

Primordial Black Hole Dark Matter

Based on Phys.Rev.Lett. 120(2018)191102 done with S. Wang, Y.F. Wang and T. Li

Phys.Rev.D 101(2020)063019 done with Y.F. Wang, T. Li and S. Liao

ApJ 864(2018)61 and arXiv:1904.02396 done with Z.C. Chen

ApJ 871(2019)97 done with Z.C. Chen and F. Huang

Phys.Rev.D 100(2019)081301(R) & 1011(2020)043019 & arXiv:1910.12239 done with Z.C. Chen and C. Yuan

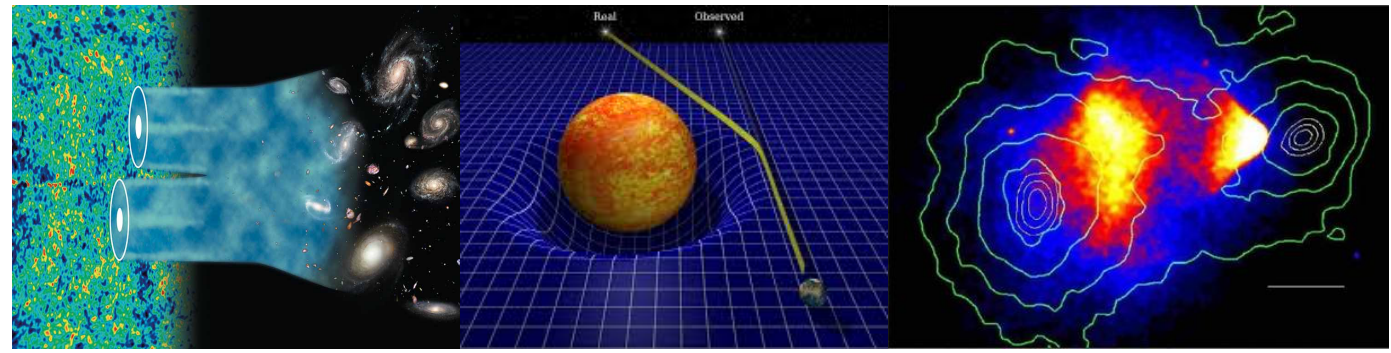
LISA
Taiji
TianQin

PPTA
IPTA
FAST
SKA

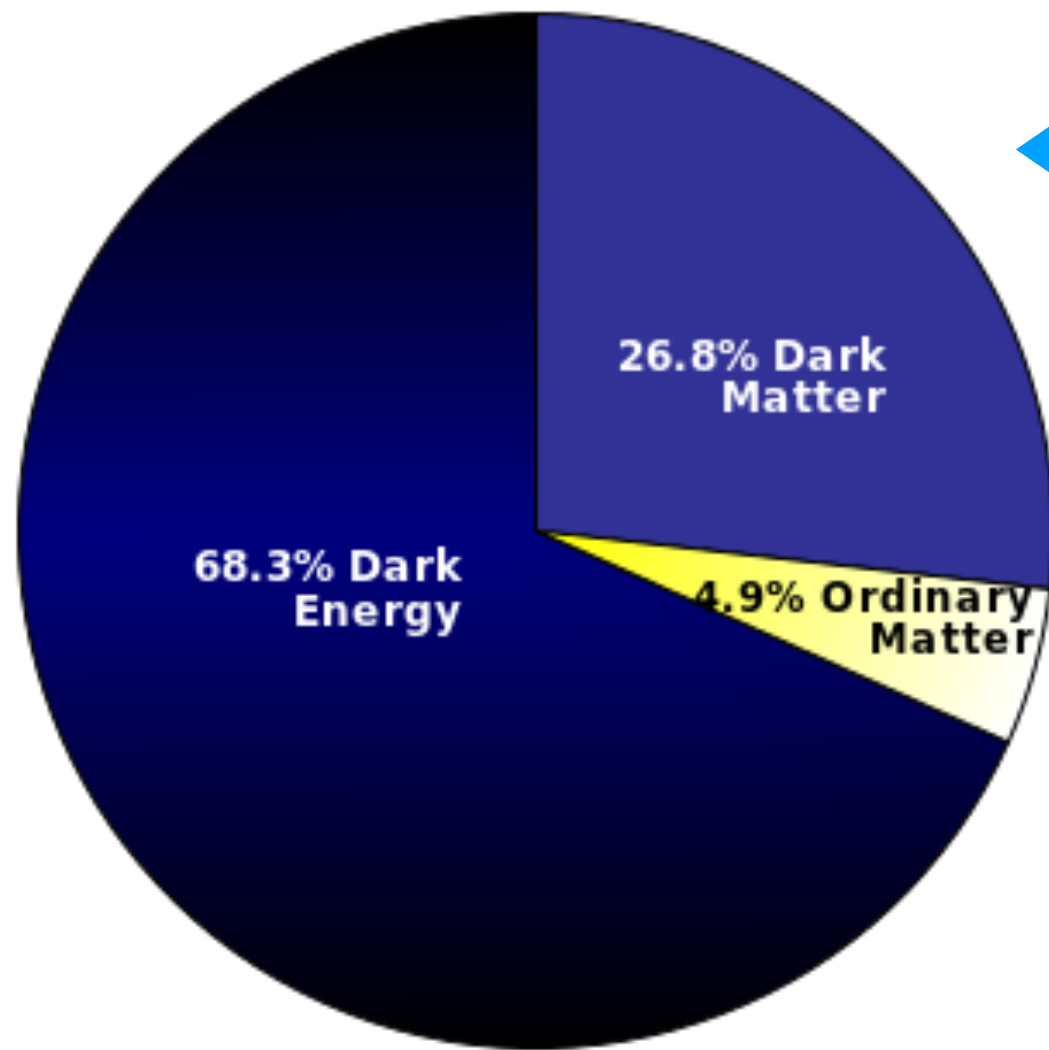
LIGO
Virgo
KAGRA

Qing-Guo Huang
Institute of theoretical physics, CAS

The nature of **Dark Matter**



Planck 18 (CMB only): $\Omega_c h^2 = 0.1200 \pm 0.0012$ (100σ)
Planck 18+BAO: $\Omega_c h^2 = 0.11933 \pm 0.00091$ (131σ)



Primordial Black Hole

$$f_{\text{pbh}} = \frac{\Omega_{\text{pbh}}}{\Omega_{CDM}}$$

$$m_{pbh} \sim \frac{t}{G} \sim 10^5 \left(\frac{t}{1 \text{ s}} \right) M_\odot$$

$$\tau(m_{pbh}) \sim G^2 m_{pbh}^3 \sim 10^{64} \left(\frac{m_{pbh}}{M_\odot} \right)^3 \text{ yr}$$

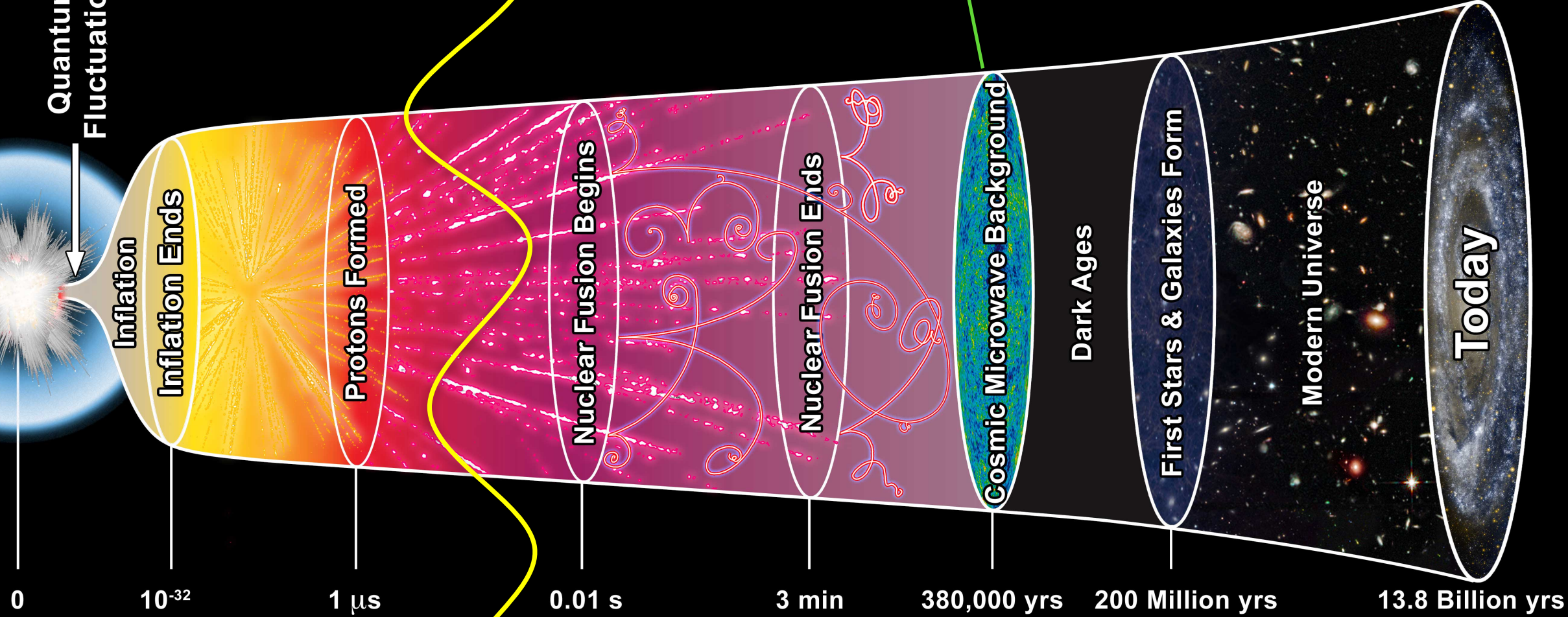
Power Spectrum

$$\delta\rho/\rho \sim 1$$

$$\delta\rho/\rho \sim 10^{-5}$$

Radius of the Visible Universe

Quantum Fluctuations

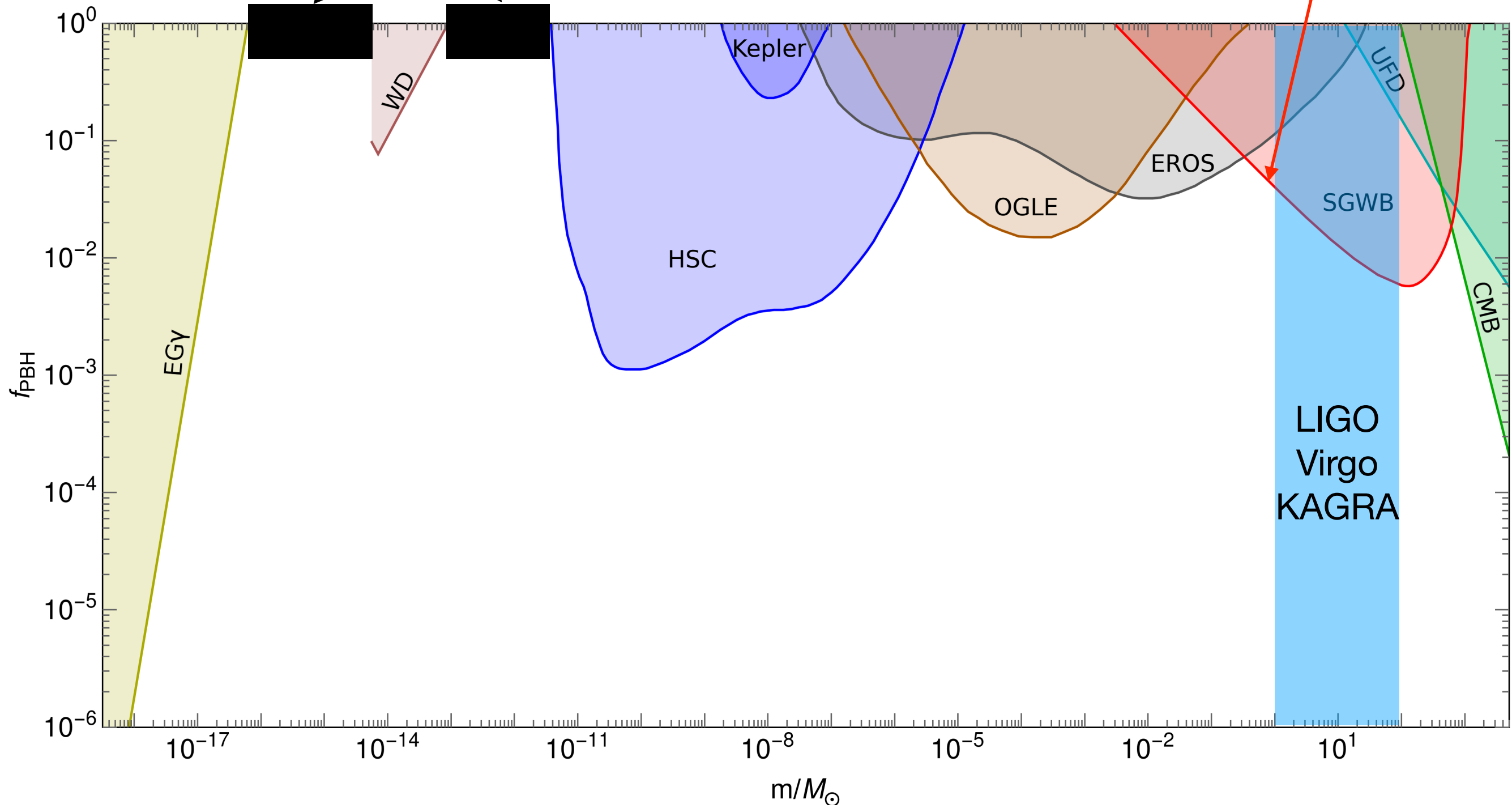


Abundance & Mass Function

Dark Matter

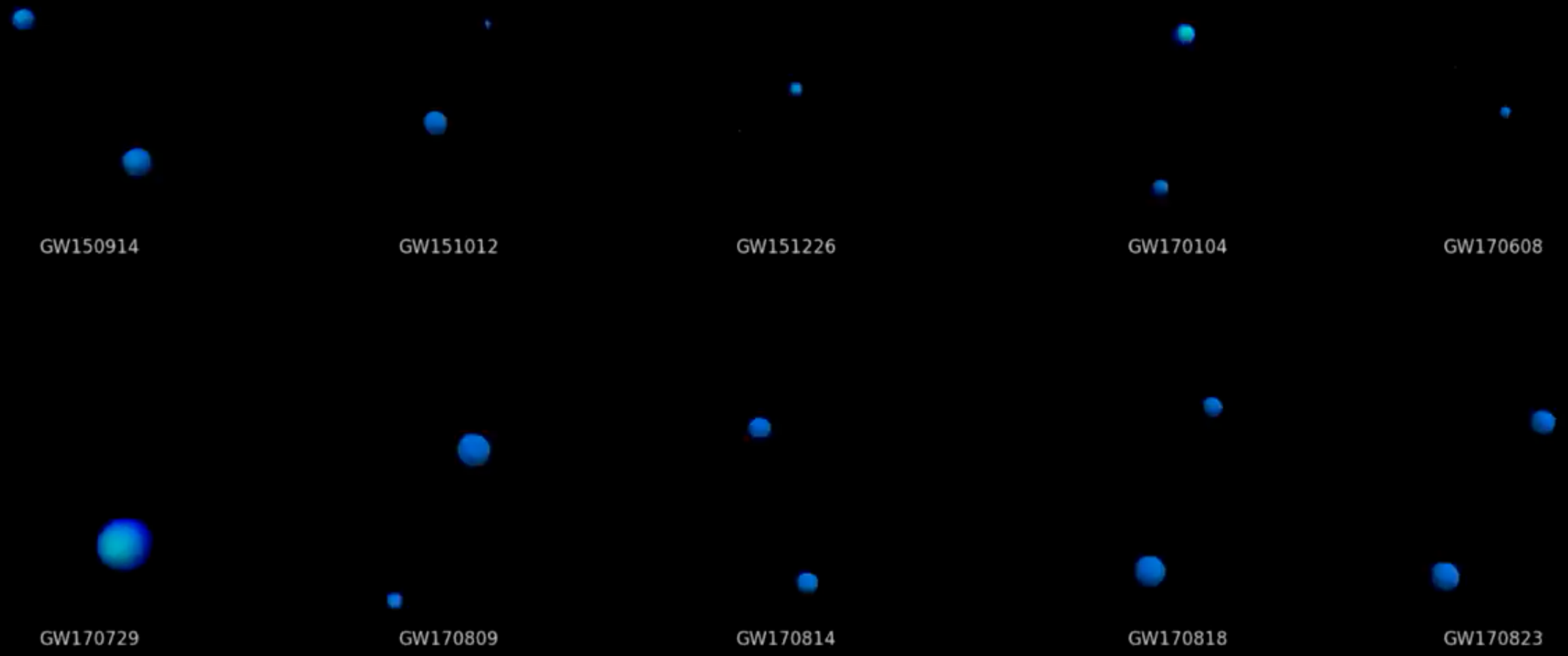
S. Wang, Y.F. Wang, QGH & T. Li, Phys.Rev.Lett. (2018)

Chen & QGH, arXiv:1904.02396

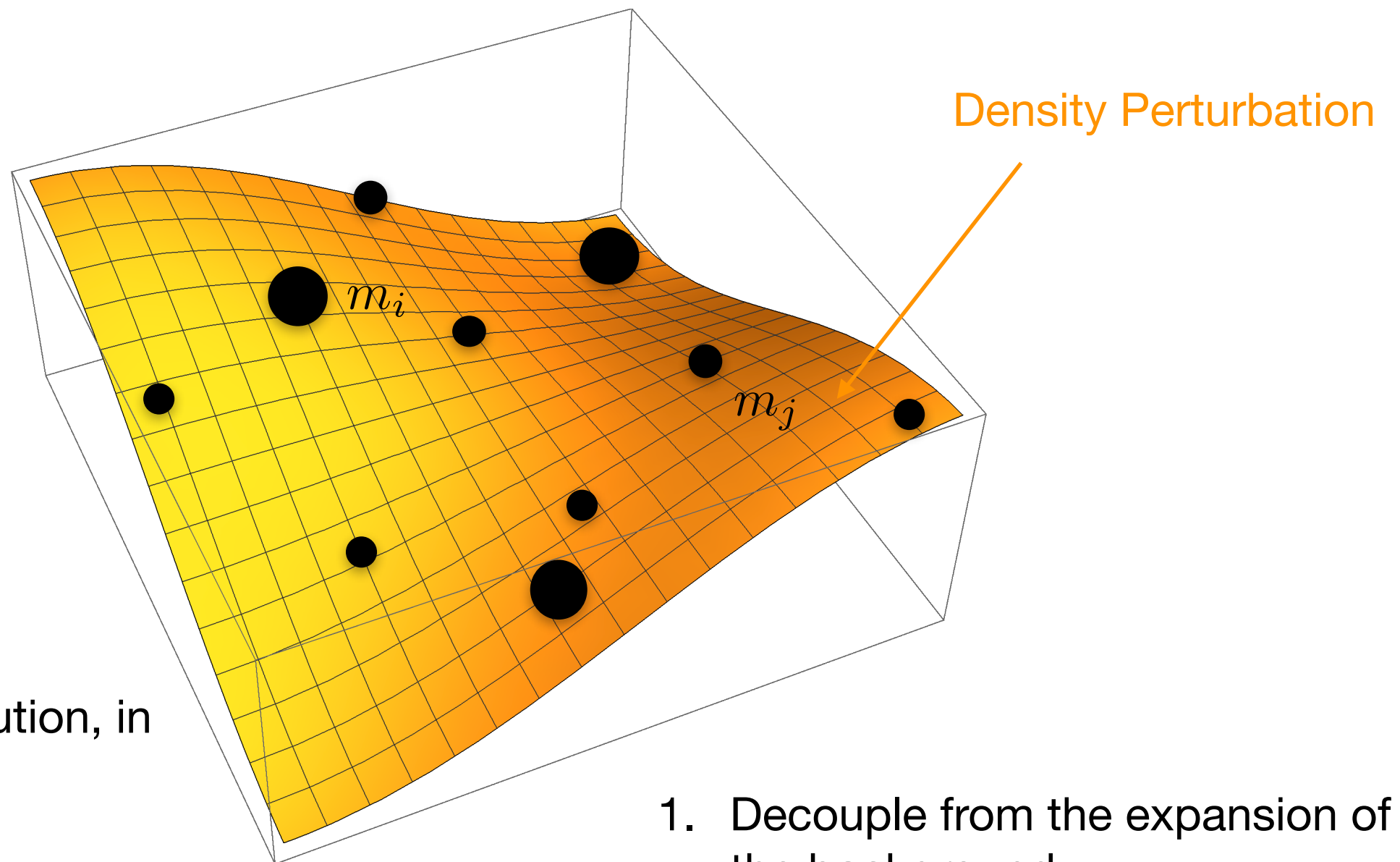




Time: -0.63 seconds



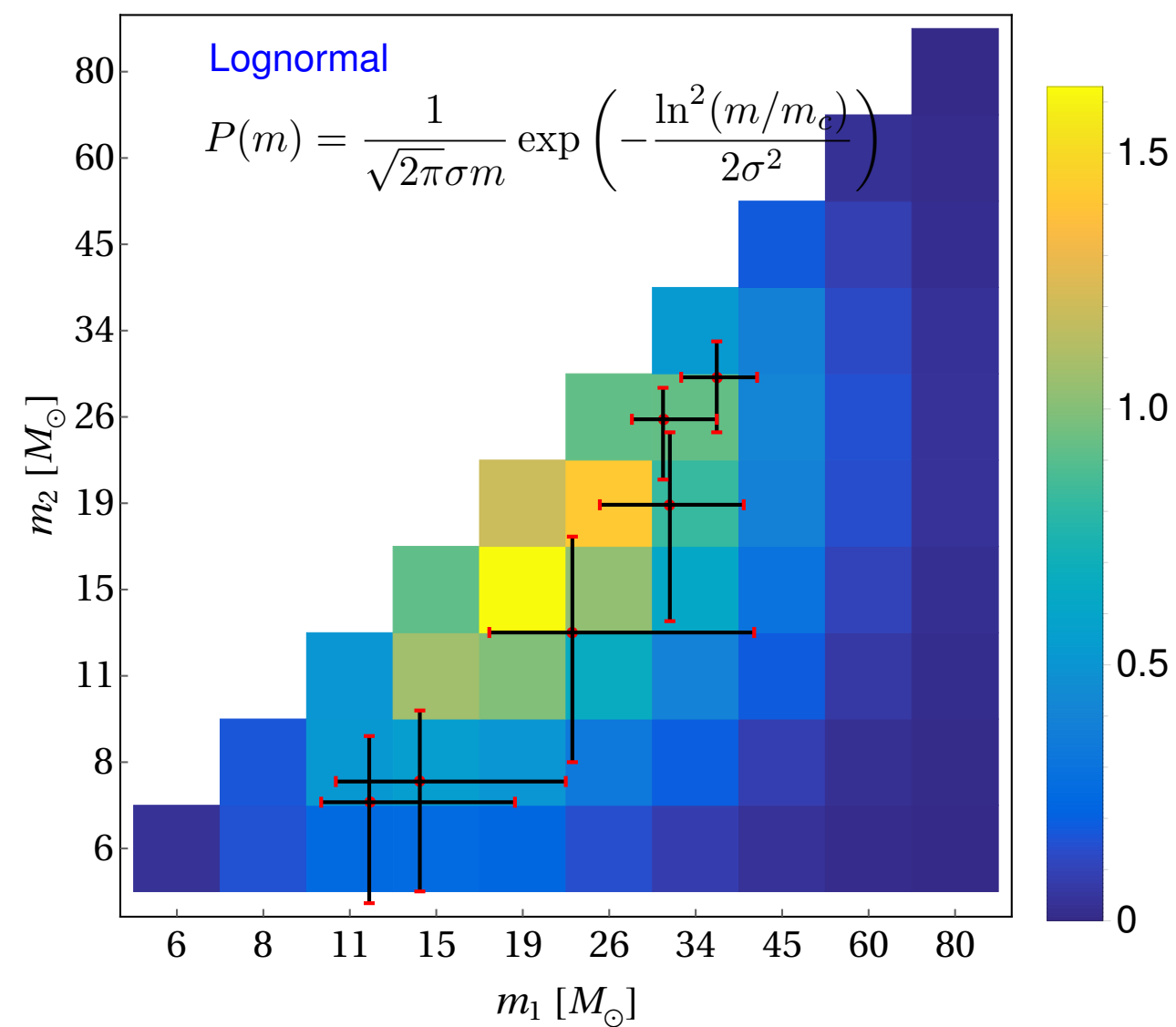
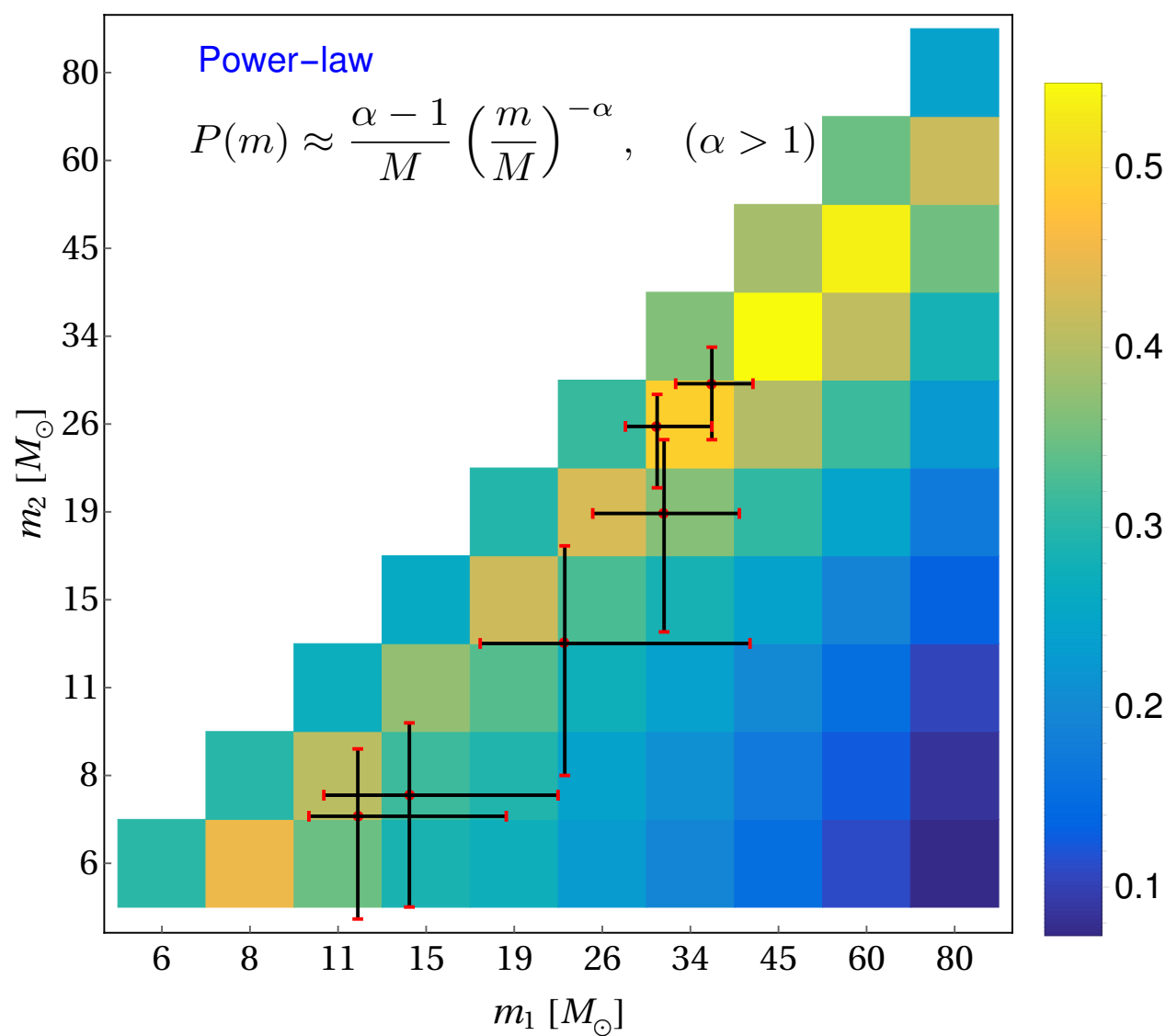
Merger rate distribution



The merger rate distribution, in unit of $\text{Gpc}^{-3}\text{yr}^{-1}$

$$R_{ij}(t) = \mathcal{R}_{ij} \Delta^2$$
$$\mathcal{R}_{ij} \simeq 3.9 \cdot 10^6 \times \frac{\rho_m}{\rho_m^0} \left(\frac{t}{t_0} \right)^{-\frac{34}{37}} f^2 (f^2 + \sigma_{eq})^{-\frac{21}{74}}$$
$$\times \min \left(\frac{P(m_i)}{m_i}, \frac{P(m_j)}{m_j} \right) \left(\frac{P(m_i)}{m_i} + \frac{P(m_j)}{m_j} \right)$$
$$\times (m_i m_j)^{\frac{3}{37}} (m_i + m_j)^{\frac{36}{37}}$$

1. Decouple from the expansion of the background
2. Torques by all other PBHs and density perturbations provides an initial angular momentum
3. Coalescence due to GW radiations

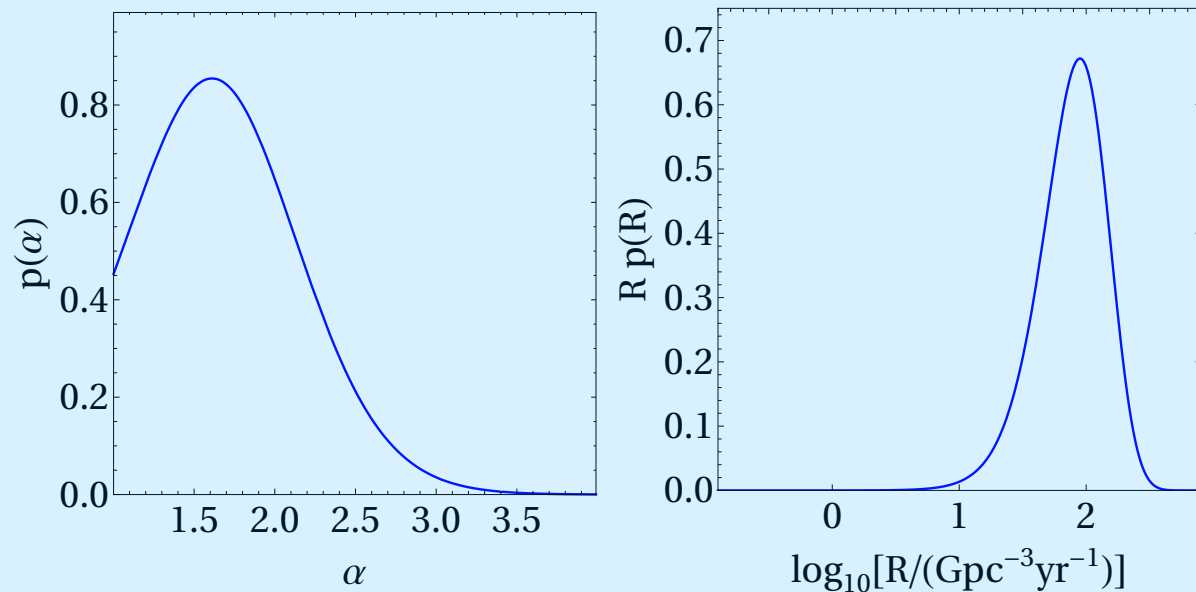


Averaged sensitive spacetime volume of LIGO

$$\Lambda_{ij} = \int_0^1 R_{ij} \frac{d\langle VT \rangle}{dz} dz$$

LIGO O1

$$P(m) = \frac{\alpha - 1}{M_{\min}} \left(\frac{m}{M_{\min}} \right)^{-\alpha} \quad \text{for } M_{\min} = 5M_{\odot}$$

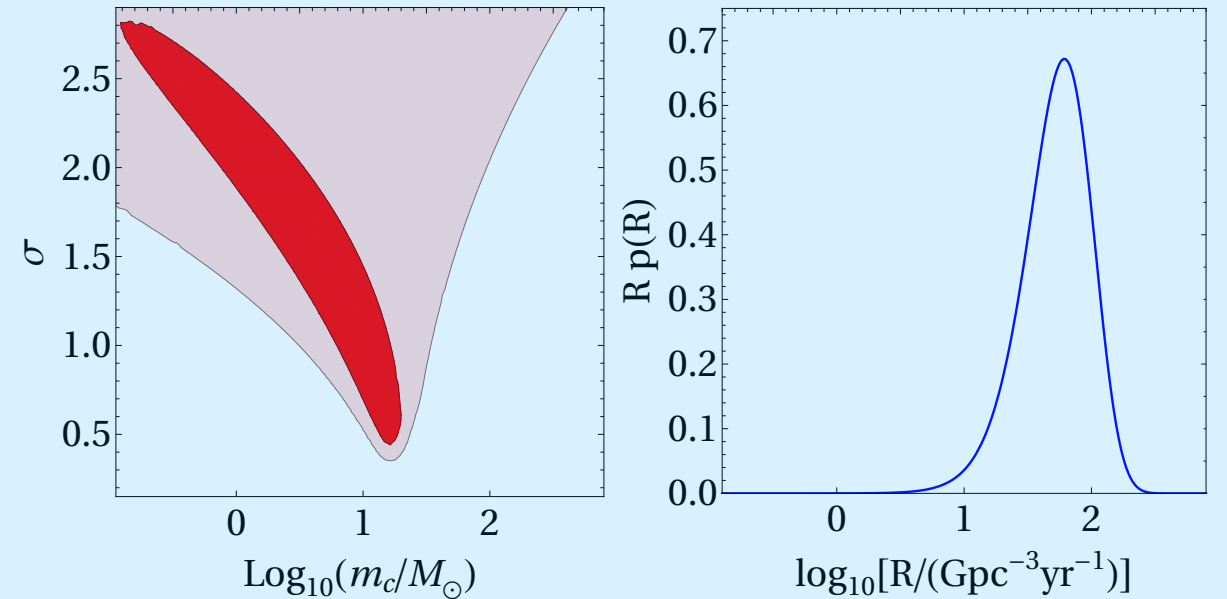


$$\alpha = 1.61 \text{ (best fit value)}$$

$$R = 80_{-56}^{+108} \text{ Gpc}^{-3}\text{yr}^{-1}$$

$$f_{pbh} = 3.8_{-1.8}^{+2.3} \times 10^{-3}$$

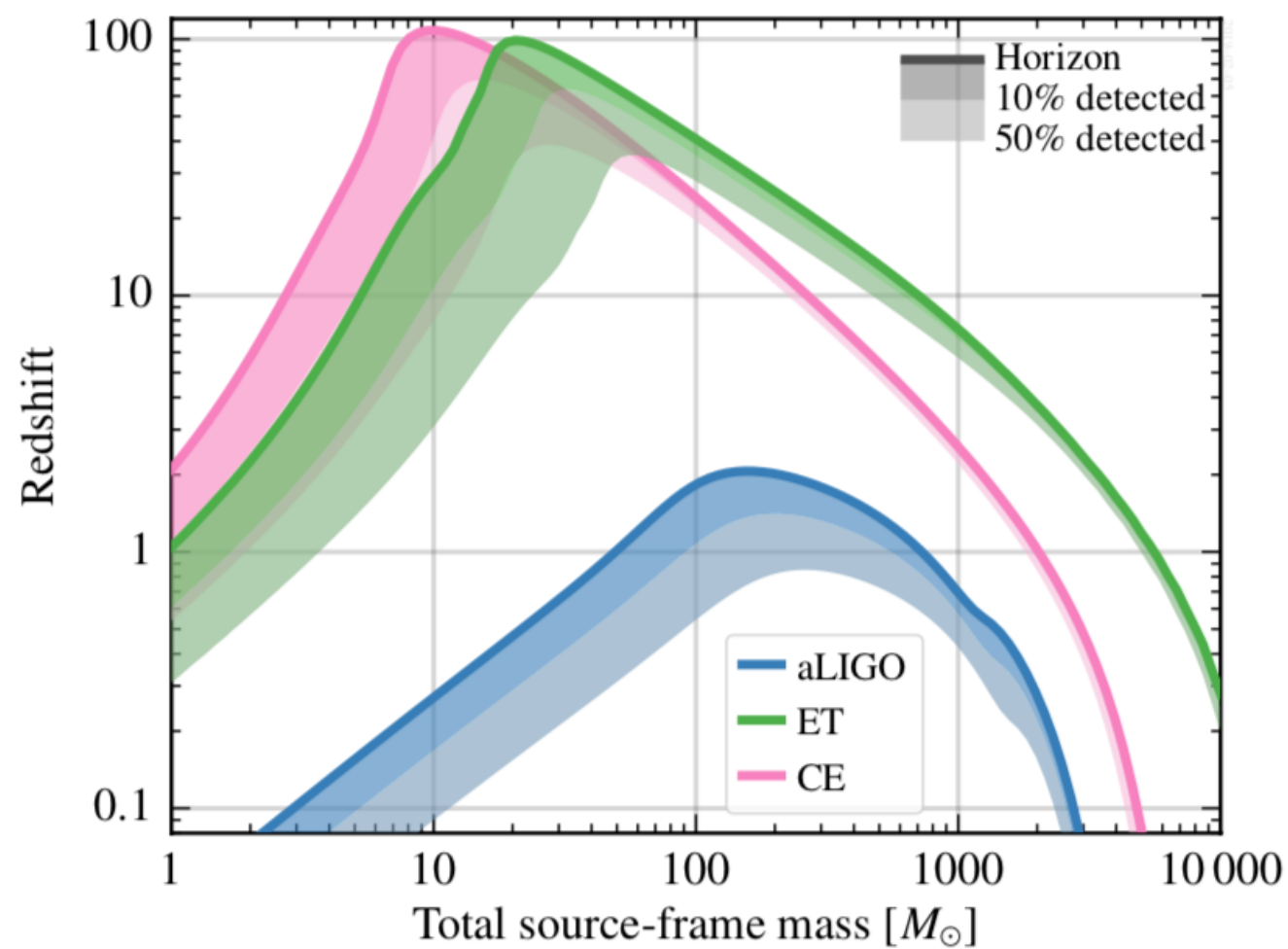
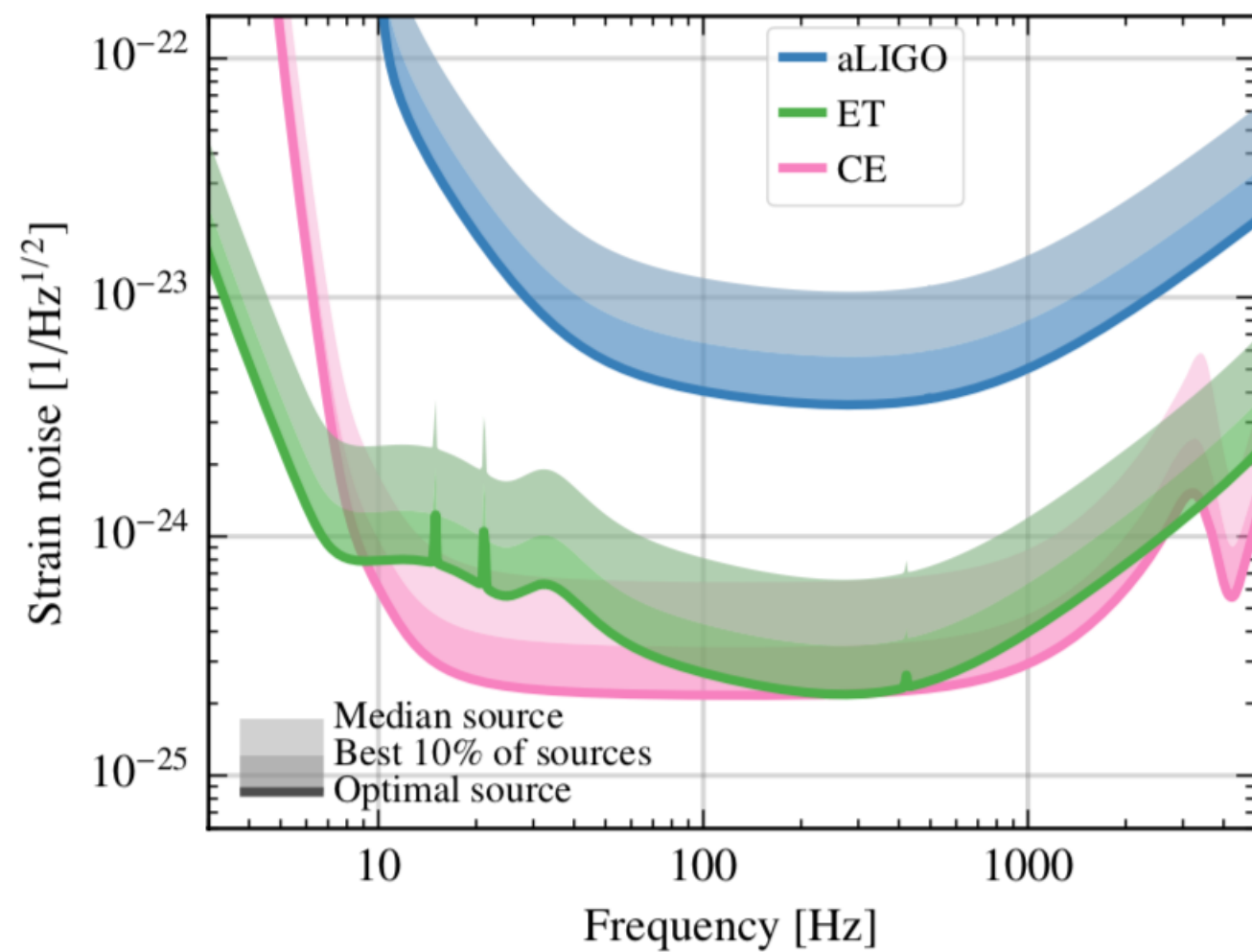
$$P(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp \left(-\frac{\log^2(m/m_c)}{2\sigma^2} \right)$$

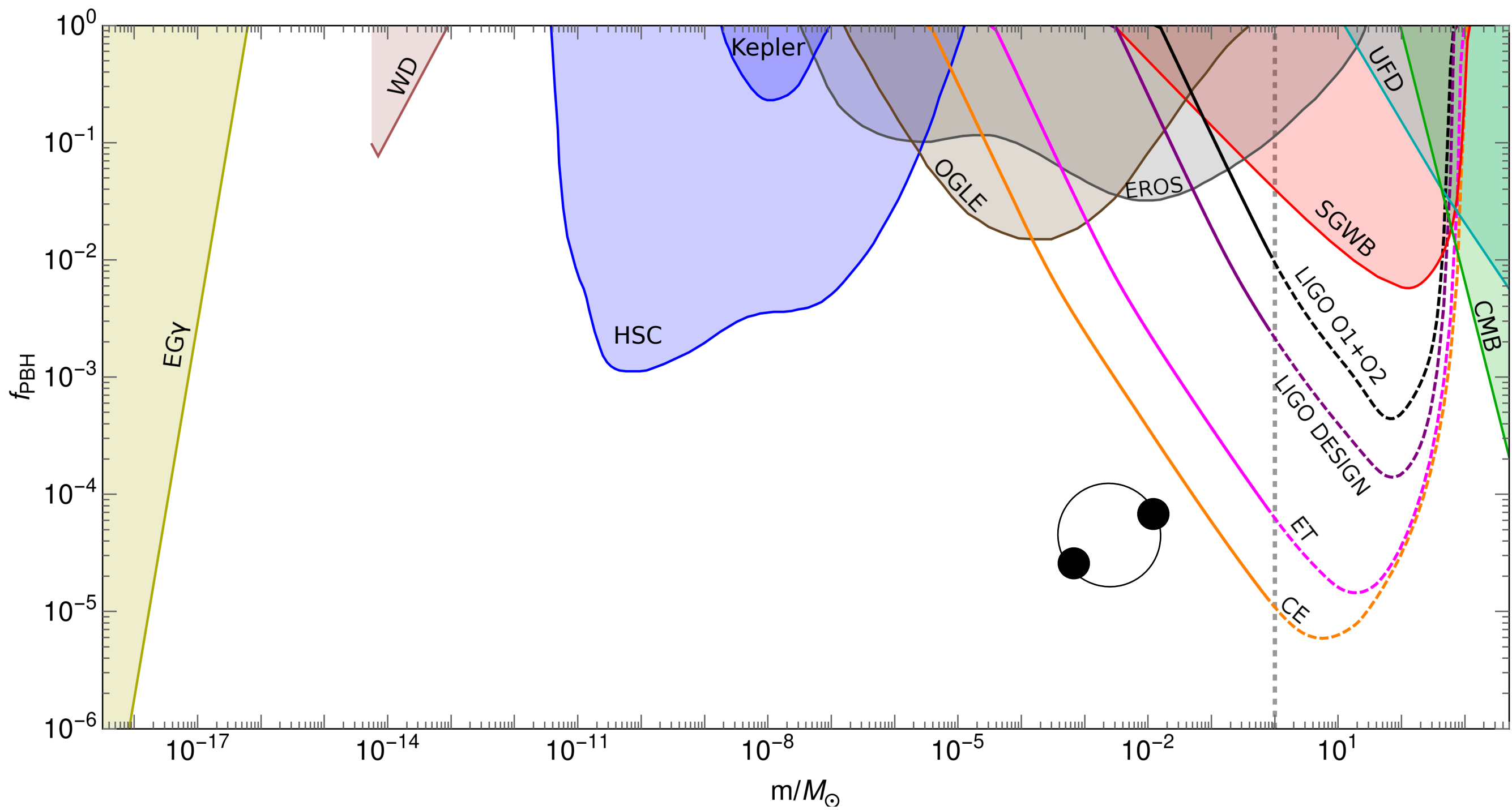


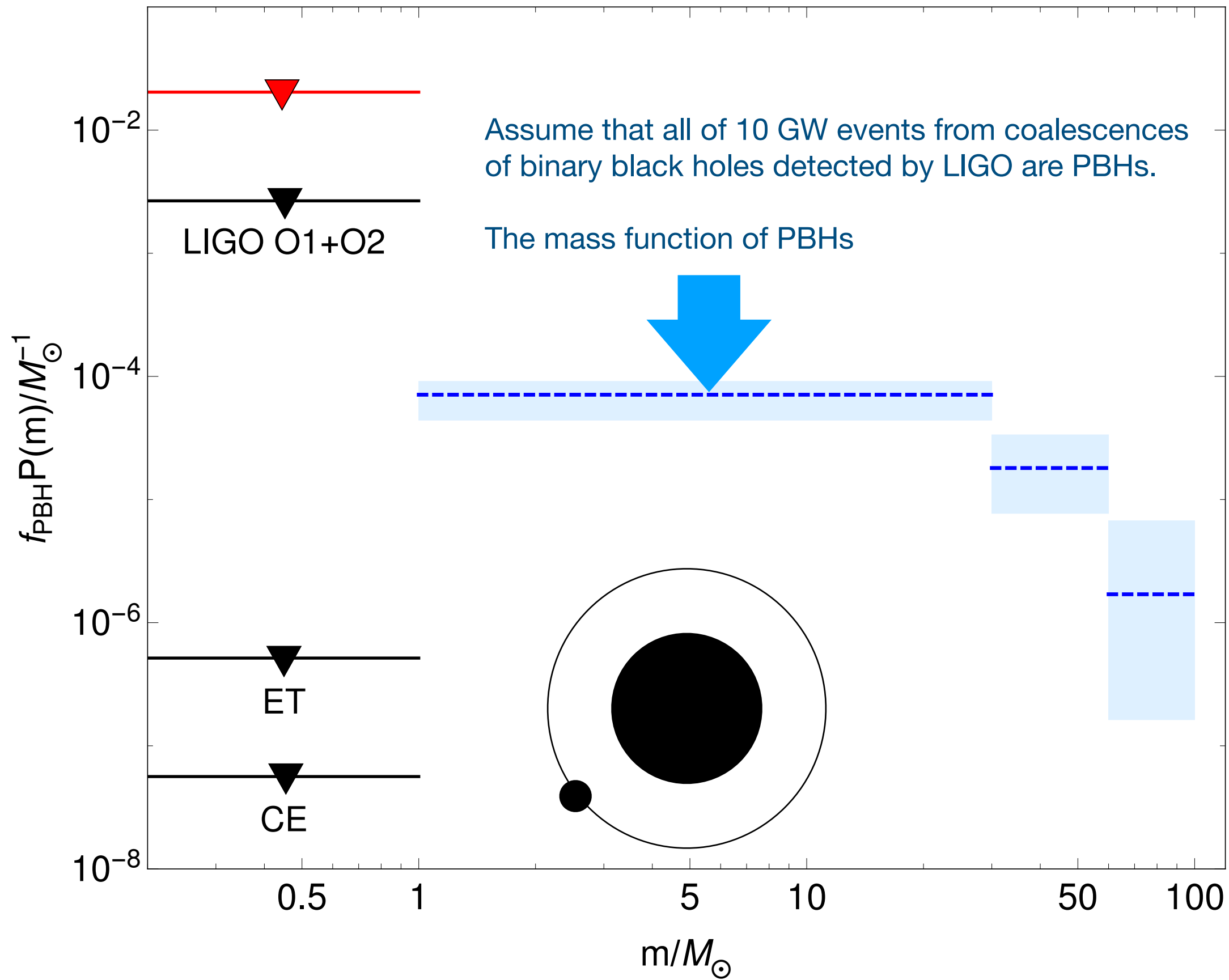
$$m_c = 14.8M_{\odot}, \sigma = 0.65 \text{ (best fit value)}$$

$$R = 55_{-38}^{+74} \text{ Gpc}^{-3}\text{yr}^{-1}$$

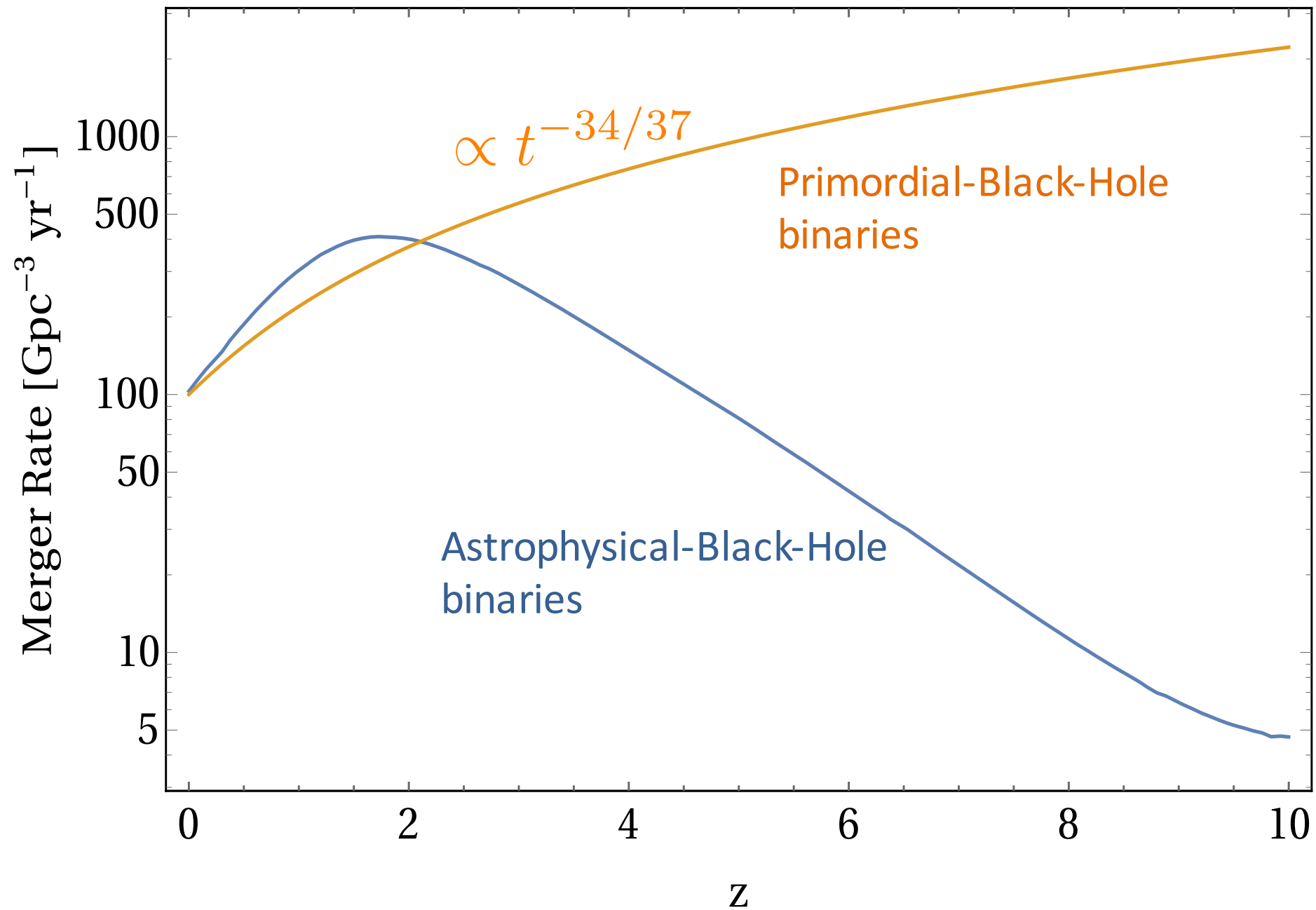
$$f_{pbh} = 2.8_{-1.3}^{+1.6} \times 10^{-3}$$



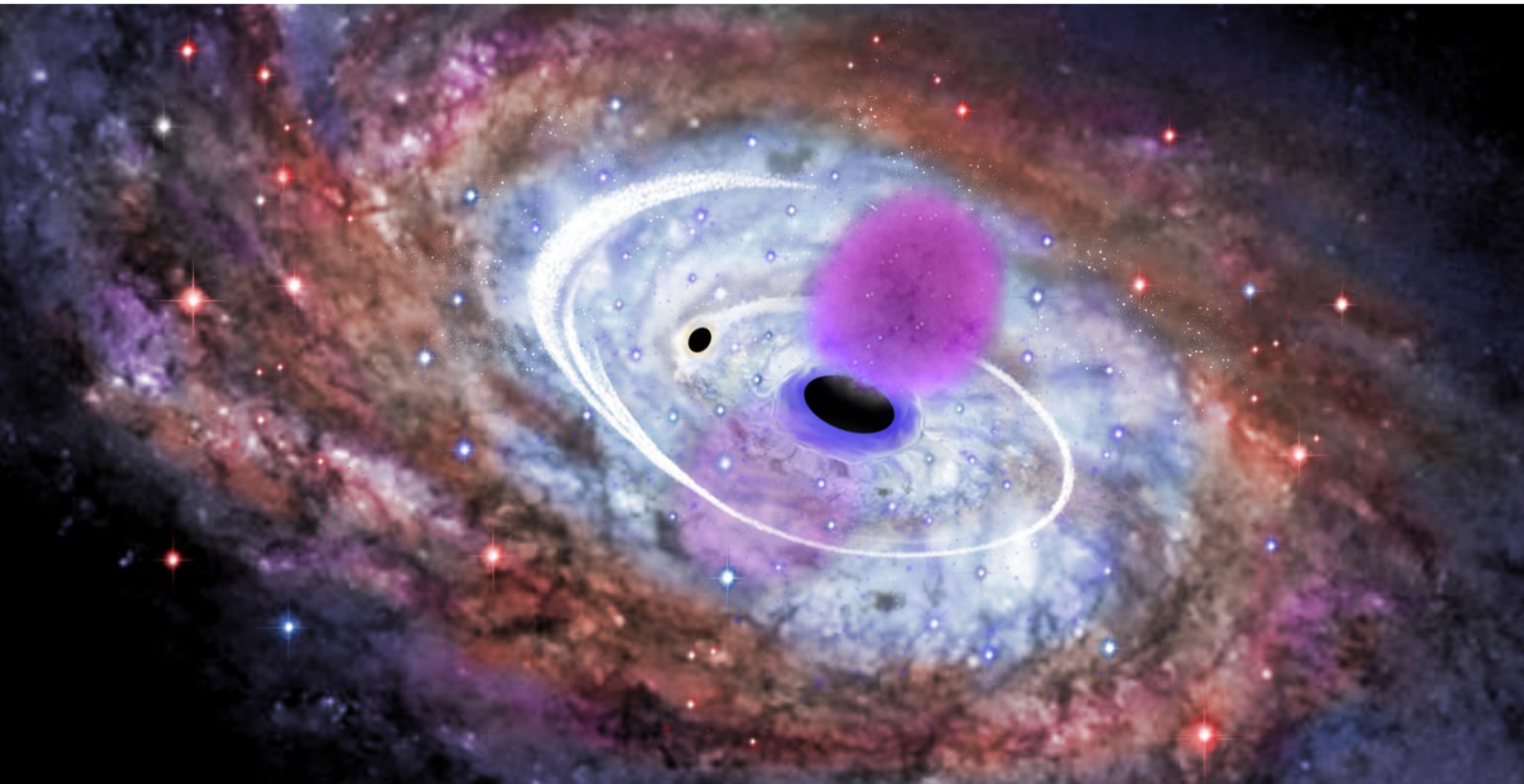




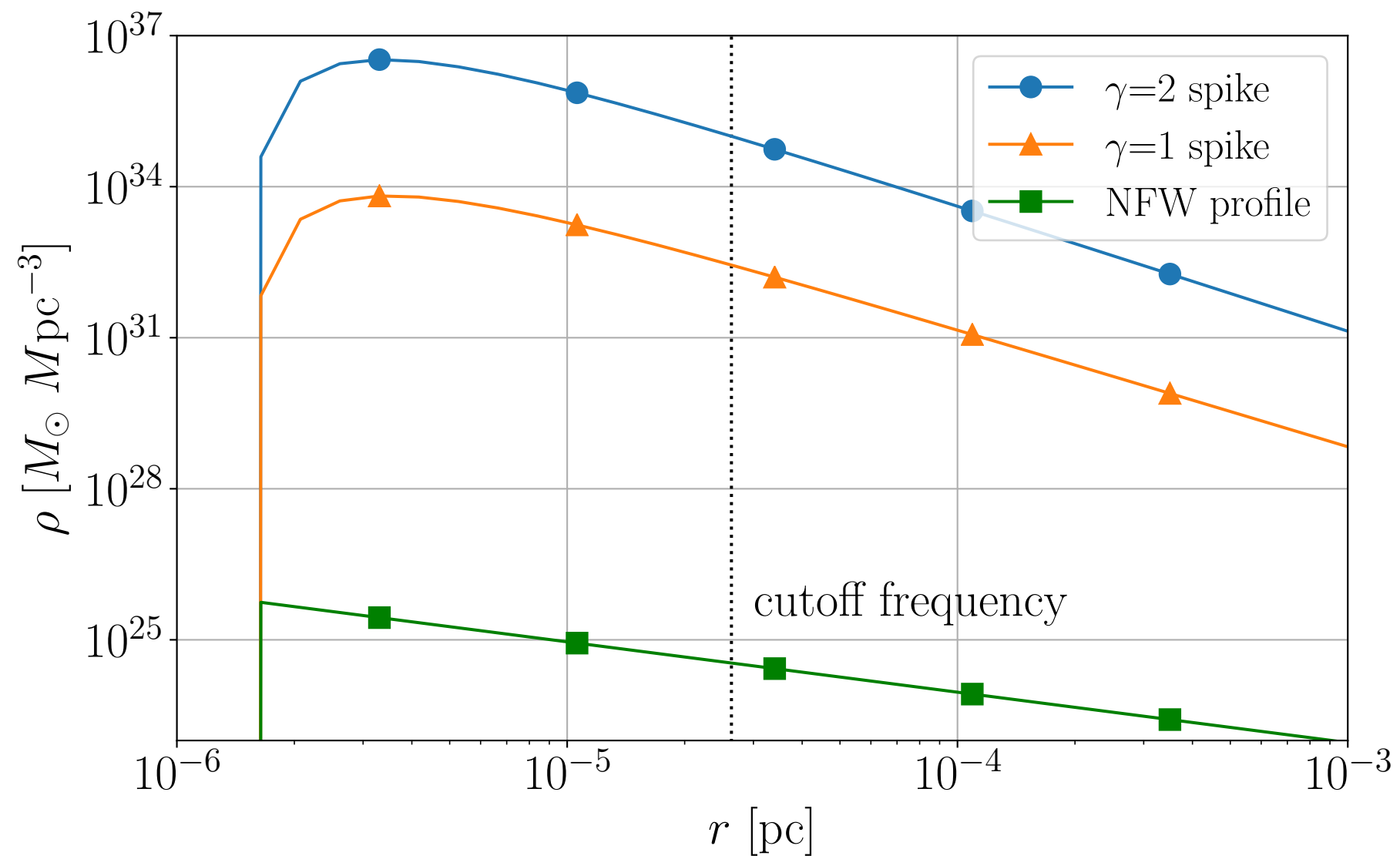
$$\mathcal{R}_{ij} \simeq 3.9 \cdot 10^6 \times \frac{\rho_m}{\rho_m^0} \left(\frac{t}{t_0} \right)^{-\frac{34}{37}} f^2 (f^2 + \sigma_{eq})^{-\frac{21}{74}} \\ \times \min \left(\frac{P(m_i)}{m_i}, \frac{P(m_j)}{m_j} \right) \left(\frac{P(m_i)}{m_i} + \frac{P(m_j)}{m_j} \right) \\ \times (m_i m_j)^{\frac{3}{37}} (m_i + m_j)^{\frac{36}{37}}$$



PBHs in the center of galaxies



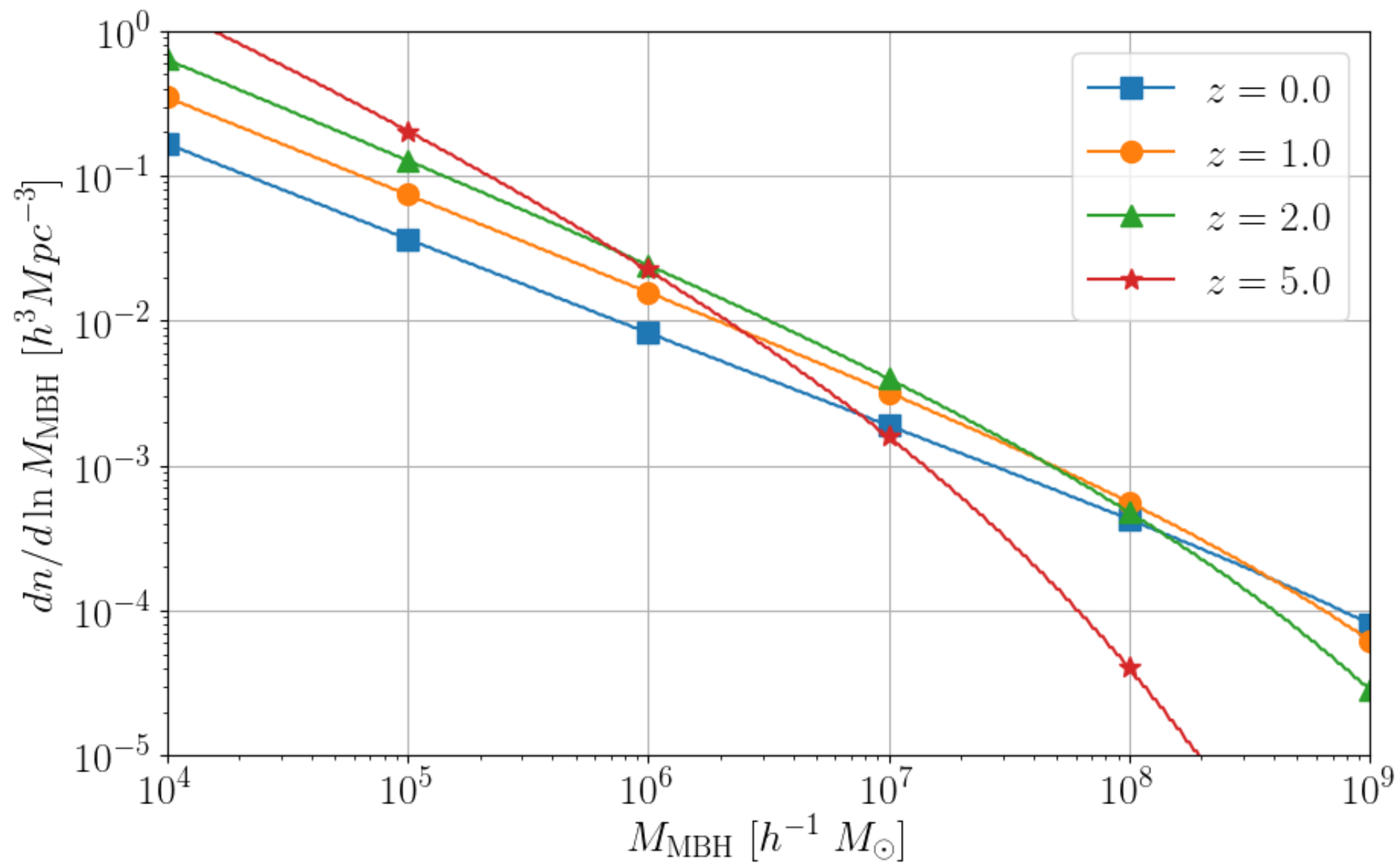
Matter distribution in the center of galaxies



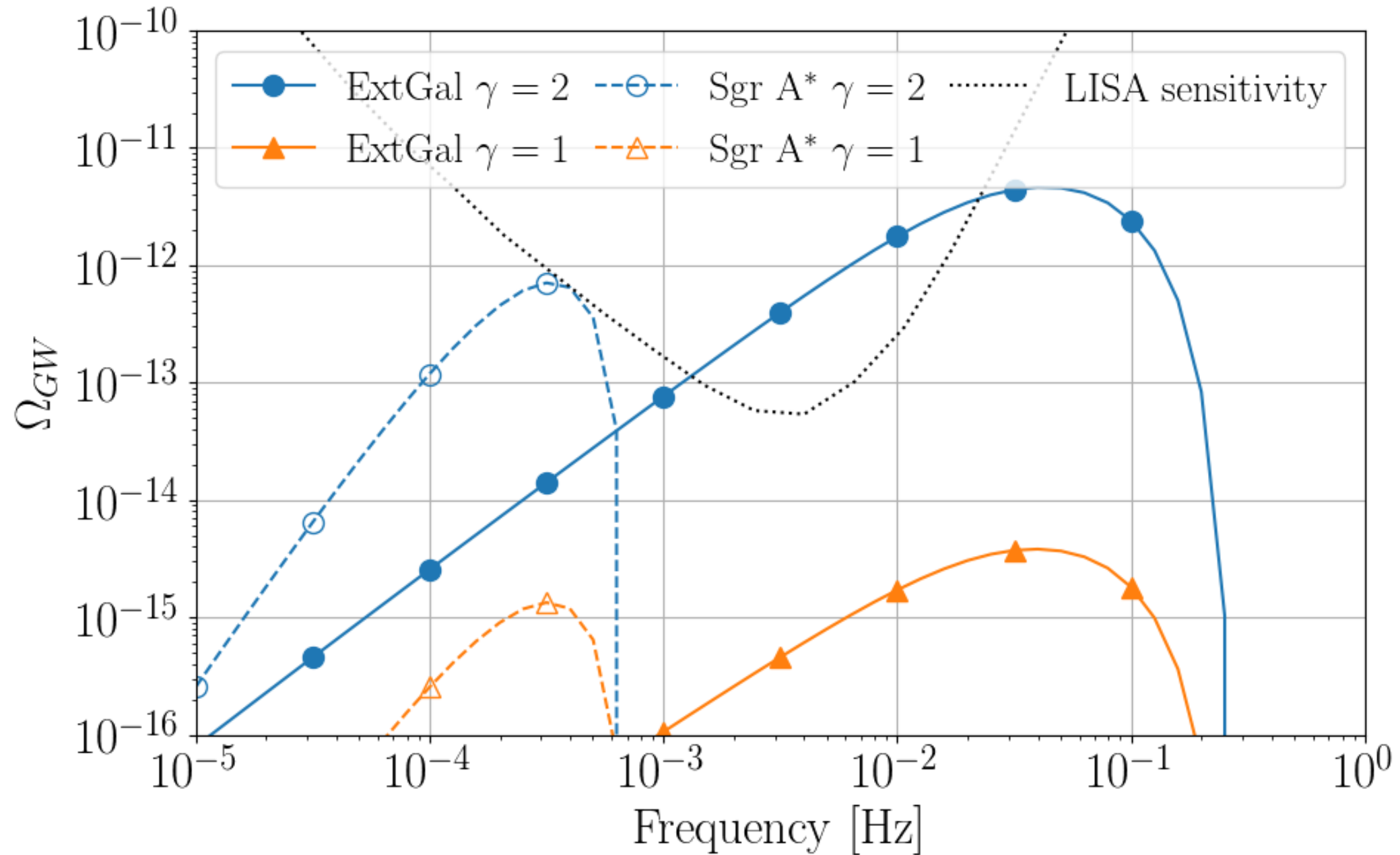
$$\rho_{sp} = \rho_R \left(1 - \frac{4R_s}{r}\right)^3 \left(\frac{R_{sp}}{r}\right)^{\gamma_{sp}}$$

$$\gamma_{sp} = (9 - 2\gamma)/(4 - \gamma)$$

Massive Black Hole Population

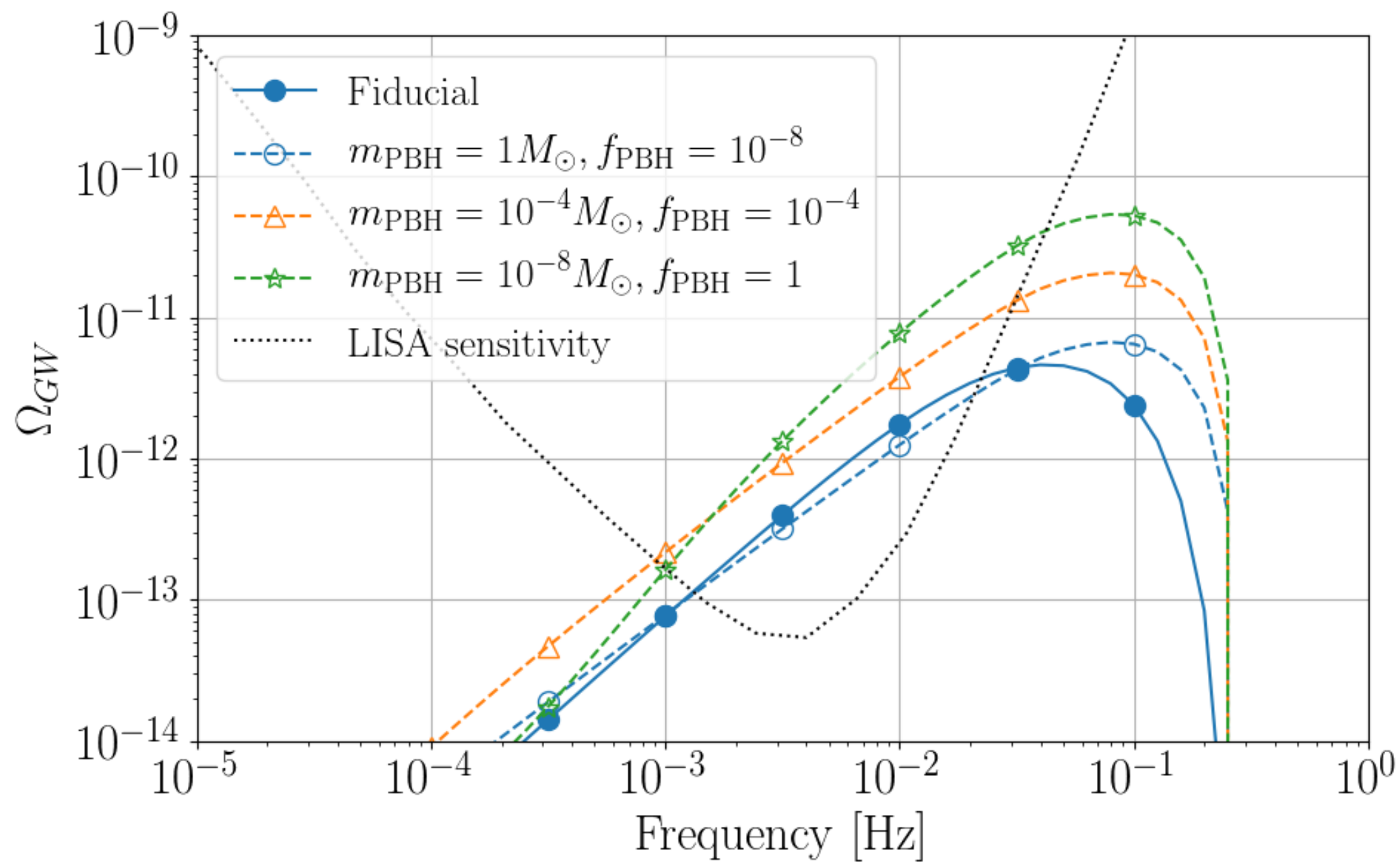


SGWB from PBHs in the galactic massive BHs

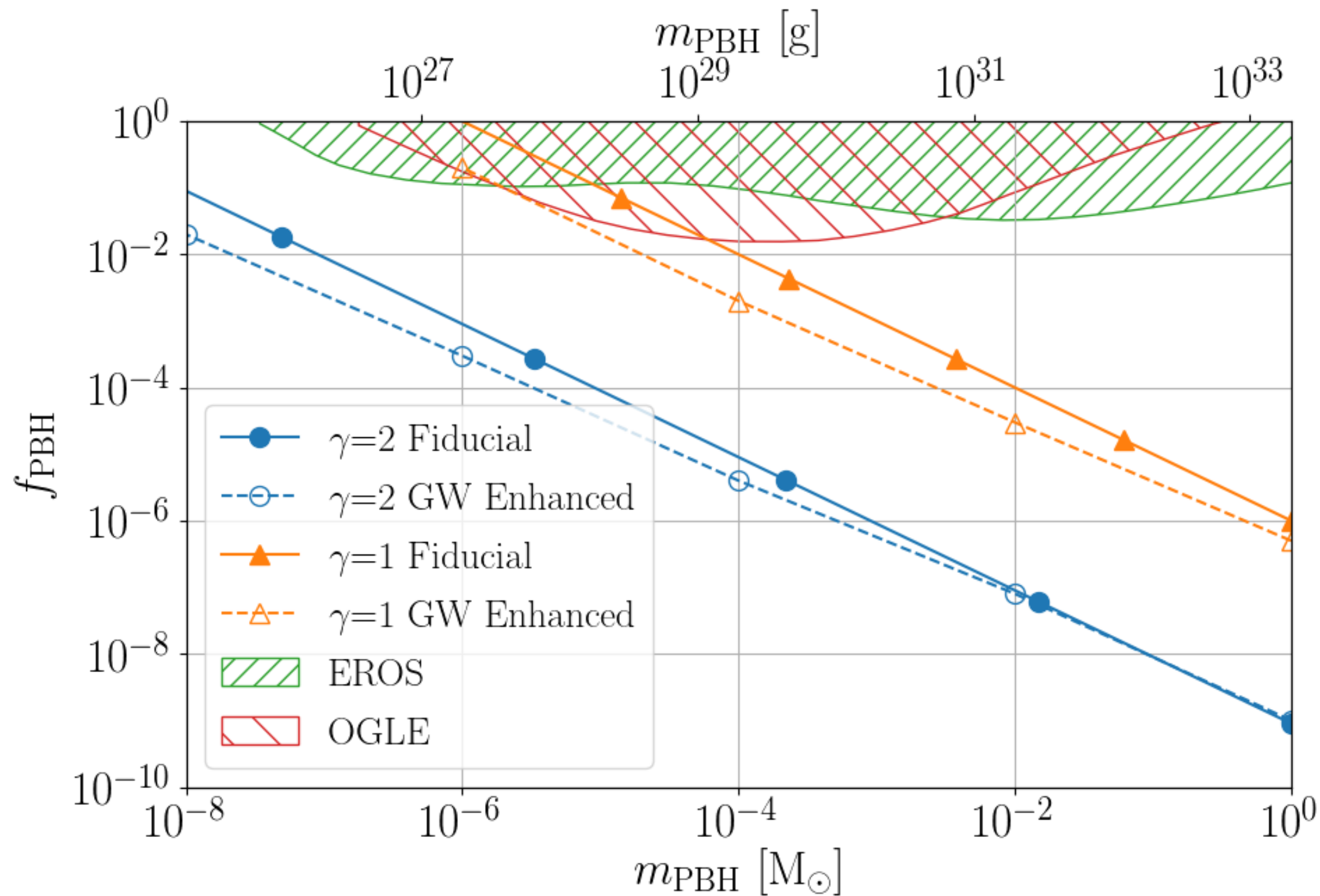


$$(m_{PBH} = 1M_{\odot}, f_{PBH} = 10^{-8})$$

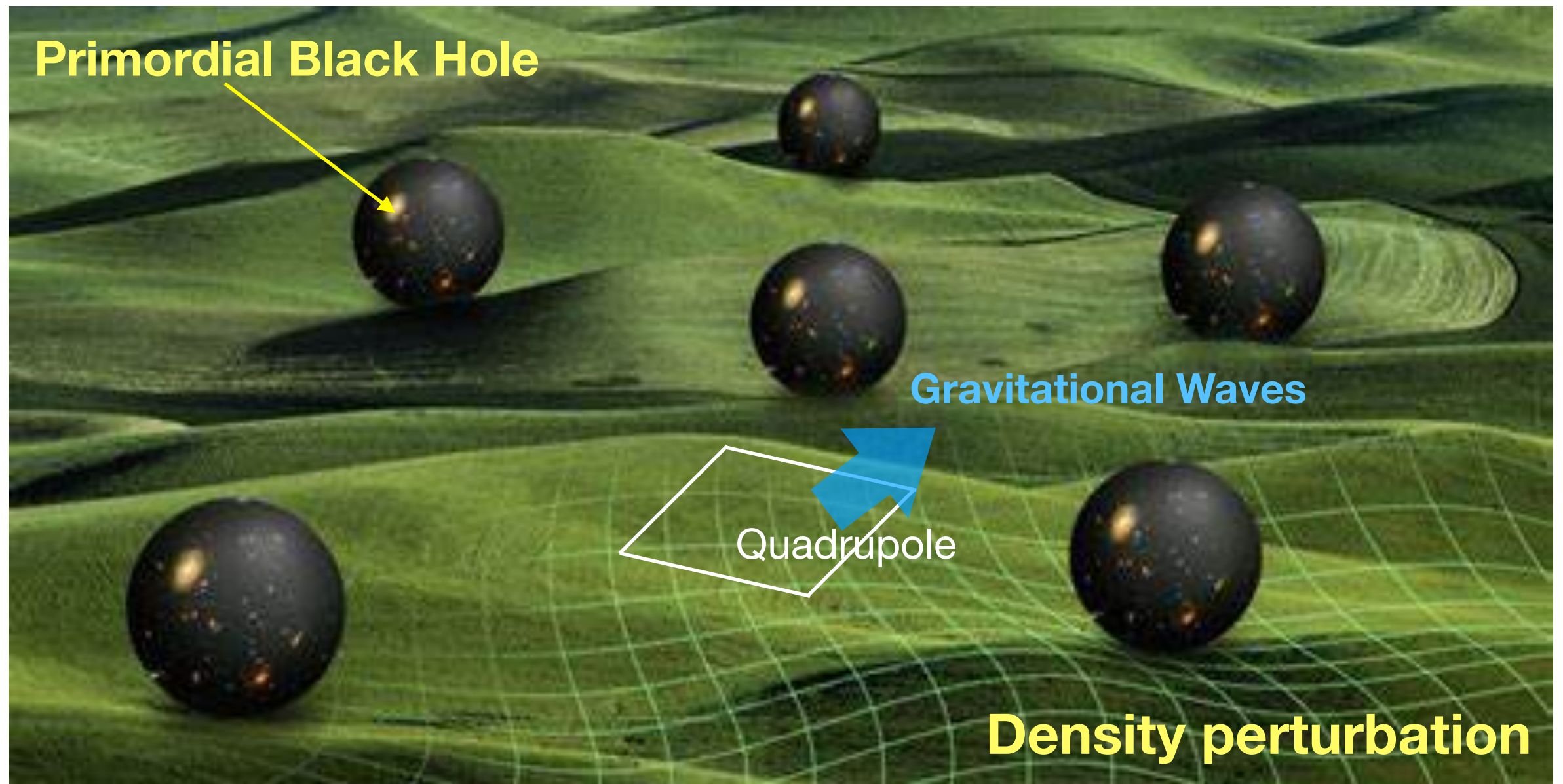
Enhancement due to GW dissipation



The projected constraints on PBH abundance in DM



Scalar induced GWs (SIGW)



$$ds^2 = a^2 \left\{ -(1 + 2\phi)d\eta^2 + \left[(1 - 2\phi)\delta_{ij} + \frac{h_{ij}}{2} \right] dx^i dx^j \right\}$$

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}^{\ell m} S_{\ell m}$$

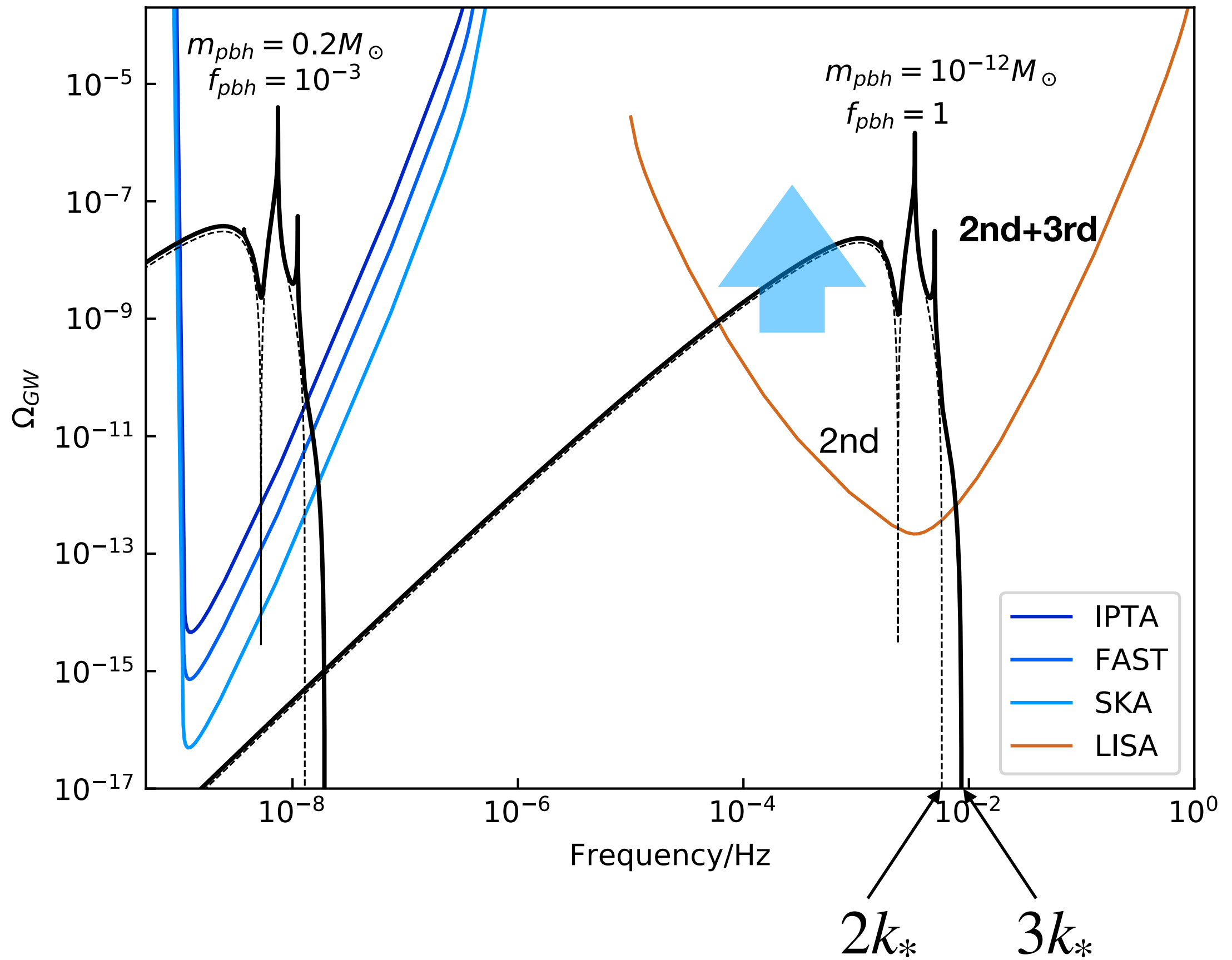
$$S_{ij}^{(2)} = 4\phi\partial_i\partial_j\phi + 2\partial_i\phi\partial_j\phi - \frac{1}{\mathcal{H}^2}\partial_i(\mathcal{H}\phi + \phi')\partial_j(\mathcal{H}\phi + \phi')$$

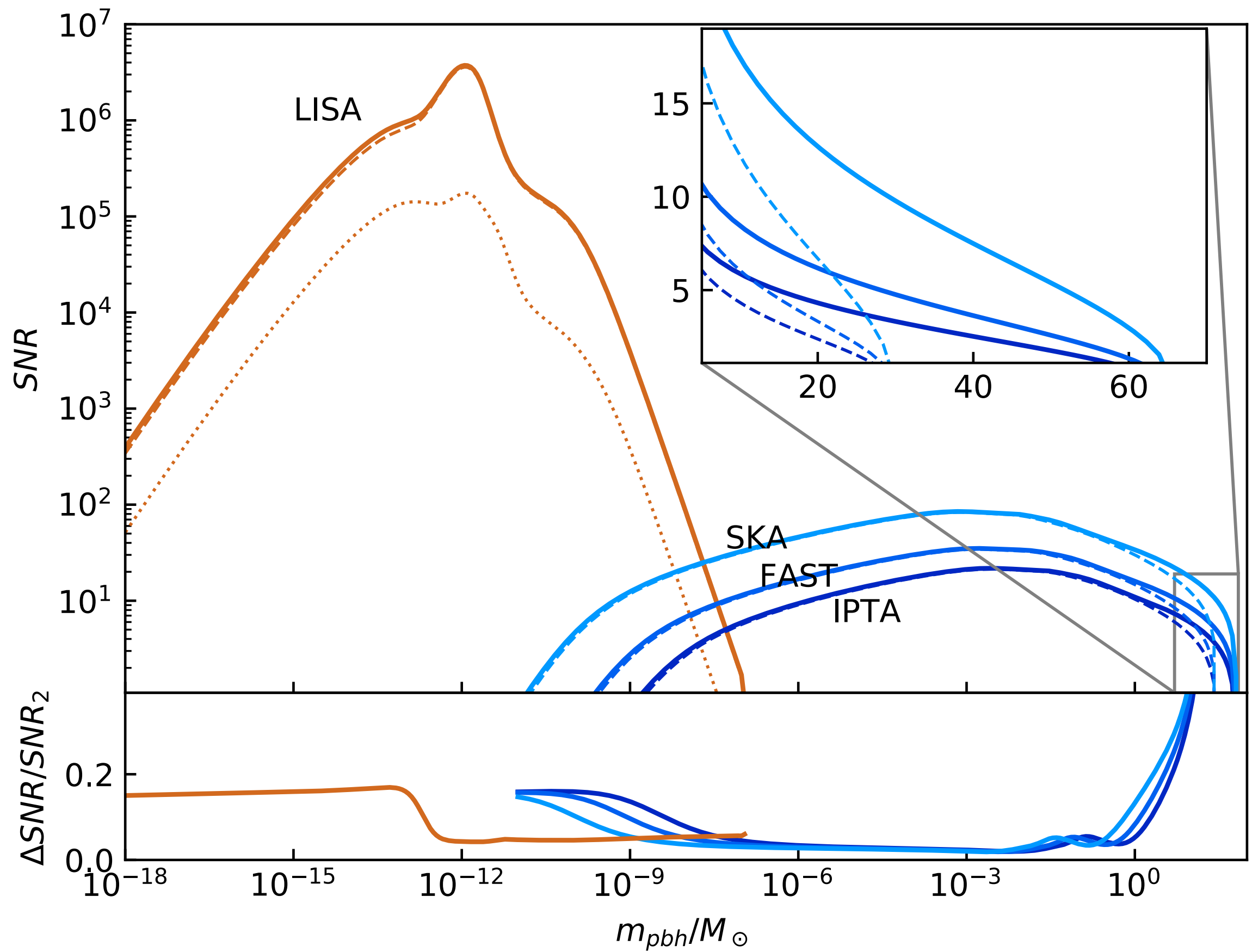
$$S = S^{(2)}(\phi^2) + S^{(3)}(\phi^3) + S^{(4)}(\phi^4)$$

$$\Omega_{GW} = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \ln f} \sim \langle S^{(2)} S^{(2)} \rangle + \langle S^{(3)} S^{(3)} \rangle + \langle S^{(2)} S^{(4)} \rangle$$

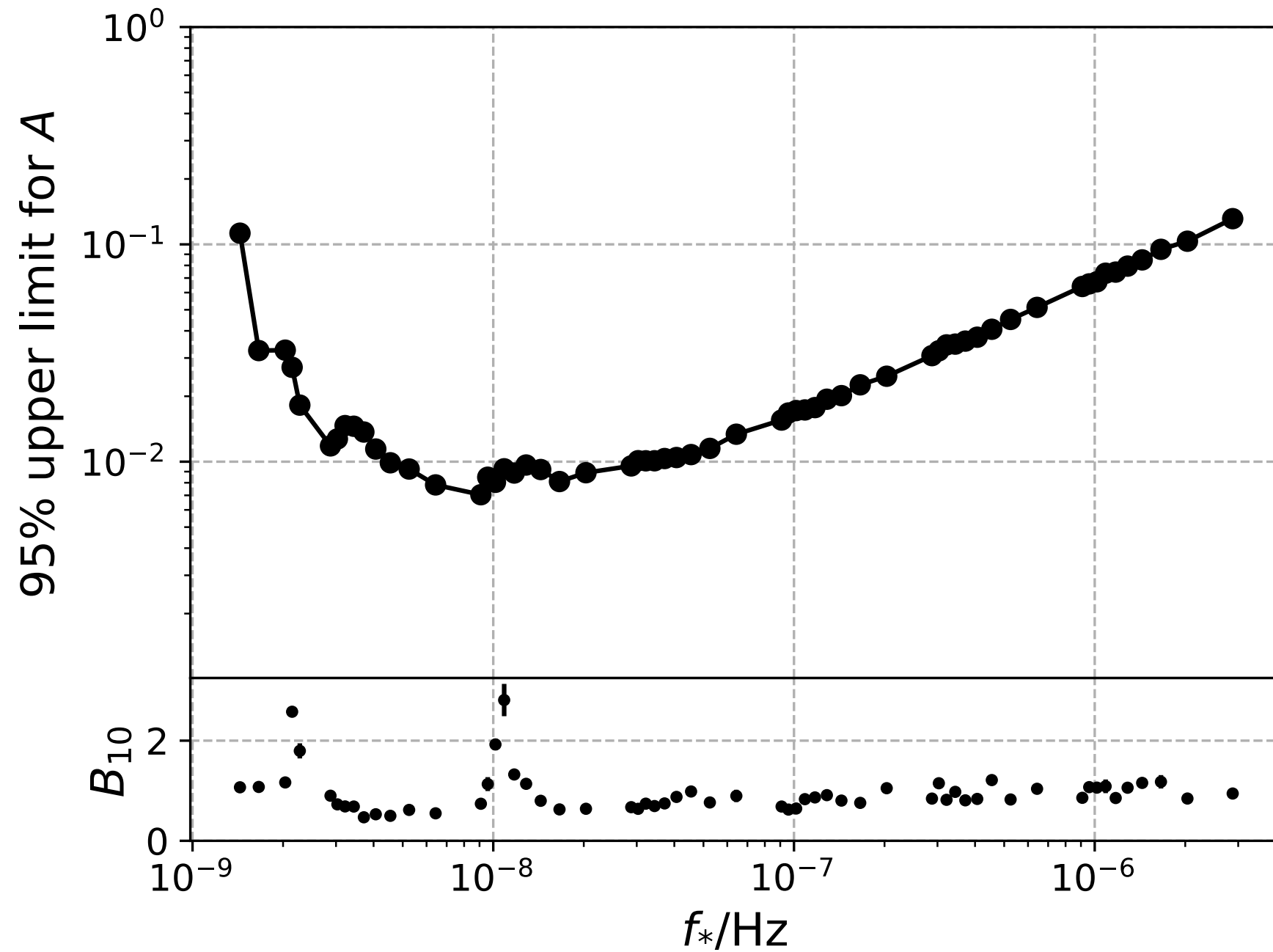
$$S_{ij}^{(3)} = \frac{1}{\mathcal{H}} (12\mathcal{H}\phi - \phi') \partial_i \phi \partial_j \phi - \frac{1}{\mathcal{H}^3} (4\mathcal{H}\phi - \phi') \partial_i \phi' \partial_j \phi' \\ + \frac{1}{3\mathcal{H}^4} (2\partial^2 \phi - 9\mathcal{H}\phi') \partial_i (\mathcal{H}\phi + \phi') \partial_j (\mathcal{H}\phi + \phi'),$$

$$S_{ij}^{(4)} = 16\phi^3 \partial_i \partial_j \phi + \frac{1}{3\mathcal{H}^3} \left[2\phi' \partial^2 \phi - 9\mathcal{H}\phi'^2 - 8\mathcal{H}\phi \partial^2 \phi \right. \\ \left. + 18\mathcal{H}^2 \phi \phi' + 96\mathcal{H}^3 \phi^2 \right] \partial_i \phi \partial_j \phi \\ + \frac{2}{3\mathcal{H}^5} \left[-\phi' \partial^2 \phi + 3\mathcal{H}\phi'^2 + 4\mathcal{H}\phi \partial^2 \phi \right. \\ \left. + 3\mathcal{H}^2 \phi \phi' - 12\mathcal{H}^3 \phi^2 \right] \partial_i \phi' \partial_j \phi' \\ + \frac{1}{36\mathcal{H}^6} \left[-16(\partial^2 \phi)^2 - 3\partial_k \phi' \partial^k \phi' + 120\mathcal{H}\phi' \partial^2 \phi \right. \\ - 6\mathcal{H}\partial_k \phi \partial^k \phi' + 144\mathcal{H}^2 \phi \partial^2 \phi - 180\mathcal{H}^2 \phi'^2 \\ \left. + 33\mathcal{H}^2 \partial_k \phi \partial^k \phi - 504\mathcal{H}^3 \phi \phi' - 144\mathcal{H}^4 \phi^2 \right] \\ \times \partial_i (\mathcal{H}\phi + \phi') \partial_j (\mathcal{H}\phi + \phi').$$

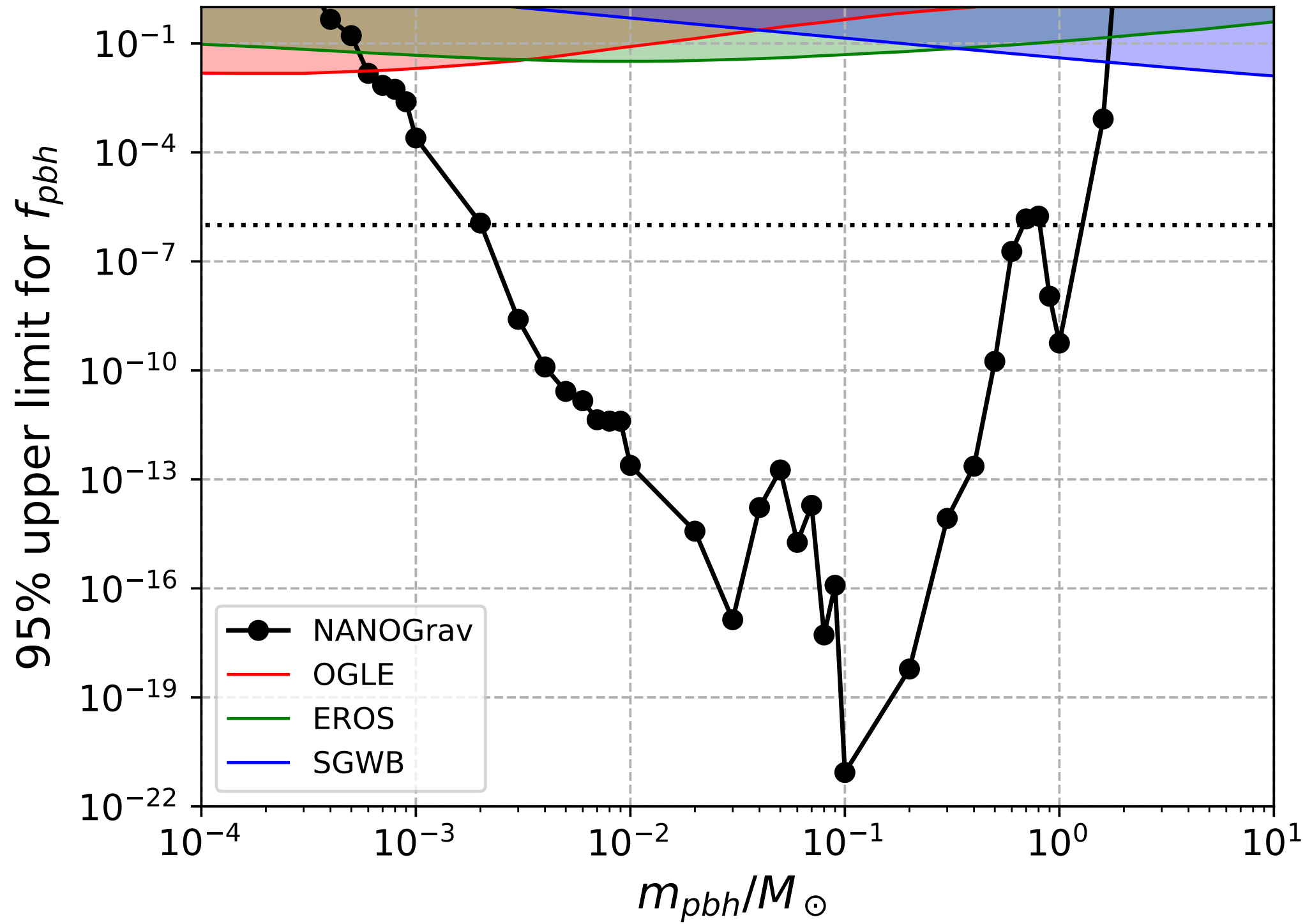




Current constraints from NANOGrav

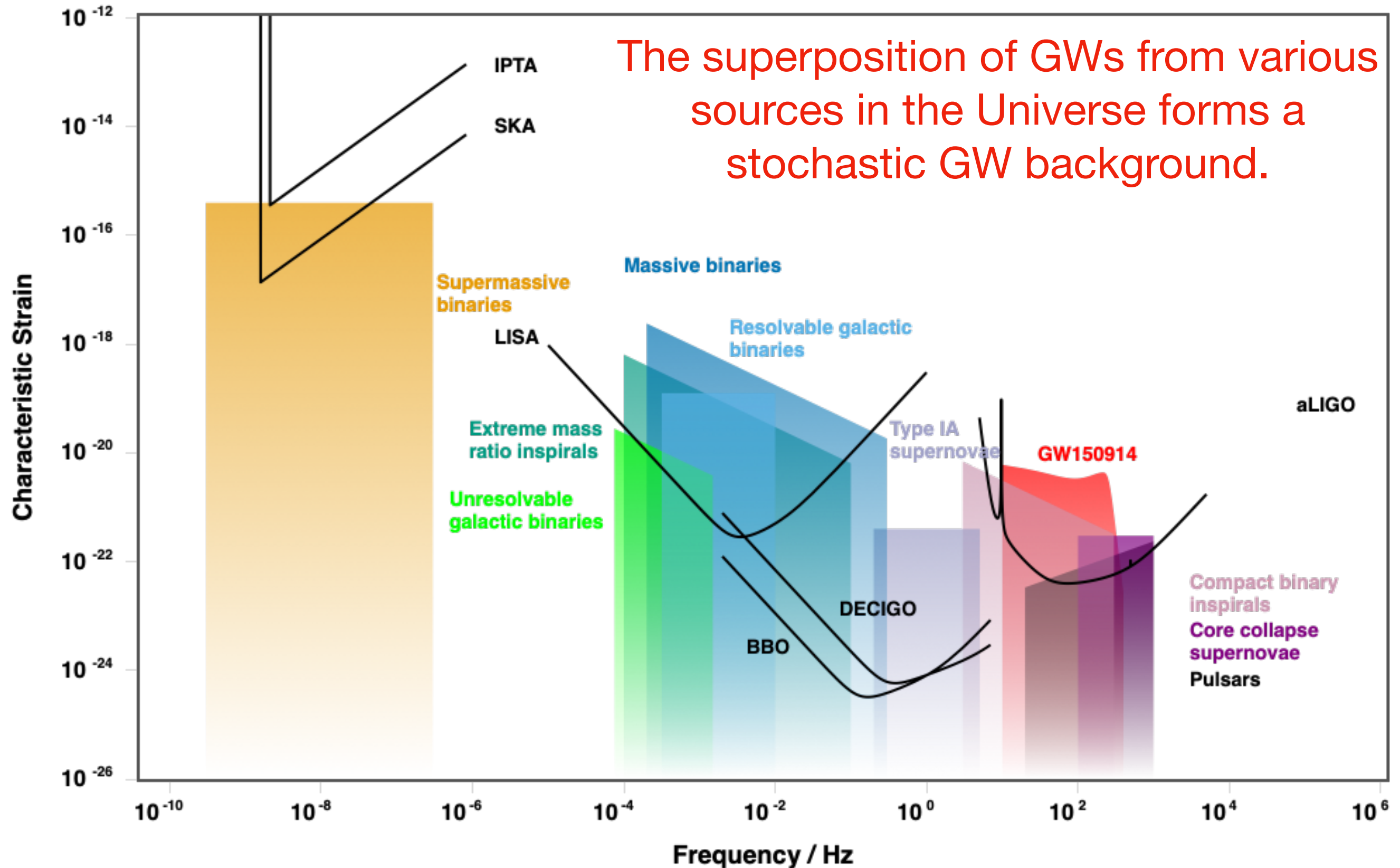


$$P_\phi(k) = Af_*\delta(f - f_*) \quad \frac{m_{\text{pbh}}}{M_\odot} \simeq 2.3 \times 10^{18} \left(\frac{H_0}{f_*} \right)^2$$



$$f_{pbh} \simeq 1.9 \times 10^7 \left(\zeta_c^2/A - 1 \right) e^{-\frac{\zeta_c^2}{2A}} \left(\frac{m_{pbh}}{M_{\odot}} \right)^{-1/2}$$

The superposition of GWs from various sources in the Universe forms a stochastic GW background.



$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d \log \rho_{\text{GW}}}{d \log f} = \frac{\pi^2}{3H_0^2} f^3 S_h(f) \propto f^{n_{\text{GW}}} \quad \text{slope}$$

GW spectral energy density

spectral density

$$n_{\text{GW}} = 2/3$$

Compact Binary Coalescences

$$n_{\text{GW}} = n_t + \alpha_t \ln(f/f_{\text{CMB}})/2$$

Primordial Gravitational Waves

$$n_{\text{GW}} = 0$$

Scale-invariant Energy

.....

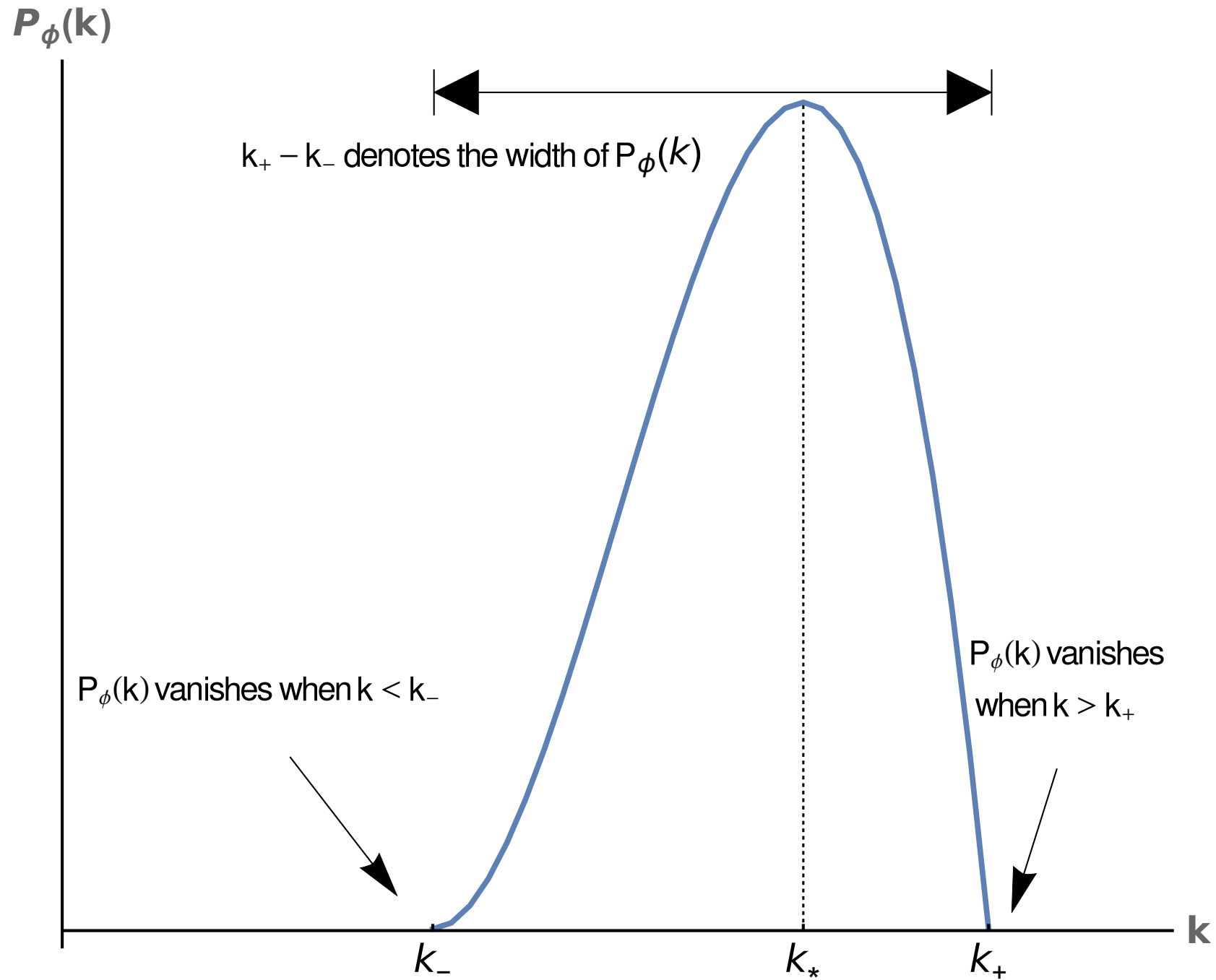
Power spectrum of scalar curvature perturbation is enhanced at small scales.

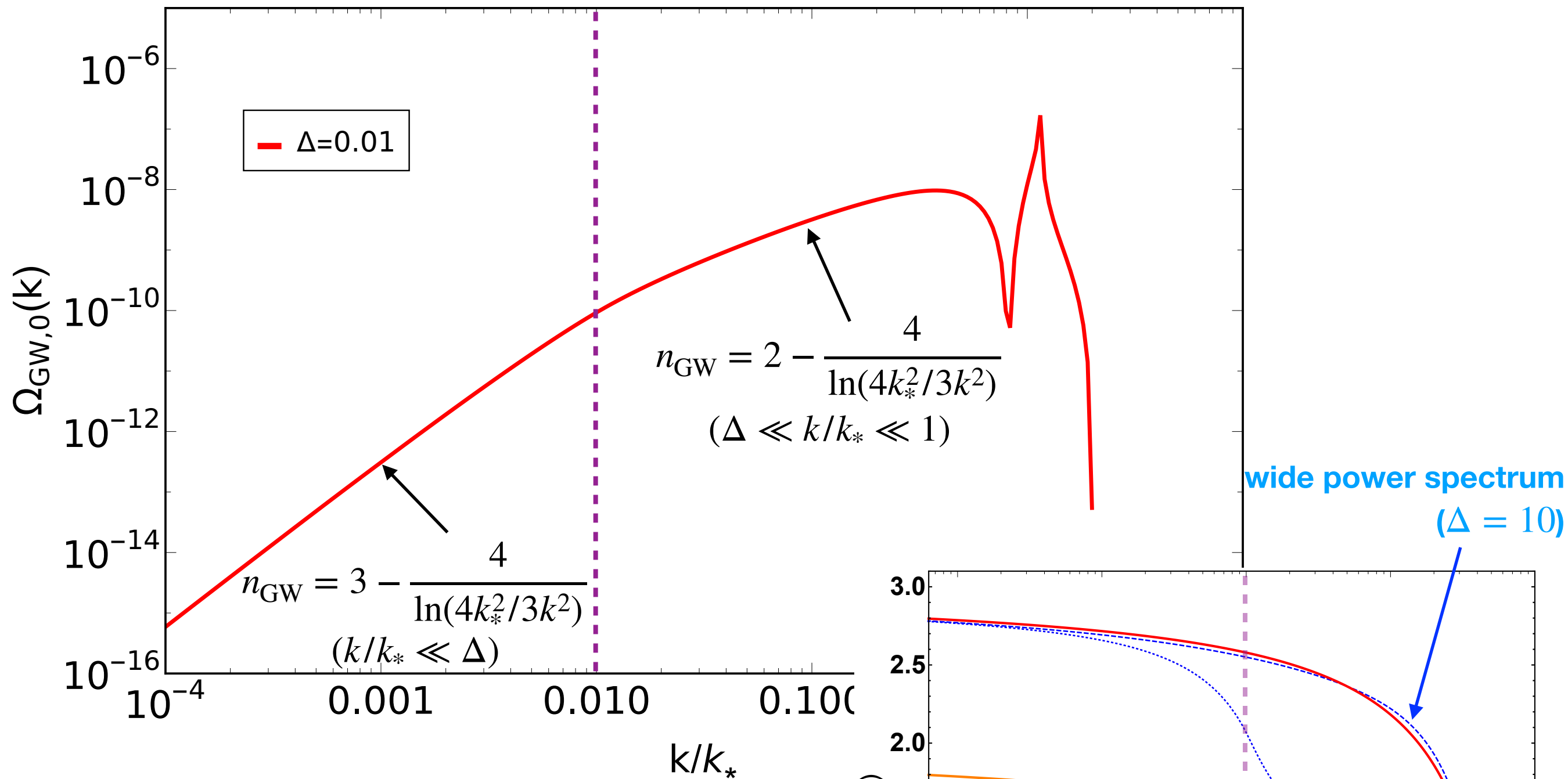
Dimensionless width parameter

$$\Delta = \frac{k_+ - k_-}{k_*}$$

For a narrow power spectrum

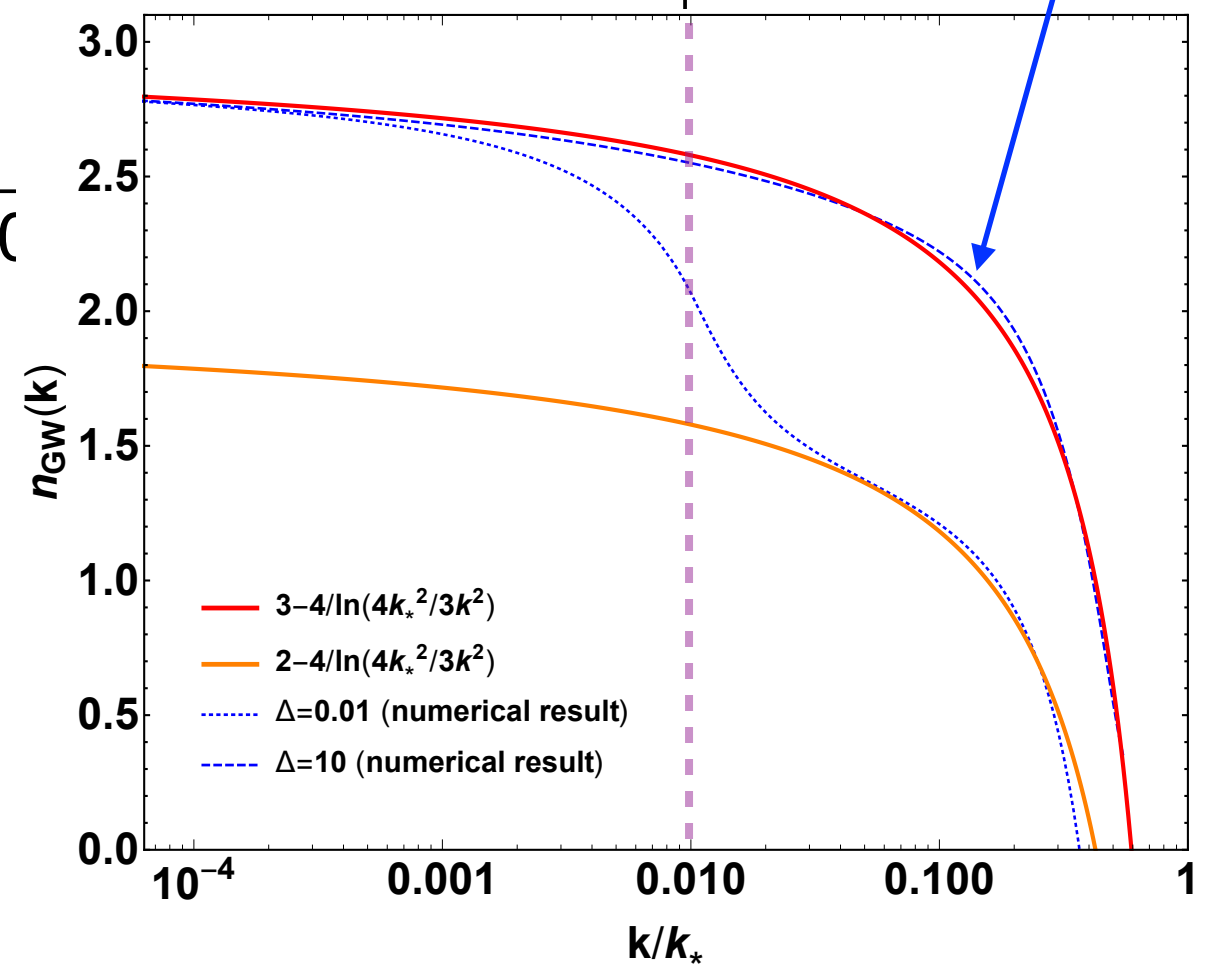
$$\Delta \ll 1$$





In the infrared limit,

$$\Omega_{\text{GW}}(k \rightarrow 0) \propto k^3$$



$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d \log \rho_{\text{GW}}}{d \log f} = \frac{\pi^2}{3H_0^2} f^3 S_h(f) \propto f^{n_{\text{GW}}} \quad \text{slope}$$

GW spectral energy density

spectral density

$$n_{\text{GW}} = 2/3$$

Compact Binary Coalescences

$$n_{\text{GW}} = n_t + \alpha_t \ln(f/f_{\text{CMB}})/2$$

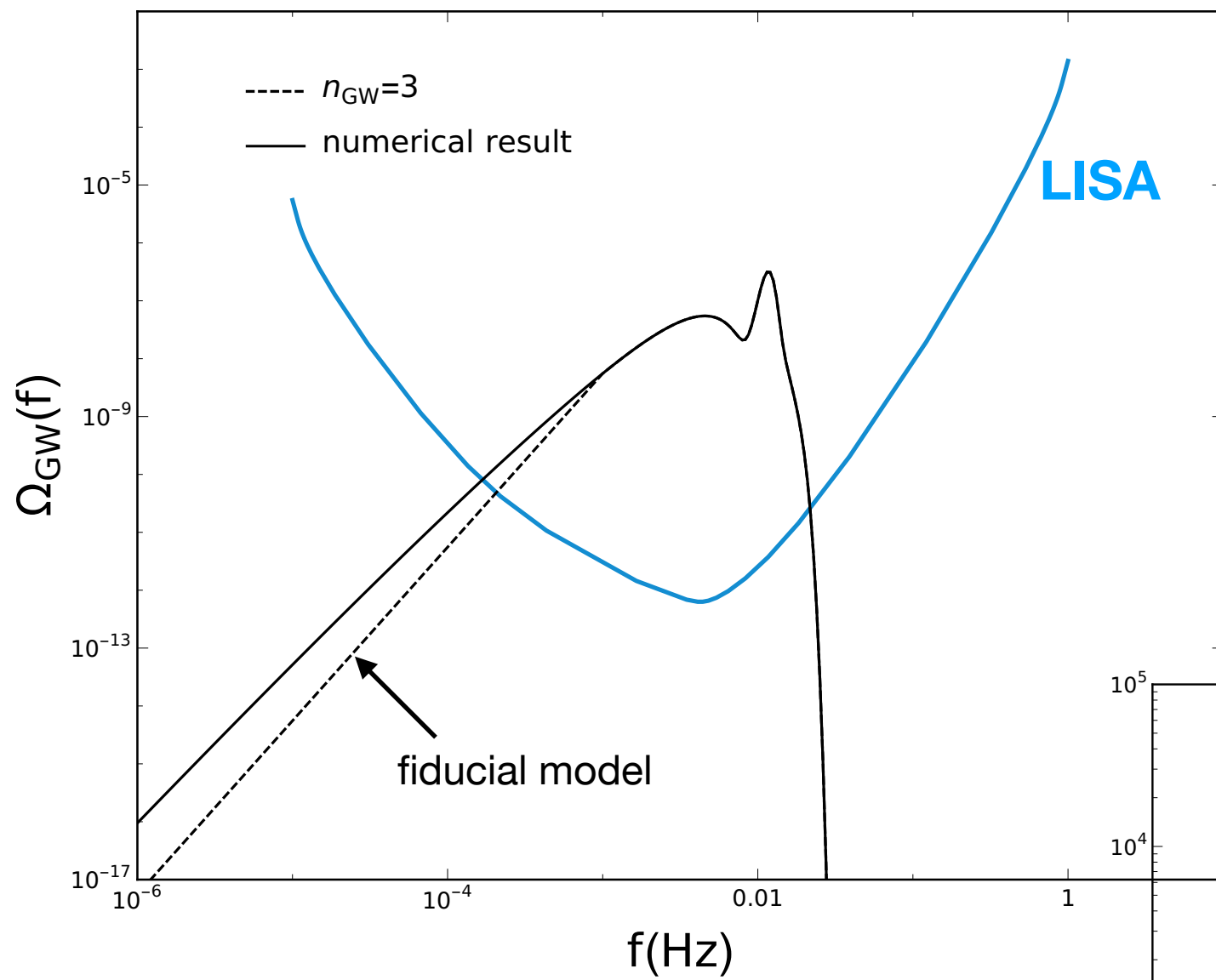
Primordial Gravitational Waves

$$n_{\text{GW}} = 0$$

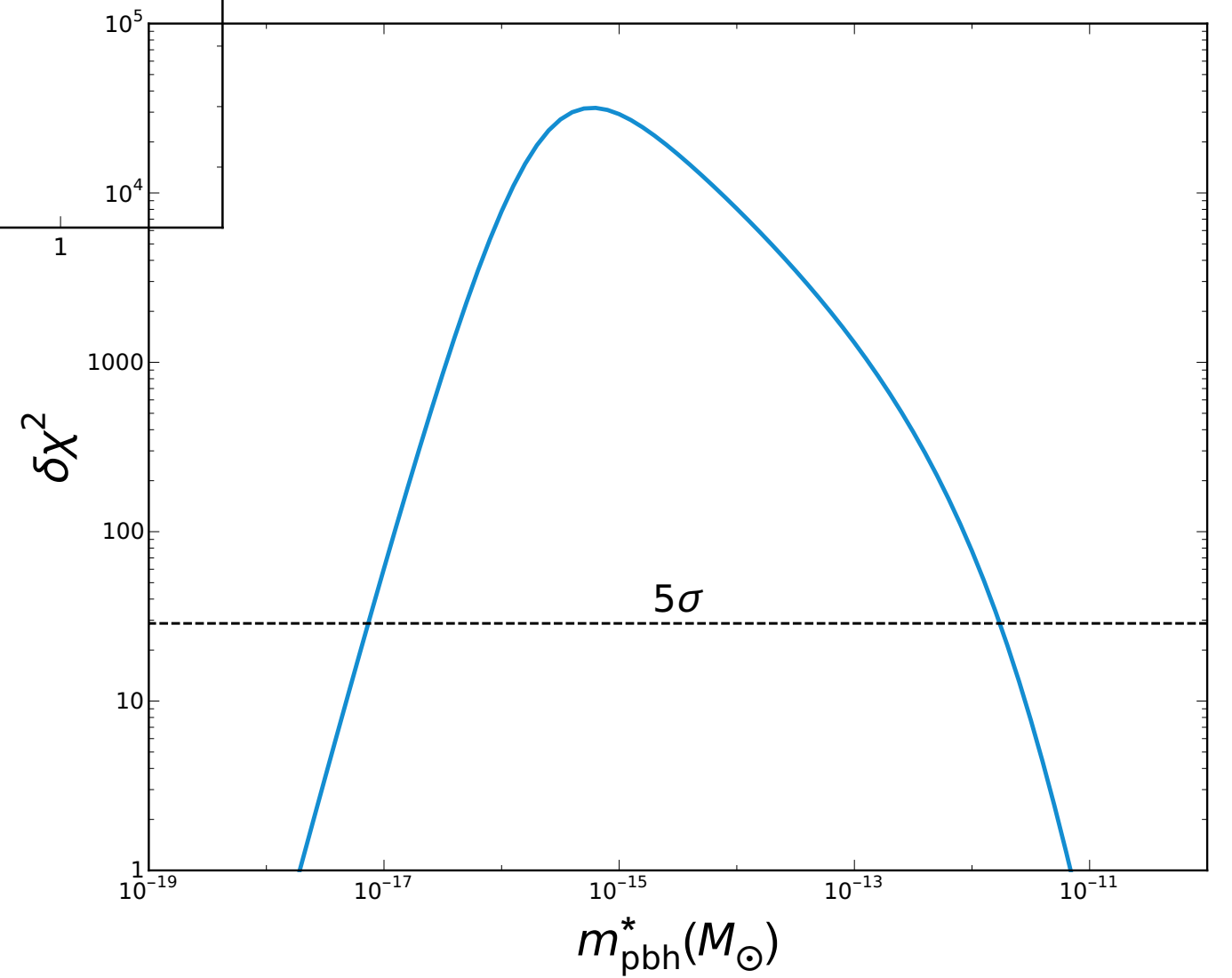
Scale-invariant Energy

$$n_{\text{GW}} = 3 \text{ or } 2 - 2/\ln(f_c/f)$$

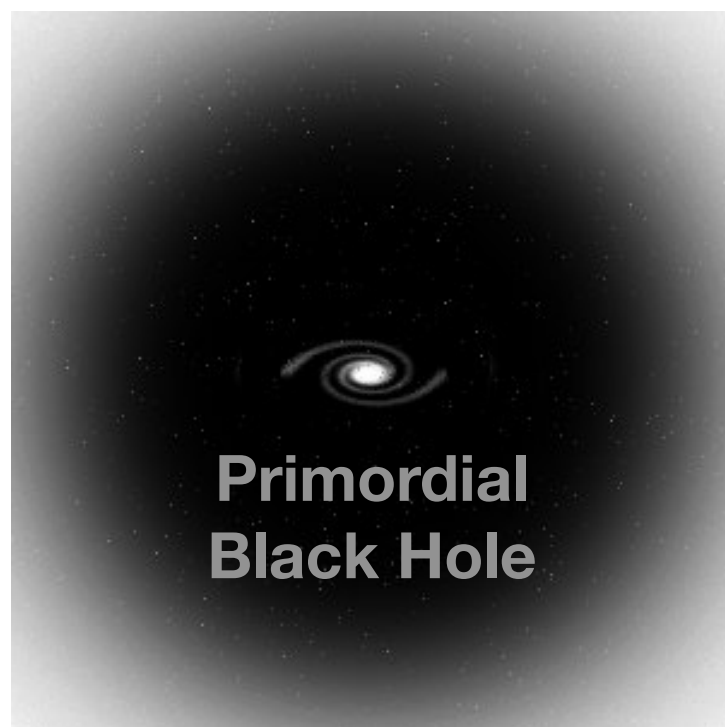
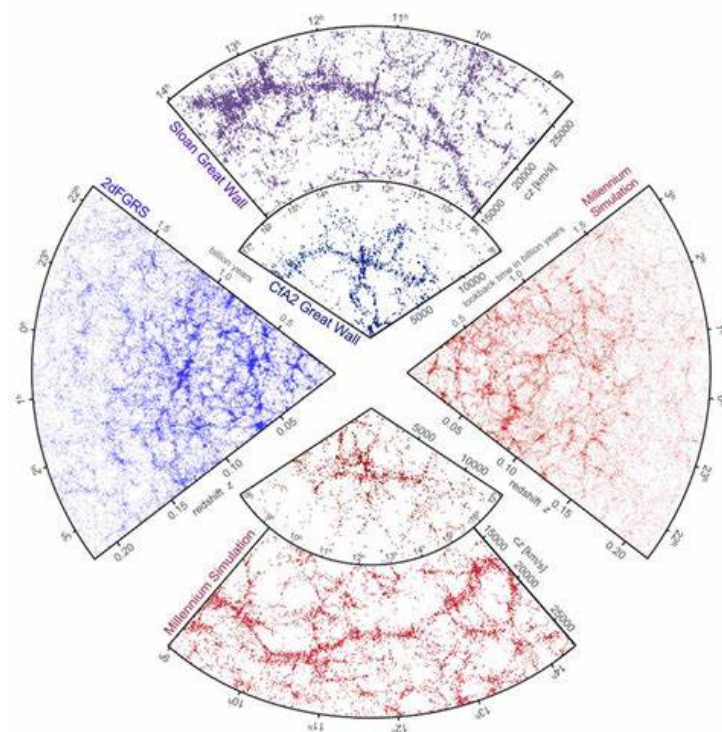
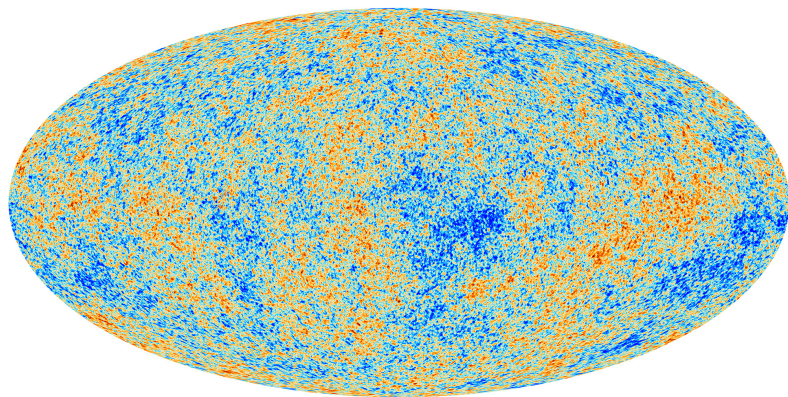
Scalar induced GWs
inevitably accompanying the
formation of PBHs



$$\delta\chi^2 \simeq T \int_0^\infty df \left(\frac{\Omega_{\text{GW}} - \Omega_{\text{GW}}^{\text{fid}}}{\Omega_{\text{GW}}^{\text{fid}} + \Omega_n} \right)^2$$



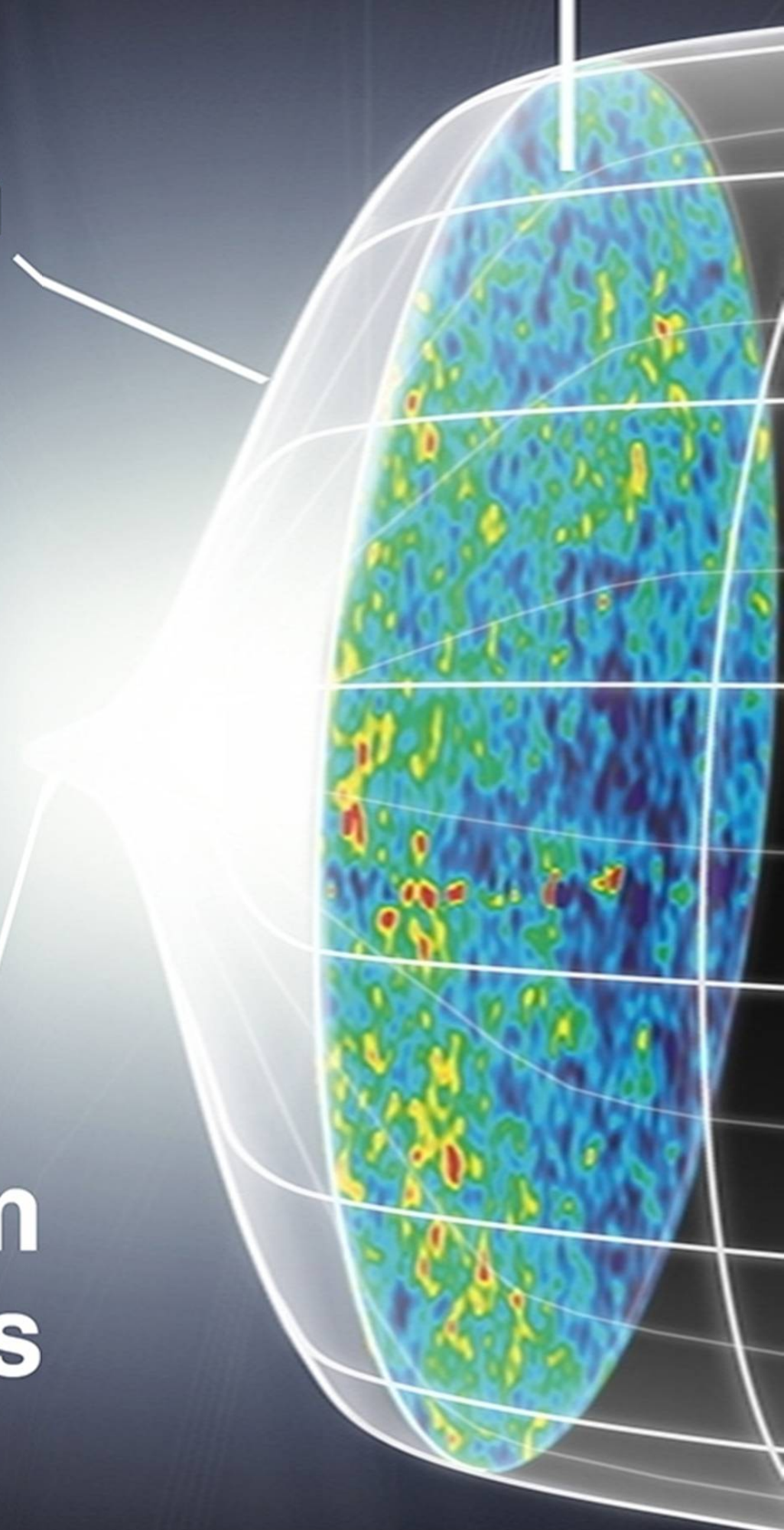
The postulation of
Primordial Black Hole Dark Matter
is testable
for
the next generation GW detectors.



Primordial
Black Hole

Inflation

Quantum Fluctuations



Thank You!