

# The Realistic Scattering of Puffy Dark Matter

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# Outline

I Motivation

II Size dark matter-dark matter elastic scattering

III Self-interaction puffy DM

IV Conclusion and discussion

# Motivation

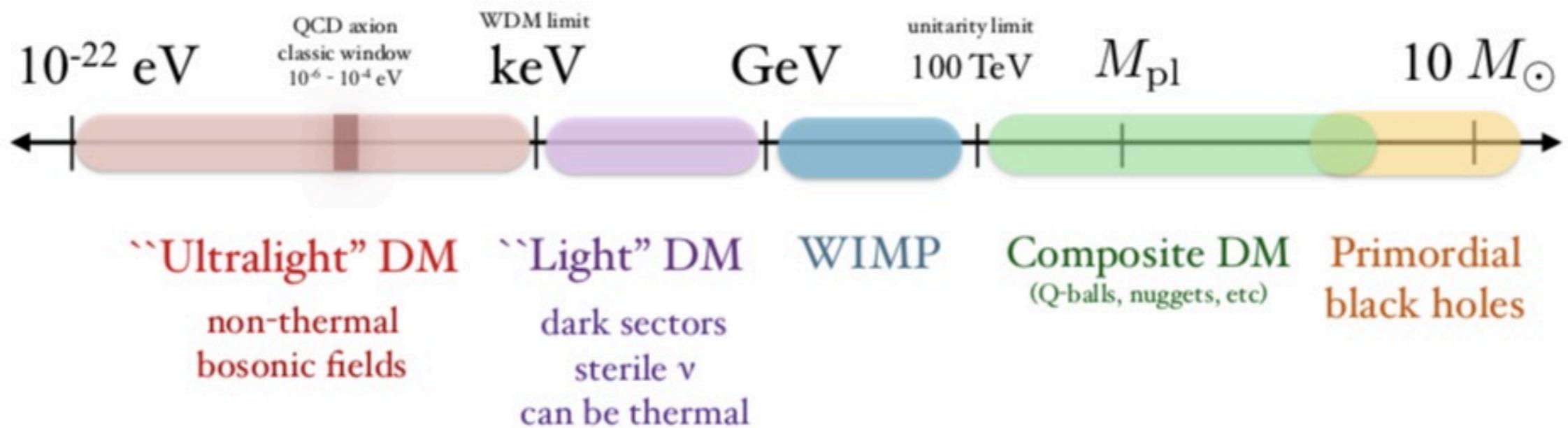
**Self-interaction DM**

**Proton-Proton elastic scattering**

**Puffy DM-DM elastic scattering**

# Motivation

## Mass scale of dark matter (not to scale)



# Motivation

## Dark matter evolution

$\Lambda$ CDM model is further in accordance with the Astronomical Data. It shows that DM is non-relativistic and collisionless in the large scale structure of the universe.

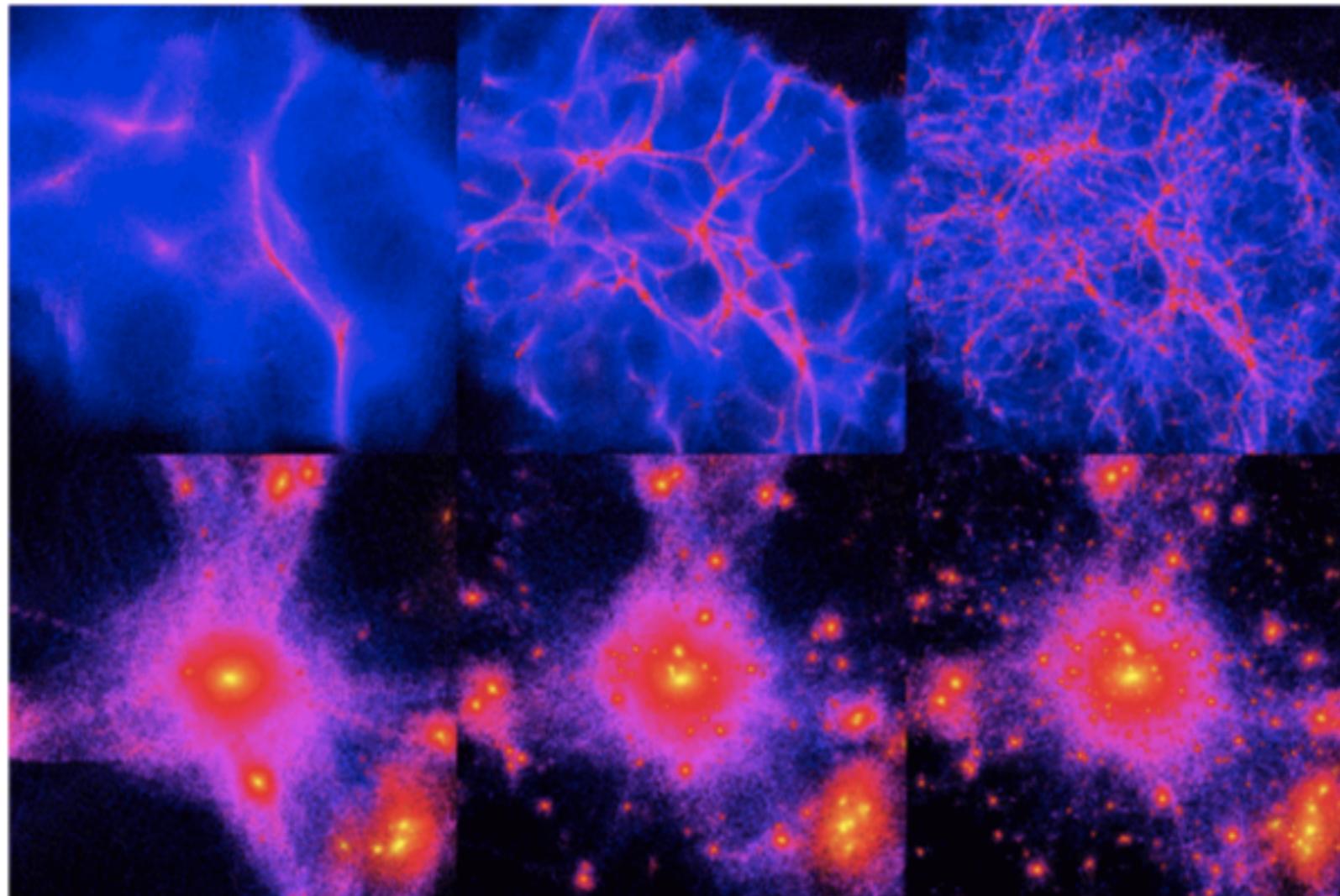


图 2.2.2: 宇宙大尺度结构的数值模拟。上图是冷暗物质模型，下图是热暗物质模型。

# Motivation

## **Self-interaction DM in the small scale**

### core-vs-cusp problem

The discrepancy is from the steep cusps predicted by the collisionless cold DM(CCDM) and the flat cores for the density profile of the DM holes

### too-big-to fail problem

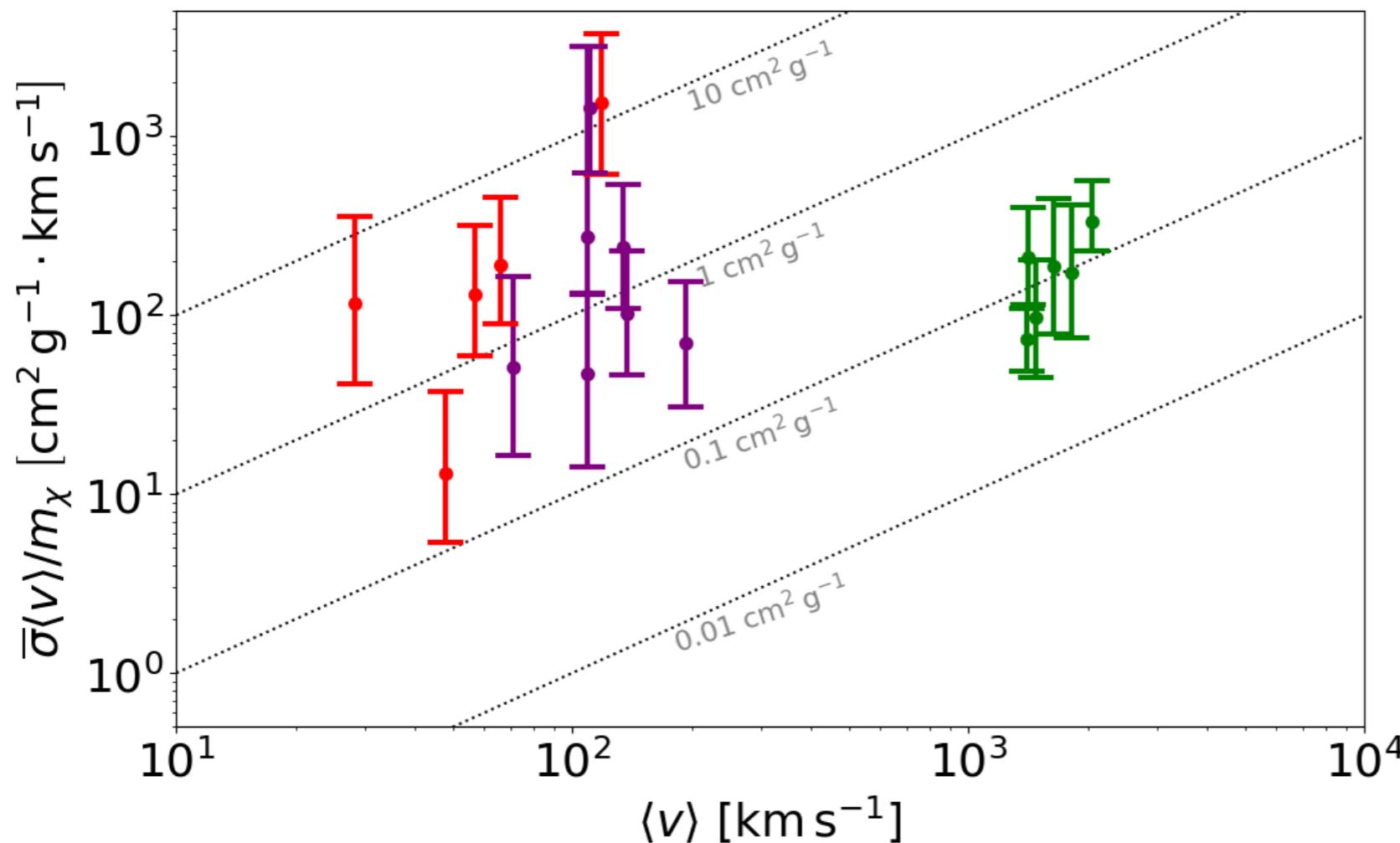
the most massive subhalos predicted by CCDM is more than that observations in the Milky Way(MW) and other dwarf galaxies

### Missing satellites problem

### Diversity problem

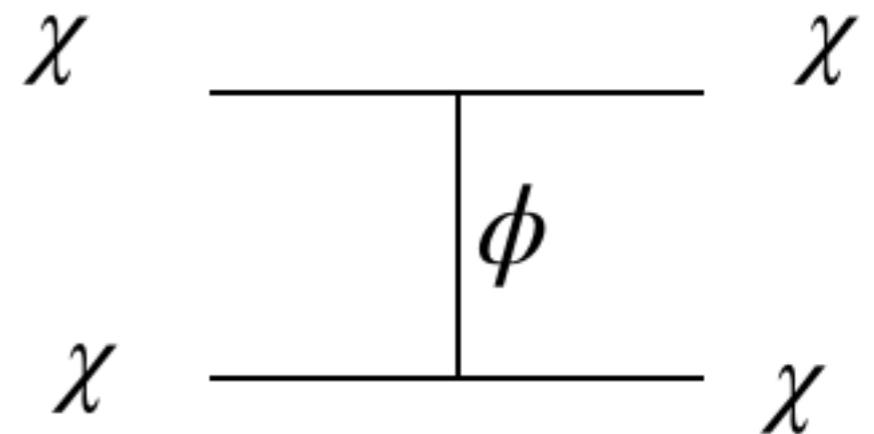
# Motivation

## Self-interaction DM



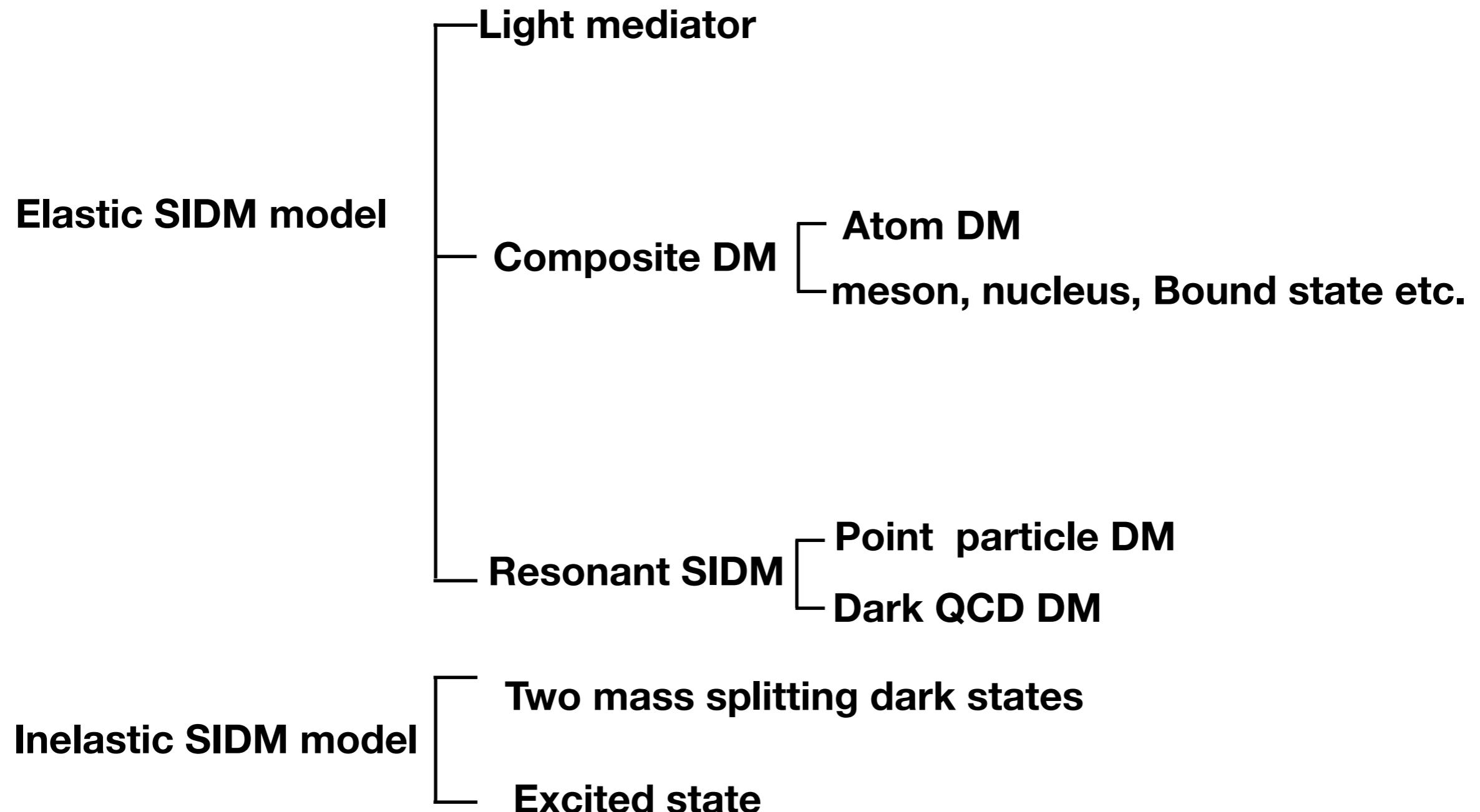
$$\frac{d\sigma}{d\Omega} = \frac{4\pi\alpha m^2}{m_\phi^4} \left( \frac{1}{1 + q^2 m_\phi^{-2}} \right)^2$$

long-range force



# Motivation

## Self-interaction DM candidates



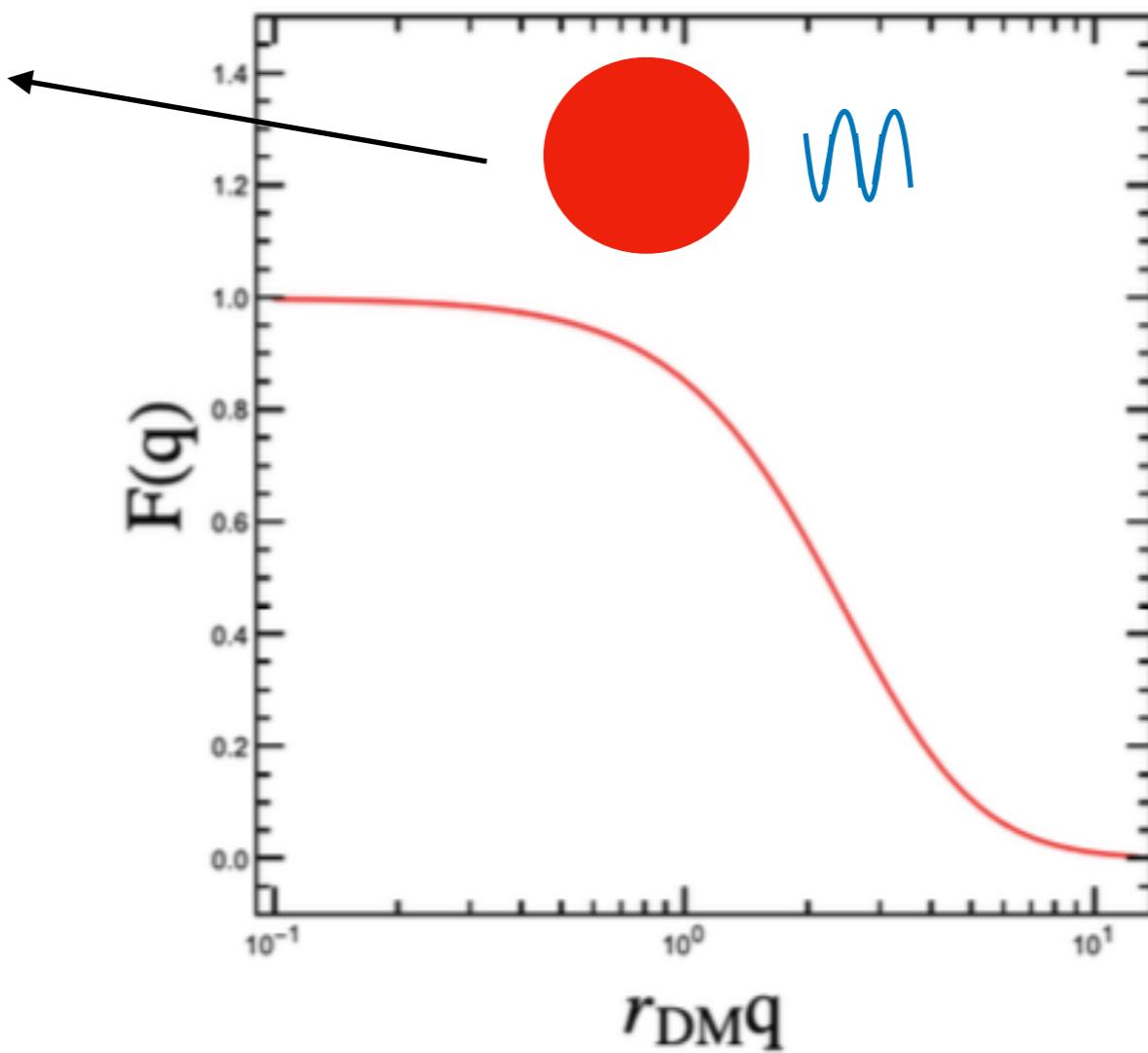
# Motivation

## Puffy DM

If dark matter has a finite size that is larger than its Compton wavelength, the corresponding self-interaction cross section decreases with the velocity.

$$F(\mathbf{q}) = \int d\mathbf{r} e^{i\mathbf{qr}} \rho(\mathbf{r})$$

$$F(q) = \frac{1}{(1 + 1/24 r_{DM}^2 q^2)^2}$$



- [2] X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

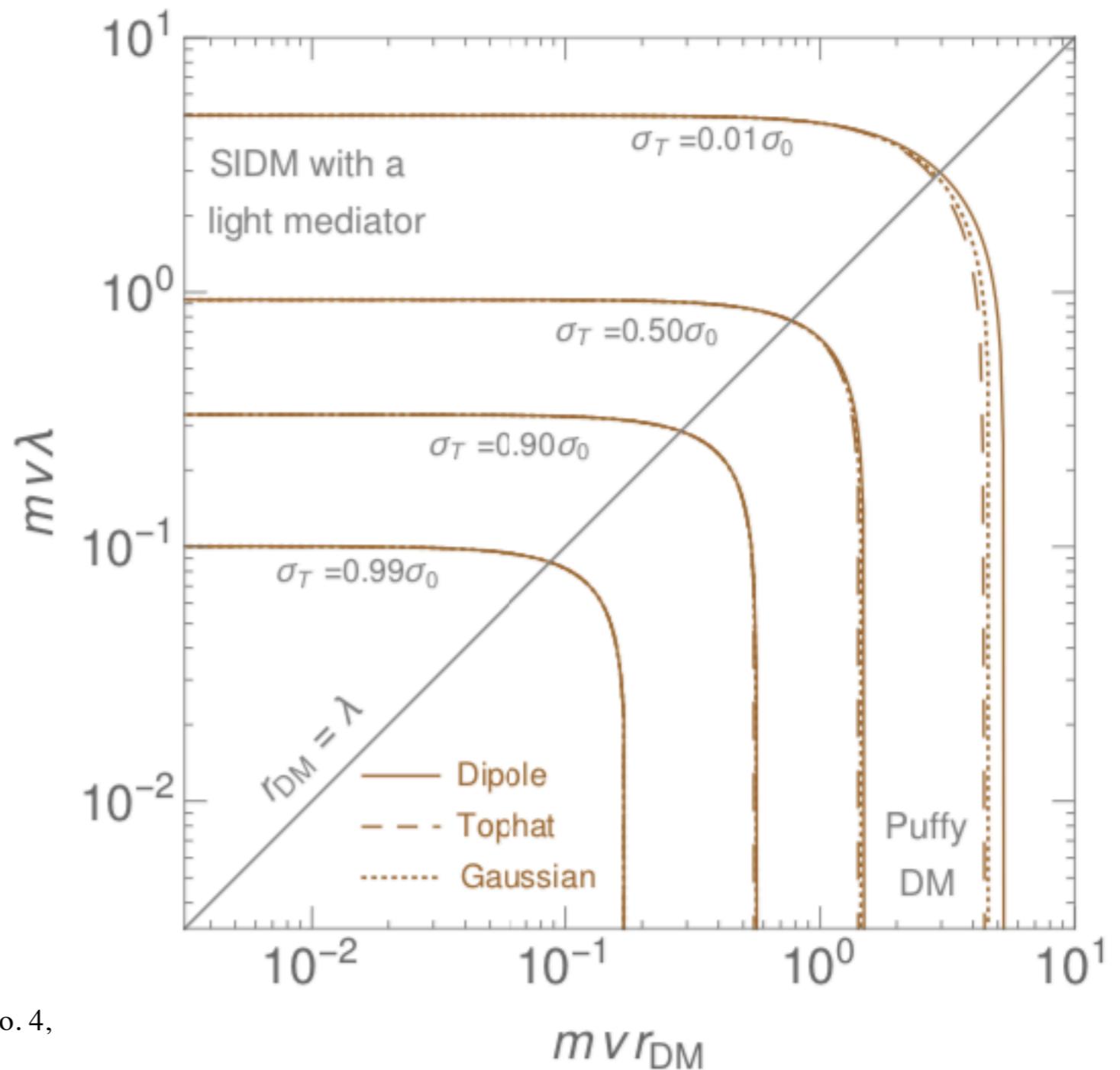
# Motivation

## Puffy DM

Even in the presence of a light particle mediating self-interactions, the finite-size effect may dominate the velocity dependence.

$$\frac{d\sigma_y}{d\Omega} = \sigma_0 \frac{F^4(q)}{((mv)^2 m_\phi^{-2} \frac{1 - \cos \theta}{2} + 1)^2}$$

$$F(q) = \frac{1}{(1 + r_0^2 m^2 v^2 \frac{1 - \cos \theta}{2})^2}$$



- [2] X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

# Self-interaction puffy DM

If dark matter has a finite size that is larger than its Compton wavelength, the corresponding self-interaction cross section decreases with the velocity.

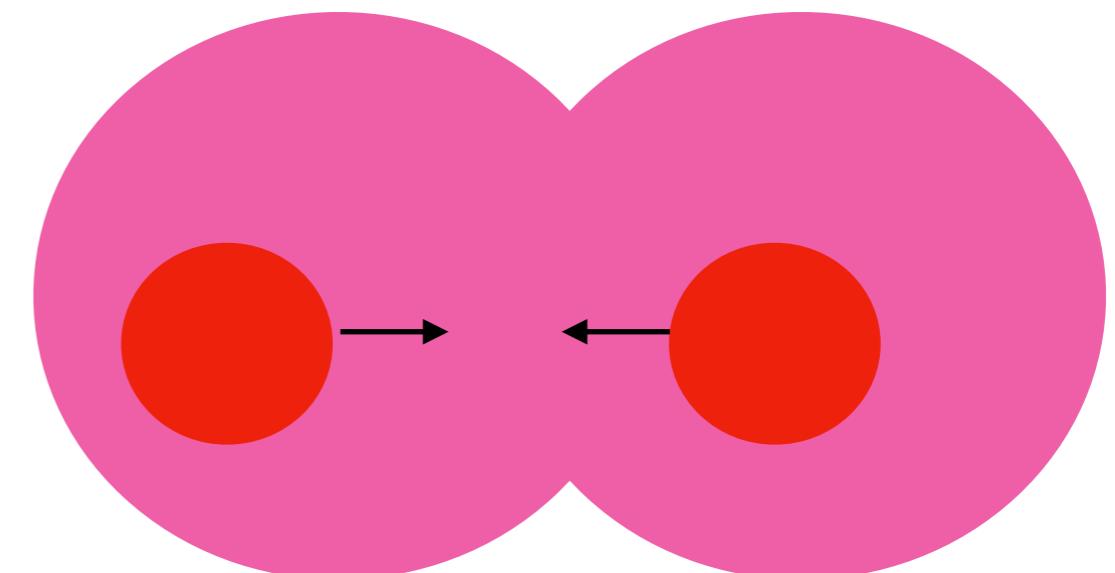
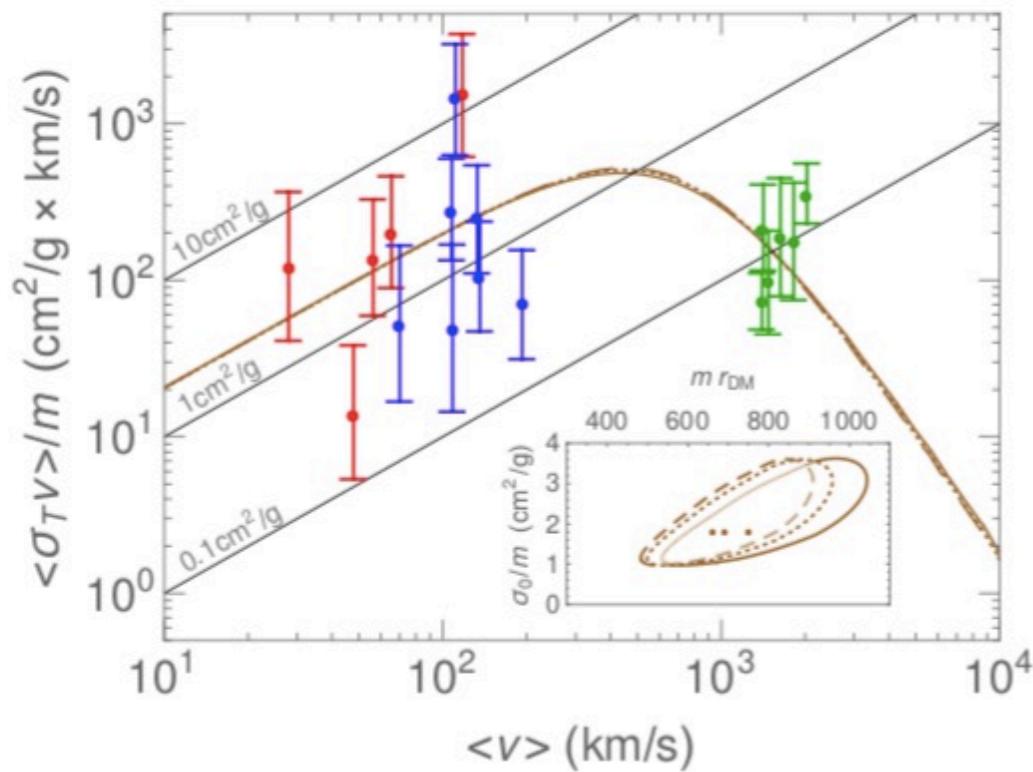


Figure 3: Velocity dependence of the transfer cross section of Puffy DM. Best-fit curves to data [41] for the dipole (solid), tophat (dashed) and the Gaussian (dotted) distributions in Table I. The inset shows the 95% C.L. contours on the parameter  $\sigma_0$  from Eq. (3) and the DM size together with the corresponding parameter sets plotted in the main figure.

**Dark strong interaction?**

# Motivation

## Proton-Proton elastic scattering

$$\frac{d\sigma}{dt} = \left| f_c(t) e^{i\alpha\phi(t)} + f_n(t) \right|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt}, \quad (1)$$

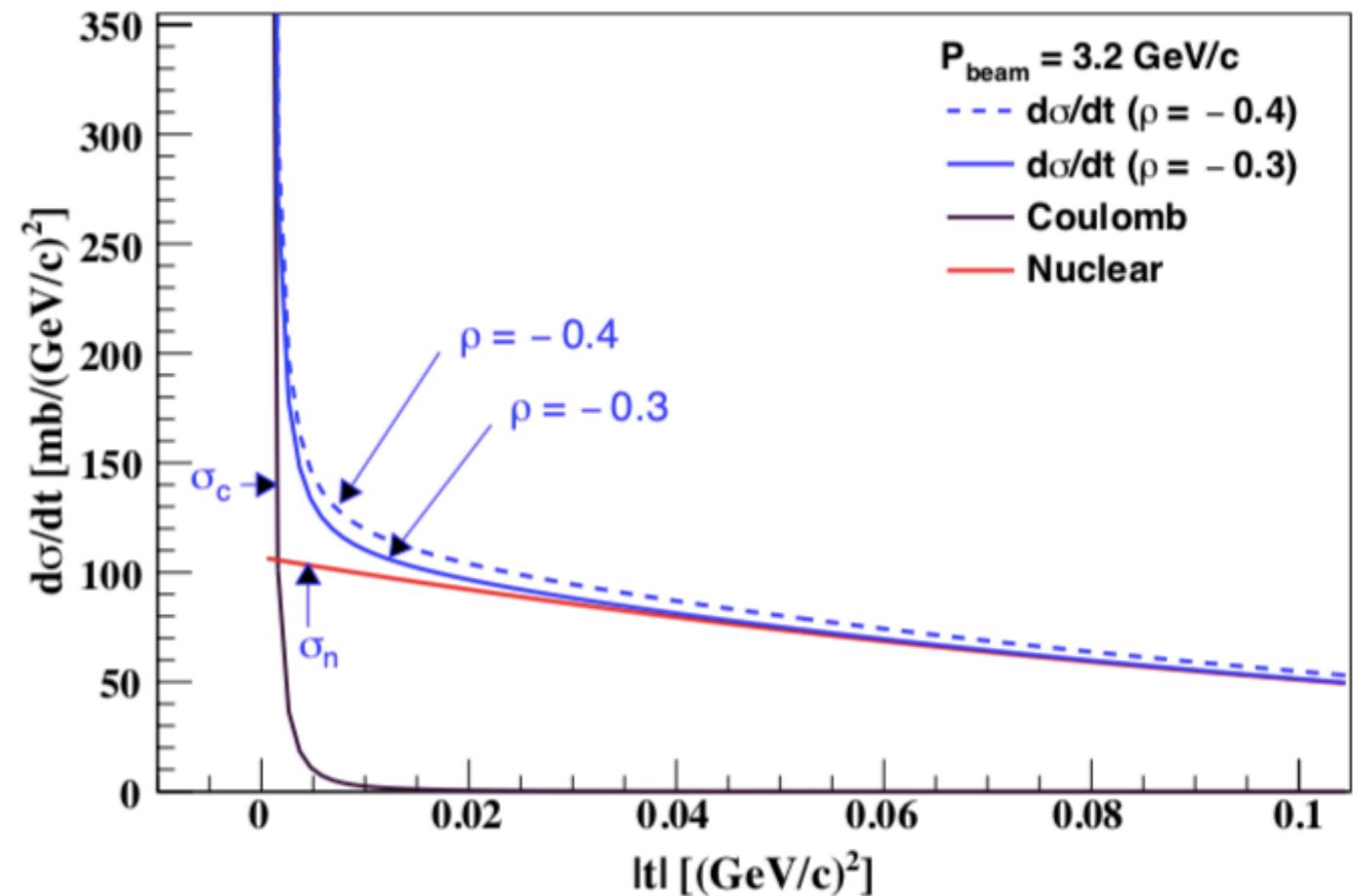
where

$$\frac{d\sigma_c}{dt} = \frac{4\pi\alpha^2 G^4(t)(\hbar c)^2}{\beta^2 t^2}, \quad (2)$$

$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{\beta |t|} G^2(t) e^{-\frac{B|t|}{2}} (\rho \cos(\alpha\phi(t)) + \sin(\alpha\phi(t))), \quad (3)$$

and

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1+\rho^2)e^{-B|t|}}{16\pi(\hbar c)^2}. \quad (4)$$



The total cross section  $\sigma_{tot}$

The relative real amplitude ratio  $\rho = Ref_n(0)/Imf_n(0)$

The slope parameter B

# dark matter-dark matter elastic scattering

$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$

where

$$\frac{d\sigma_c}{dt} = \frac{\pi}{v^2} \frac{\alpha^2 F^4(t)}{(t + m_\phi^2)^2} \quad (3)$$

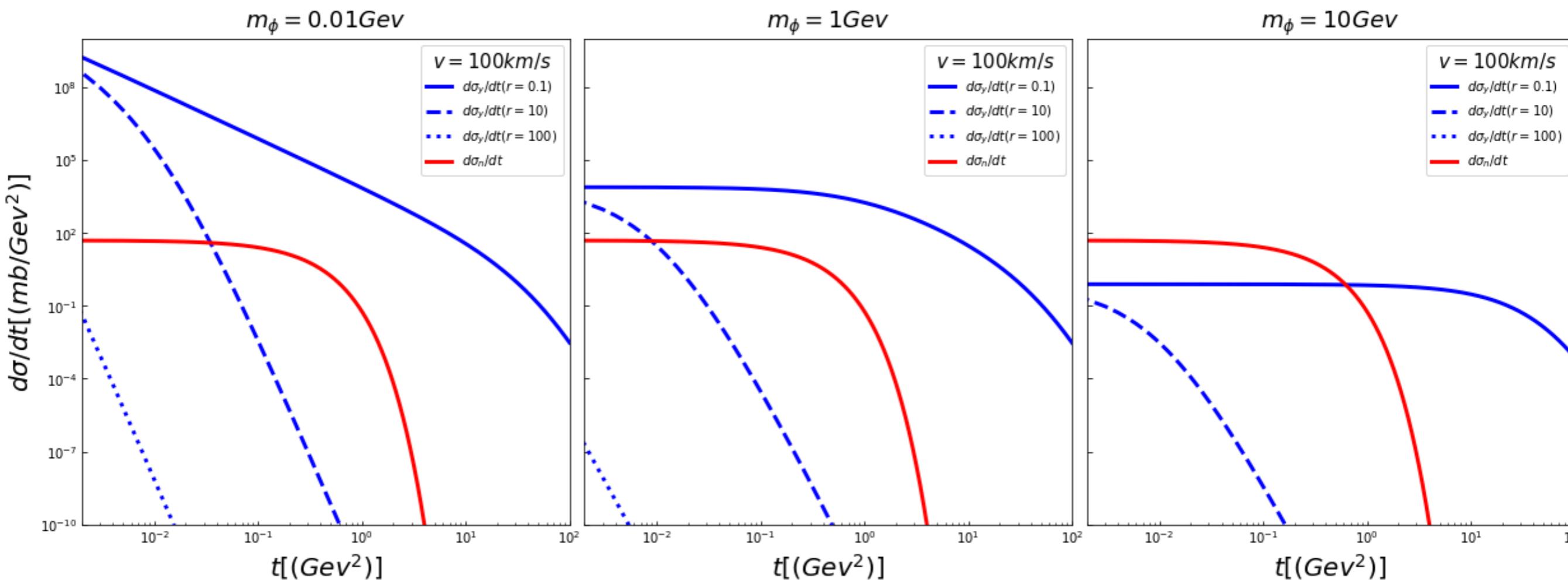
$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{2v(t + m_\phi^2)} F^2(t) e^{-\frac{B\times|t|}{2}} (\rho \cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \quad (4)$$

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1 + \rho^2)e^{-B\times|t|}}{16\pi}, \quad (5)$$

$$\text{where } \phi(t) = -[\gamma + \ln(\frac{B\times|t|}{2}) + \ln(1 + \frac{8\times r0^2}{B}) + \ln(4 \times |t| \times r0^2) \cdot (4 \times |t| \times r0^2) + 2|t| \times r0^2] \\ -$$

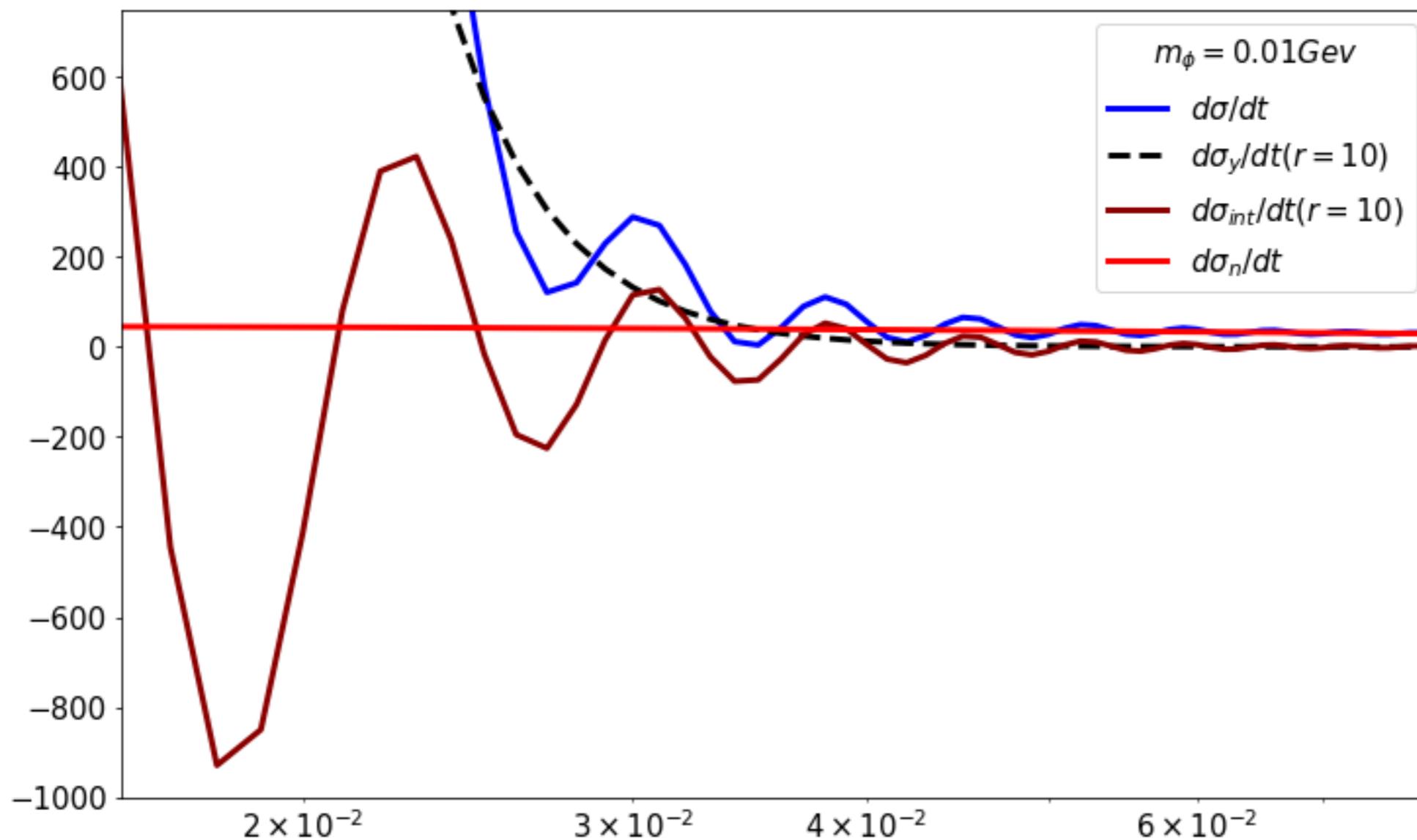
# dark matter-dark matter elastic scattering

$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$



# dark matter-dark matter elastic scattering

## Interference term



# dark matter-dark matter elastic scattering

$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$

where

$$\frac{d\sigma_c}{dt} = \frac{\pi}{v^2} \frac{\alpha^2 F^4(t)}{(t + m_\phi^2)^2} \quad (3)$$

$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{2v(t + m_\phi^2)} F^2(t) e^{-\frac{B\times|t|}{2}} (\rho \cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \quad (4)$$

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1 + \rho^2)e^{-B\times|t|}}{16\pi}, \quad (5)$$

$$\text{where } \phi(t) = -[\gamma + \ln(\frac{B\times|t|}{2}) + \ln(1 + \frac{8\times r0^2}{B}) + \ln(4 \times |t| \times r0^2) \cdot (4 \times |t| \times r0^2) + 2|t| \times r0^2] \\ -$$

# Self-interaction puffy DM

The standard partial-wave expansion of the scattering amplitude is

$$f_{c.m.}(s, t) = \frac{1}{k} \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) a_l(k) \quad a_l(k) = \frac{e^{2i\delta_l} - 1}{2i}$$

$$l \rightarrow \infty \quad \text{Polynomials} \quad p_l(\cos \theta) \rightarrow J_0[(2l + 1)\sin(\theta/2)]$$

**eikonal approximation condition:** de Broglie wavelength of the heavy dark matter are much smaller than the size of the dark matter  $mvr_{DM} \gg 1$

$$f_n(s, t) = 2k \int b db J_0(qb) a(b, s) \quad bk = l + 1/2 \quad J_0(t) = \frac{1}{2\pi} \int d\phi \exp(-it \cos \phi)$$

$$a_l \rightarrow a(b, s) \quad a(b, s) = \frac{1}{4\pi k} \int d^2 b \exp(iq \cdot b) f_{c.m.}(s, t)$$

Traditionally,  $\sigma_{tot}$  and the slope parameter  $B$  are given as, using the approximation of the impact-parameter representation, pure imaginary  $a(b, s)$  are always adopted for the forward high-energy scattering, there are several profiles such as disk, parabolic form, Gaussian shape and Chou-Yang model etc

$$\sigma_{tot} = \frac{4\pi}{k} \text{Im} f_{cm}(s, 0) = 4 \int d^2 b \text{Im} a(b, s)$$

$$B = \frac{\int d^2 b b^2 a(b, s)}{2 \int d^2 b a(b, s)}$$

# Self-interaction puffy DM

## Chou-Yang model

Chou-Yang model consider that the attenuation of two hadron going through each other is denoted by evaluating the opaqueness at the impact parameter b [5]. The density of opaqueness can be seen as the charge distribution inside the hadron.

$$a(b, s) = \frac{1}{2i}(e^{2i\delta})^{-1} = \frac{i}{2}(1 - e^{-\Omega(b)})$$

$$\Omega(b) = A \frac{1}{8}x^3 K_3(x) \quad x = b/r_0$$

$\sigma_A(A, r_0)$  are rewritten as

$$\sigma_A = \sigma_0 + \sigma_a$$

$$\sigma_a = r_0^2 \sigma_{tot} \left( \frac{1}{r_0} \right)^2 = 4 \times 2\pi \times r_0^2 \int dx (xa)$$

# SIDM in Astrophysical halos

When the DM-DM scattering is proceeded in the Astrophysical halos, the transfer cross section

$$\sigma_T = \int d\Omega (1 - \cos\theta) \frac{d\sigma}{d\Omega}$$

The total transfer cross section is

$$\begin{aligned} \sigma_T &= \int d\Omega (1 - |\cos\theta|) \frac{d\sigma}{d\Omega} \\ &= \int_{-1}^1 -2\pi d\cos\theta (1 - |\cos\theta|) \left( \frac{d\sigma_c}{d\Omega} + \frac{d\sigma_{int}}{d\Omega} + \frac{d\sigma_n}{d\Omega} \right), \end{aligned} \quad (7)$$

where

$$\frac{d\sigma_y}{d\Omega} = \frac{m^2}{4} \frac{\alpha^2 G^4(t)}{\left((mv)^2 \frac{1-\cos\theta}{2} + m_\phi^2\right)^2} \quad (8)$$

$$\frac{d\sigma_{int}}{d\Omega} = -\frac{m^2 v}{4\pi} \frac{\alpha \sigma_{tot}}{2\left((mv)^2 \frac{1-\cos\theta}{2} + m_\phi^2\right)} G^2(t) e^{-\frac{B \times |t|}{2}} (\rho \cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \quad (9)$$

$$\frac{d\sigma_n}{d\Omega} = \frac{(mv)^2}{4\pi} \frac{\sigma_{tot}^2 (1 + \rho^2) e^{-B \times |t|}}{16\pi}. \quad (10)$$

# SIDM in Astrophysical halos

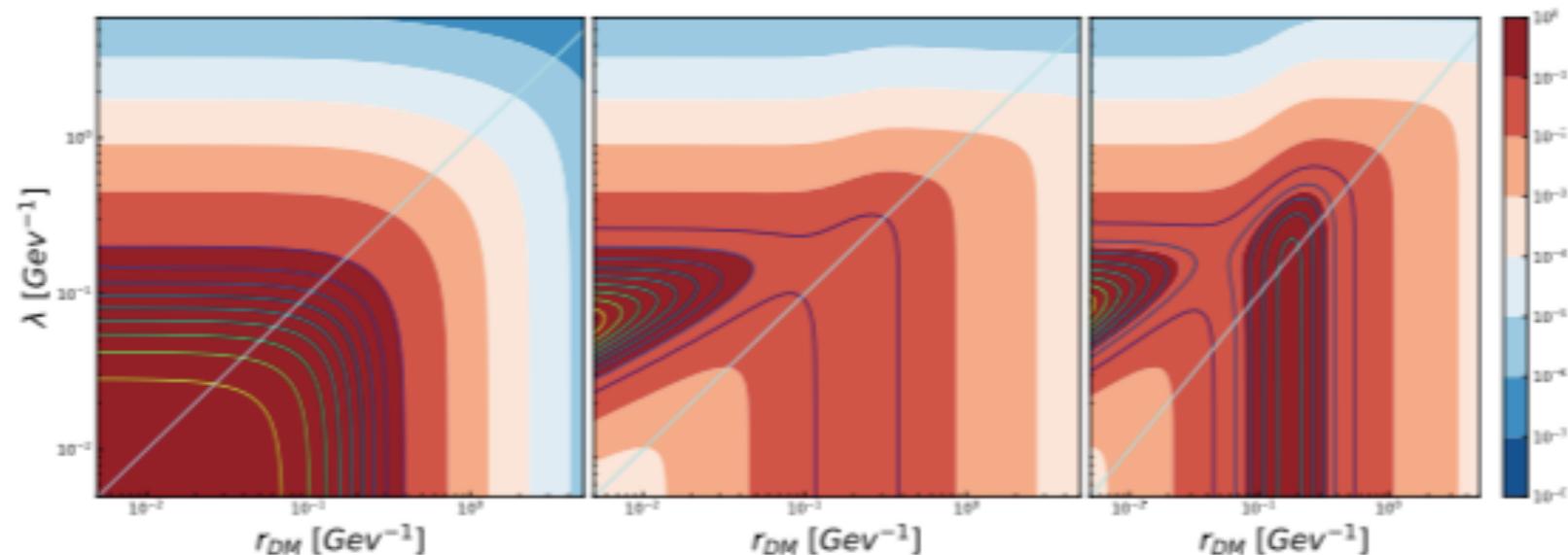


FIG. 1: Ratio  $\sigma_T/\sigma_A$  in  $r_{DM}$  and  $\lambda$  space with different absorption parameter  $A = 0, 1, 10$ , respectively. The other parameters are the same which is  $m_{DM} = 100\text{GeV}$ ,  $\alpha = 0.01$  and  $v/c = 0.1$ .

# SIDM in Astrophysical halos

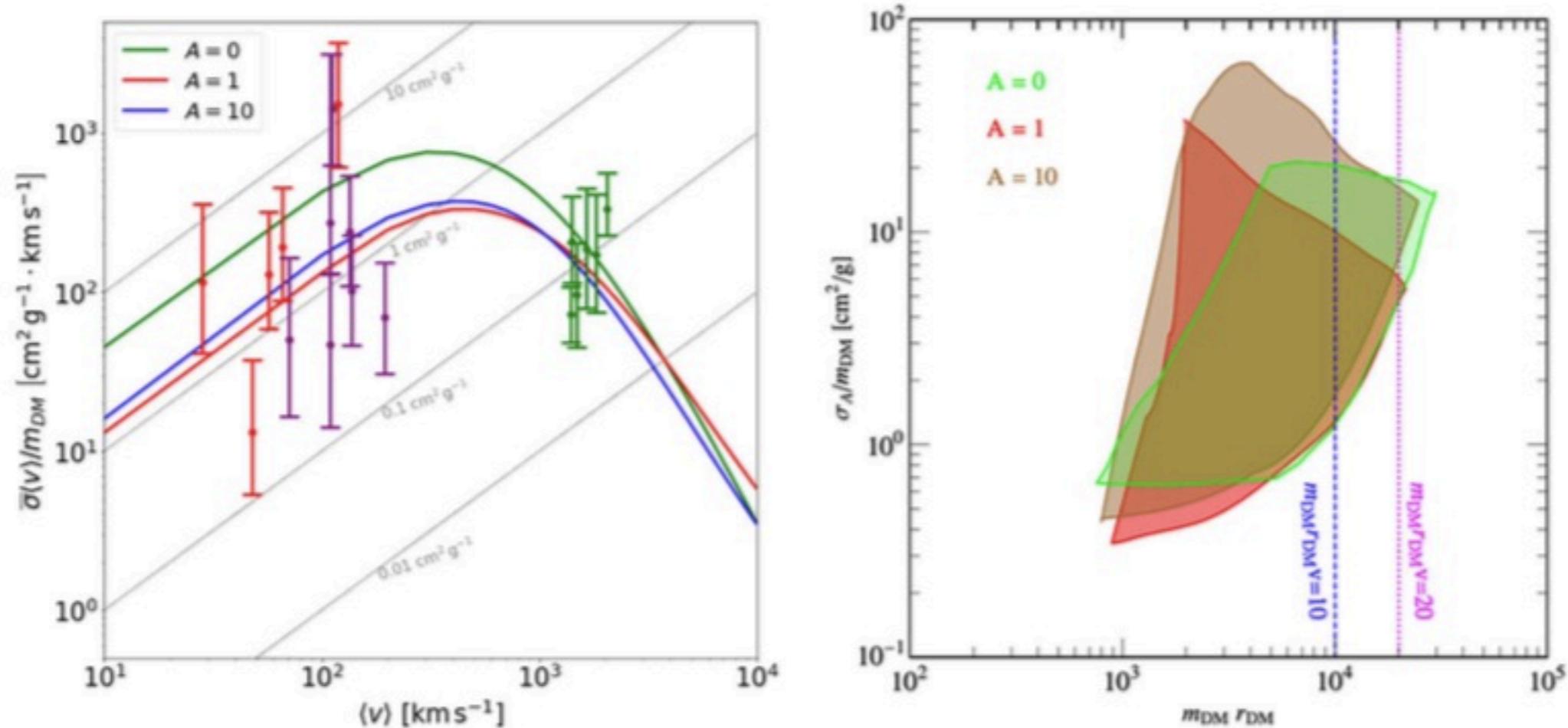


FIG. 2: Left panel: The best fit point for the velocity dependence the the puffy dark matter, Right panel: the fitted  $\sigma_A/m_{DM}$  versus  $m_{DM}r_{DM}$  in the  $\lambda < r_{DM}$  case.

# Conclusion

The sketch map of the calculation of the cross-section and a more realistic realization of the matter and charge distribution, Chou-Yang model, are shown in this work. With the participation of the strong interaction, the small scale problems can be solved.

# **Discussion**

**Puffy dark matter's production mechanism.**

**The detection of puffy DM.**