Trace anomaly in neutron stars

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Reference:

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, arXiv:2207.06753

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Outline of this talk

Q. Dense (conformal) quark matter in neutron stars (NSs)?

→ Trace anomaly can be a useful measure

1. Rapid approach to the conformal limit of the trace anomaly, giving rise to the sound velocity peak

2. Strongly-interacting conformal matter inside NS?

3. Is the trace anomaly positive semi-definite at finite density?

Outline of the talk

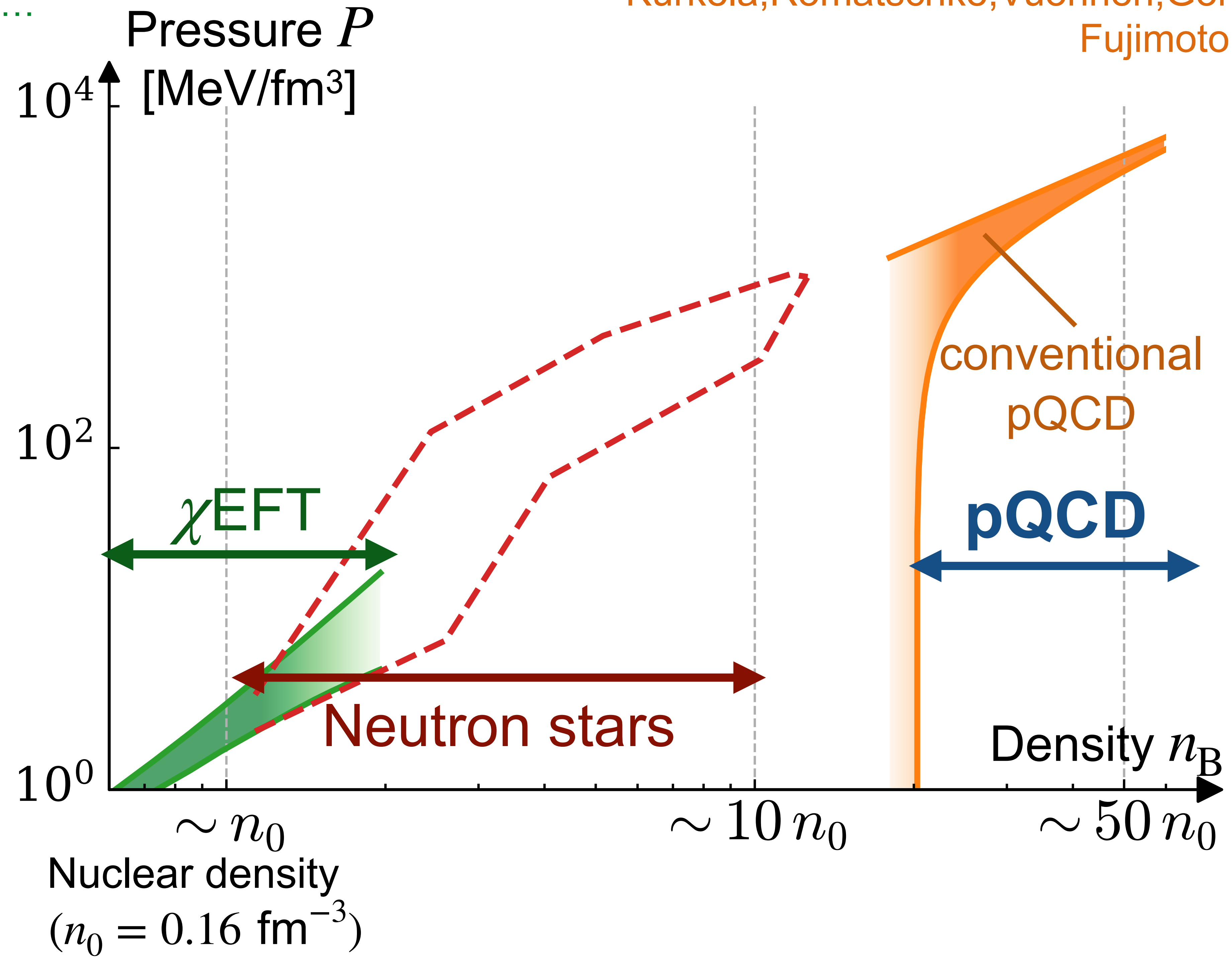
- Introduction
- Trace anomaly from actual data, its relation to a peak in the sound velocity
- Strongly-interacting conformal matter and positivity of the trace anomaly
- Possible implications from field theory and NS observations?
- Summary

QCD point of view on the EoS

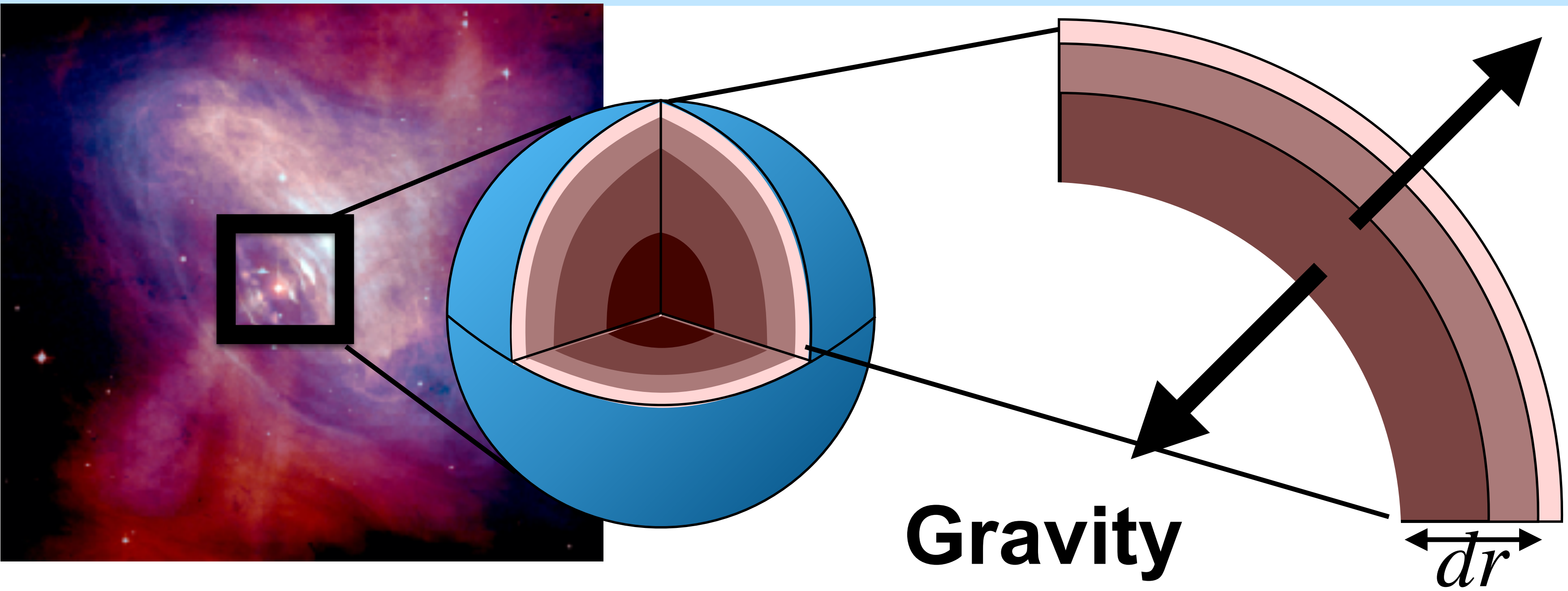
ab initio QCD calculations:

χEFT: Drischler,Furnstahl,Melendez,Philips(2020)
Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)
& many others…

pQCD: Freedman,McLerran (1978); Baluni (1979);
Kurkela,Romatschke,Vuorinen,Gorda,Sappi,+ (2009-)
Fujimoto,Fukushima (2020)



Structure equation for neutron stars



Hydrostatic equilibrium (pressure = gravity)

Tolman (1939)
Oppenheimer, Volkoff (1939)

$$\frac{dP(r)}{dr} = -G\frac{m(r)\varepsilon(r)}{r^2} \times \underbrace{\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi r^3 P}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}}_{\text{General relativistic correction}}$$

← **TOV equation**

$$m(r) = \int_0^r dr 4\pi r^2 \varepsilon(r)$$

Unknown variables:
$P(r)$, $m(r)$ and $\varepsilon(r)$

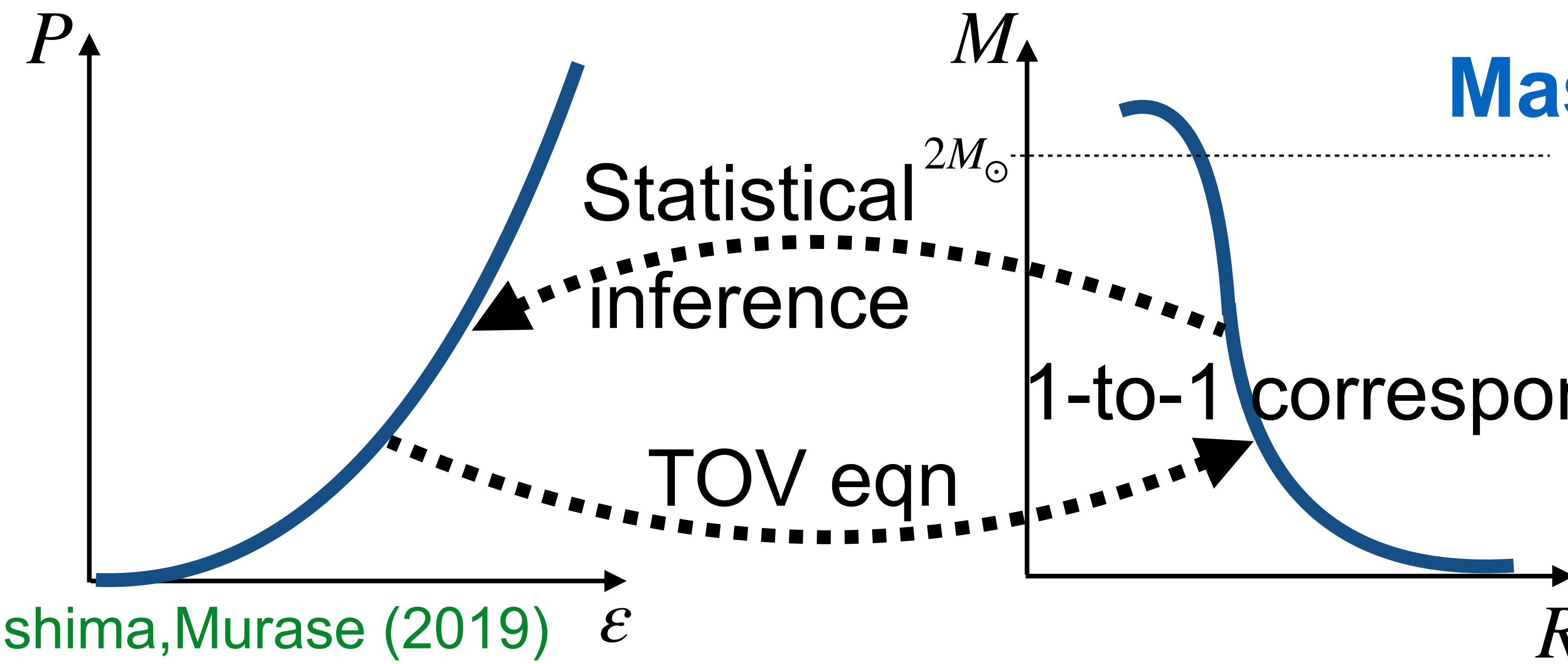
One condition missing →

Equation of State (EoS)

$$P = P(\varepsilon)$$

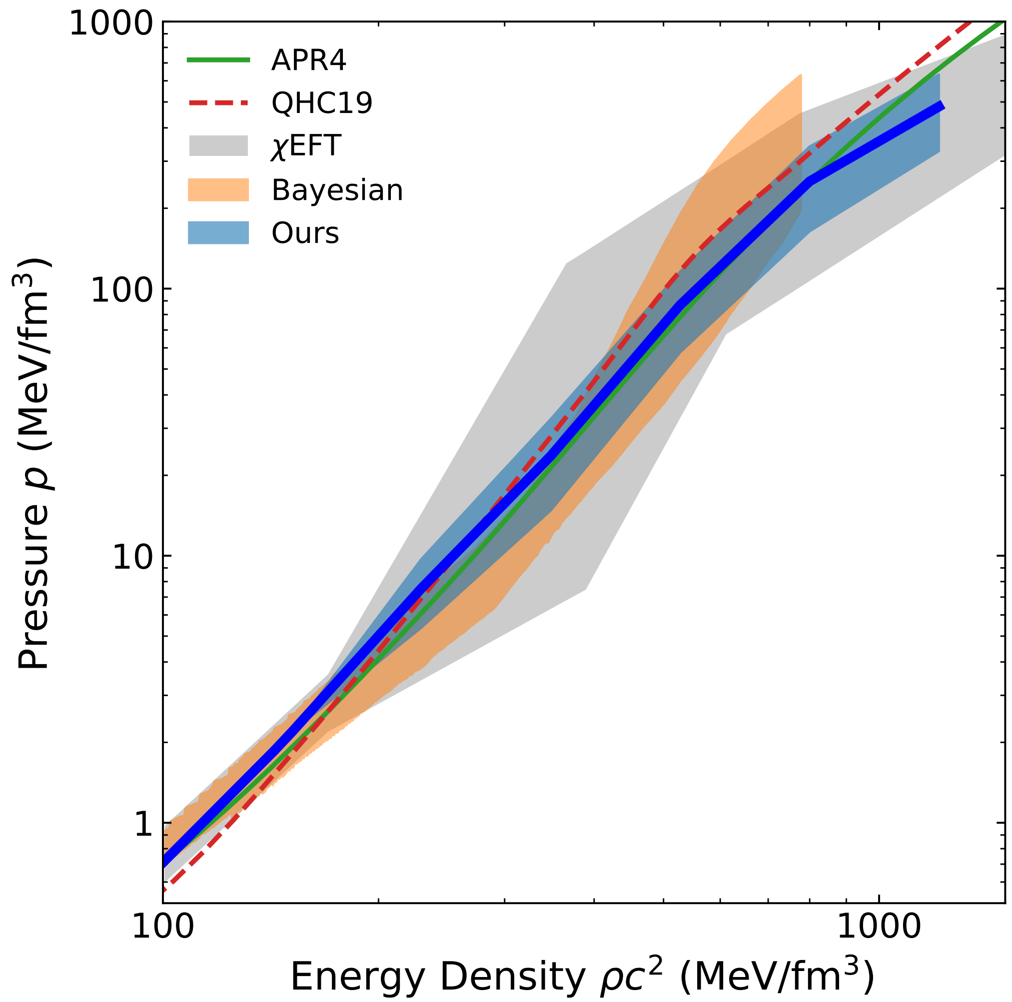
EoS and mass-radius relation

EoS

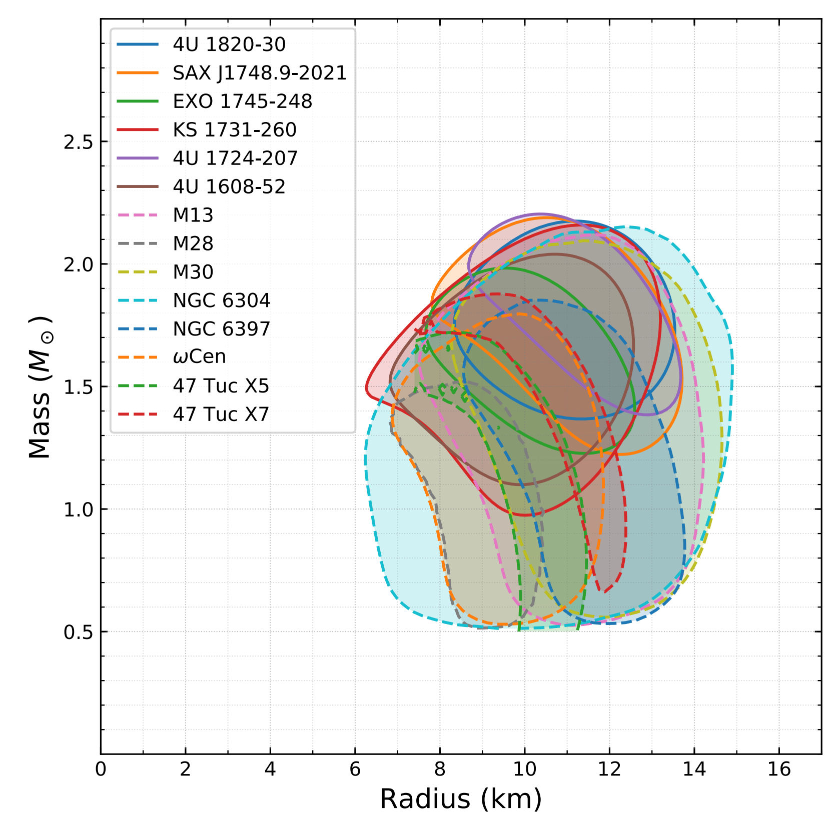


Mass-Radius

Fujimoto,Fukushima,Murase (2019)



Deep learning



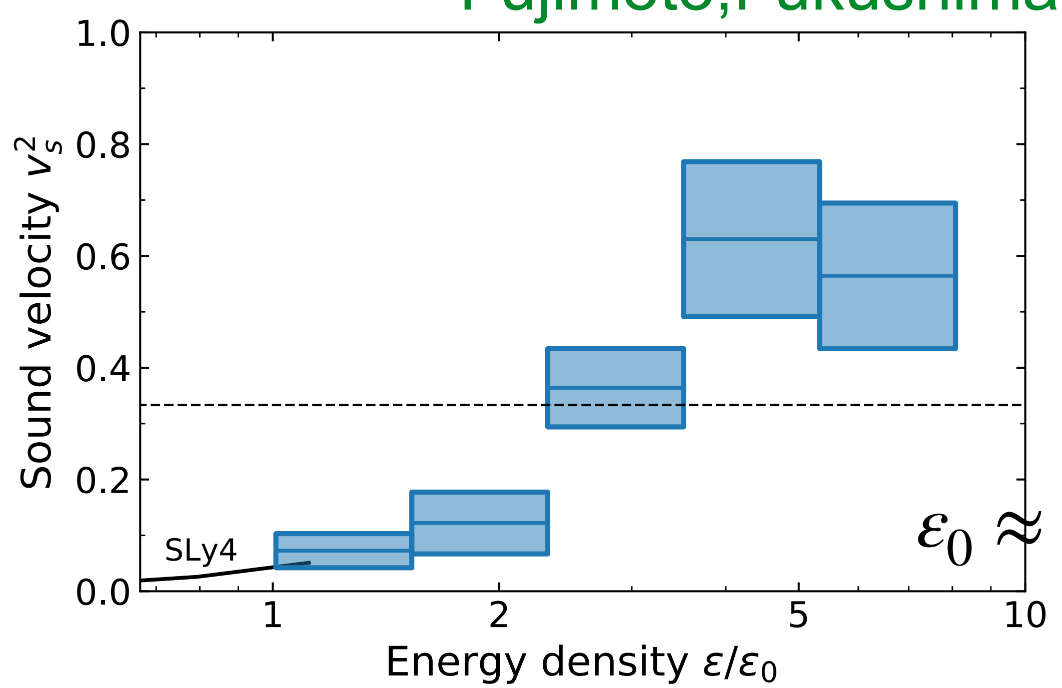
Özel et al. (2015); NICER (2019)

Sound velocity peak in the EoS

- NS data favors rapid increase in sound velocity, accompanied by a peak structure

$$v_s^2 \equiv \frac{dP}{d\varepsilon} = \frac{P'(\mu_B)}{\varepsilon'(\mu_B)}$$

Fujimoto,Fukushima,Murase (2019)



$\varepsilon_0 \approx 150\,\mathrm{MeV/fm^3}$

Cherman,Cohen,Nellore (2009); cf. Hohler,Stephanov (2009)

- There was a hypothesis that the conformal limit, $v_s^2 \leq 1/3$, is the absolute bound on the sound velocity

$\rightarrow$ In strong tension with the heavy ($M \gtrsim 2M_\odot$) pulsars!

Bedaque,Steiner (2015); Tews,Carlson,Gandolfi,Reddy (2018); Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020); Altiparmak,Ecker,Rezzolla (2022); Gorda,Komoltsev,Kurkela (2022) & many others

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- **Trace anomaly from actual data, its relation to a peak in the sound velocity**

- Strongly-interacting conformal matter and positivity of the trace anomaly

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Trace anomaly equation

- The trace of the QCD energy-momentum tensor $T^{\mu}_{\ \mu}$ is a measure of scale invariance, or conformality:

$$T^{\mu}_{\ \mu} = \frac{\beta}{2g} F^a_{\mu\nu} F^{\mu\nu}_a + (1+\gamma_m) \sum_f m_f \bar{q}_f q_f$$

- Finite-μ_B part of the trace anomaly (interaction measure):

$$\langle T^{\mu}_{\ \mu} \rangle_{\mu_B} = \varepsilon - 3P$$

$$\langle T^{\mu}_{\ \mu} \rangle = \langle T^{\mu}_{\ \mu} \rangle_{\mu_B} + \langle T^{\mu}_{\ \mu} \rangle_0$$

- We consider the normalized trace anomaly:

$$\boxed{\Delta \equiv \frac{\langle T^{\mu}_{\ \mu} \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}}$$

$$-\frac{2}{3} \lesssim \Delta \leq \frac{1}{3}$$

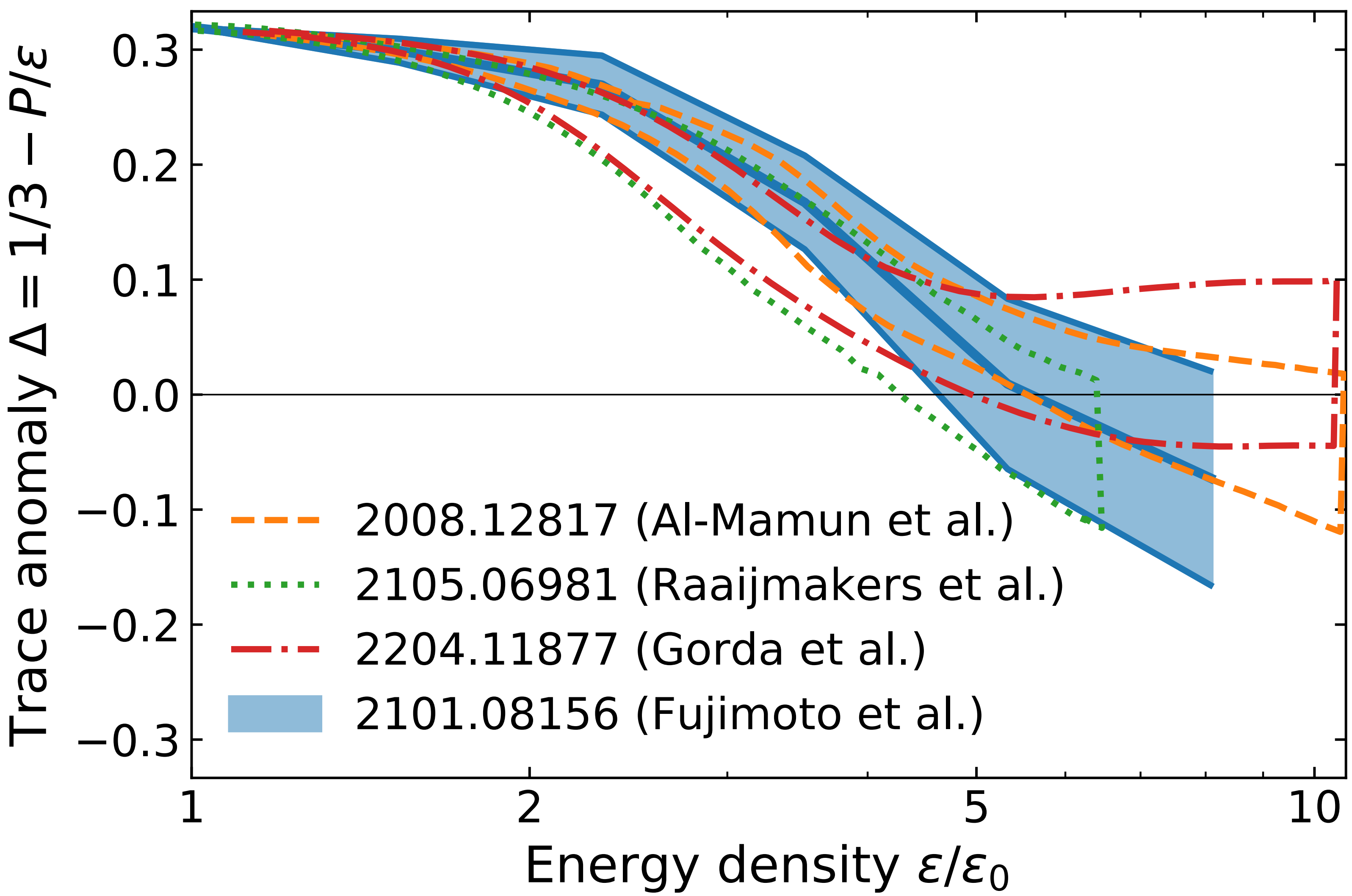
cf. Gavai,Gupta,Mukherjee (2004)

Trace anomaly from neutron star data

Fujimoto,Fukushima,McLerran,Praszalowicz (2022)

$$\Delta \equiv \frac{\langle T^\mu_\mu \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

- Inferred from neutron star data:



$\Delta \sim 0$ already at $\sim 5\varepsilon_0$

$\rightarrow$ **rapid approach to conformality**

Suggests strongly-coupled conformal matter with $P \approx \varepsilon/3$

Decomposition of sound velocity

- Sound velocity can be decomposed into Δ and its derivative

$$v_s^2 = \varepsilon \frac{d}{d\varepsilon}\left(\frac{P}{\varepsilon}\right) + \frac{P}{\varepsilon}$$

$$= \underbrace{\varepsilon \frac{d\Delta}{d\varepsilon}}_{\text{Derivative component}} + \underbrace{\left(\frac{1}{3} - \Delta\right)}_{\text{Non-derivative component}}$$

- Two bounds put by conformal limit: $\Delta \geq 0$ and $v_s^2 \leq 1/3$

This decomposition explains why $\Delta \geq 0$ and $v_s^2 > 1/3$ are possible simultaneously

Decomposition of sound velocity

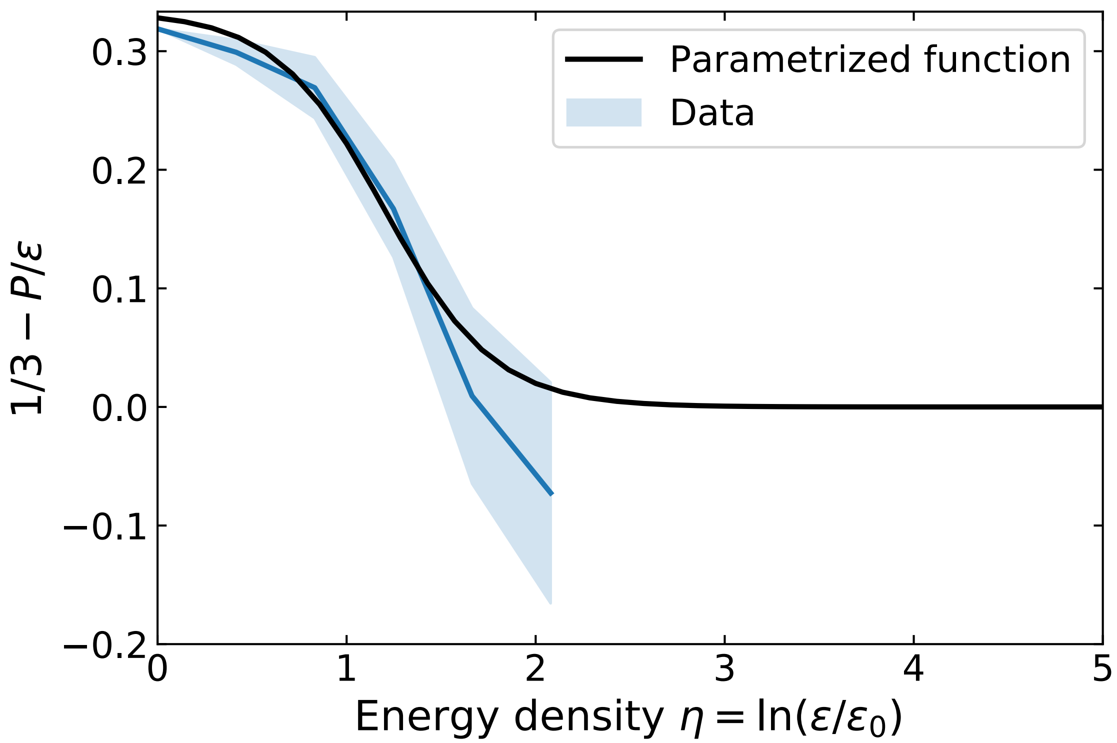
Fujimoto,Fukushima,McLerran,Praszalowicz (2022)

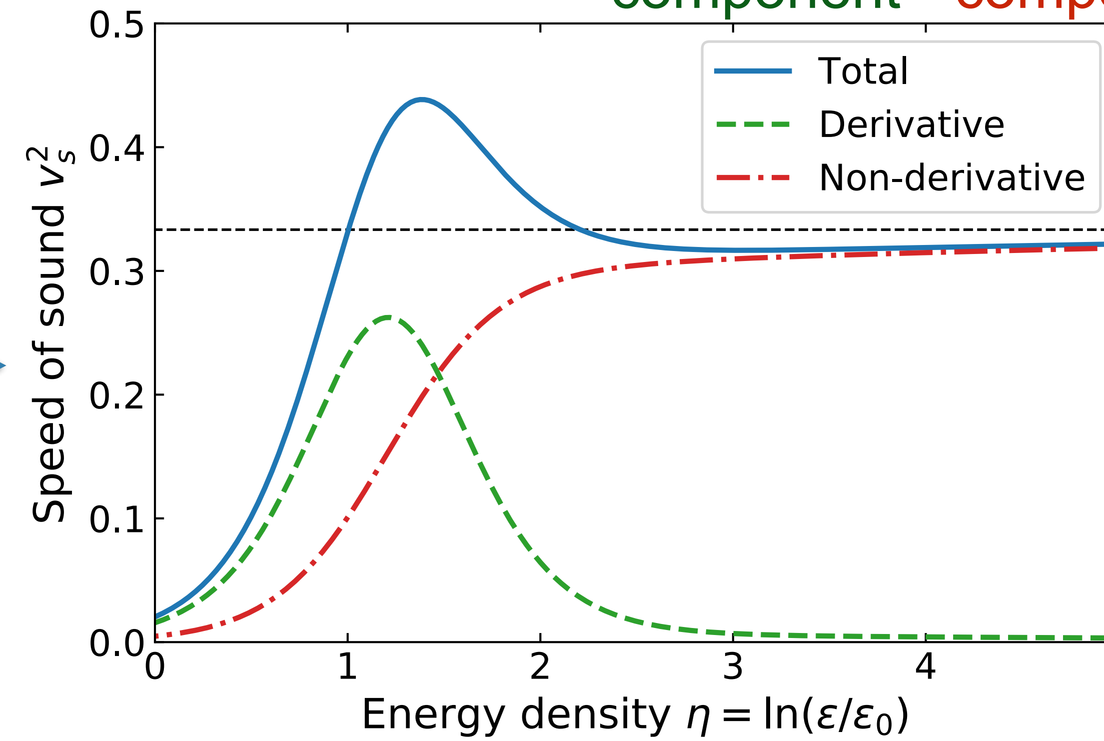
Rapid approach to $\Delta \to 0$ naturally spikes v_s^2

Sound velocity $v_s^2 = \varepsilon \dfrac{d\Delta}{d\varepsilon} + \left(\dfrac{1}{3} - \Delta\right)$

Derivative component Non-derivative component

Trace anomaly $\Delta = \dfrac{1}{3} - \dfrac{P}{\varepsilon}$





$\Delta \geq 0$ $v_s^2 > 1/3$

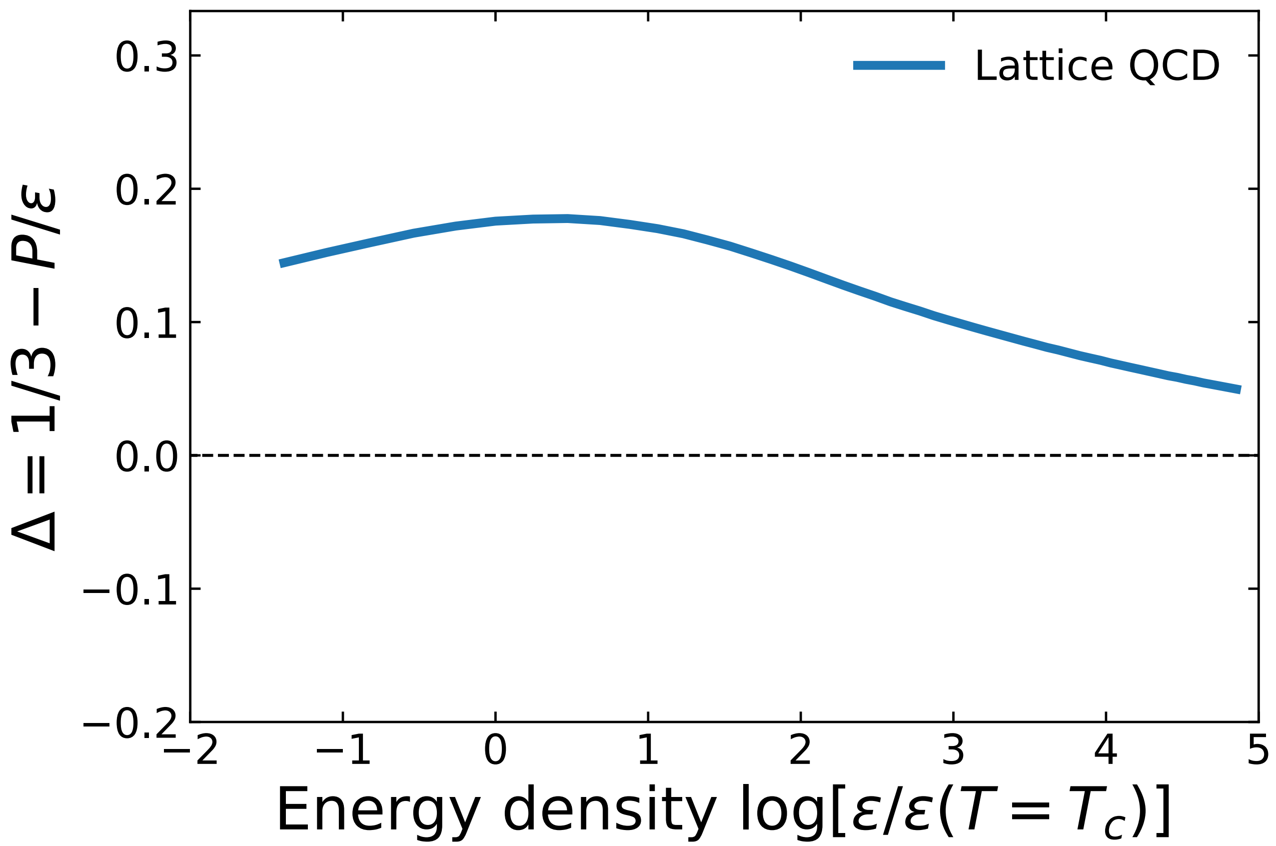
Monotonic Δ gives rise to non-monotonic v_s^2 and violation of $v_s^2 \leq 1/3$

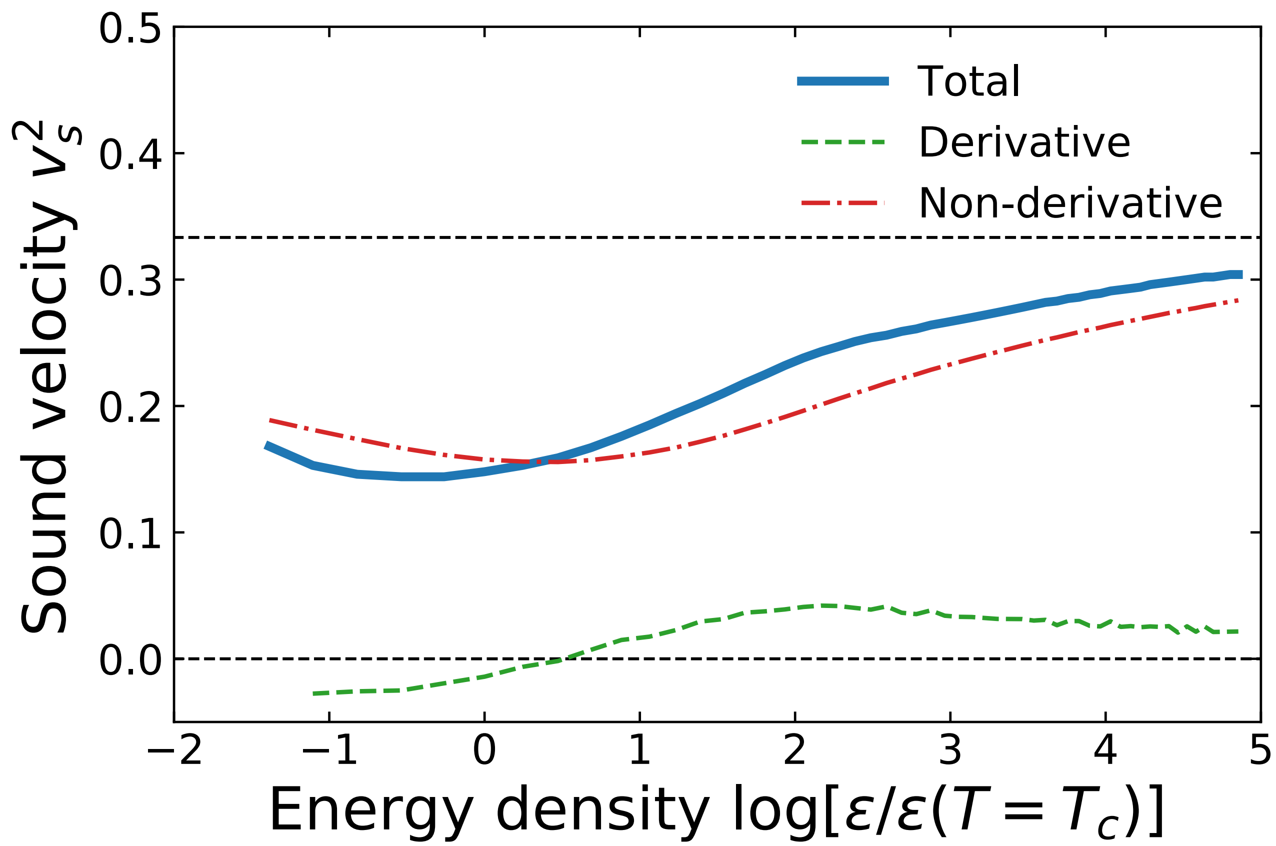
Derivative component creates the peak

Cf. Sound velocity at finite-T

Sound velocity $v_s^2 = \varepsilon \underbrace{\frac{d\Delta}{d\varepsilon}}_{\text{Derivative component}} + \underbrace{\left(\frac{1}{3} - \Delta\right)}_{\text{Non-derivative component}}$

Trace anomaly $\Delta = \frac{1}{3} - \frac{P}{\varepsilon}$

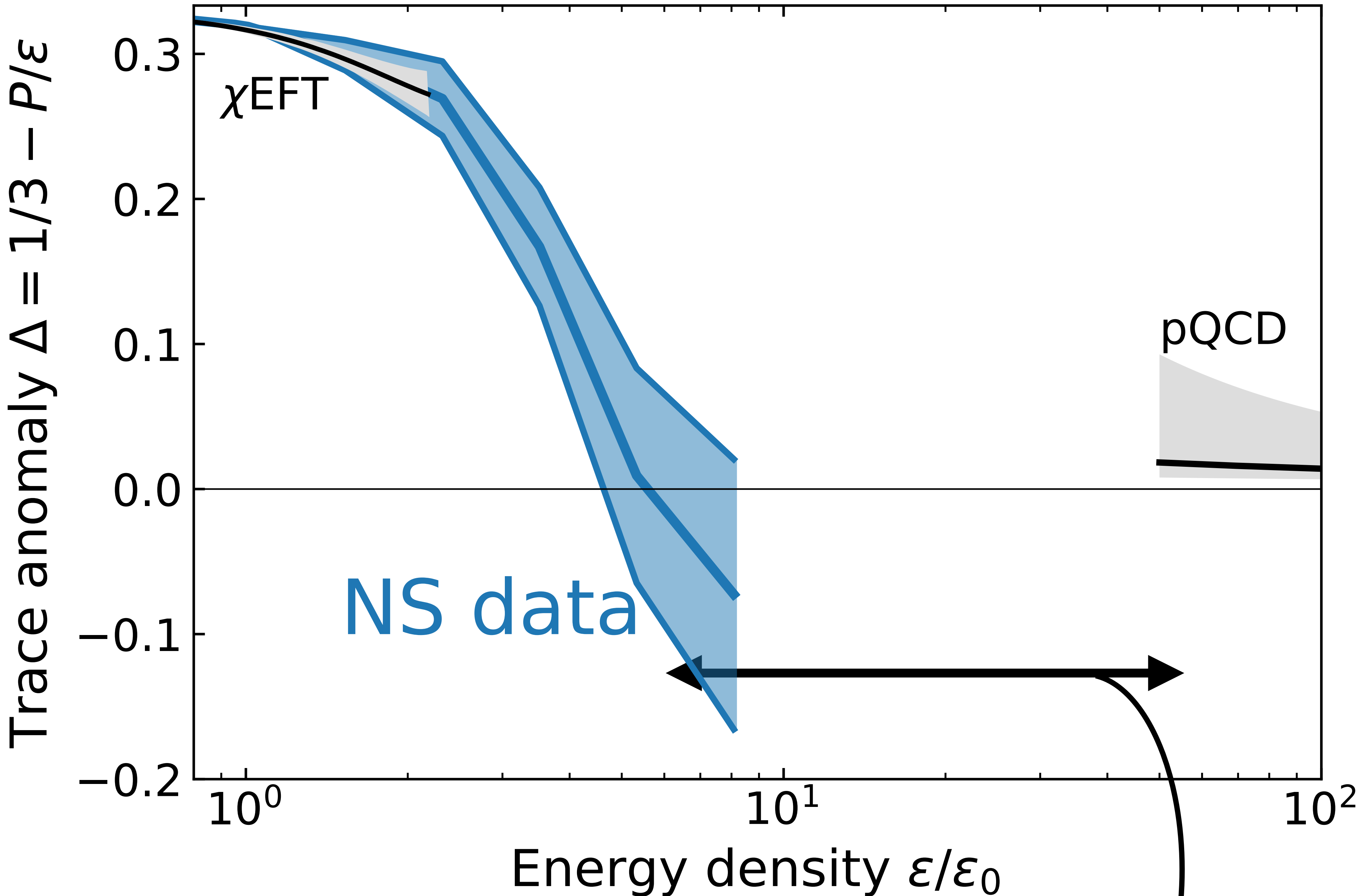




$v_s^2 \approx P/\varepsilon$ HotQCD (2014)

Derivative component does not contribute

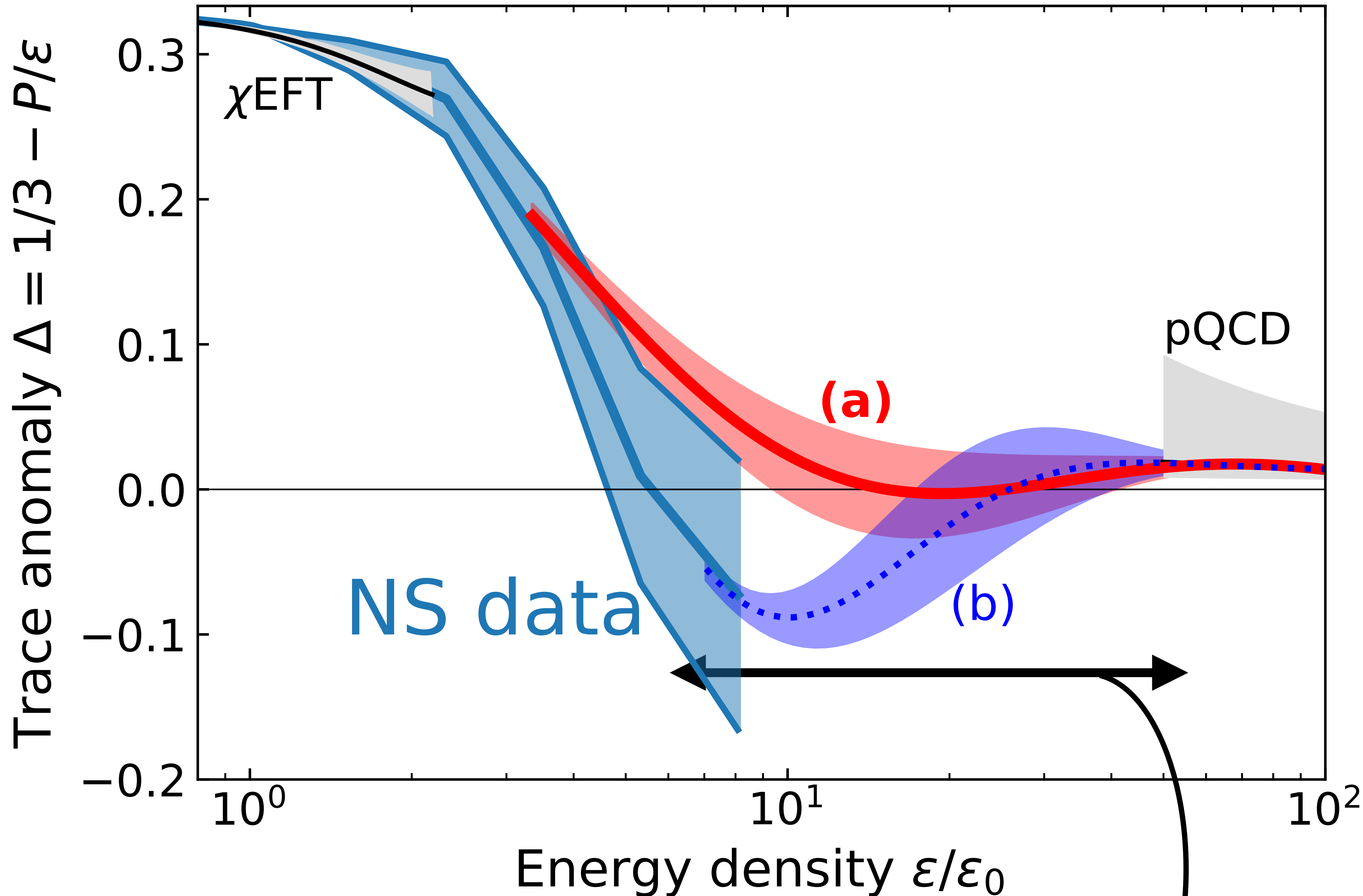
Strongly-coupled conformal matter



not well-constrained by data
→ QCD tells us something?

Strongly-coupled conformal matter

Interpolation by Gaussian process:



not well-constrained by data
$\rightarrow$ QCD tells us something?

Is the trace anomaly positive?

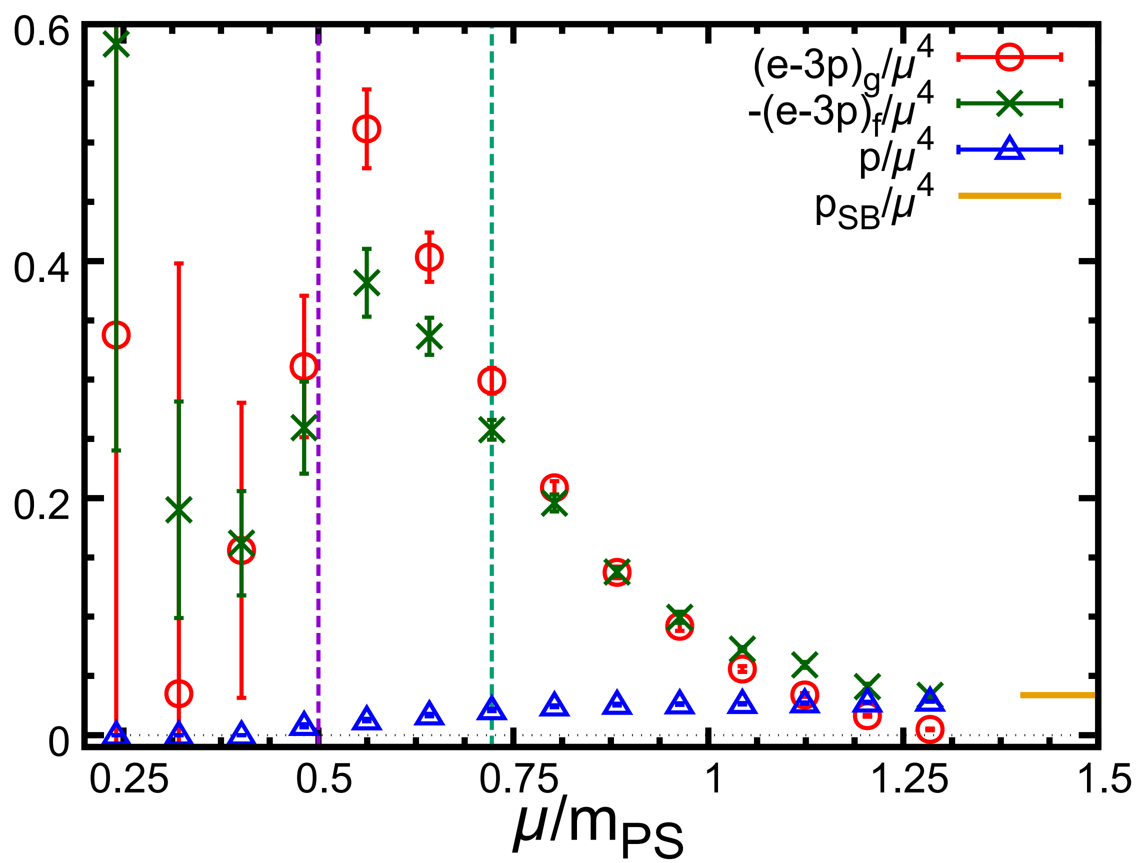
- **It may well be positive, but it doesn't have to be.**

  cf) single-particle matrix element: $\langle p \,|\, T^{\mu}_{\ \mu} \,|\, p \rangle \sim p^2 = m^2 \geq 0$

- Several known examples of negative trace anomaly:
  - Two-color QCD e.g., Cotter,Giudice,Hands,Skullerud (2012); Iida,Itou (2022)
  - QCD at finite isospin chemical potential Son,Stephanov (2001); Brandt,Endrodi+ (2018-)…

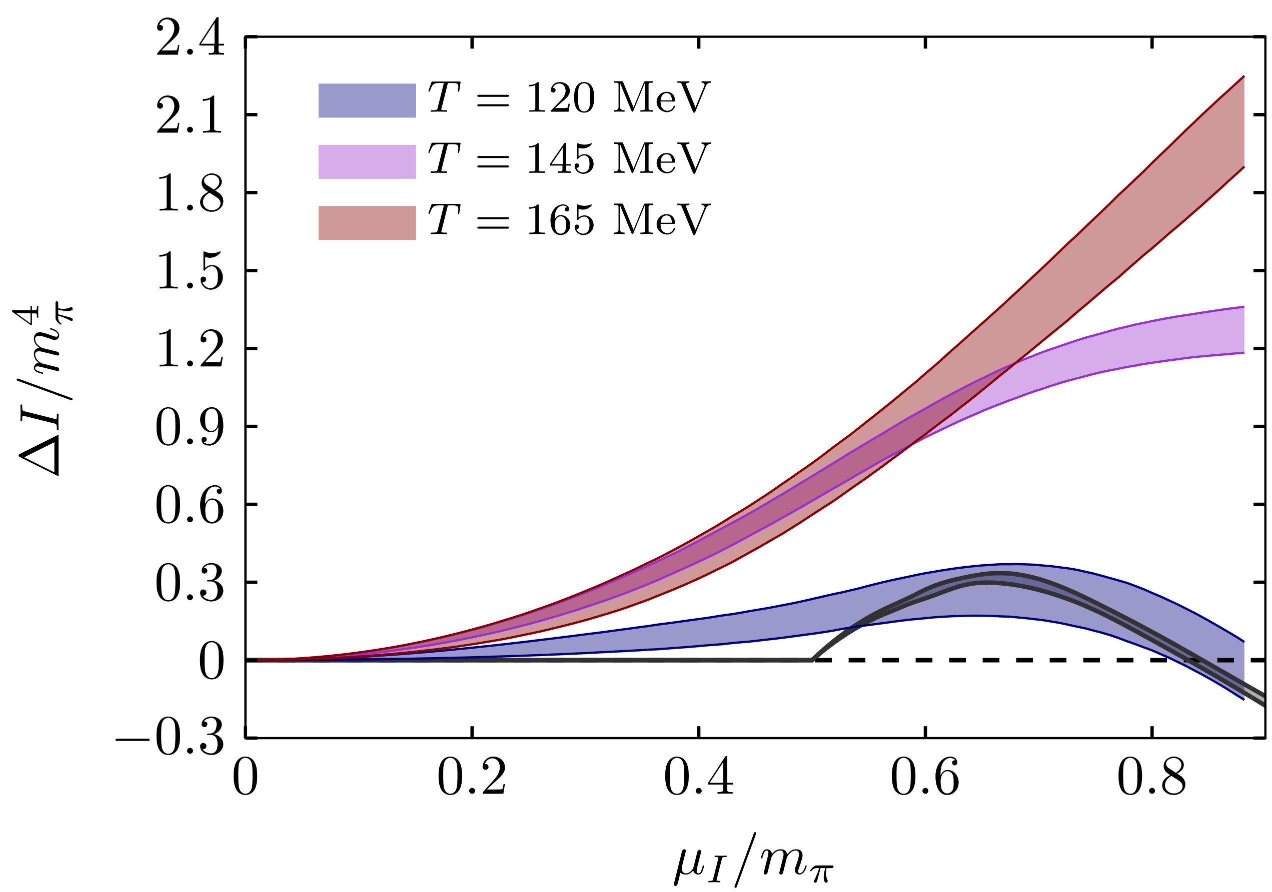
2-color QCD & finite-μ_I lattice data

$N_c = 2$ QCD



Iida,Itou (2022)

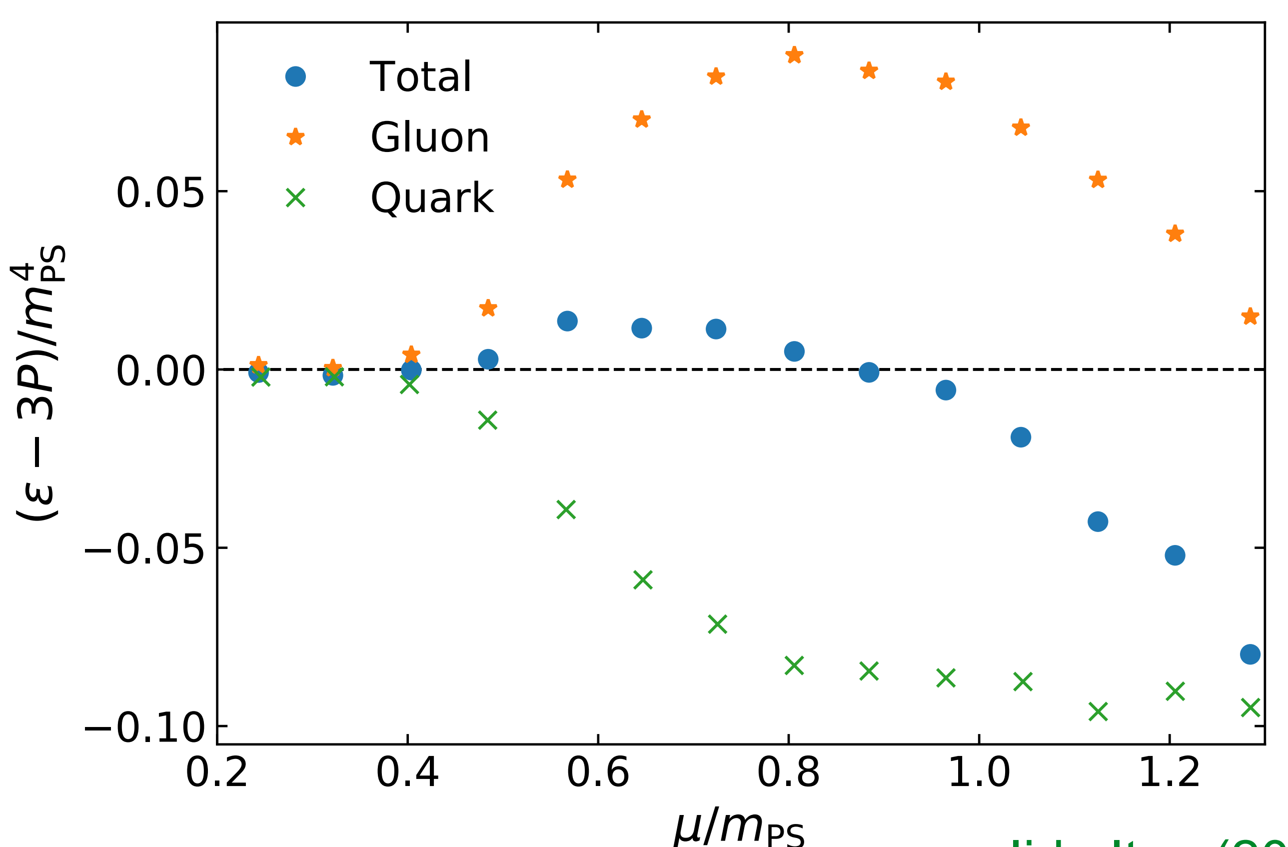
QCD at finite isospin chemical potential



Brandt,Cuteri,Endrodi (2021)

2-color QCD & finite-μ_I lattice data

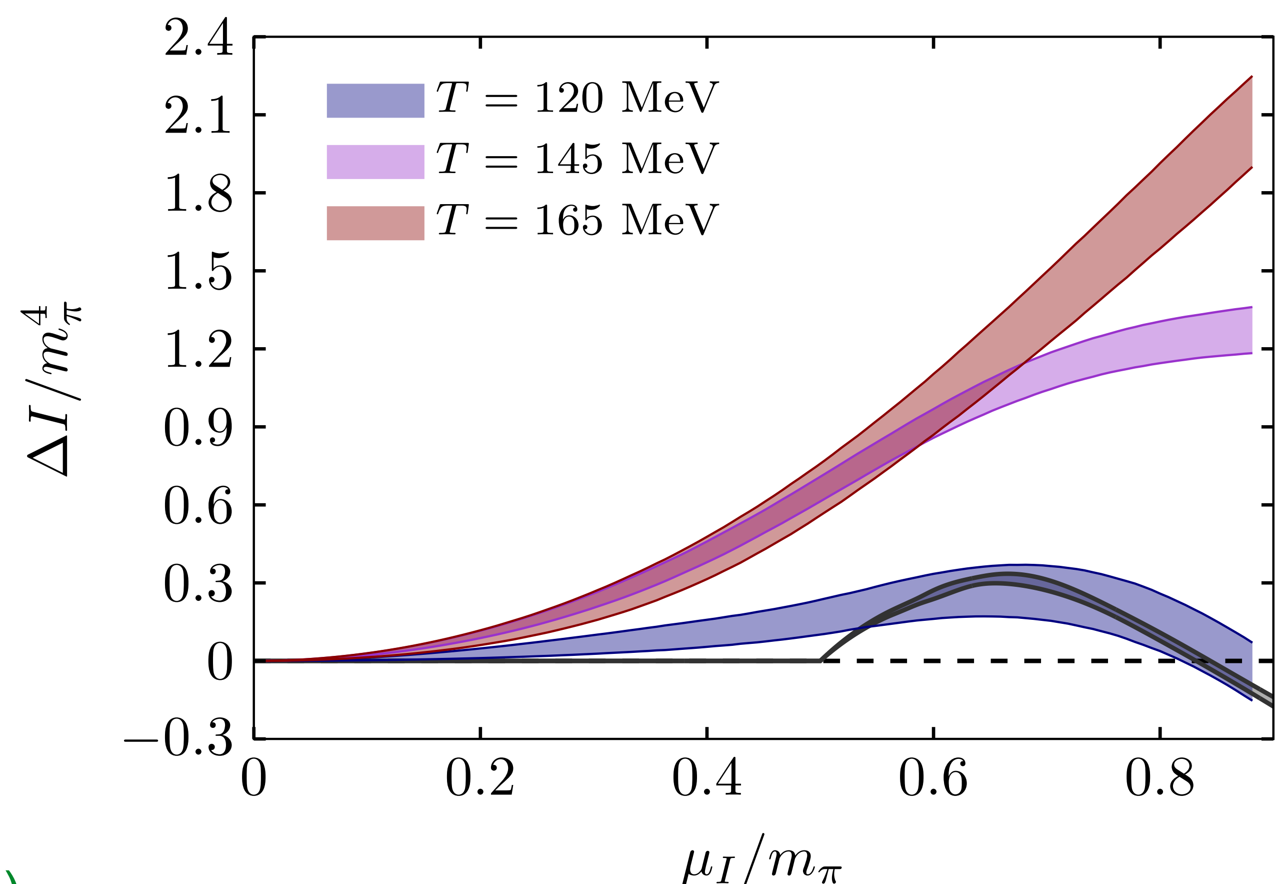
$N_c = 2$ QCD



Iida,Itou (2022)

(replotted their data by myself, I simply discard error bars)

QCD at finite isospin chemical potential



Brandt,Cuteri,Endrodi (2021)

These data shows that the trace anomaly is negative

Is the trace anomaly positive?

- **It may well be positive, but it doesn't have to be.**

  cf) single-particle matrix element: $\langle p \,|\, T^{\mu}_{\ \mu} \,|\, p \rangle \sim p^2 = m^2 \geq 0$

- Several known examples of negative trace anomaly:
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- Consider the following simple expression:

$$\varepsilon(n) = mn + \frac{C}{\Lambda^2} n^2$$

$\Delta \to 0$ is reached at $n = \dfrac{m\Lambda^2}{2C}$,

$m = m_N$ for finite-μ_B

$m = m_\pi$ for finite-μ_I

Is the trace anomaly positive?

- In the chiral limit, $\langle \theta \rangle_{\mu_B} \equiv \langle T^{\mu}_{\ \mu} \rangle_{\mu_B} = \frac{\beta}{2g} \langle F^a_{\mu\nu} F^{\mu\nu}_a \rangle_{\mu_B}$

- Trace anomaly is related to the counting of the degrees of freedom in pressure, $\nu \equiv P/\mu_B^4$:

$$\frac{\langle \theta \rangle_{\mu_B}}{\mu_B^4} = \mu_B \frac{d\nu}{d\mu_B} \geq 0$$

If ν keeps increasing, we get $\Delta \geq 0$

Open question: what if we have color superconductivity?
interplay between trace anomaly and diquark condensate?

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Bounds from theory?

Consideration based on the low-energy theorem

$$G^E(0,\mathbf{0}) = \lim_{q\to 0}\int d^4x e^{iqx}\langle T\theta(x),\theta(0)\rangle = \left(\mu\frac{\partial}{\partial\mu} - 4\right)\langle\theta\rangle \geq 0$$

Using the relation $\langle\theta\rangle = \langle\theta\rangle_{\mu_B} + \langle\theta\rangle_0 = \varepsilon - 3P + \langle\theta\rangle_0$,

then integrate w.r.t. μ from m_N to μ_B :

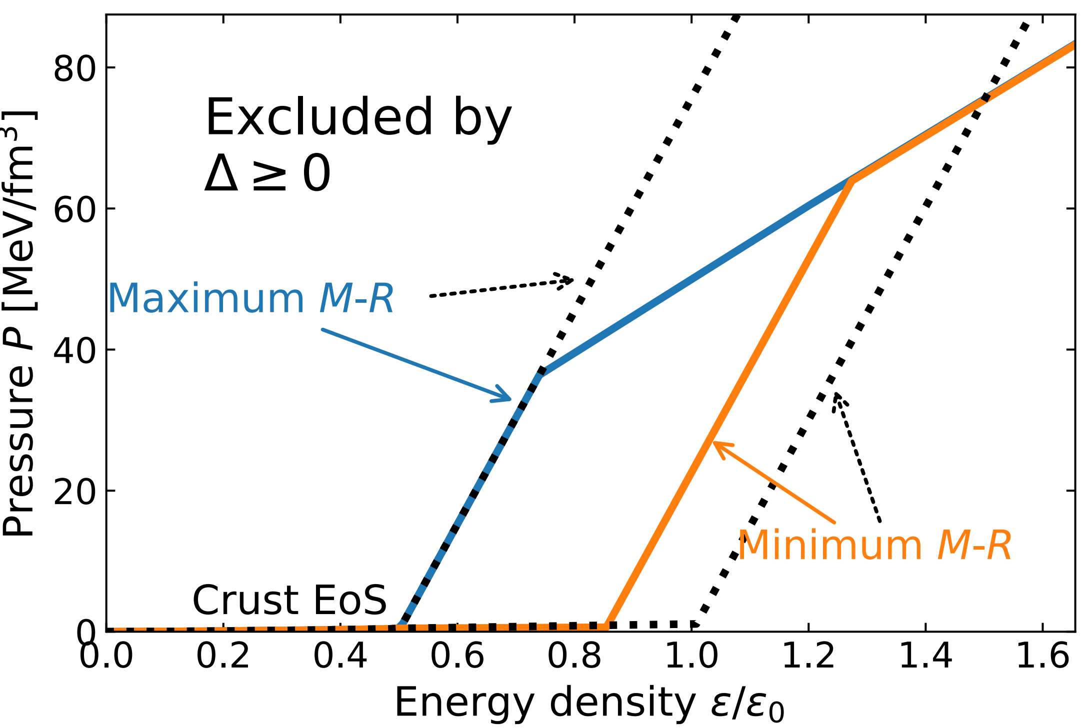
$$\Delta \geq -\frac{\langle\theta\rangle_0}{3\varepsilon}\left(1 - \frac{\mu_B^4}{m_N^4}\right) = \frac{4\,|\,\varepsilon_v\,|}{3\varepsilon}\left(1 - \frac{\mu_B^4}{m_N^4}\right)$$

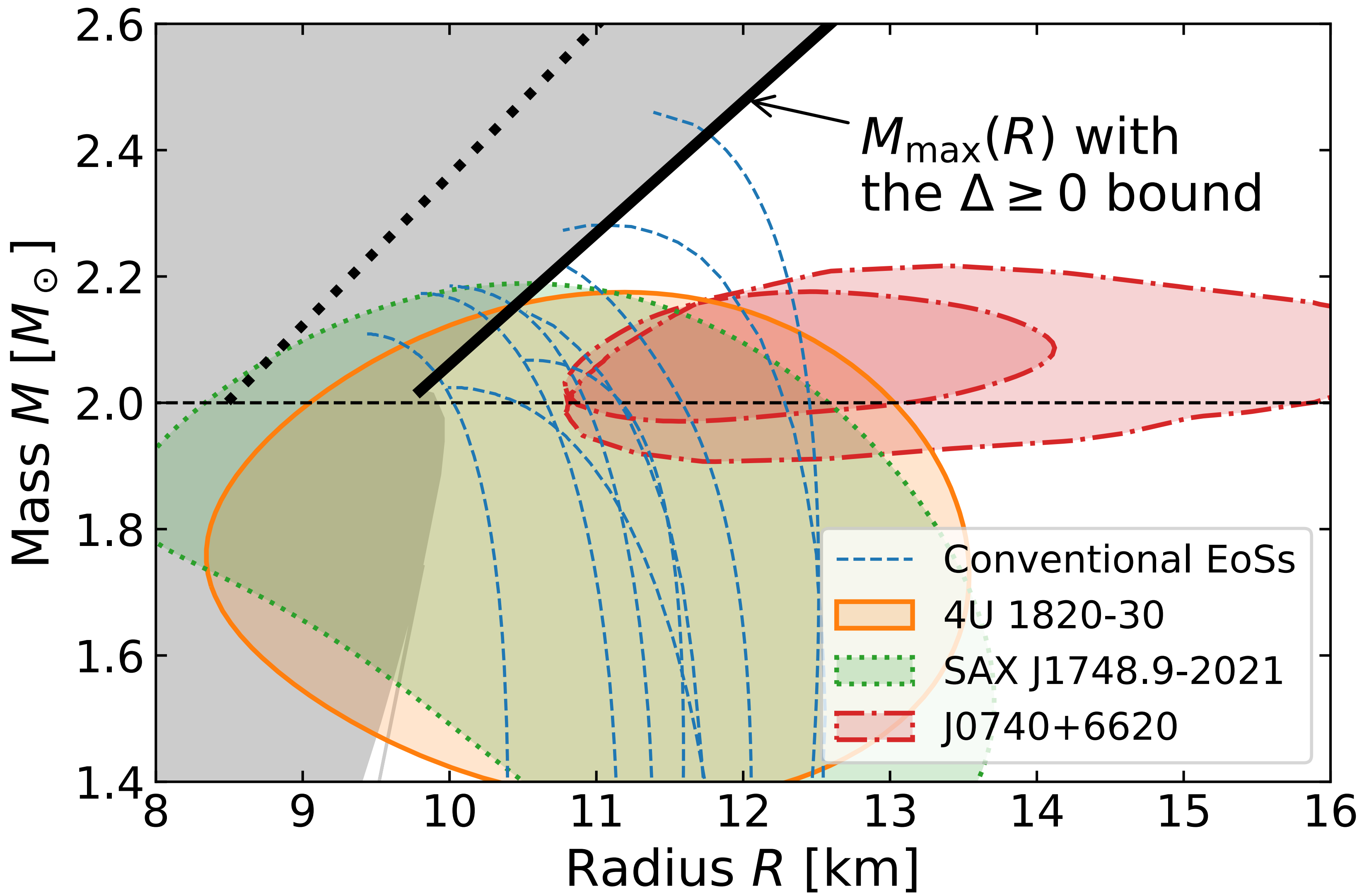
It depends on the vacuum value $\langle\theta\rangle_0 = -4\,|\,\varepsilon_v\,|$.

RHS quickly decreases, so tells nothing about positivity…

Testing $\Delta \geq 0$ by NS observation

- One example: $\Delta \geq 0$ put the bound on the maximum mass





The maximally large M-R: the stiffest EoS Rhoades Jr.,Ruffini (1974)

The most massive and compact M-R: soft at low density and stiff at high density

Koranda,Stergioulas,Friedman (1995)

See also: Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)

Summary

- Trace anomaly Δ measures conformality, is is a complement to the speed of sound v_s^2

- NS data suggest Δ rapidly approach to the conformal limit

- $\Delta \to 0$ gives rise to the sound velocity peak

  Consistent with microscopic pictures, e.g., Masuda,Hatsuda,Takatsuka (2013); McLerran,Reddy (2018); Pisarski (2021); Kojo (2021) & many others

- Strongly-interacting conformal matter may be inside NSs

- The trace anomaly may be positive (not proven). It can be tested by, e.g., the bound on the maximum mass of NSs