Spin alignments of vector mesons - new frontier of spin dynamics

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Outline

- **Introduction**
- **Global polarization of hyperons in HIC**
- **Global spin alignment of vector mesons in HIC**
- **Relativistic Spin Boltzmann Equation (for spin alignments of vector mesons) in Closed-Time-Path formalism (CTP) from Kadanoff-Baym equation (KBE)**
- **Questions and discussions**

Barnet effects and Einstein-de Hass effects

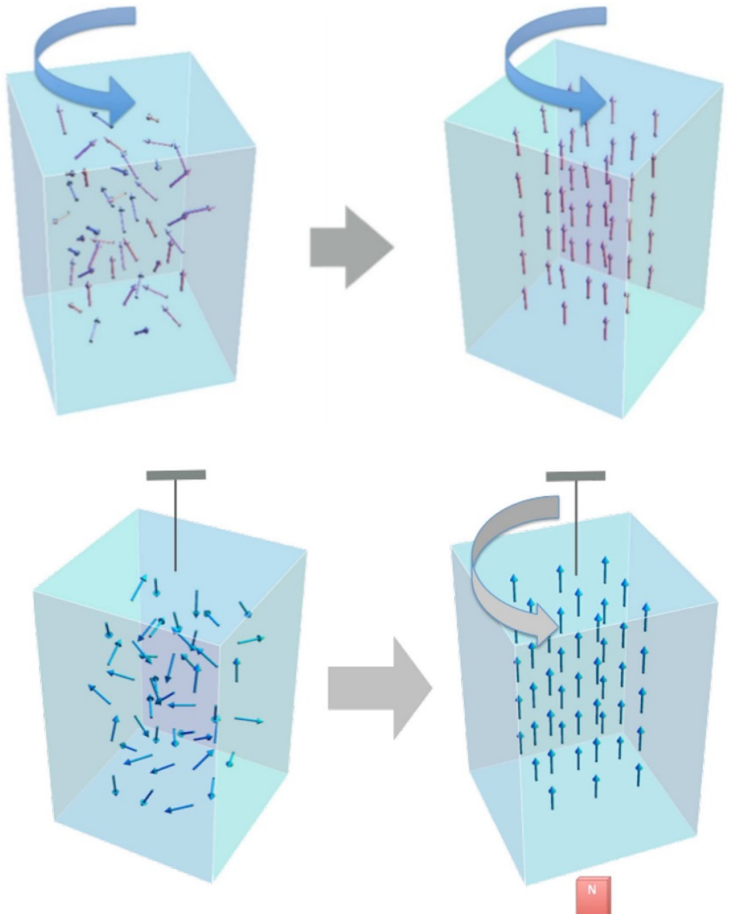
Barnett effect:

Barnett, Magnetization by rotation, Phys Rev. 6, 239-270 (1915).

Spin-orbit (LS) coupling!

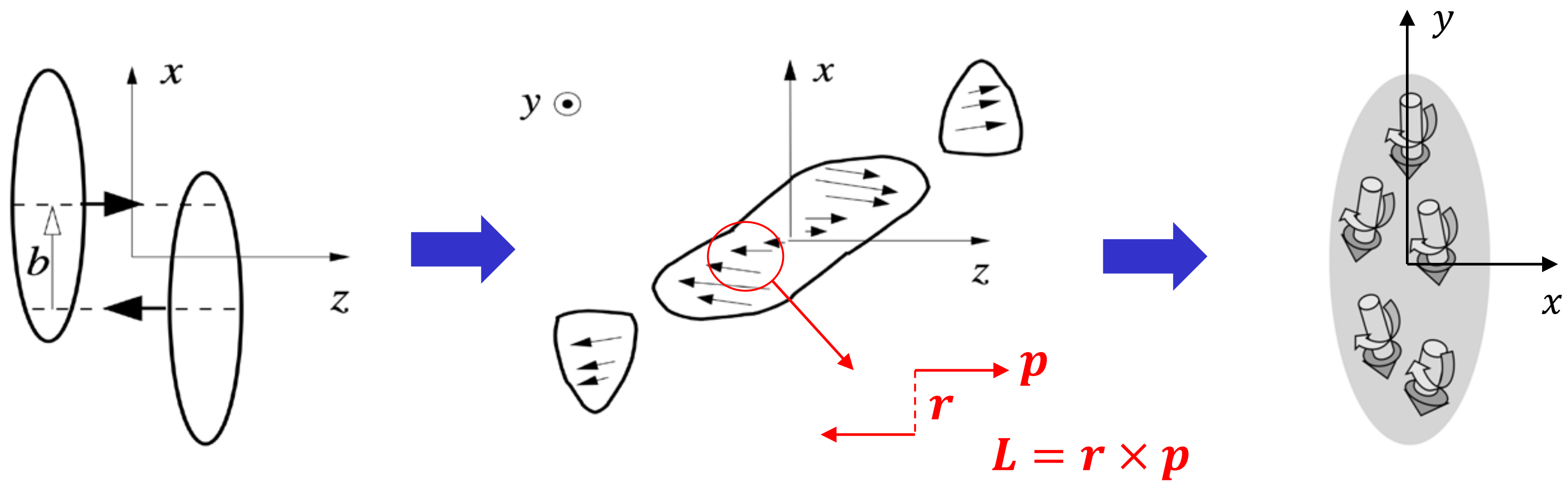
Einstein-de Haas effect:

Einstein, de Haas, Experimental proof of the existence of Ampere’s molecular currents, Verhandl. Deut. Phys. Ges. 17, 152–170 (1915).



Pictures from doi:10.3389/fphy.2015.00054

Global OAM and polarization in HIC



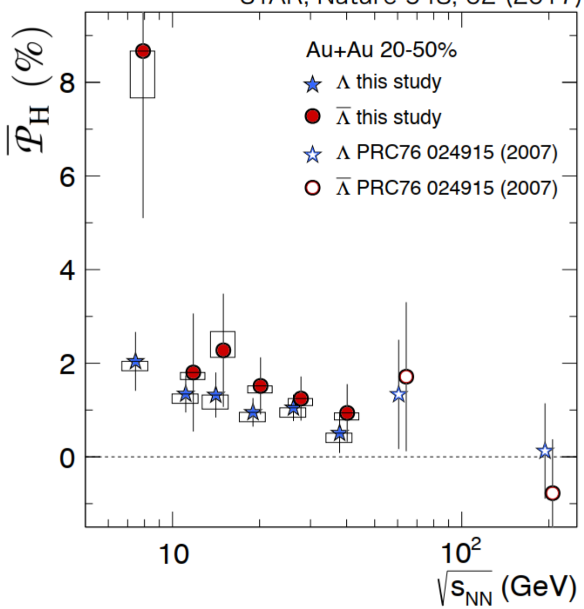
Global OAM leads to global polarization of Λ hyperons and spin alignment of vector mesons through spin-orbit coupling

Liang and Wang, PRL 94,102301(2005); PLB 629, 20(2005)
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization of hyperons in HIC

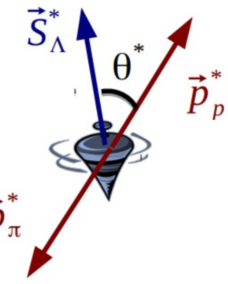
STAR results: Hyperon Polarization

STAR, Nature 548, 62 (2017)



parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

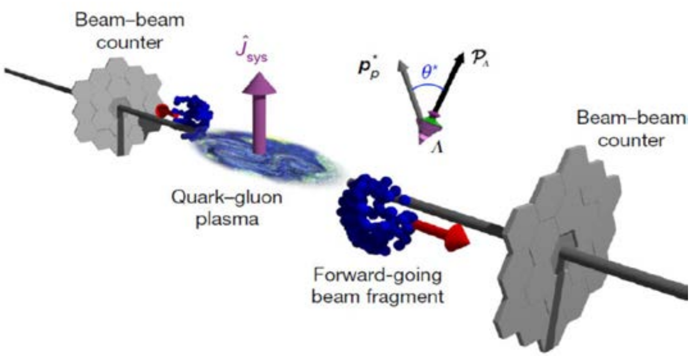


$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

Updated by BES III, PRL129, 131801 (2022)

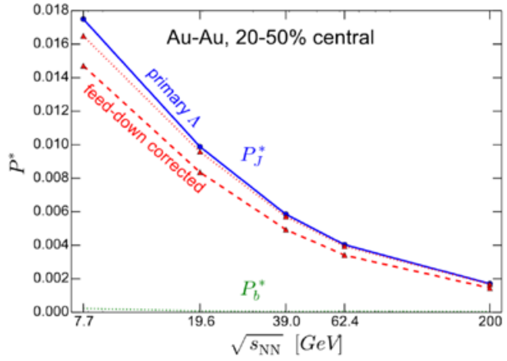
$\Lambda \rightarrow p + \pi^+$
(BR: 63.9%, cτ~7.9 cm)



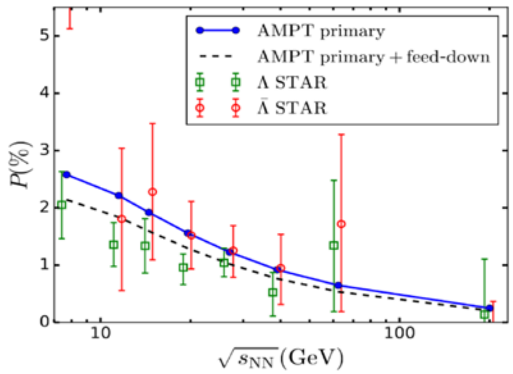
ω = (9 ± 1)x10[21]/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)

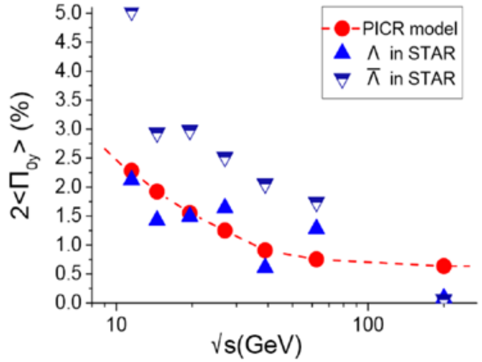
Global polarization in HIC: model calculation



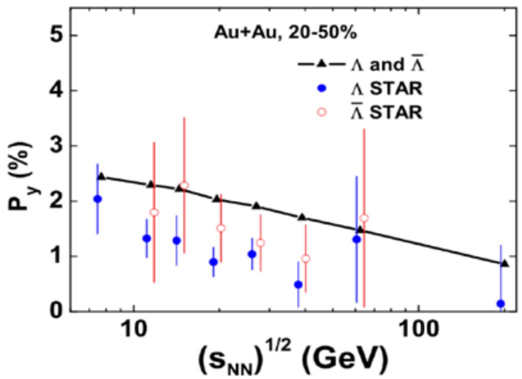
Karpenko, Becattini, EPJC(2017)



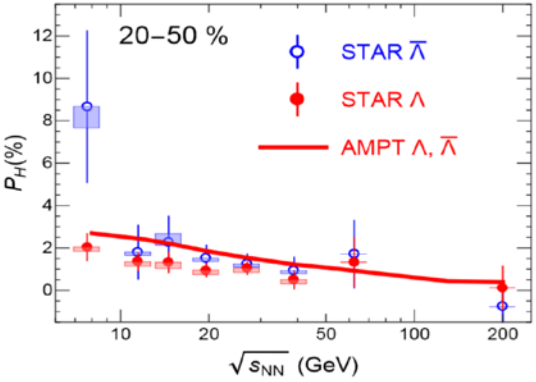
Li, Pang, Wang, Xia PRC(2017)



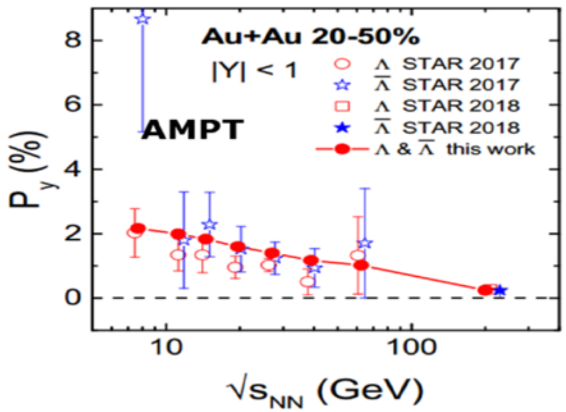
Xie, Wang, Csernai, PRC(2017)



Sun, Ko, PRC(2017)

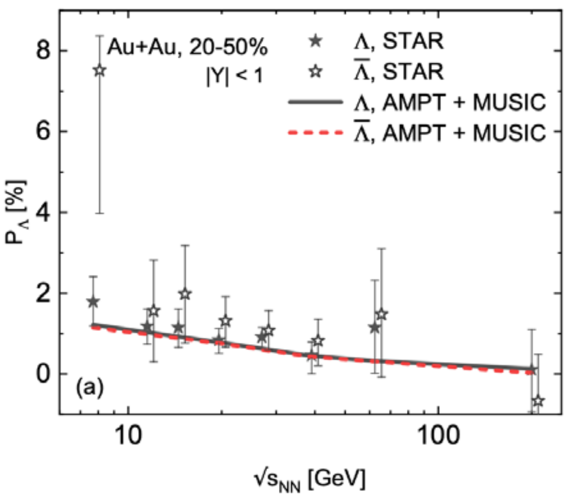


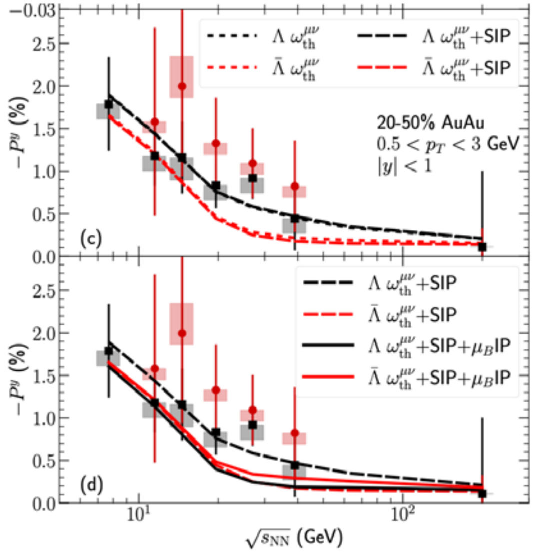
Shi, Li, Liao, PLB(2018)

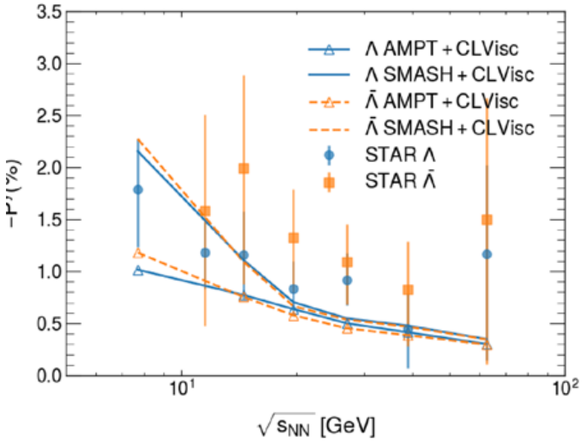


Wei, Deng, Huang, PRC(2019)

Global polarization in HIC: model calculation







B.C. Fu, K. Xu, X.G. Huang, H.C. Song, Phys. Rev. C 103, 024903 (2021)

S. Ryu, V. Jupic, C. Shen, arXiv:2106.08125

Y.X. Wu, C. Yi, G.Y. Qin, S.Pu, arXiv:2204.02218

Local spin polarization from QKT

- **The polarization vector is connected to the axial current in phase space by (modified) Cooper-Frye formula [Karpenko, Becattini, EPJC. (2017); Fang, Pang, QW, Wang, PRC (2016)]**

$$\mathcal{S}^{\mu}(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p,X)}{2m_{\Lambda} \int d\Sigma \cdot \mathcal{N}(p,X)},$$

- **All possible contributions to the polarization vector [Hidaka, Pu, Yang, PRD (2018); Yi, Pu, Yang, PRC(2021); Yi, Pu, Gao, Yang, PRC (2022)]**

$$\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N} \int d\Sigma^{\sigma} p_{\sigma} f_V^{(0)}(1-f_V^{(0)}) \epsilon^{\mu\nu\alpha\beta} p_{\nu} \partial_{\alpha} \frac{u_{\beta}}{T}$$

Thermal vorticity

$$\mathcal{S}^{\mu}_{\text{shear}}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N} \int d\Sigma \cdot p f_V^{(0)}(1-f_V^{(0)}) \frac{\epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta}}{(u\cdot p)T} \frac{1}{2} \{p^{\sigma}(\partial_{\sigma} u_{\nu} + \partial_{\nu} u_{\sigma}) - Du_{\nu}\}$$

Shear viscous tensor

$$\mathcal{S}^{\mu}_{\text{accT}}(\mathbf{p}) = -\frac{\hbar}{8m_{\Lambda}N} \int d\Sigma \cdot p f_V^{(0)}(1-f_V^{(0)}) \frac{1}{T} \epsilon^{\mu\nu\alpha\beta} p_{\nu} u_{\alpha} (Du_{\beta} - \frac{1}{T}\partial_{\beta} T),$$

Fluid acceleration

$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N} \int d\Sigma \cdot p f_V^{(0)}(1-f_V^{(0)}) \frac{1}{(u\cdot p)} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} \partial_{\nu} \frac{\mu}{T},$$

Gradient of chemical potential

- **Also see from other approaches: Fu, Liu, Pang, Song, Yin (JHEP2021, PRL 2021); Becattini (PRD 2021, PRL 2021);**

Spin alignment of vector mesons

Vector meson spin alignment: strong decays

- Vector mesons K^{*0} and ϕ decay mainly through strong decays (parity is conserved), the polarization cannot be measured

$$\begin{aligned} K^{*0} &\rightarrow K^+ + \pi^-, \quad (\sim 100\%) \\ \phi &\rightarrow K^+ + K^-, \quad (\sim 49\%) \end{aligned}$$

Kaons and pions are (pseudo)scalar mesons

- These decays are in p-wave (L=1). For ϕ meson the decay amplitude has the form

$$\langle K^+, K^- | \mathscr{S} | \phi; S_z \rangle = Y_{1,S_z}(\theta, \varphi)$$

$S_z = -1, 0, 1$

angles of one particular kaon

- Angular distribution of decay products

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta, \quad Y_{1,\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\varphi}$$

$$\frac{dN}{d\Omega} = \left| \langle K^+, K^- | \mathscr{S} | \phi; S_z \rangle \right|^2 = \left| Y_{1,S_z}(\theta, \varphi) \right|^2$$

symmetric for $\theta \rightarrow \pi - \theta$

Vector meson polarization: strong decays

- **The spin ensemble of ϕ vector meson is described by spin density matrix ρ (3×3 complex matrix). The angular distribution of one decay product (kaon) is**

$$\frac{dN}{d\Omega} = \sum_{S_z} \rho_{S_z} \left| \langle K^+, K^- | \mathscr{S} | \phi; S_z \rangle \right|^2$$

$$= \sum_{S_z} \langle K^+, K^- | \mathscr{S} | \phi; S_z \rangle \rho_{S_z} \langle \phi; S_z | \mathscr{S} | K^+, K^- \rangle$$

$$\rightarrow \sum_{S_{z1}, S_{z2}} \langle K^+, K^- | \mathscr{S} | \phi; S_{z1} \rangle \rho_{S_{z1} S_{z2}} \langle \phi; S_{z2} | \mathscr{S} | K^+, K^- \rangle$$

$$= \sum_{S_{z1}, S_{z2}} \rho_{S_{z1} S_{z2}} Y_{1,S_{z1}}(\theta, \varphi) Y^*_{1,S_{z2}}(\theta, \varphi)$$

Spin density matrix $\mathrm{Tr}(\rho) = 1$

8 real indep. variables

$$\begin{pmatrix} \rho_{-1,-1} & \rho_{-1,0} & \rho_{-1,1} \\ \rho^*_{-1,0} & \rho_{00} & \rho_{01} \\ \rho^*_{-1,1} & \rho^*_{01} & \rho_{11} \end{pmatrix}$$

5 real variables

Re	Re+Im	Re+Im
ρ_{00}	$\rho_{-1,1}$	$(\rho_{-1,0} - \rho_{01})$

- **The polarization of vector meson is related to some elements (not all) of spin density matrix.**

$$\vec{\mathcal{P}} = [\mathcal{P}_1, \mathcal{P}_3, \mathcal{P}_3]$$

$$= \left[\sqrt{2}\mathrm{Re}\left(\rho_{-1,0} + \rho_{01}\right), \sqrt{2}\mathrm{Im}\left(\rho_{-1,0} + \rho_{01}\right), \left(\rho_{11} - \rho_{-1,-1}\right) \right]$$

3 real variables

Polarization in spin quantization direction

Vector meson polarization: strong decays

- **By integrating over φ, we get the polar angle distribution**

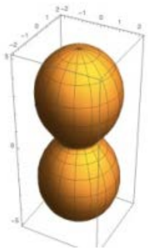
$$\frac{dN}{d\cos\theta} = \int_0^{2\pi} d\varphi \frac{dN}{d\Omega} = \frac{3}{4}\left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta\right]$$

The angle between decay product and spin direction of the vector meson in its rest frame

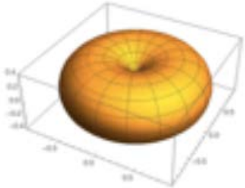
polar angle dependence is related to ρ_{00}

- **One cannot measure the polarization of vector mesons by strong decays. One can only know if vector mesons are polarized or not (polar angle dependence disappears or not) by comparing ρ_{00} and 1/3.**

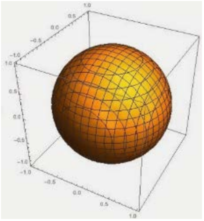
Longitudinal polarization $\rho_{00} > \frac{1}{3}$



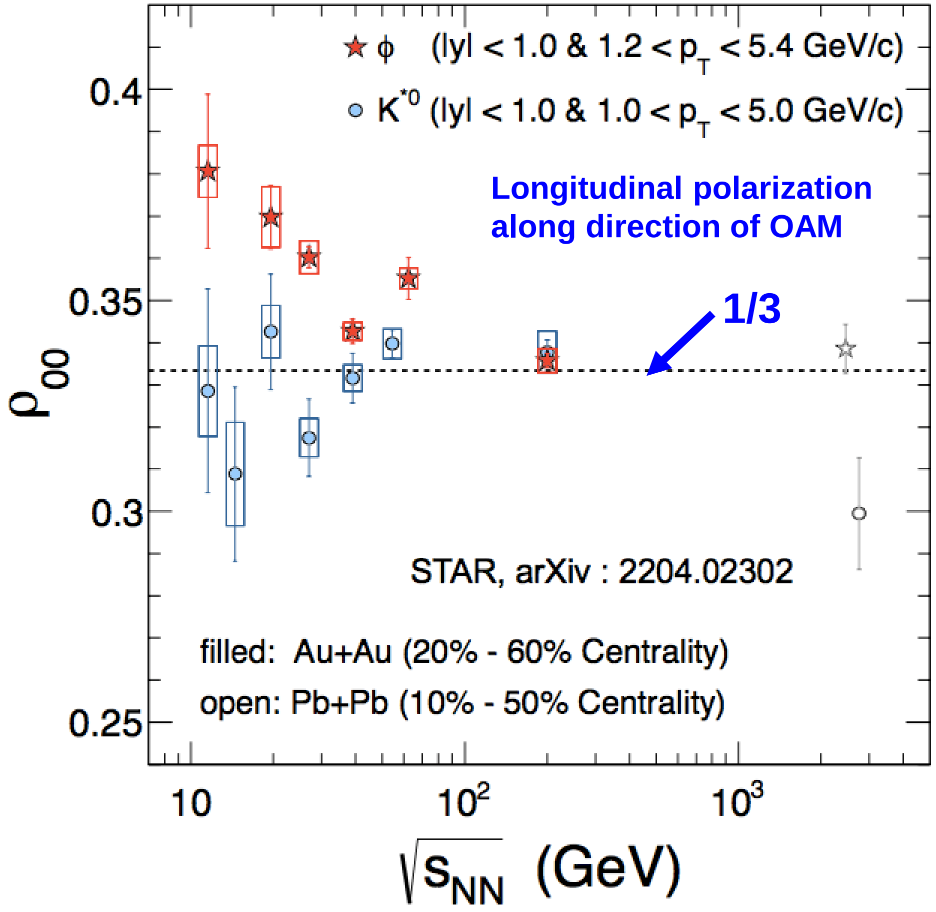
Transverse polarization $\rho_{00} < \frac{1}{3}$



No polarization (no angle depend.) $\rho_{00} = 1/3$



STAR results on global spin alignments of vector mesons



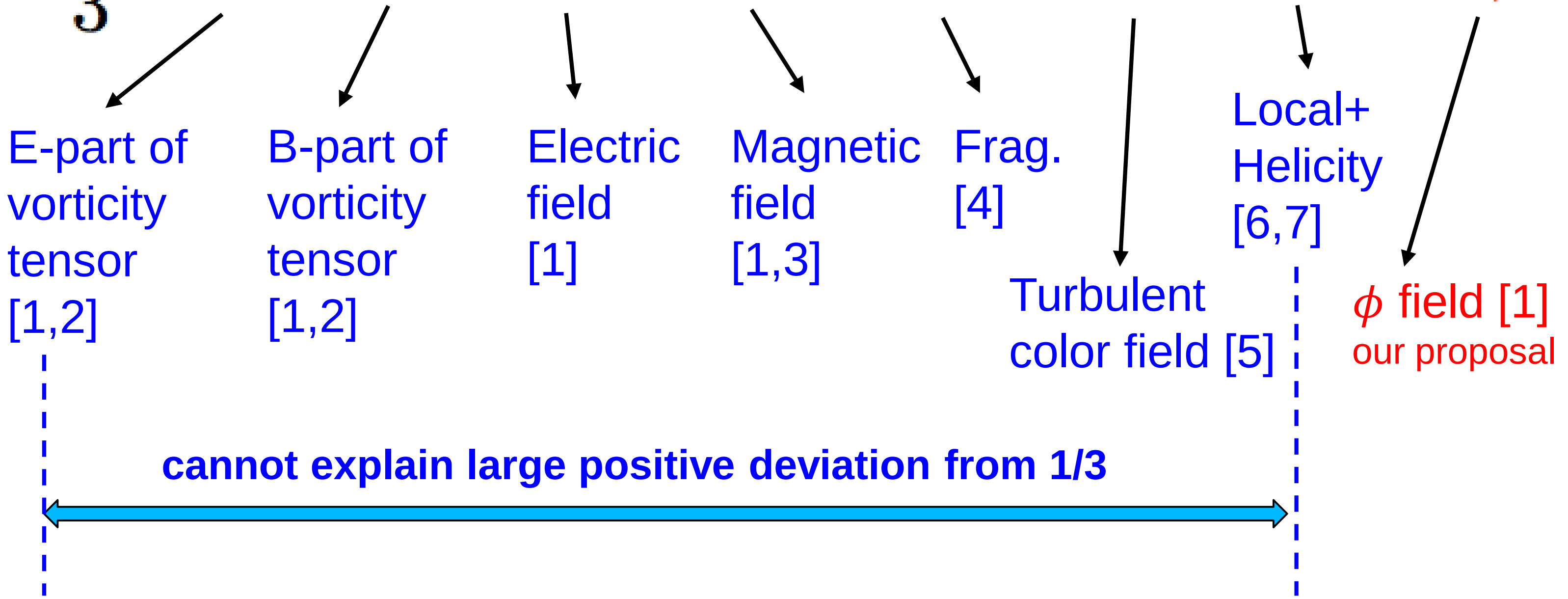
STAR Collab., 2204.02302
[to appear in Nature]

"the global spin alignment for phi unexpectedly large, while that for K^{*0} is consistent with zero. The observed spin-alignment pattern and magnitude for the phi cannot be explained by conventional mechanisms, while a model with strong force fields [2,3] accommodates the current data. "

[2] Sheng, Oliva, QW (2020, Erratum 2022)
[3] Sheng, QW, Wang (2020)

Possible contributions to ρ_{00}^{ϕ}

$$\rho_{00}^{\phi}=\frac{1}{3}+c_{\varepsilon}+c_{\omega}+c_{E}+c_{B}+c_{F}+c_{A}+c_{L}+c_{\phi}$$



[1] Sheng, Luica, QW (2019);
[2] Becattini, Csernai, Wang (2013);
[3] Yang, Fang, QW, Wang (2018);
[4] Liang, Wang (2005);
[5] Muller, Yang (2022);
[6] Xia, Li, Huang, Huang (2021);
[7] Gao (2021);

Polarization of strange quarks by ϕ vector fields (non-relativistic model)

- Like electric charges in motion can generate an EM field, s and $\bar{s}$ quarks in motion can generate an **effective ϕ vector field** [Sheng, Oliva, QW (2020)].
- The ϕ vector field can polarize s and $\bar{s}$ with a large magnitude due to strong interaction, in analogy to how EM field polarize (anti)quarks.

$$\vec{\mathcal{P}}_{s/\bar{s}} = \frac{1}{2}\omega + \frac{1}{2m_s}\varepsilon \times \mathbf{p}_{s/\bar{s}} \pm \frac{Q_s}{2m_s T}\mathbf{B} \pm \frac{Q_s}{2m_s^2 T}\mathbf{E} \times \mathbf{p}_{s/\bar{s}} \pm \frac{g_\phi}{2m_s T}\mathbf{B}_\phi \pm \frac{g_\phi}{2m_s^2 T}\mathbf{E}_\phi \times \mathbf{p}_{s/\bar{s}}$$

$$\langle \vec{\mathcal{P}}_{\Lambda/\bar{\Lambda}} \rangle = \langle \vec{\mathcal{P}}_{s/\bar{s}} \rangle$$

Sheng, Oliva, QW (2020)

Electric part of spin polarization corresponds to spin-orbit couplings (spin-Hall effects) not accessible via Λ polarization:

$$\mathbf{E} \times \mathbf{p} \sim -\frac{1}{r}\frac{d\Phi}{dr}(\mathbf{r} \times \mathbf{p})$$

Spin ⟷ Local OAM

ρ_{00}^{ϕ} from ϕ fields in non-relativistic coalescence model

- Assuming the spin quantization direction is y-direction (OAM), in a non-relativistic coalescence model [Greco, Ko, Levai (2003); Fries, Muller, Nonaka, Bass (2003); Hua, Yang (2003)], ρ_{00} has the form

$$\rho_{00}^{\phi}(t,\mathbf{x}) \approx \frac{1}{3} - \frac{4}{9}\int \frac{d^3\mathbf{p}}{(2\pi)^3} \left|\psi_{\phi}(\mathbf{p})\right|^2 \times \left\{ P_s^y(\mathbf{p}) P_{\bar{s}}^y(-\mathbf{p}) - \frac{1}{2}\left[P_s^z(\mathbf{p}) P_{\bar{s}}^z(-\mathbf{p}) + P_s^x(\mathbf{p}) P_{\bar{s}}^x(-\mathbf{p})\right] \right\}$$

$$\approx \frac{1}{3} + \frac{g_\phi^2}{9 m_s^2 T_{\text{eff}}^2}\left[\left\langle B_{\phi,y}^2\right\rangle - \frac{1}{2}\left\langle B_{\phi,x}^2 + B_{\phi,z}^2\right\rangle\right] + \frac{g_\phi^2 \left\langle \mathbf{p}^2\right\rangle_\phi}{27 m_s^4 T_{\text{eff}}^2}\left[\left\langle E_{\phi,y}^2\right\rangle - \frac{1}{2}\left\langle E_{\phi,x}^2 + E_{\phi,z}^2\right\rangle\right] \equiv \frac{1}{27 m_s^2 T_{\text{eff}}^2} G_s^{(y)}$$

ϕ meson's non-relativistic wave function

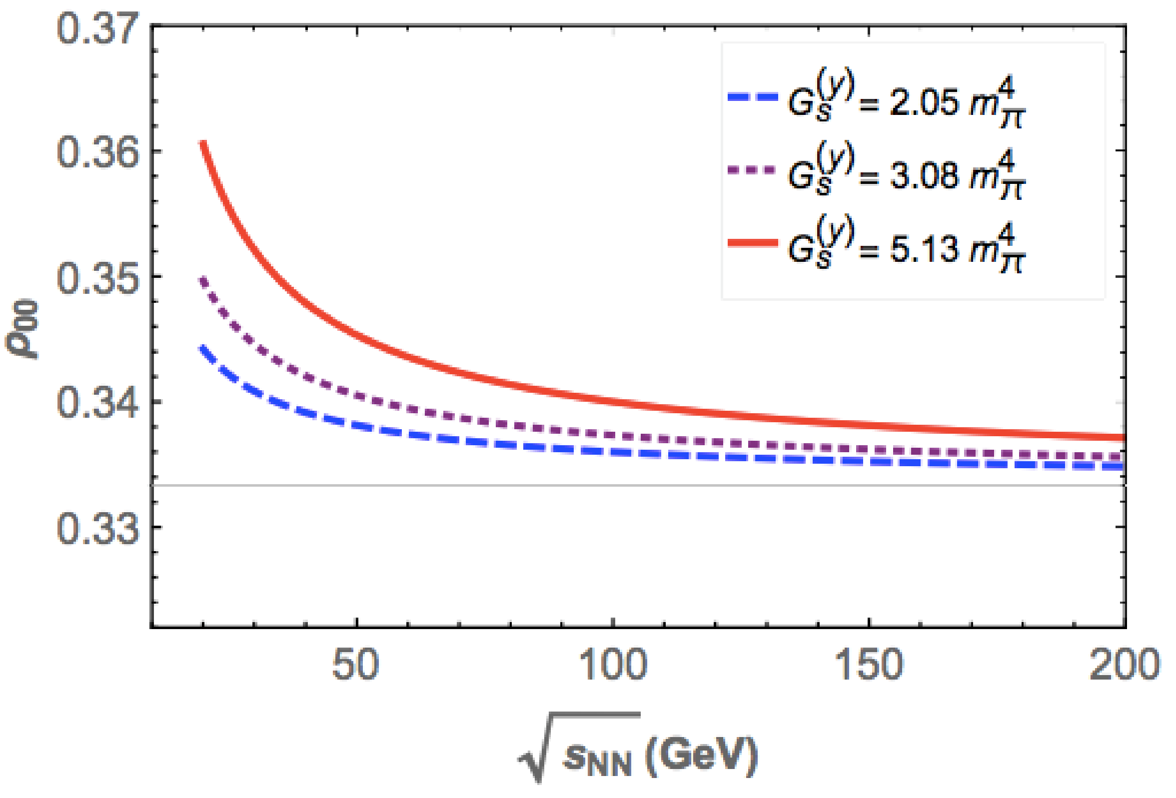
in products

ϕ meson is static

average p^2 for s or $\bar{s}$ in ϕ-meson's WF

Sheng, Oliva, QW (2020)

Prediction for ρ_{00} from ϕ field (non-relativistic coalescence model)



Sheng, Oliva, QW (2020)

Shortcomings of non-relativistic coalescence model for ρ_{00}^{ϕ}

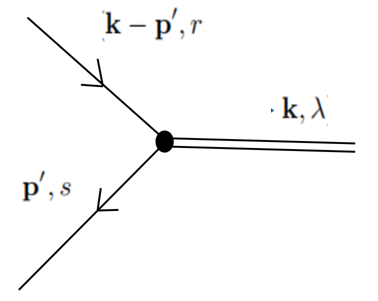
- **Spins are decoupled from momenta in spin density matrix: too simple to account for spin dynamics. The sign of anti-quark's momentum is not easy to determine (easy to make a mistake)**
- **No Lorentz covariance, only valid for quasi-static ϕ mesons, cannot be applied to ϕ mesons with non-vanishing momenta with confidence**
- **It is not a model based on relativistic quantum field theory**
- **The deeper implication of ϕ field cannot be explored**

→

- **To solve above problems, it is necessary to develop a relativistic spin transport theory for ϕ mesons, which can describe the relativistic fusion process $s\bar{s} \rightarrow \phi$ with spin dof**

Relativistic spin Boltzmann equation for fusion process

- **A phenomenological approach to Relativistic Spin Boltzmann Equation (RSBE) for fusion process**



$$
\begin{aligned}
&k\cdot\partial_x f_\lambda^V(x,\mathbf{k})\\
\Rightarrow\ &\sum_{r,s=\pm1/2}\int \frac{d^3\mathbf{p}'}{E_{\mathbf{p}'}^{\bar q}E_{\mathbf{k}-\mathbf{p}'}^{q}}\,\delta\left(E_{\mathbf{k}}^V-E_{\mathbf{k}-\mathbf{p}'}^q-E_{\mathbf{p}'}^{\bar q}\right)\\
&\times\left|M(\mathbf{k}-\mathbf{p}',r;\mathbf{p}',s\to\mathbf{k},\lambda)\right|^2\\
&\times\left\{f_r^q(\mathbf{k}-\mathbf{p}')f_s^{\bar q}(\mathbf{p}')\left[1+f_\lambda^V(\mathbf{k})\right]-f_\lambda^V(\mathbf{k})\left[1-f_r^q(\mathbf{k}-\mathbf{p}')\right]\left[1-f_s^{\bar q}(\mathbf{p}')\right]\right\}
\end{aligned}
$$

Gain term **Loss term**

- **It is more rigorous to derive RSBE from CTP (SK) or KBE in terms of Matrix Valued Spin Dependent Distributions (MVSD) for quarks and vector mesons**

$$f_r^q \to f_{r_1 r_2}^q \qquad f_\lambda^V \to f_{\lambda_1\lambda_2}^V$$
$$f_s^{\bar q} \to f_{s_1 s_2}^{\bar q}$$

MVSD: Sheng, Weickgenannt, Speranza, Rischke, QW (2021); Sheng, QW, Rischke (2022)

Spin density matrix: diagonal ⇒ **diagonal + off-diagonal elements**

Matrix Valued Spin Dependent Distributions (MVSD)

- **MVSD in phase space**

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q\cdot x\right) \delta(p\cdot q) \left\langle a^\dagger(s,\mathbf{p}_2) a(r,\mathbf{p}_1)\right\rangle$$

- **MVSD can be parameterized in terms un-polarized distributions and polarization distributions**

$$f_{rs}^{(+)}(x,\mathbf{p}) = \frac{1}{2} f_q(x,\mathbf{p}) \left[\delta_{rs} - P_\mu^q(x,\mathbf{p}) n_j^{(+)\mu}(\mathbf{p}) \tau_{rs}^j\right],$$

$$f_{rs}^{(-)}(x,-\mathbf{p}) = \frac{1}{2} f_{\bar{q}}(x,-\mathbf{p}) \left[\delta_{rs} - P_\mu^{\bar{q}}(x,-\mathbf{p}) n_j^{(-)\mu}(\mathbf{p}) \tau_{rs}^j\right],$$

Pauli matrices in spin space (rs-space)

Un-polarized dist. **Polarization dist.**

Four-vectors of three basis directions in rest frame of q and $\overline{q}$ (one is the spin quantization direction)

MVSD:
Sheng, Weickgenannt, et al. (2021); Sheng, QW, Rischke (2022)

RSBE in MVSD from CTP or KBE

- **A general RSBE based on relativistic quantum field theory (CTP or KBE) for fusion process**

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{16}\sum_{\lambda_1',\lambda_2'}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_1',\mathbf{k})\delta_{\lambda_2\lambda_2'} + \delta_{\lambda_1\lambda_1'}\epsilon^*_\mu(\lambda_2',\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\right]\mathcal{C}^{\mu\nu}_{\lambda_1'\lambda_2'}(x,\mathbf{k}),$$

polarization vector for vector meson

collision kernel

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\lambda_1'\lambda_2'}(x,\mathbf{k}) &= \sum_{r_1,s_1,r_2,s_2}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\overline{q}}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\overline{q}}_{\mathbf{p}'} - E^q_{\mathbf{k}-\mathbf{p}'}\right)\\ &\times\left\{f^{\overline{q}}_{r_1s_1}(\mathbf{p}')f^q_{r_2s_2}(\mathbf{k}-\mathbf{p}')\left[\delta_{\lambda_1'\lambda_2'} + f^V_{\lambda_1'\lambda_2'}(\mathbf{k})\right]\right.\\ &\left. - \left[\delta_{r_1s_1} - f^{\overline{q}}_{r_1s_1}(\mathbf{p}')\right]\left[\delta_{r_2s_2} - f^q_{r_2s_2}(\mathbf{k}-\mathbf{p}')\right]f^V_{\lambda_1'\lambda_2'}(\mathbf{k})\right\}\\ &\times \mathrm{Tr}\left[\Gamma^\nu v_{s_1}(\mathbf{p}')\overline{v}_{r_1}(\mathbf{p}')\Gamma^\mu u_{r_2}(\mathbf{k}-\mathbf{p}')\overline{u}_{s_2}(\mathbf{k}-\mathbf{p}')\right], \qquad (2)\end{aligned}$$

collision kernel

gain term

loss term

$$\Gamma^\alpha \approx g_V B(\mathbf{k}-\mathbf{p}',\mathbf{p}')\gamma^\alpha$$

Bethe-Salpeter wave function for vector meson [Roberts et al (2019, 2021)]

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Fusion and dissociation process

- **In the dilute gas limit**

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

$$f^V_{\lambda_1\lambda_2} \sim f^q_{rs} \sim f^{\overline{q}}_{rs} \ll 1.$$

- **RSBE for fusion (coalescence) and dissociation process $q\overline{q} \leftrightarrow V$ can be simplified as**

Coalescence collision kernel

n_x, n_y, n_z are three basis directions in rest frame of vector meson

$$k \cdot \partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k}) - \mathcal{C}_{\text{diss}}(\mathbf{k})f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\right],$$

Dissociation collision kernel

$$\epsilon^\mu(\lambda,\mathbf{k}) = \left(\frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V}, \epsilon_\lambda + \frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V(E^V_\mathbf{k}+m_V)}\mathbf{k}\right) \Longrightarrow k_\mu \epsilon^\mu(\lambda,\mathbf{k}) = 0$$

$$\begin{aligned} \epsilon_0 &= \mathbf{n}_y \\ \epsilon_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x) \\ \epsilon_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x) \end{aligned}$$

- **The coalescence part depends on MVSDs of q and $\overline{q}$, while the dissociation part does not.**

MVSD or spin density matrix element for vector mesons

- **Forml solution to MVSD (spin density matrix) for vector mesons**

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1}{\mathcal{C}_{\mathrm{diss}}(\mathbf{k})}\left[1-e^{-\mathcal{C}_{\mathrm{diss}}(\mathbf{k})\Delta t}\right] \times \epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})$$

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

- **where the coalescence collision kernel $C^{\mu\nu}_{coal}$ is given by**

$$\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) = \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^{q}_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\bar{q}}_{\mathbf{p}'} - E^{q}_{\mathbf{k}-\mathbf{p}'}\right) \times \mathrm{Tr}\left\{\Gamma^\nu\left(p'\cdot\gamma - m_{\bar{q}}\right)\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right] \times \Gamma^\mu\left[(k-p')\cdot\gamma + m_q\right]\left[1+\gamma_5\gamma\cdot P^{q}(x,\mathbf{k}-\mathbf{p}')\right]\right\} \times f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}'),$$

BS wave function for vector meson

polarization distributions in phase space for q and $\bar{q}$

un-polarized quark distribution functions

Spin density matrix element for vector mesons

- **Spin density matrix (normalized MVSD) for vector mesons**

$$f^V_{\lambda_1\lambda_2} \propto \rho^V_{\lambda_1\lambda_2} = \frac{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}$$

- **Focus on ϕ meson, polarization distributions for s and $\bar{s}$ appear in the collision kernel are in the form**

$$P^\mu_s(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

$$P^\mu_{\bar{s}}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

Field strength tensor of ϕ field

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Spin density matrix element for vector mesons

- **The fusion (coalescenece) collision kernel $C_{coal}^{\mu\nu}$ can be evaluated in the rest frame of ϕ meson, which gives ρ_{00}^{ϕ}**

$$\rho_{00}(x,\mathbf{0}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right],$$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2+2m_s^2)},$$

$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2+2m_s^2)}.$$

All fields with prime are defined in the rest frame of ϕ meson

spin quantization direction

- **Features: (1) Perfect factorization of x and p dependence; (2) Perfect cancellation for mixing terms (protected by symmetry): all fields appear in squares, i.e. ρ_{00}^{ϕ} measures fluctuations of fields. Surprising results!**

Lorentz transformation for ϕ fields

- We can express ρ^{ϕ}_{00} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons

$$\mathbf{B}'_{\phi} = \gamma \mathbf{B}_{\phi} - \gamma \mathbf{v} \times \mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$

$$\mathbf{E}'_{\phi} = \gamma \mathbf{E}_{\phi} + \gamma \mathbf{v} \times \mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

- where $\gamma = E^{\phi}_{\mathbf{k}}/m_{\phi}$ and $\mathbf{v} = \mathbf{k}/E^{\phi}_{\mathbf{k}}$

- In terms of lab-frame fields we obtain (factorization of x and p)

$$\overline{\rho}^{\phi}_{00}(x,\mathbf{k}) \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} I_{B,i}(\mathbf{k})\left[\boldsymbol{\omega}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}(\mathbf{B}_i^{\phi})^2\right] + \frac{1}{3}\sum_{i=1,2,3} I_{E,i}(\mathbf{k})\left[\boldsymbol{\varepsilon}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}(\mathbf{E}_i^{\phi})^2\right]$$

three basis directions in lab frame

Momentum functions

Parameters and comparison with data

- **Two parameters (transverse and longitudinal field squares)**

$$\left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle = F^2$$

$$\left\langle (g_\phi \mathbf{B}^\phi_z)^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_z)^2 \right\rangle = r_z F^2 < F^2$$

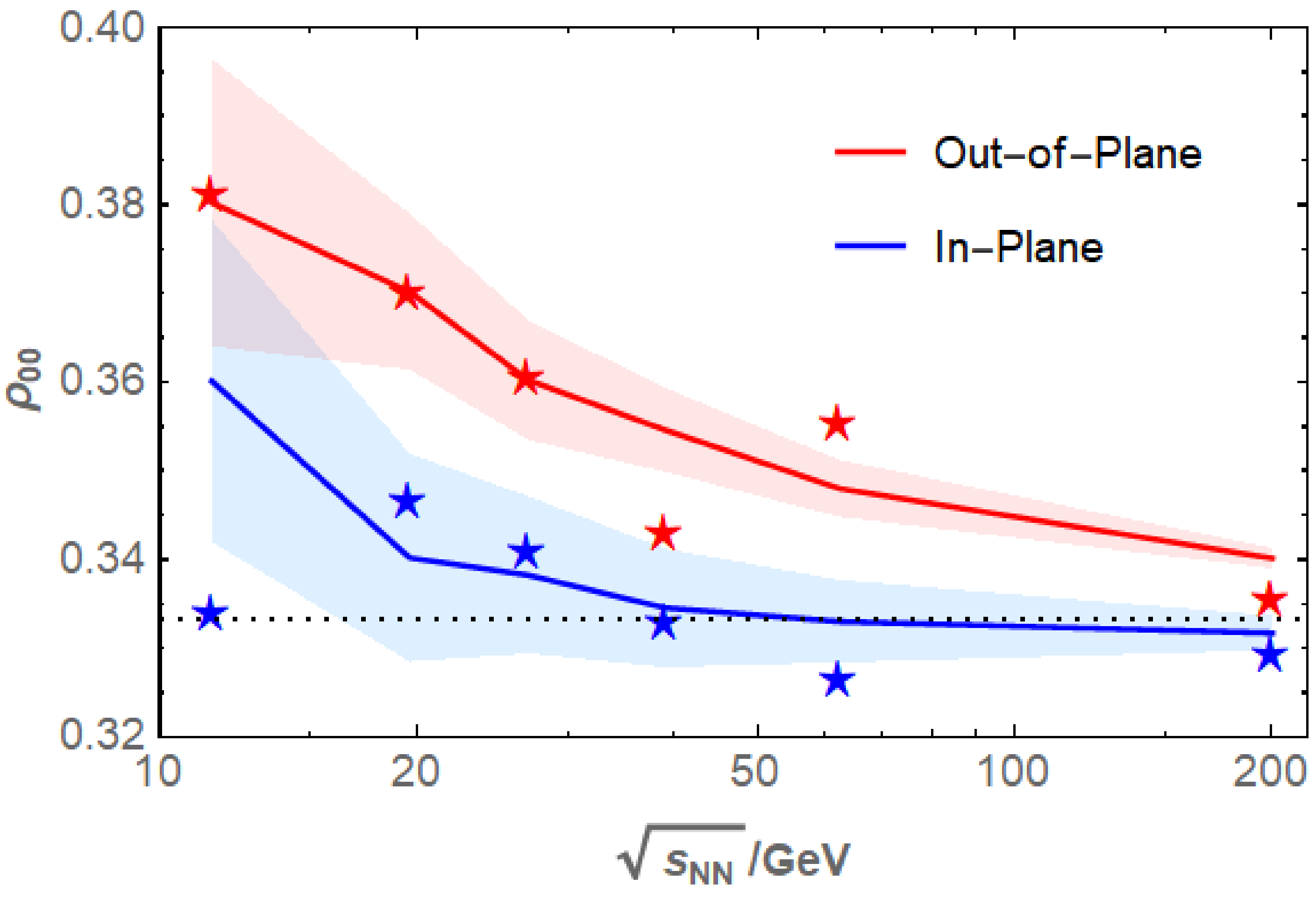
- **Two sets of values for parameters give the same result**

$$F^2 = 0.45 m_\pi^4,\ m_s = 170\,\mathrm{MeV}$$
$$F^2 = 5.02 m_\pi^4,\ m_s = 530\,\mathrm{MeV}$$

$$r_z = 0.79$$

- **The magnitude of electric field's contribution decreases with increasing m_s**

Collision energy dependence



Transverse momentum dependence

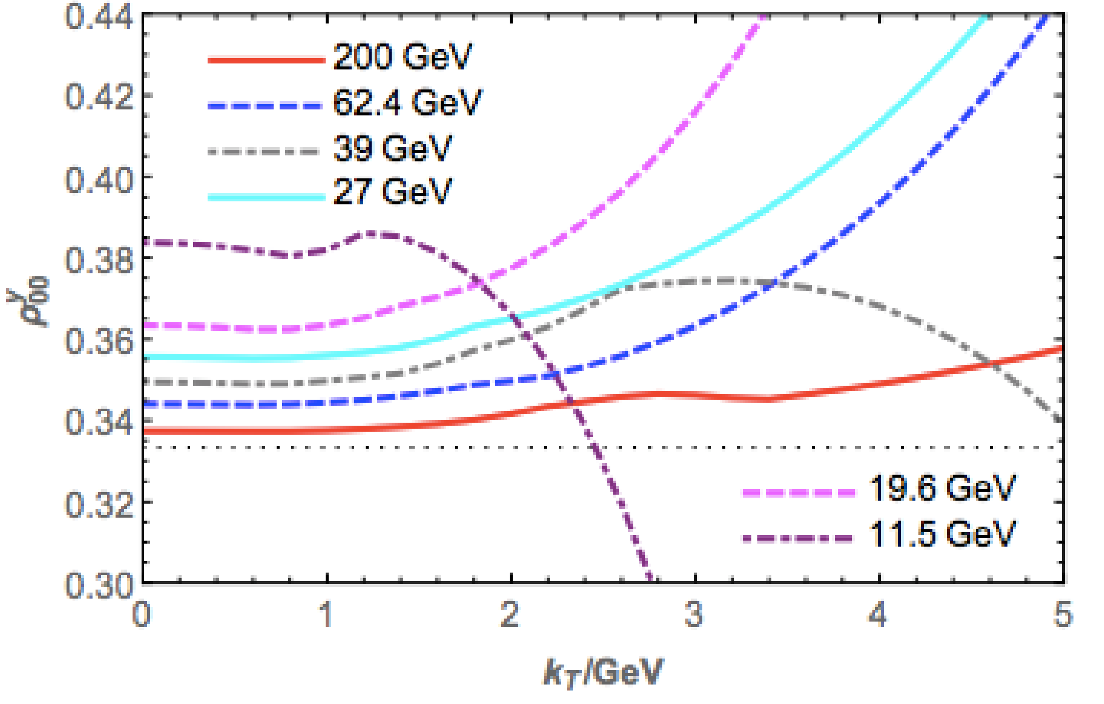


Figure 2. The ϕ meson's ρ_{00}^{y} as functions of transverse momenta at different collision energies.

Centrality dependence

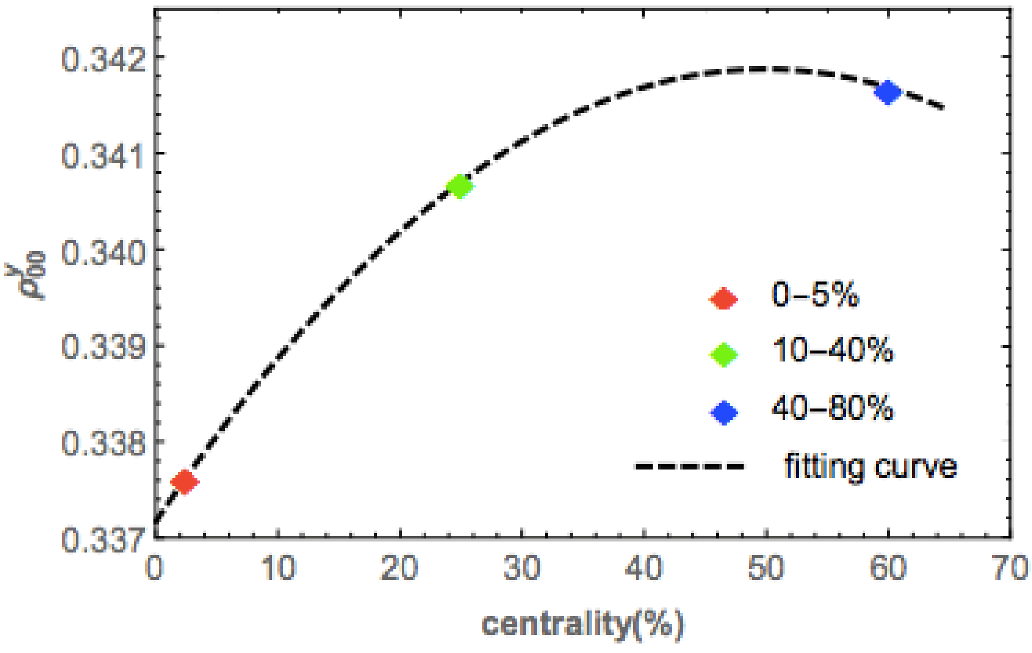


Figure 4. The ϕ meson's ρ_{00}^{y} as a function of centrality at 200 GeV. Red, green, and blue diamond points are our results for centrality ranges 0-5%, 10-40%, and 40-80%, respectively. The dashed line is the fitting curve using a second order polynomial.

Take-home message and Questions for discussions

- **Take-home message: P_Λ measures the fields (net mean field), ρ_{00}^{ϕ} measures field squares (field fluctuation).**

Questions to be answered in the future:

- **What are particles? What are fields? Particle-field duality?**
- **What is the nature of vector meson fields? Are they real entities? Can we calculate field squares on Lattice?**
- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Any implication for J/Psi polarization (gluon fields)?**

Some discussions

Particles and fields

- **Particles and fields are most fundamental forms of matter: particle-like matter, field-like matter**

  Particle-wave duality ⇨ **particle-field duality**

  Quantization of fields: fields ⇨ **particle's quantum states**

  Inverse mapping: particle's quantum states ⇨ **fields**

$$\underbrace{\Phi(x)}_{\text{field}} \Longrightarrow \underbrace{|\mathbf{p}\rangle = \sqrt{2E_p}a^\dagger(\mathbf{p})\,|0\rangle}_{\text{particle's momentum state}}$$

Yang Li, et al, 2206.12903

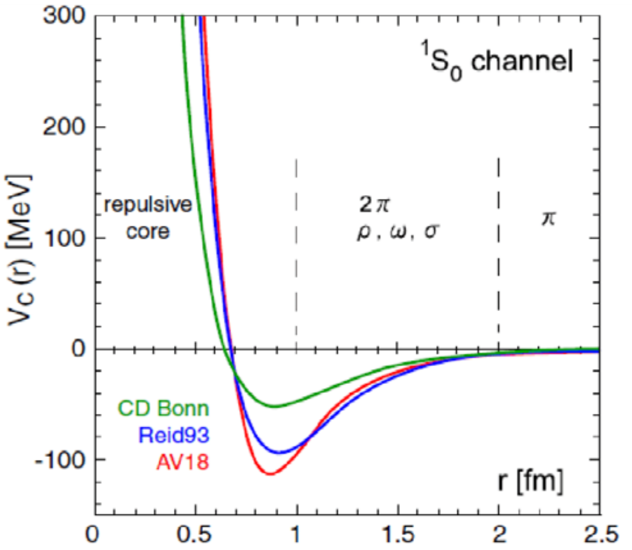
$$\underbrace{|\Phi\rangle}_{\text{particle's state as wave packet}}, \underbrace{|\mathbf{p}\rangle}_{\text{particle's momentum state}} \Longrightarrow \underbrace{\Phi(x)}_{\text{field}} = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} \underbrace{\langle \mathbf{p}\,|\Phi\rangle}_{\text{wave packet's momentum amplitude}} e^{-ip\cdot x}$$

- **Which are more fundamental: particles and fields? "The reason that our field theories work so well is not that they are fundamental truths, but that any relativistic quantum theory will look like a field theory when applied to particles at sufficiently low energy"**
  -- Steven Weinberg

Particles and fields

- In analogy with electromagnetic fields Yukawa proposed in 1935 the existence of mesons that mediate nuclear forces to bind protons and neutrons into atomic nuclei [Nobel Prize 1949].
- π : Powell and Perkins, 1947 [Nobel prize 1950]
- ρ, ω : Alvarez et al, 1961 [Nobel prize 1968]

- Nuclear force or NN potential: one boson exchange potential (OBEP)
- In low-energy nuclear reactions other meson fields may exist which carry strangeness quantum number such as K, ϕ etc.



Chiral quark model

Nuclear Physics B234 (1984) 189–212

Citations: 2201 (till Nov 14, 2022)

CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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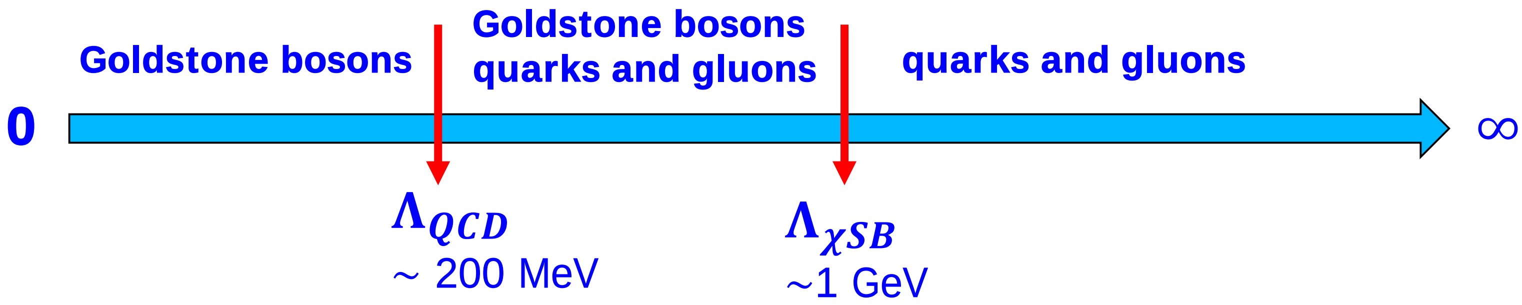
Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

Chiral quark model

- **Scale for strong interaction in dynamical process**



- **SU(3) Goldstone bosons by 3×3 matrix Σ and ξ,**

$$\Sigma = \exp\left(i\frac{2\chi}{f}\right) = \exp\left(i\frac{\chi}{f}\right)\exp\left(i\frac{\chi}{f}\right) = \xi\xi$$

f = **93 MeV**

$$\chi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Chiral quark model

- **Σ and ξ transform under $SU_L(3) \times SU_R(3)$ as**

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger$$

- **A set of color and flavor triplet quarks** $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad \psi = U\psi$

- **Lagrangian**

$$\begin{aligned} \mathcal{L} &= \overline{\psi}\left[i\gamma_\mu(\partial^\mu + igG^\mu) + g_V\gamma_\mu V^\mu\right] + g_A\overline{\psi}\gamma_\mu A^\mu\psi \\ &\quad + \frac{1}{4}f^2 \mathrm{Tr}\left(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right) - \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} \end{aligned}$$

$$\begin{aligned} V^\mu &= \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right) \\ A^\mu &= \frac{1}{2}i\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right) \end{aligned}$$

→ **Vector field induced by Goldstone boson fields**

Scale anomaly in QCD

- **The quantum effects (loop diagrams) modify the expression for the trace of the energy-momentum tensor**

$$T_\mu^\mu = H_m + \left(\gamma_m H_m + \frac{\beta}{2g}F^2\right), \qquad M_H = \left\langle T_\mu^\mu \right\rangle_H$$

$$\langle O \rangle_H = \langle H|O|H \rangle$$

$$H_m = \sum_q m_q \bar{q} q \qquad = (1+\gamma_m)\left\langle H_m \right\rangle_H + \frac{\beta}{2g}\left\langle F^2 \right\rangle_H$$

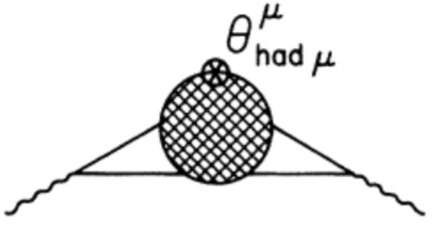
- **Running coupling → dimensional transmutation → mass scale**

$$\beta(g) = -b\frac{g^3}{16\pi^2} + \ldots, \quad b = 9 - \frac{2}{3}n_h,$$

Gross, Wilczek, Politzer (1973)

- **At small momentum transfer, heavy quarks decouple:**

$$\sum_h m_h \bar{Q}_h Q_h \to -\frac{2}{3}\, n_h \, \frac{g^2}{32\pi^2} G^{\alpha\beta a} G^a_{\alpha\beta} + \ldots$$



Shifman (1978)