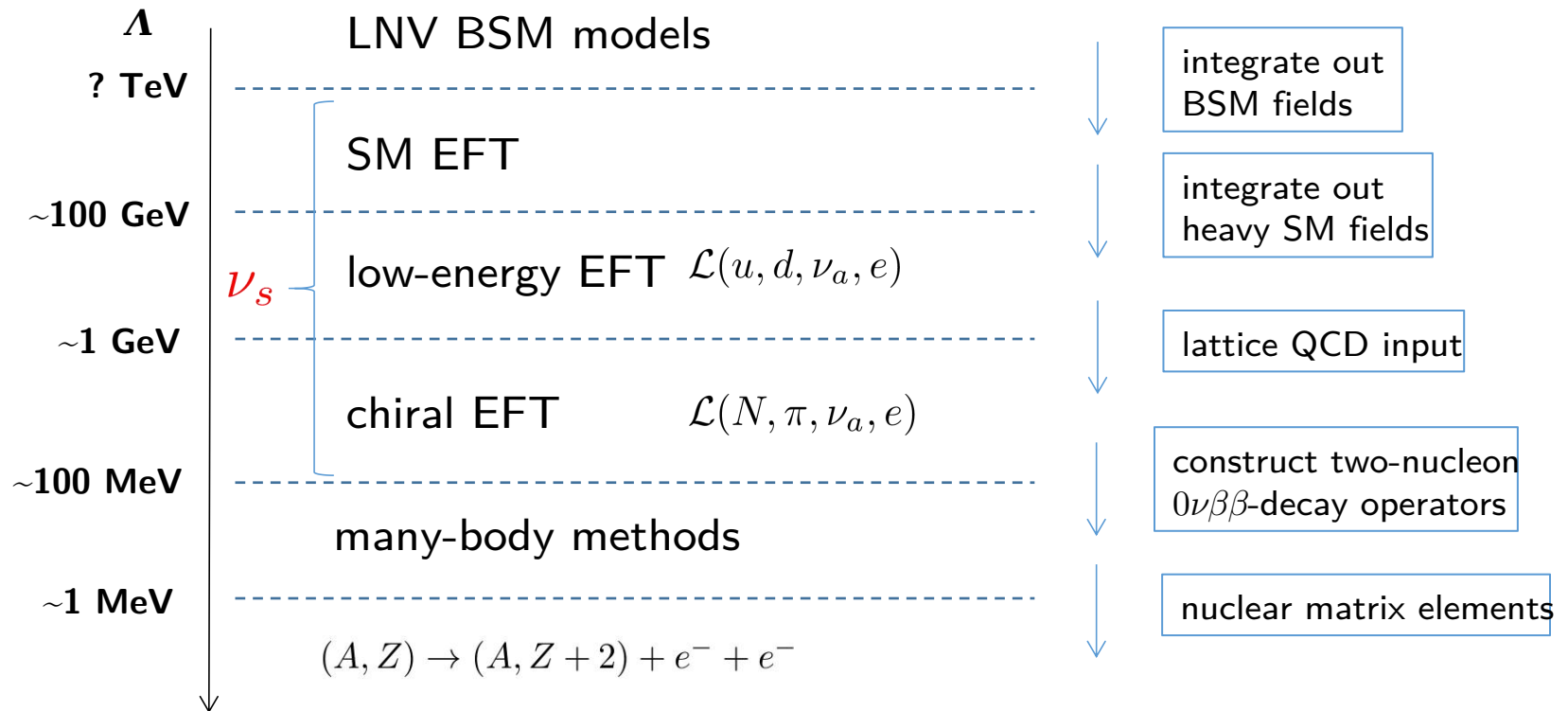
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EFT approach to $0\nu\beta\beta$ decay

Describe contributions to $0\nu\beta\beta$ decay systematically and consistently

Λ		
? TeV	LNV BSM models	integrate out BSM fields
~100 GeV	SM EFT	integrate out heavy SM fields
~1 GeV	low-energy EFT $\mathcal{L}(u,d,\nu_a,e)$	lattice QCD input
~100 MeV	chiral EFT $\mathcal{L}(N,\pi,\nu_a,e)$	construct two-nucleon $0\nu\beta\beta$-decay operators
~1 MeV	many-body methods	nuclear matrix elements
	$(A,Z) \to (A,Z+2) + e^- + e^-$	

ν_s

SMEFT, LEFT, ν SMEFT, ν LEFT, ... which EFT?

EFT approach to $0\nu\beta\beta$ decay

Describe contributions to $0\nu\beta\beta$ decay systematically and consistently

Λ		
? TeV	LNV BSM models	integrate out BSM fields
	SM EFT	integrate out heavy SM fields
~100 GeV		
	low-energy EFT $\mathcal{L}(u, d, \nu_a, e)$	
~1 GeV		lattice QCD input
	chiral EFT $\mathcal{L}(N, \pi, \nu_a, e)$	
~100 MeV		construct two-nucleon $0\nu\beta\beta$-decay operators
	many-body methods	
~1 MeV		nuclear matrix elements
	$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$	

ν_s

We always **keep** ν_s, and deal with RGE, LECs and NMEs

EFT approach to $0\nu\beta\beta$ decay

We construct the effective Lagrangian in the mass basis

$$\mathcal{L}_{6,\nu\mathrm{LEFT}} = \frac{2G_F}{\sqrt{2}} \left\{ \bar{u}_L \gamma_\mu d_L \left[\bar{e}_L \gamma^\mu C_{\mathrm{VLL}}^{(6)} \nu + \bar{e}_R \gamma^\mu C_{\mathrm{VLR}}^{(6)} \nu \right] + \bar{u}_R \gamma_\mu d_R \, \bar{e}_R \gamma^\mu C_{\mathrm{VRR}}^{(6)} \nu \right\}$$

$$
\begin{aligned}
C_{\mathrm{VLL}}^{(6)}(m_W) &= -2V_{ud}\, PU\,, \\
C_{\mathrm{VLR}}^{(6)}(m_W) &= V_{ud} \left(v^2 C_L^{(6)}(m_{W_R}) \right) P_s U^*\,, \\
C_{\mathrm{VRR}}^{(6)}(m_W) &= \left(v^2 C_R^{(6)}(m_{W_R}) \right) P_s U^*\,.
\end{aligned}
\qquad
\begin{aligned}
C_L^{(6)}(m_{W_R}) &= 2\frac{\xi e^{-i\alpha}}{1+\xi^2} \frac{C_R^{(6)}}{V_{ud}^R} \\
\\
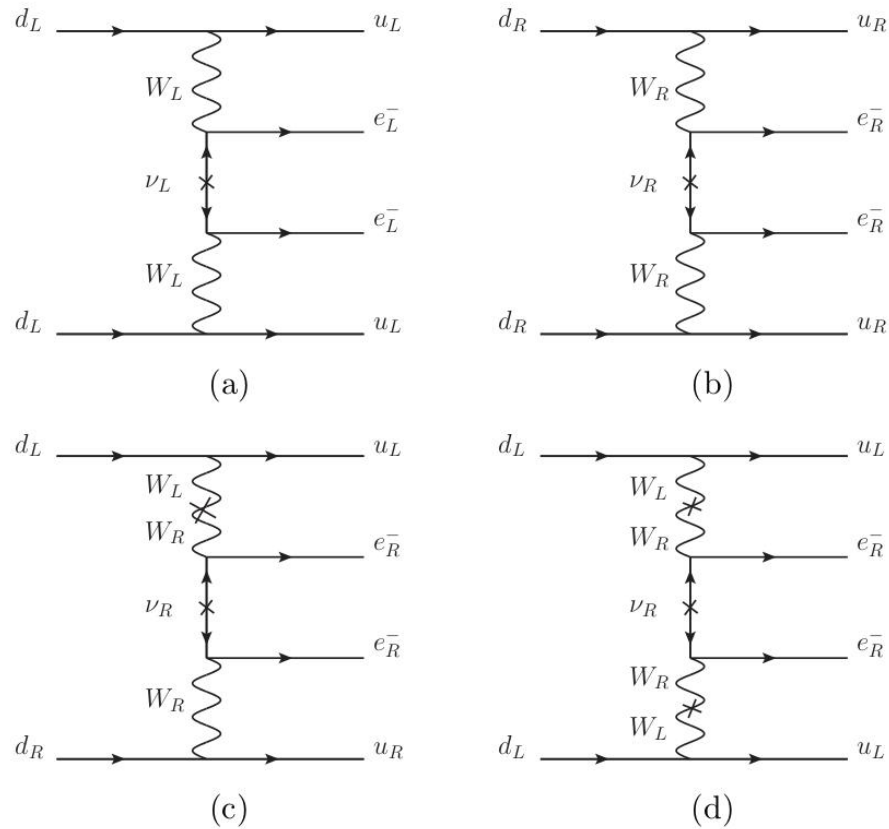C_R^{(6)}(m_{W_R}) &= -\frac{1}{v_R^2} V_{ud}^R
\end{aligned}
$$

$$PU = (U_{\mathrm{PMNS}}, S) \qquad\qquad P_s U^* = (T^*, U_R^*)$$

Interestingly, all contributions to $0\nu\beta\beta$ decay in the mLRSM can be described by these three Wilson coefficients

J. de Vries, **GL**, M. J. Ramsey-Musolf, J. C. Vasquez, 2209.03031 (JHEP)
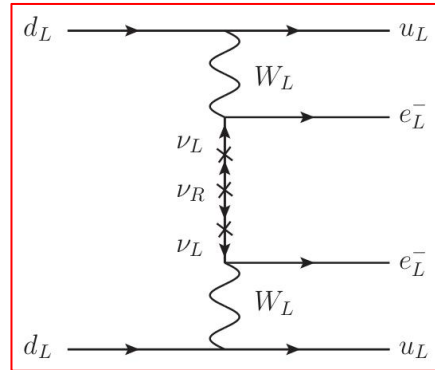
Diagrams

Type-II:

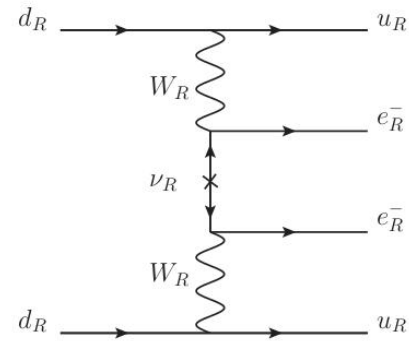(a): $P_L \dfrac{\not{q} + m_i}{q^2 - m_i^2} P_L = P_L \dfrac{m_i}{q^2 - m_i^2} P_L$ (b)(c)(d): $P_R \dfrac{\not{q} + m_i}{q^2 - m_i^2} P_R = P_R \dfrac{m_i}{q^2 - m_i^2} P_R$
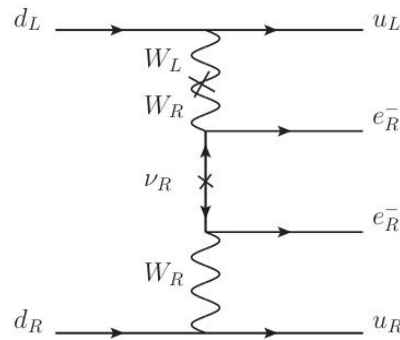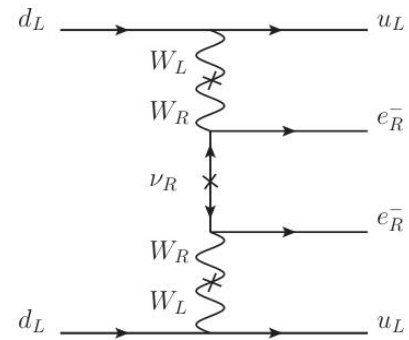
Diagrams

Type-I:
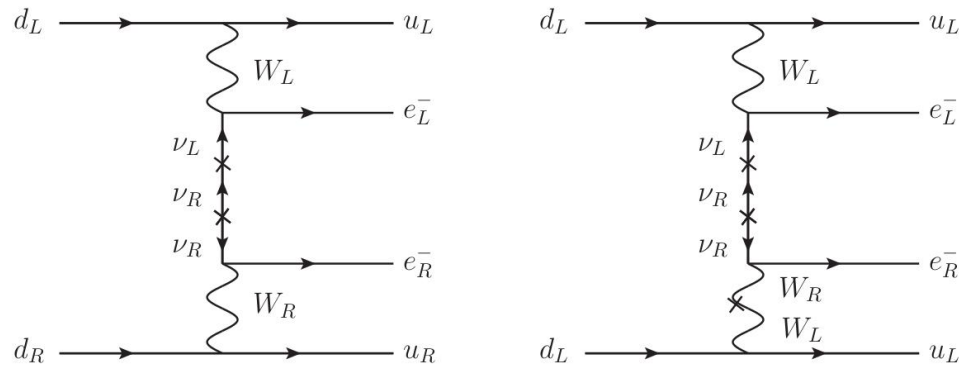

(a): $P_L \dfrac{\not{q} + m_i}{q^2 - m_i^2} P_L = P_L \dfrac{m_i}{q^2 - m_i^2} P_L$

(b)(c)(d): $P_R \dfrac{\not{q} + m_i}{q^2 - m_i^2} P_R = P_R \dfrac{m_i}{q^2 - m_i^2} P_R$

Diagrams

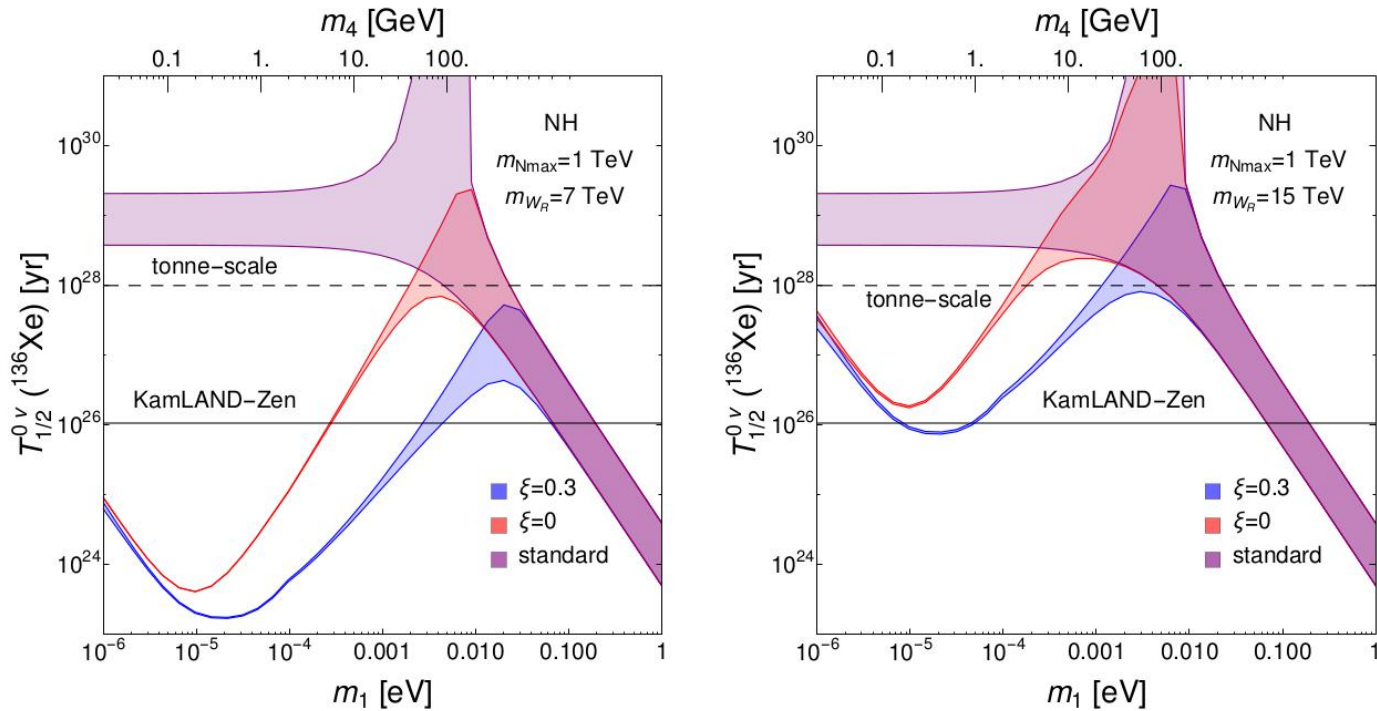**Type-I:** λ **and** η **diagrams** (Doi et al., 1983)

$$P_L \frac{\slashed{q} + m_i}{q^2 - m_i^2} P_R = P_L \frac{\slashed{q}}{q^2 - m_i^2} P_R$$

A **complete** EFT approach to $0\nu\beta\beta$ decay half-life of the mLRSM for **any** sterile neutrino mass

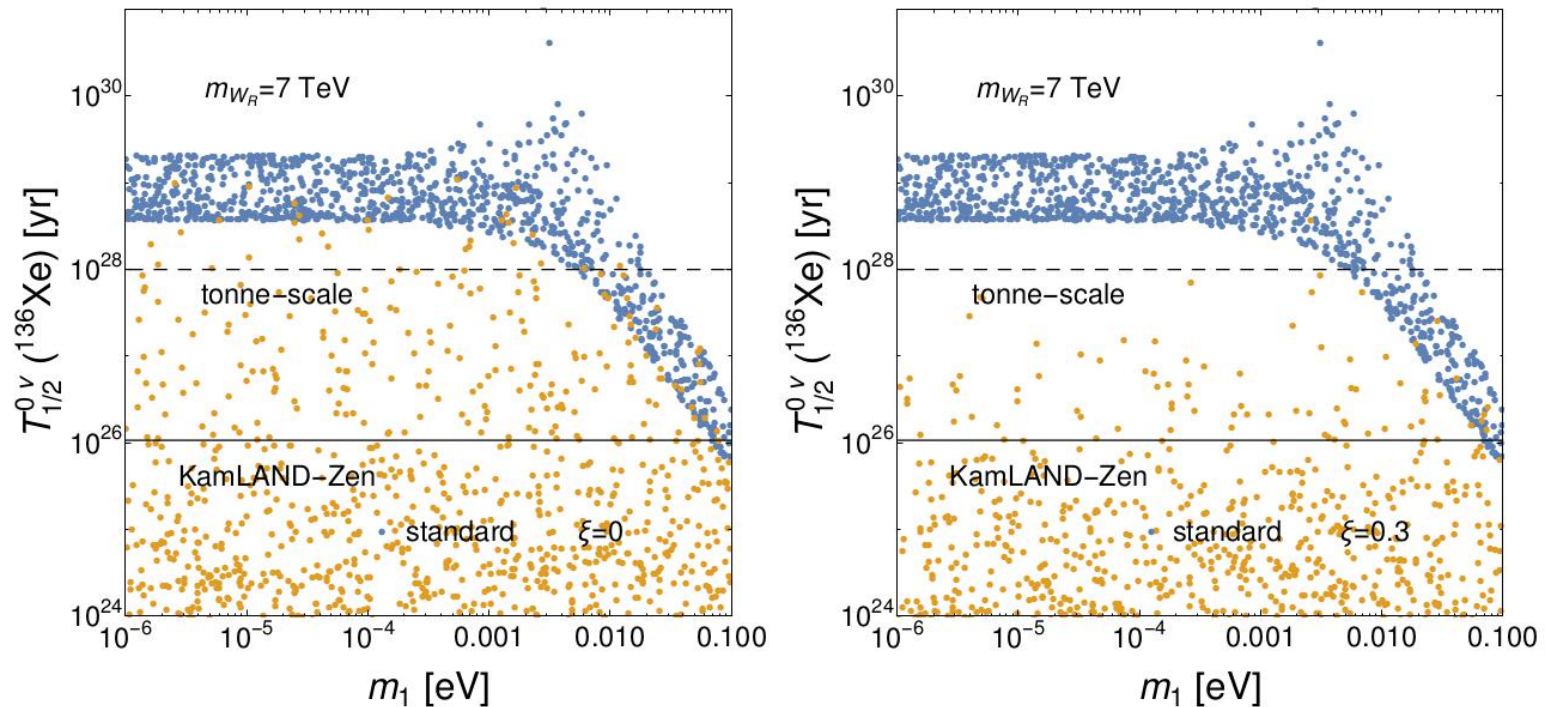J. de Vries, **GL**, M. J. Ramsey-Musolf, J. C. Vasquez, 2209.03031 (JHEP)

Results

Type-II: sterile neutrino and active neutrino masses are related

second turning point for $m_4 \sim 200$ MeV $\qquad \dfrac{m_4}{\mathbf{q}^2} \leftrightarrow \dfrac{1}{m_4}$

Results

Type-I: sterile neutrino masses are varied within $[10 \text{ MeV}, 1 \text{ TeV}]$

broder parameter space compared to type-II:
cancellation between two lighter sterile neutrinos

Summary

- $0\nu\beta\beta$ decay in the mLRSM is considered with particluar attention to light sterile neutrinos, which are motivated by the strong CP problem

- A general EFT approach is developed, where all contributions to $0\nu\beta\beta$ decay are described by a few Wilson coefficients

- This formalism is suitable for $0\nu\beta\beta$ decay experimental benchmarks and can be easily extended to other neutrino mass models

work in progress

Sterile neutrinos

Current constraints:

Bolton, Deppisch, Bhupal Dev, 1912.03058 (JHEP)
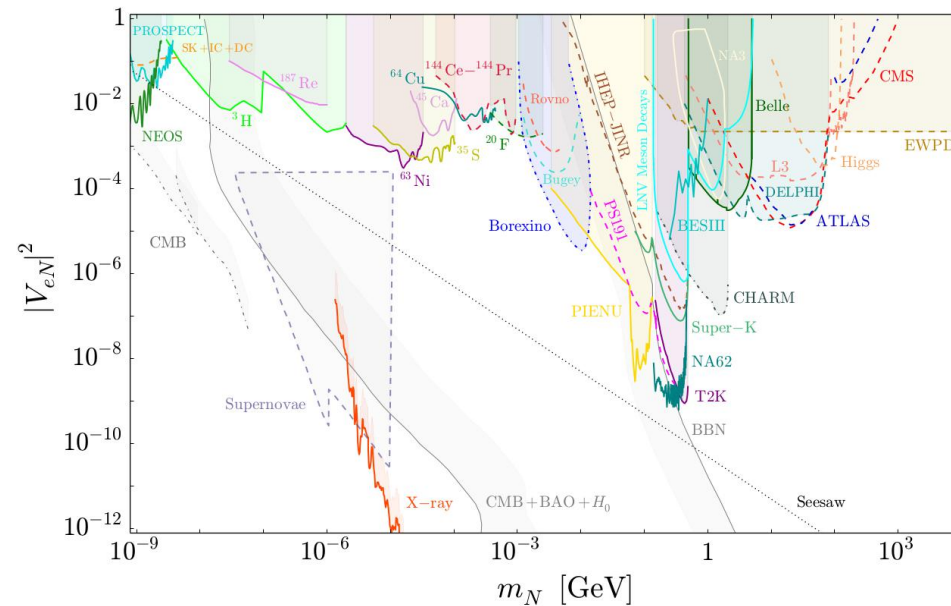
The coupling of ν_s to W boson is proportional to S_{ei} $(i = 1,2,3)$ for ν_4, ν_5, ν_6, respectively

$$\boxed{S = RU_R} \qquad \parallel R \parallel \lesssim \sqrt{0.1 \text{ eV}/10 \text{ MeV}} = 10^{-4}$$