g-mode Oscillations in Neutron Stars

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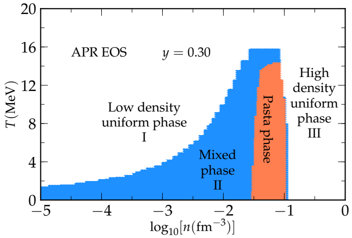
	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
Baryon Density(n_0)	$10^{-8} - 10$	$10^{-8} - 10$	$10^{-8} - 10$
Temperature(MeV)	$0 - 30$	$0 - 50$	$0 - 100$
Entropy(k_B)	$0.5 - 10$	$0 - 10$	$0 - 100$
Proton Fraction	$0.35 - 0.45$	$0.01 - 0.3$	$0.01 - 0.6$



- Phase diagram corresponding to the APR EOS at a lepton fraction of 0.3. From Eur. Phys. J. A (2019) 55 :10

Supranuclear EOS

Phenomenological Approaches:

- Skyrme(-like): $\hat{V}_{NN} = \sum_{i<j} \hat{V}_{ij} + \sum_{i<j<k} \hat{V}_{ijk}$, zero-range.
  Evaluated in the Hartree-Fock approximation $\Rightarrow \mathcal{H} = \frac{\hbar^2}{2m^*}\tau + V(n)$.
- Relativistic meson exchange in the mean-field approximation (= negligible meson-field fluctuations, uniform and static system).
- Momentum-dependent interactions of the Yukawa type, borrowed from heavy-ion physics.

Microscopic Approaches:

- High-precision interactions fitted to NN scattering data
  - meson-exchange models
    e.g. Nijmegen, Paris, Juelich-Bonn
  - sums of local operators
    e.g. Urbana, Argonne
- Interactions from chiral EFT
- RG-evolved potentials

Extension of the above to bulk matter by a variety of techniques: SCGF, BHF, variational, etc.

Laboratory Equilibrium Constraints

- Near the Nuclear Equilibrium Point $(n = n_0, \alpha = 0)$,

  $E(n,\alpha) \simeq E_0(n) + S_2(n)\alpha^2 + \mathcal{O}(\alpha^4)$
  $E_0(n) \simeq \mathcal{E}_0 + \frac{1}{2}K_0\left(\frac{n-n_0}{3n_0}\right)^2 + \ldots$
  $S_2(n) \simeq S_v + L\left(\frac{n-n_0}{3n_0}\right) + \ldots$

- Saturation density, $n_0 = 0.16 \pm 0.01\ \mathrm{fm}^{-3}$
  High-energy electron scattering: $r_0 \propto \pi/qR$, $n_0 = \left(\frac{4}{3}\pi r_0^3\right)^{-1}$

- Energy per particle, $\mathcal{E}_0 = -16 \pm 1$ MeV
  Fits to masses of atomic nuclei :
  $B(N,Z) = \mathcal{E}_o A - b_{surf} A^{2/3} - S_v \frac{(N-Z)^2}{A} - b_{Coul} Z^2 A^{-1/3}$

- Symmetry energy, $S_v = 30 - 35$ MeV
  (fits to masses of atomic nuclei)

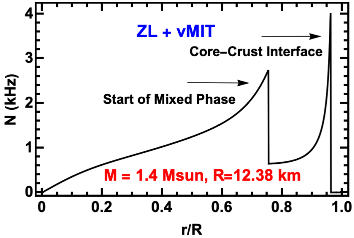
- Slope of S_2, $L = 40 - 70\mathrm{MeV}$
  (variety of experiments)

- Compression modulus, $K_0 = 240 \pm 30$ MeV
  Giant monopole resonances : $E_{GMR} = \left(\frac{K_A}{m<r^2>}\right)^{1/2}$
  $K_A = K_0 + K_{surf} A^{-1/3} + K_\tau \frac{(N-Z)^2}{A^2} + K_{Coul} \frac{Z^2}{A^{4/3}}$

Neutron Stars

- Matter in β-equilibrium supported against gravitational collapse by neutron degeneracy.
- Structure determined by simultaneous solution of:
  - Interior mass, $m(r) = 4\pi \int_0^r \epsilon(r')r'^2 dr'$
  - Hydrostatic equilibrium, $\frac{dp}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{m(r)}\right]\left[1 - \frac{2Gm(r)}{r}\right]^{-1}$
  - EOS, $p = p(\epsilon)$
- Constraints
  - Largest observed mass, (binaries) $M \simeq 2\ M_\odot$
  - Largest observed frequency, (pulsars) $\Omega = 114$ rad/s
  - Inferred radius range, (photospheric emission, thermal spectra) $10\text{ km} \leq R \leq 14\text{ km}$

- Relativistic binaries not in equilibrium : Gravitational wave (GW) emission leads to orbital decay.
- Early stage: only gravitational interactions, GW signal contains information for the masses of the components.
- Coalescence stage: Tidal disruption of the lower-mass star, mass transfer onto the more massive one. Mass transfer rate depends on $\mathcal{C} = M_{NS}/R_{NS}$ and reflected in GW signal. Ejected matter is very neutron-rich and can lead to heavy element formation via the r-process.
- Late stage: Black hole or hypermassive neutron star formation.
- EOS relevance
  - Tidal disruption of NS during coalesence of BH-NS binary depends on the stiffness of the EOS. GW frequency sensitive to orbital frequency at disruption.
  - r-process production rates and abundances depend on the composition of the ejecta and thus the EOS.
  - Tidal deformability, $\Lambda = \frac{2}{3} k_2 \left(\frac{R\, c^2}{G\, m} \right)^5$.
  - g-mode frequencies: $N^2 = g^2 \left(\frac{1}{c_e^2} - \frac{1}{c_s^2} \right) \mathrm{e}^{\nu - \lambda}$
    $g = -\nabla [p/(\epsilon + p)]$



Motivation: Hybrid Stars (?)

- The size of nucleons (uncertain as it may be) implies that deconfined quark matter can exist in the cores of NSs.
- However, such a possibility lacks observational and theoretical support:
  - Measurements of M, R, Λ cannot differentiate normal and hybrid stars.
  - LQCD and PQCD not applicable to NS conditions.
- Possible solution: identify an observable with strong dependence on composition.
- Enter g-modes!

- Global, long-lived, nonradial fluid oscillations resulting from fluid-element perturbations in a stratified environment.
- Slow chemical equilibration generates buoyancy forces to oppose dispacement.
- In stably-stratified systems the opposing force sets up oscillations with a characteristic frequency (Brunt-Väisälä) which depends on both the equilibrium and the adiabatic sound speeds.
- g-mode oscillations couple to tidal forces; they can be excited in a NS merger and provide information on the interior composition.
- Detection remains a challenge; but within sensitivity of 3rd generation detectors.

- In linearized GR, the calculation of g-mode frequencies, damping times, and amplitudes requires the solution of 4 coupled ODEs.
- The relativistic Cowling approximation neglects metric perturbations that must accompany matter perturbations in a GR treatment reducing complexity:

$$\frac{dU}{dr} = \frac{g}{c_{\rm ad}^2} U + e^{\lambda/2}\left[\frac{l(l+1)e^{\nu}}{\omega^2} - \frac{r^2}{c_{\rm ad}^2}\right] V$$

$$\frac{dV}{dr} = e^{\lambda/2-\nu}\,\frac{\omega^2 - N^2}{r^2} U + g\Delta(c^{-2}) V$$

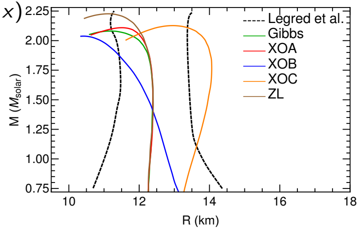
where $U = r^2 e^{\lambda/2}\,\xi_r$, $V = \omega^2 r\,\xi_h$, $\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$,
$N^2 = g^2 \Delta(c^{-2}) e^{\nu-\lambda}$, $g = -\nabla P/(\varepsilon + P)$,
and λ, ν are Schwarzchild metric functions.

- Accurate to a few % compared to GR.
- Cannot compute imaginary part of eigenfrequeny (damping time).

- Nucleons: Zhao - Lattimer

$$\epsilon_B = \sum_{h=n,p} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{M_B^2 + k^2}\, dk + n_B V(u,x)$$

$$V = 4x(1-x)(a_0 u + b_0 u^{\gamma}) + (1-2x)^2 (a_1 u + b_1 u^{\gamma_1})$$

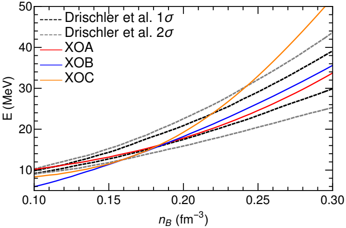


- Quarks: vMIT

$$\mathcal{L} = \sum_{q=u,d,s} \left[\bar{\psi}_q \left(i\partial\!\!\!/ - m_q - B\right) \psi_i + \mathcal{L}_{\rm int}\right] \Theta$$

$$\mathcal{L}_{\rm int} = -G_v \sum_q \bar{\psi}\gamma_\mu V^\mu \psi + \left(m_V^2/2\right) V_\mu V^\mu$$

$$\epsilon_Q = \sum_q \epsilon_{\rm FG,q} + \frac{1}{2}\left(\frac{G_v}{m_V}\right)^2 n_Q^2 + B$$



- Leptons: noninteracting, relativistic fermions

$$\epsilon_L = \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{m_L^2 + k^2}\, dk$$

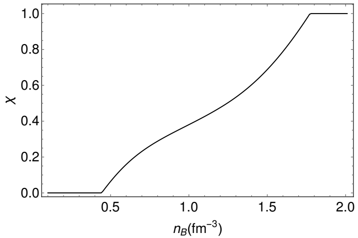
- Gibbs

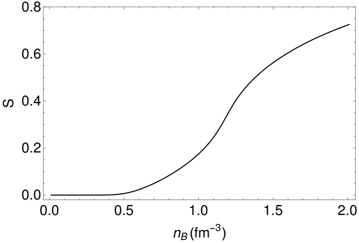
$$\varepsilon^* = (1-\chi)\varepsilon_H + \chi\varepsilon_Q \quad ; \quad 0 \le \chi \le 1$$
$$P_Q = P_H$$

- Crossover (Kapusta-Welle)

$$P_B = (1-S)P_H + S\ P_Q$$
$$S = \exp\left[-\left(\frac{\mu_0}{\mu}\right)^4\right]$$
$$\mu_0 \sim 2\ \mathrm{GeV}$$

- Neutron-star matter
  - Strong equilibrium:
    $\mu_n = 2\mu_d + \mu_u \;\; ; \;\; \mu_p = 2\mu_u + \mu_d$
  - Weak equilibrium:
    $\mu_n = \mu_p + \mu_e \;\; ; \;\; \mu_e = \mu_\mu \;\; ; \;\; \mu_d = \mu_s$
  - Charge neutrality:
    $n_p^* + (2n_u^* - n_d^* - n_s^*)/3 - (n_e + n_\mu) = 0$
  - Baryon number conservation:
    $n_n^* + n_p^* + (n_u^* + n_d^* + n_s^*)/3 - n_B = 0$





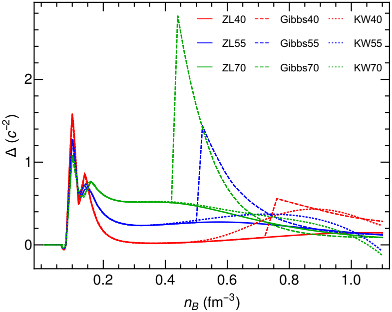
- $c^2_{\rm eq}(n_B) = \frac{dP}{d\varepsilon} = \frac{dP_\beta}{dn_{\rm B}}\left(\frac{d\varepsilon_\beta}{dn_{\rm B}}\right)^{-1}$
  mechanical equilibrium restored instantaneously.

- $c^2_{\rm ad}(n_{\rm B}, x) = \left(\frac{\partial P}{\partial \varepsilon}\right)_x = \frac{\partial P}{\partial n_{\rm B}}\Big|_x \left(\frac{\partial \varepsilon}{\partial n_{\rm B}}\Big|_x\right)^{-1}$
  $c^2_{{\rm ad},\beta}(n_{\rm B}) = c^2_{\rm ad}[n_{\rm B}, x_\beta(n_{\rm B})]$
  $\tau_\beta \gg \tau_{\rm oscillation}$.

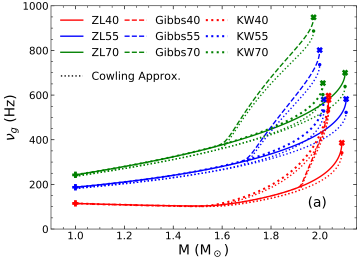
- The difference $\Delta(c^{-2}) = c^{-2}_{\rm eq} - c^{-2}_{\rm ad}$ drives the restoring force for g-mode oscillations. For example, in *npe* matter

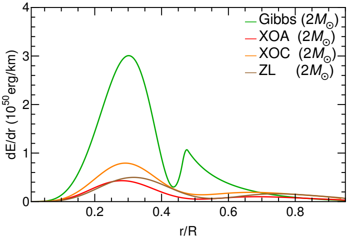
$$c^2_{\rm ad} = c^2_{\rm eq} + \frac{\left[n_B\left(\frac{\partial \tilde{\mu}}{\partial n_B}\right)_x\right]^2}{\mu_n\left(\frac{\partial \tilde{\mu}}{\partial x}\right)_{n_B}}$$

$$\tilde{\mu} = \mu_e + \mu_p - \mu_n \xrightarrow{\beta-{\rm eq.}} 0$$



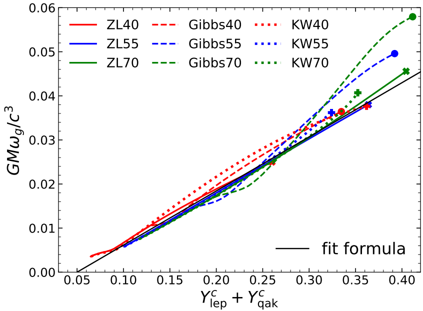
- g-modes in Gibbs hybrid matter have a larger frequency range compared to the pure-nucleon and crossover cases corresponding to the behavior of $\Delta(c^{-2})$ in the mixed phase.
- Dramatic changes in ν_g require new particle species not merely a smooth change in composition.
- The Cowling approx. is qualitatively similar to GR but underestimates ν_g by up to 10%; does better for low-mass stars.
- Energy per unit radial distance in oscillatory motion: The Gibbs energy scale is one order of magnitude larger than ZL and KW once quark matter appears ($\sim 10^{51}$ ergs/km vs. $\sim 10^{50}$ ergs/km).





Universal relation: Ω_g vs. Y^c

- Universal relations depend weakly on the EOS and can be used to break degeneracies and otherwise constrain difficult-to-access observables.
- Given the sensitivity of g-modes to departures from chemical equilibrium, it is likely that N and ν_g depend strongly on composition
- $\Omega_g = GM\omega_g/c^3 = 1.228(Y^c - 0.05)$



Summary

- First calculation of g-mode properties under Gibbs phase rules and for the KW model (both with the Cowling approximation as well as linearized GR).
- g-modes can detect nonnucleonic matter in the cores of NS; assuming quark matter (by some other means), g-modes can distinguish between a first-order phase transition and a crossover.
- Universal relation between Ω_g and Y^c.
- (Near) Future:
  - Extend KW to finite T.
  - Applications to protoneutron stars (cooling, superfluidity)
  - Other signals?
  - Construct EOS that uses the same underlying description for quarks and hadrons; explore hybrid matter microscopically.
  - Explore 1st-order phase transitions with intermediate surface tension.