

# Statistical properties of repeating FRBs, in comparison with earthquakes and solar flares

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- based on the recent paper in MNRAS, 526, 2795 (2023)

## Fast radio bursts trigger aftershocks resembling earthquakes, but not solar flares

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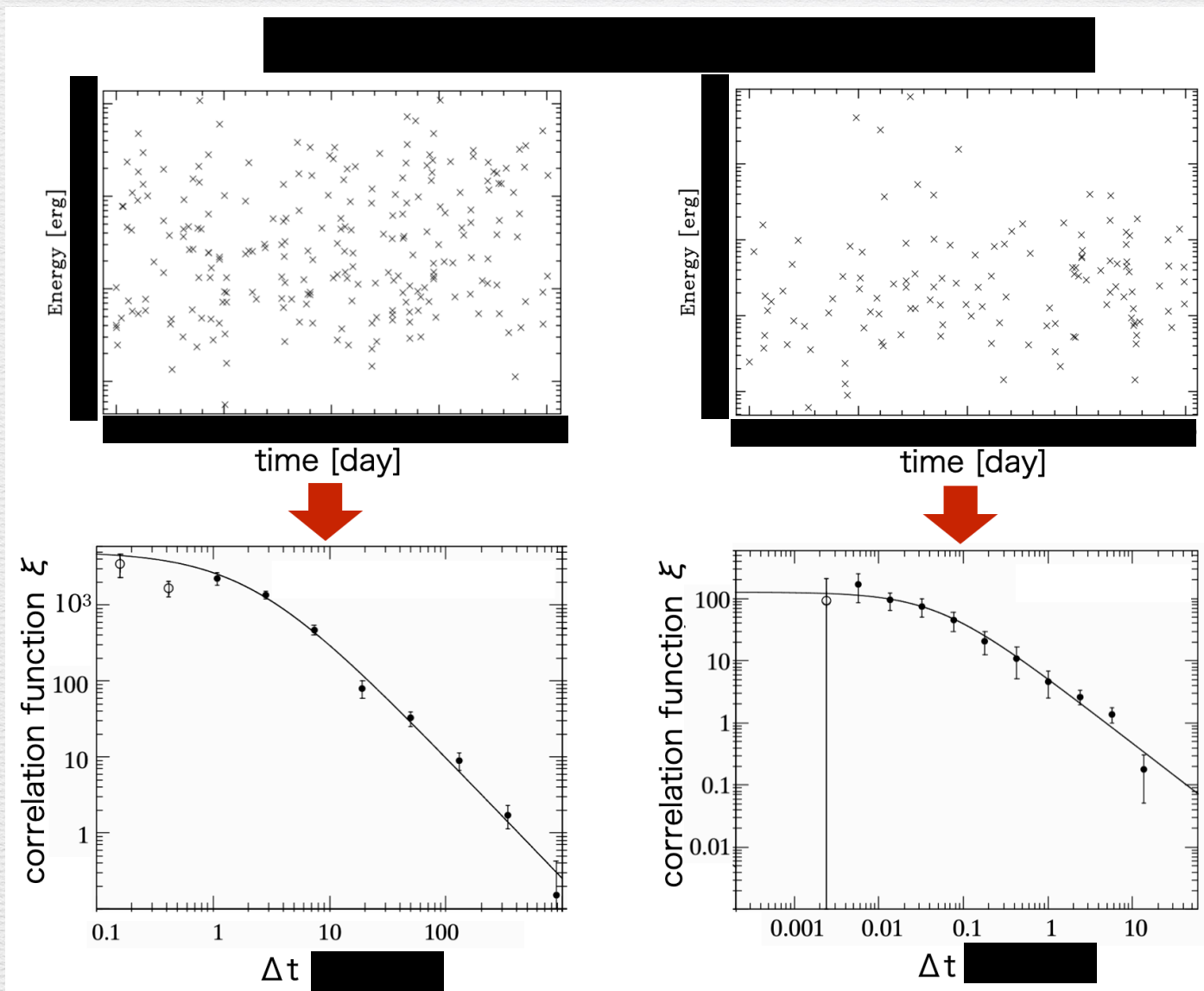
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### ABSTRACT

The production mechanism of repeating fast radio bursts (FRBs) is still a mystery, and correlations between burst occurrence times and energies may provide important clues to elucidate it. While time correlation studies of FRBs have been mainly performed using wait time distributions, here we report the results of a correlation function analysis of repeating FRBs in the 2D space of time and energy. We analyse nearly 7,000 bursts reported in the literature for the three most active sources of FRB 20121102A, 20201124A, and 20220912A, and find the following characteristics that are universal in the three sources. A clear power-law signal of the correlation function is seen, extending to the typical burst duration ( $\sim 10$  msec) towards shorter time intervals ( $\Delta t$ ). The correlation function indicates that every single burst has about a 10–60 per cent chance of producing an aftershock at a rate decaying by a power law as  $\propto (\Delta t)^{-p}$  with  $p = 1.5\text{--}2.5$ , like the Omori–Utsu law of earthquakes. The correlated aftershock rate is stable regardless of source activity changes, and there is no correlation between emitted energy and  $\Delta t$ . We demonstrate that all these properties are quantitatively common to earthquakes, but different from solar flares in many aspects, by applying the same analysis method for the data on these phenomena. These results suggest that repeater FRBs are a phenomenon in which energy stored in rigid neutron star crusts is released by seismic activity. This may provide a new opportunity for future studies to explore the physical properties of the neutron star crust.

**Key words:** Sun: flares – stars: neutron – radio continuum: transients – fast radio bursts.

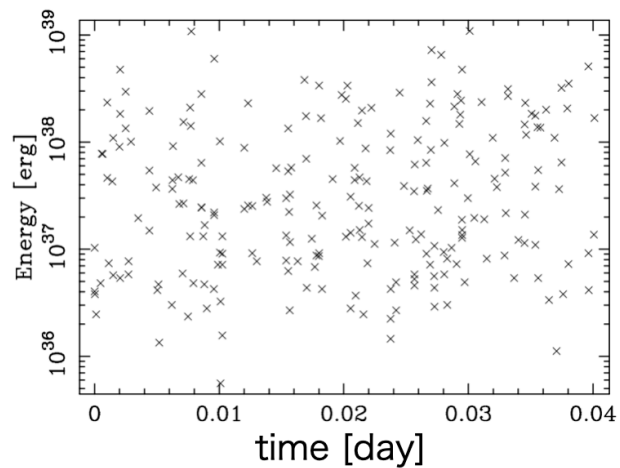
# FRB & earthquake... which is which?



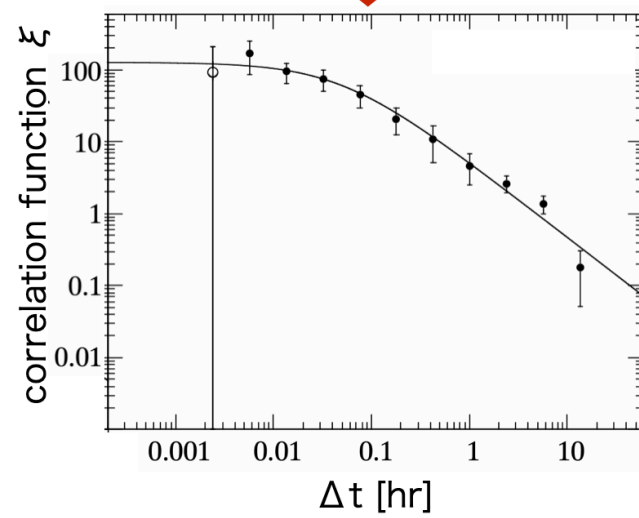
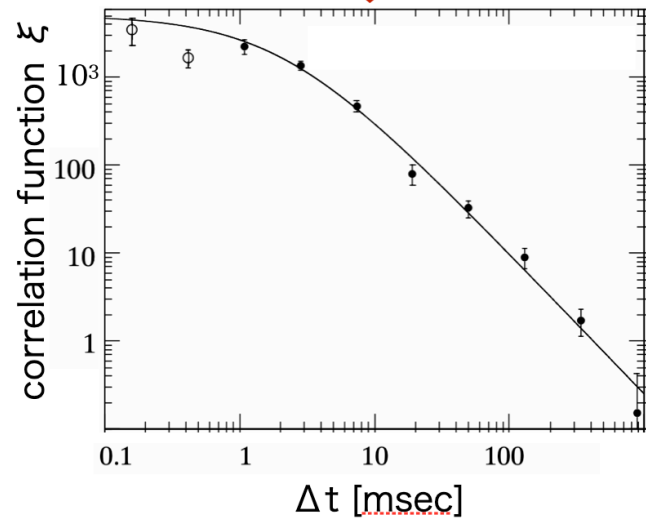
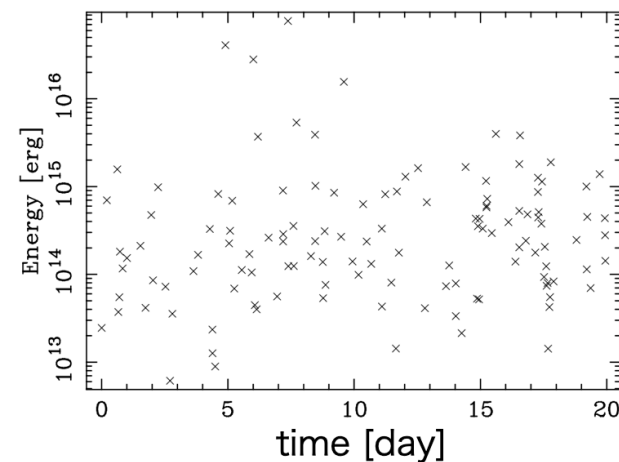


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FRB

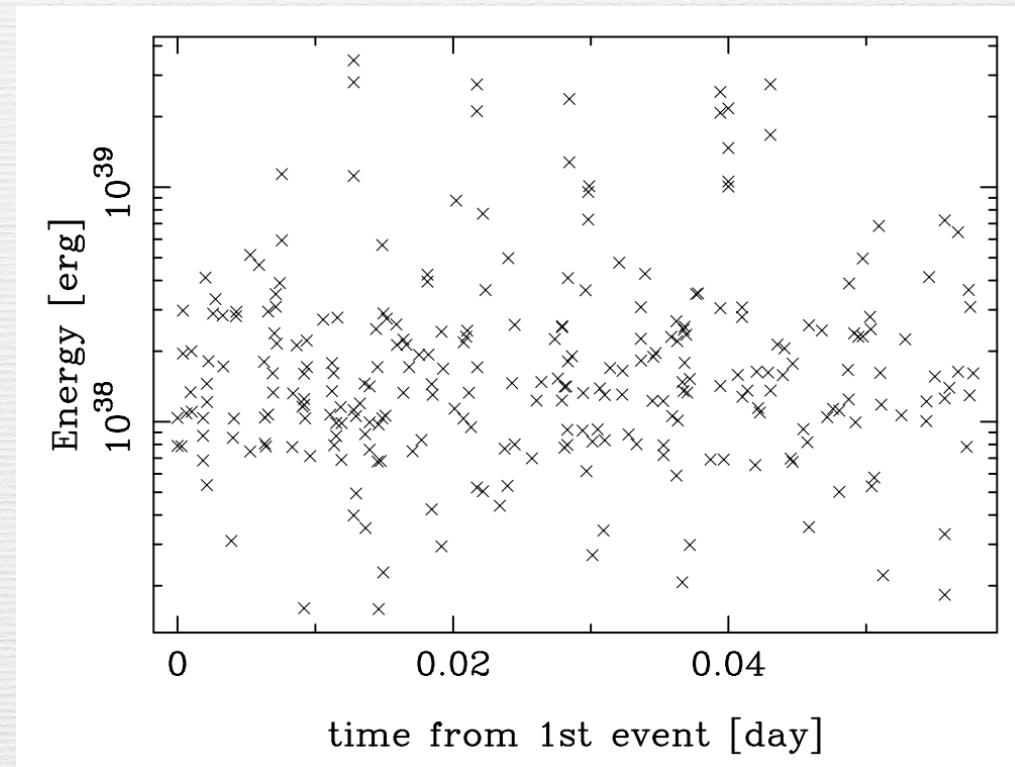


earthquake



# So many bursts from repeater FRBs!

- Thousands of bursts from a few repeater FRB sources
- $> 100$  bursts in 1 hr
- Detailed studies on the statistical nature of these bursts now possible

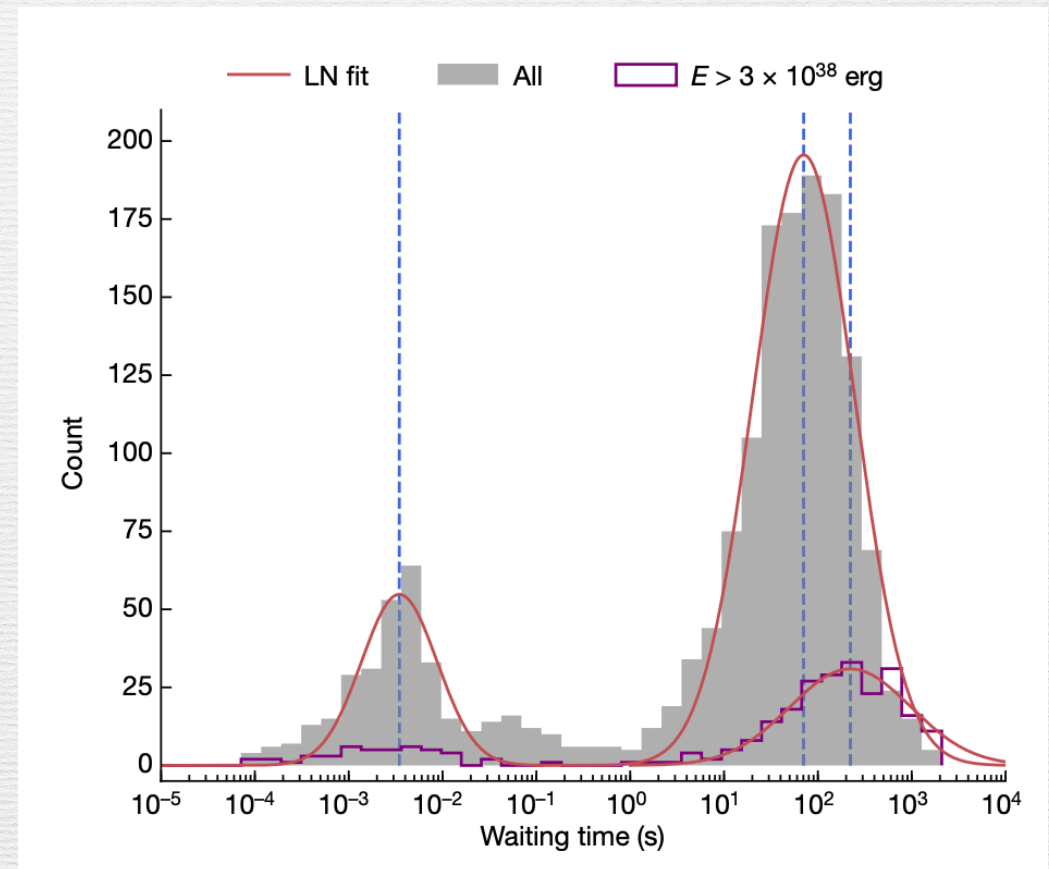


A part of Jahns+'23 data  
for FRB 20121102A



# Statistical properties of repeating FRB occurrence time?

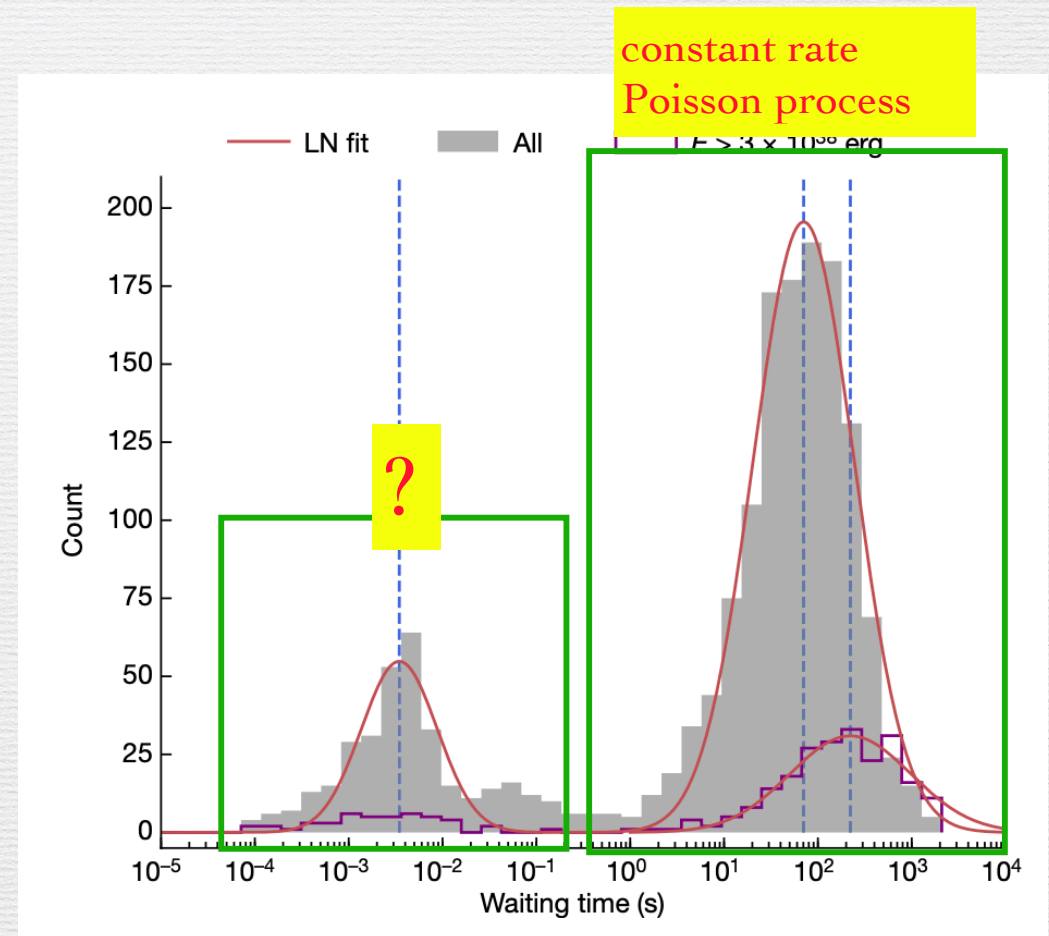
- bimodal wait-time distribution found universally for all repeater FRB sources
- The peak at longer wait times is consistent with a Poisson process with a constant event rate
- The origin of short wait-time peak is unknown.
  - peaks at 1-10 msec, close to the duration of one FRB.
  - Related to radiative process/source activity?



Li+'21

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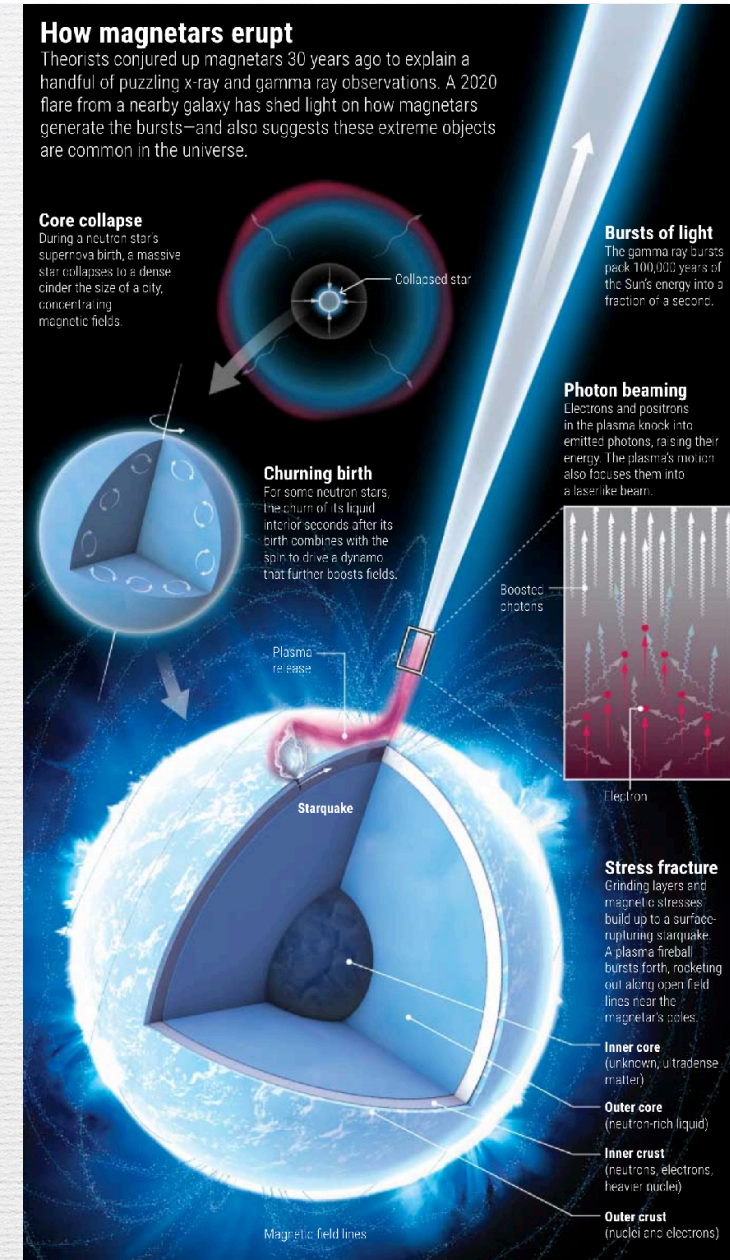


Li+'21



# FRBs vs. earthquakes and solar flares

- FRB statistical properties may be similar to earthquakes or solar flares
  - FRBs related to magnetars (e.g. SGR 1935+2154)
  - magnetar flares thought to occur by starquakes in the surface solid crust of a neutron star, induced by magnetic energy
  - similarity between magnetars, earthquakes, and solar flares investigated in the literature
    - e.g. the power-law energy distribution are common for magnetar bursts and earthquakes (Wadati-Gutenberg-Richter law of earthquakes)





# Wadati? Gutenberg-Richter law?

- The power-law distribution of earthquake magnitudes (energies) often called “Gutenberg-Richter” (1944) law
  - $\log N(>M) \propto M^{-b}$ ,  $dN/dE \propto E^{-1-2b/3}$ ,  $b \sim 1$
  - $M$  = magnitude,  $E$  = energy
- But...
- 和達清夫 (WADATI Kiyoo, 1902-1995, famous by Wadati-Benioff zone) found this law earlier in 1932
  - Wadati, K. “On the frequency distribution of earthquakes.” Journal of the Meteorological Society of Japan. Ser. II 10, 559–568 (1932) (in Japanese).

氣象集誌第二輯第十卷第十號

論 文

地震回数の分布に就いて

(中央氣象臺臺報)

和 達 清 夫

On the Frequency Distribution of Earthquakes.  
by K. Wadati.

一、緒 言

昭和七年六月の地震研究所談話會に於いて、寺田教授は「地震の分布と観測所の分布」と題する研究を發表された。本文は之と同様な構想に依つてなされたものである。

一般に観測される地震の回数は、観測所の数が多い程増加し、観測所の地震計の感度が鋭敏なる程又増加する。即ち勢力の小さな地震は観測されない場合があるからである。今地震の發現回数が到る所同じである、換言すれば地震發現回数密度が一定であると假定する。かかる場合に於いても、ある一個所の観測所に依つて観測される地震の分布は、観測所附近に密であり、観測所を距るに従ひ疎となる。

此の密疎の状態は何に起因するかと言へば、第一に地震波動が震源より四圍に傳播するに際して、如何に減衰して行くかと云ふ事、第二に地震發現回数が地震のエネルギーと如何なる關係を有するかと云ふ事に因る。


實際の場合に於いては前述の假定即ち地震發現回数が到る所同じであると云ふ事は成立して居ない。地震の發現回数は震源の深さ及び震央地域に大なる關係を有する。よく知られて居る様に地震は數十軒の深さに最も多く發現し、又震央は所謂地震帯或は頻發地域と稱せられる所に最も多く存する。従つて観測された地震の分布はかなり複雑なものが期待される筈であるが、併し適當な考慮を加へる時は、この観測結果から

一、地震發現回数の地震エネルギーに對する分布を知る事が出来る。本文に於ては其の方法、並びに東京の地震觀測結果より調査したる實例に就いて述べられて居る。

二、理

地震のエネルギー、震源よ

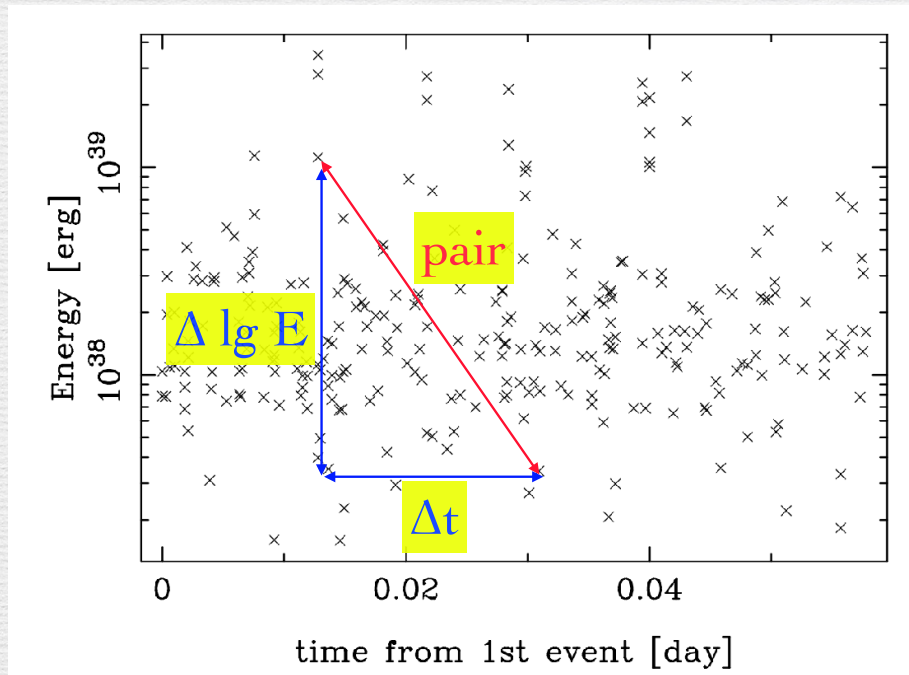
観測された地



ため観



## What we did: correlation function $\xi$ in time-energy space



data for FRB 20121102A  
from Jahns+'23

- two-point correlation function  $\xi$  in the space of  $\Delta t$  and  $\Delta \lg E$  ( $= \Delta \log_{10} E$ )

$$dn_p = (1 + \xi) \bar{n}_p d(\Delta t) d(\Delta \lg E) ,$$

- $\xi$  is the excess of pair counts compared with the case of no correlation ( $\bar{n}_p$ )
- random data (no correlation) is produced assuming “constant event rate” and “constant energy distribution” during one-day observation (~ a few hours)



# 7 FRB data sets for 3 sources

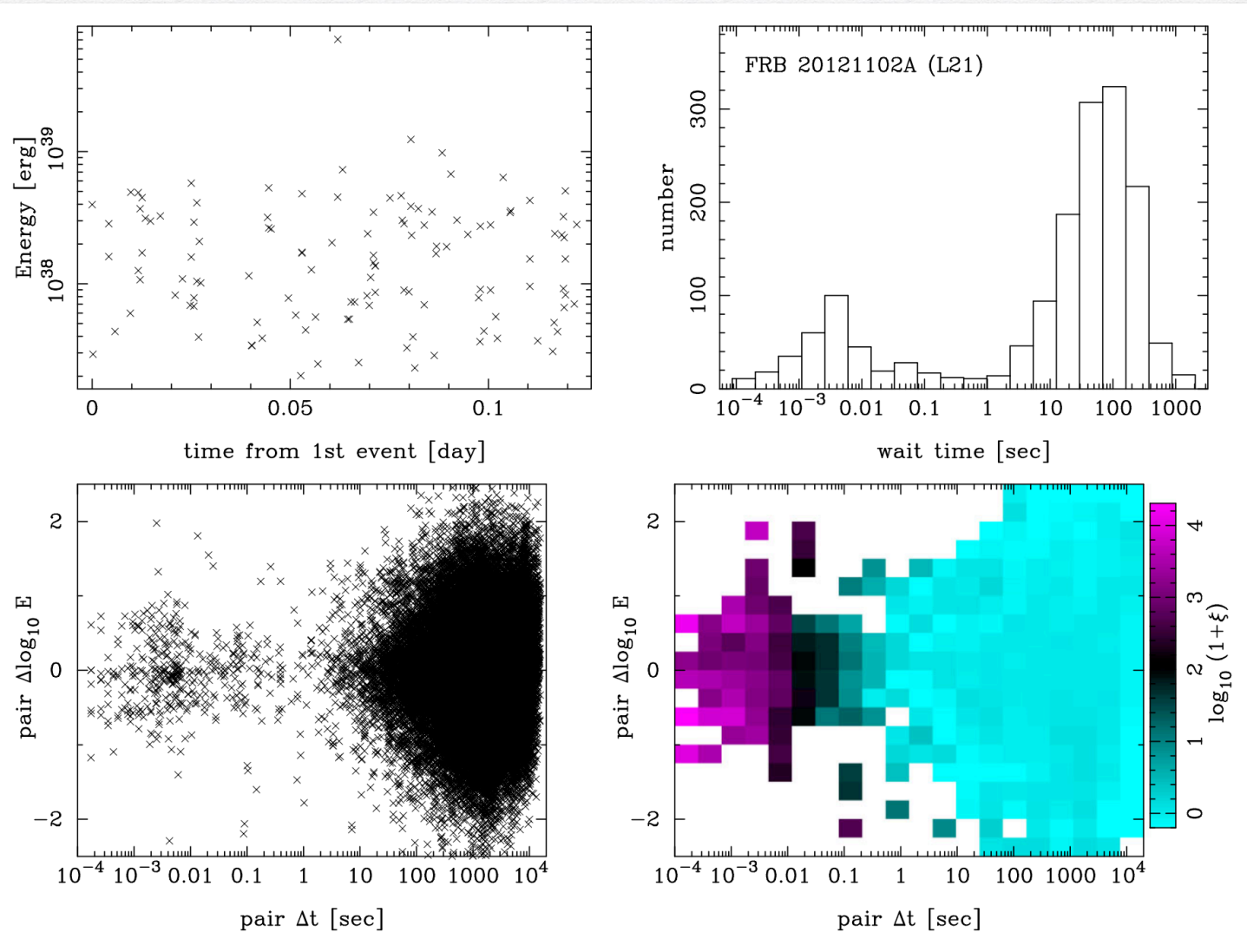
- nearly 7,000 events in total, from Arecibo & FAST
- 3 sources (FRBs 20121102A, 20201124A, 20220912A)

Data set name	Telescope	Period (MJD)	Days <sup>a</sup>	$t_{\text{obs}}$ <sup>b</sup> (day)	Events	$r_m$ <sup>c</sup> (/day)	$C_{\text{best}}$ <sup>d</sup> $C_{-1\sigma}$ $C_{+1\sigma}$	$p_{\text{best}}$ $p_{-1\sigma}$ $p_{+1\sigma}$	$\tau_{\text{best}}$ $\tau_{-1\sigma}$ $\tau_{+1\sigma}$	$n$ <sup>e</sup>
FRB 20121102A (L21) (5)	FAST	58724.87–58776.88	39	1.76	1651	1500	5100 3100 9700	1.6 1.4 1.8	0.0020 0.0009 0.0033	0.28
FRB 20121102A (H22) (6)	Arecibo	57510.80–57666.42	18	0.733	475	870	490 280 1200	9.1 2.1 $\infty$	0.28 0.019 1.4	0.17
FRB 20121102A (J23) (9)	Arecibo	58409.35–58450.28	8	0.272	1027 (849) <sup>f</sup>	4900	770 500 1100	2.3 1.8 3.5	0.012 0.0063 0.028	0.40
FRB 20201124A (X22) (7)	FAST	59307.33–59360.18	45	3.13	1863	840	340 250 500	28.3 4.5 $\infty$	1.3 0.13 1.5	0.16
FRB 20201124A (Z22 D3) (8)	FAST	59484.81–59484.86	1	0.040	232	5800	270 83 $\infty$	3.4 1.5 $\infty$	0.071 0 1.7	0.54
FRB 20201124A (Z22 D4) (8)	FAST	59485.78–59485.83	1	0.040	542	14000	54 35 60	4.2 2.1 $\infty$	0.19 0.058 1.9	0.50
FRB 20220912A (Z23) (10)	FAST	59880.49–59935.39	17	0.32	1076	6900	70 50 170	5.7 2.4 $\infty$	0.26 0.043 1.8	0.30

<sup>a</sup>Total number of days on which observations were made



# Example of $\xi$ calculation



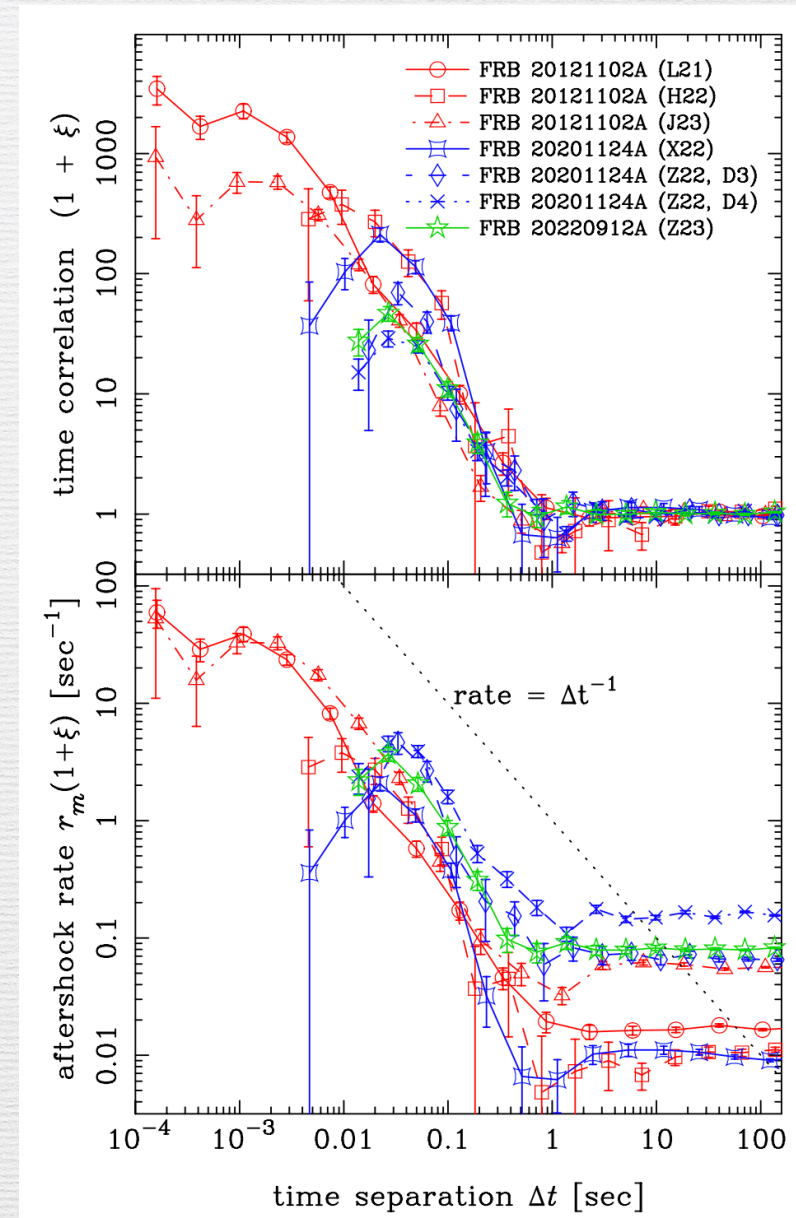


## time correlation $\xi(\Delta t)$

- power-law signal at  $\Delta t < 1$  sec
- flat at  $\Delta t \sim$  FRB duration(<10 msec)
  - Note: different sub-burst treatments by different authors
- can be fit by  $\xi \propto (\Delta t + \tau)^{-p}$ 
  - similar to the Omori-Utsu law of aftershock rate of earthquakes
- “aftershock rate” after one event is given as  $r_m(1 + \xi)$ , where  $r_m$  is the mean event rate
- expected number of aftershocks following one event:

$$n = \int r_m \xi(\Delta t) d(\Delta t)$$

- $n = 0.1$ -0.5 for FRBs
- multiple aftershocks to one event are rare
- stable against change of mean rate  $r_m$  or different sources





# The Omori-Utsu law for earthquake aftershocks

- 大森 房吉 (OMORI, Fusakichi, 1868-1923)
  - Omori law: Omori, F. “On the after-shocks of earthquakes.” The journal of the College of Science, Imperial University, Japan 7, 111–200 (1894).
  - aftershock rate  $\propto (\Delta t + \tau)^{-1}$
- 宇津徳治 (UTSU, Tokuji, 1923-2004)
  - modified Omori law (Omori-Utsu law) Utsu+ 1957, 1961
  - aftershock rate  $\propto (\Delta t + \tau)^{-p}$



## On the After-shocks of Earthquakes.

by

F. Ōmori, *Rigakushi*.

### I. General Considerations.

§ 1. A strong earthquake is almost invariably followed by weaker ones and when it is violent and destructive the number of minor shocks following it may amount to hundreds or even thousands. When after-shocks are not reported to have happened it is probably because they were deemed unimportant to record. Or it may be that the seat of origin of the earthquake being very deep or far out under the ocean-bed, the after-shocks did not reach the observer.

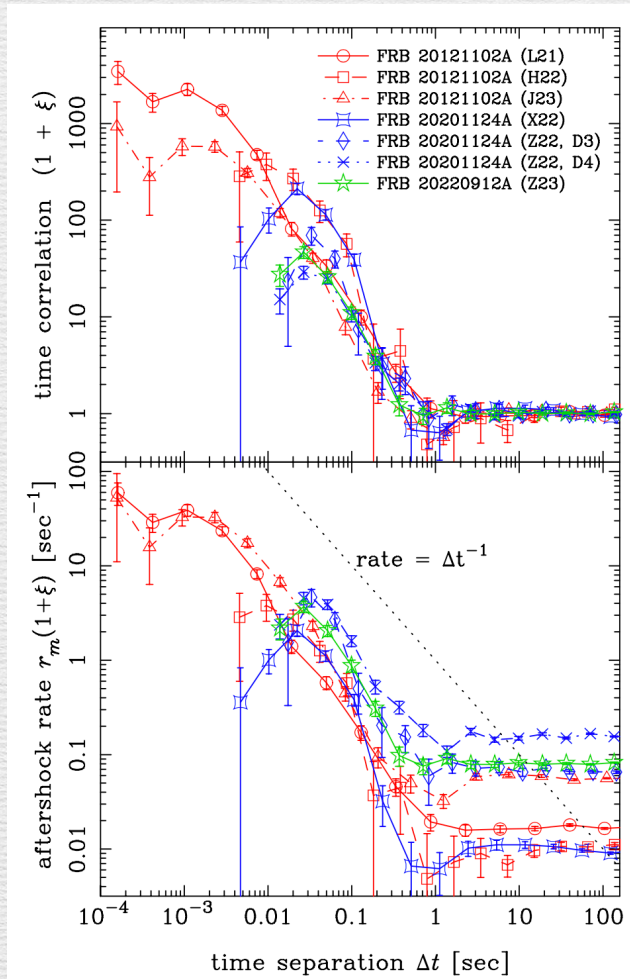
Complete records of after-shocks were obtained, I believe, for the first time in the cases of the three recent great earthquakes in Japan; namely, those of Kumamoto in 1889, of Mino and Owari in 1891,



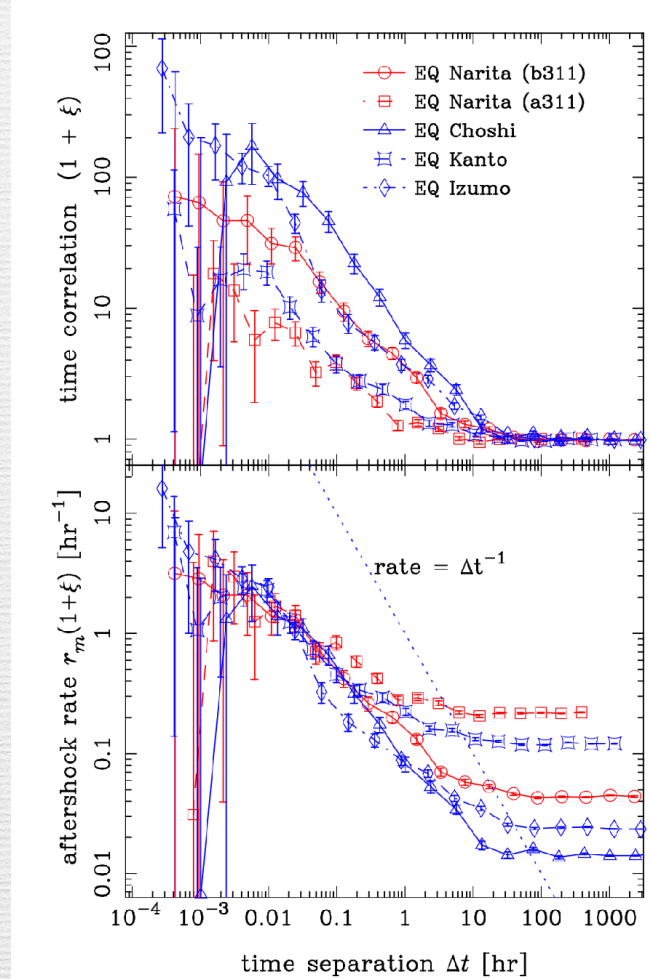


# time correlation function: FRBs vs. earthquakes vs. solar flares

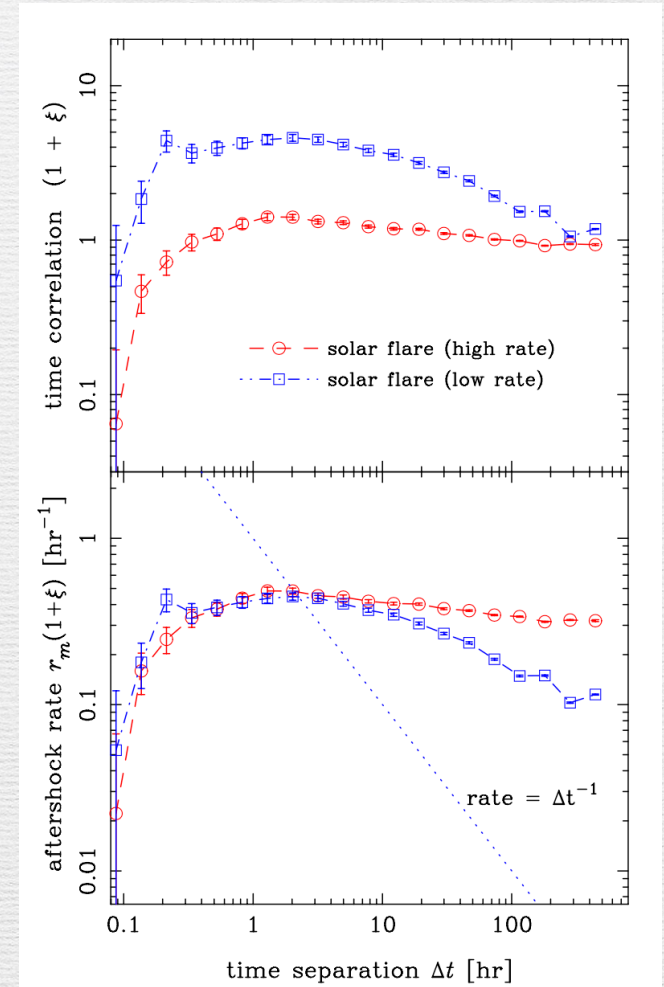
## FRBs



## Earthquakes



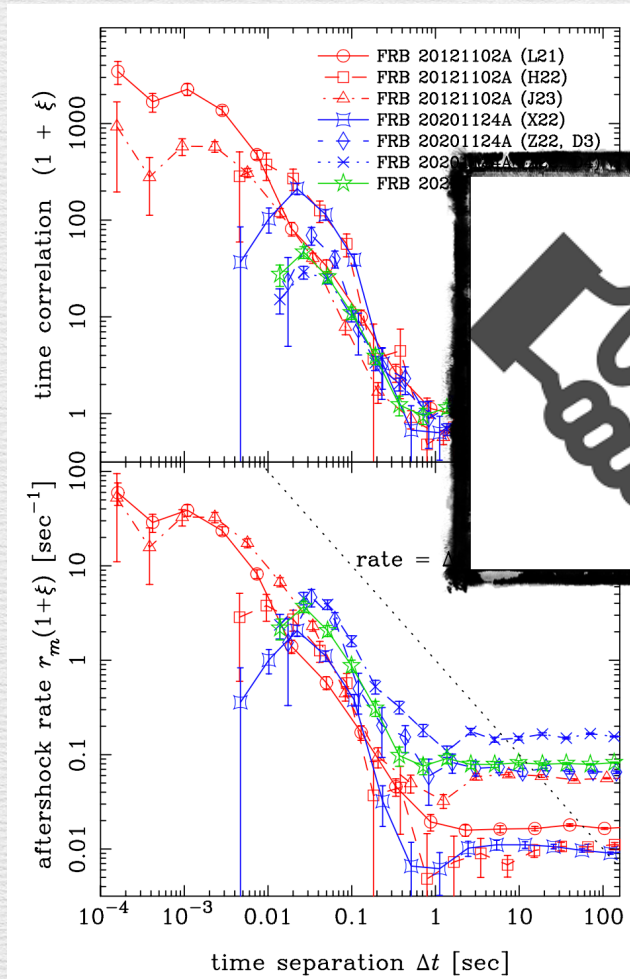
## solar flares



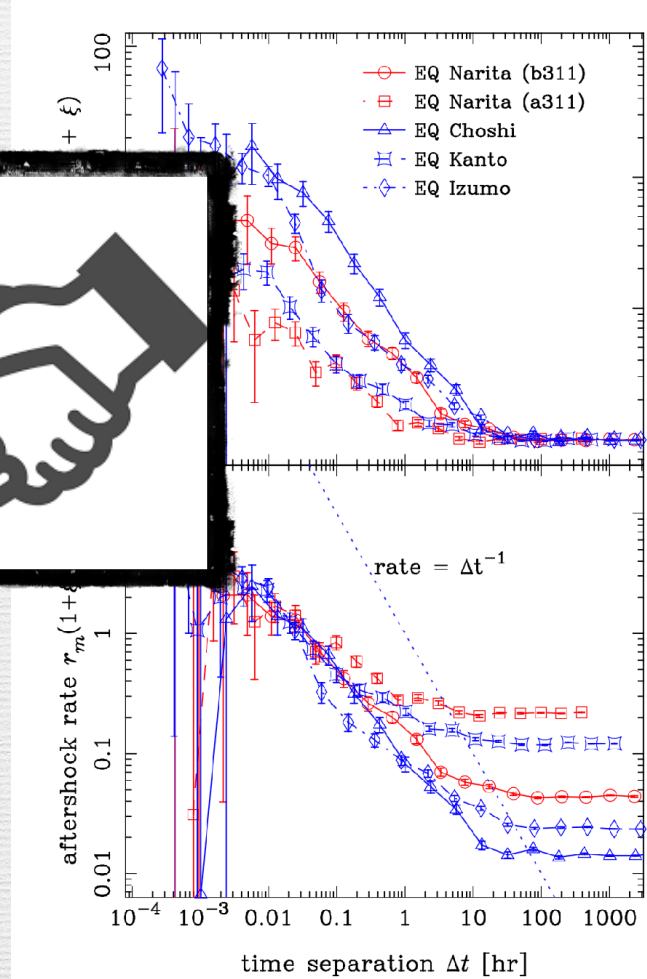


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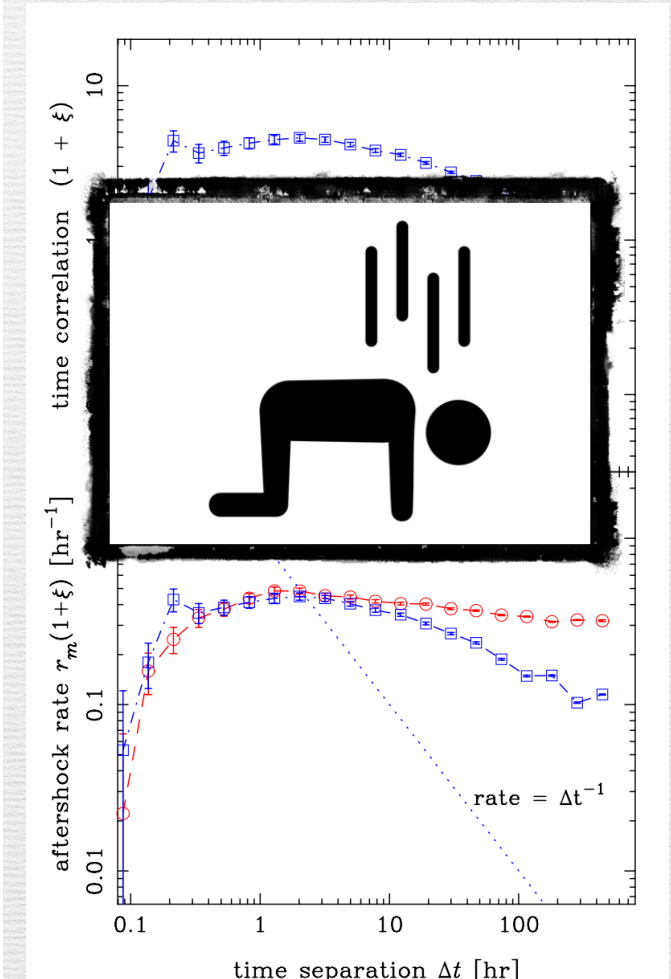
FRBs



Earthquakes



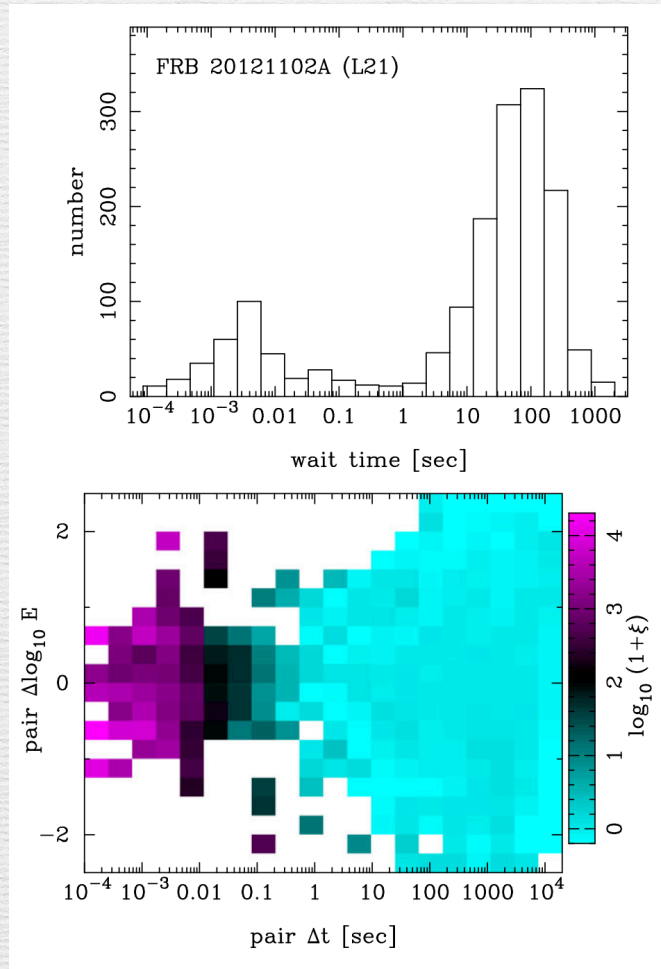
solar flares



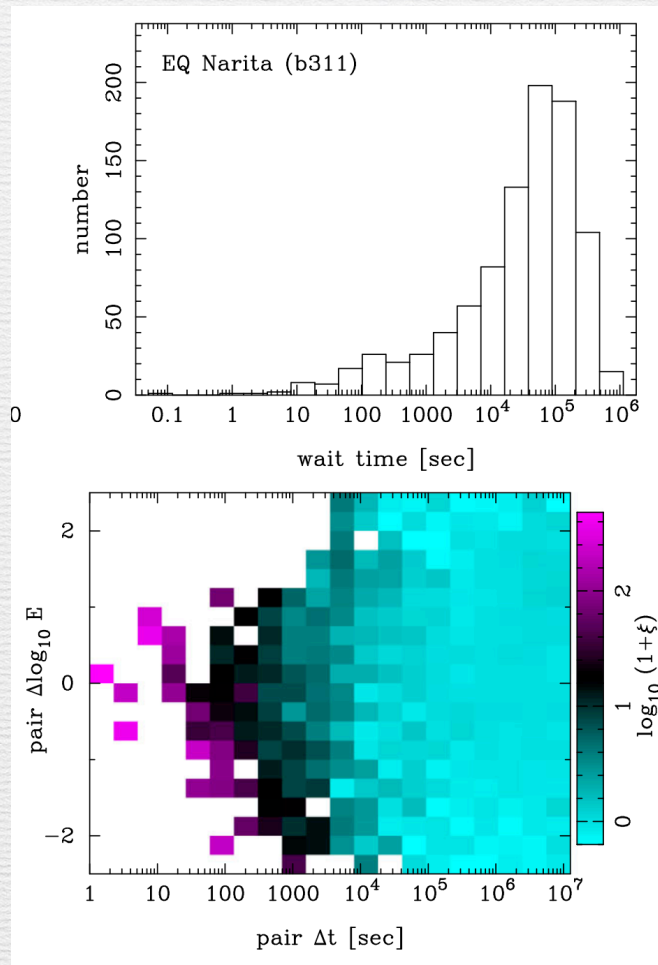


# time-energy correlation: FRBs vs. earthquakes vs. solar flares

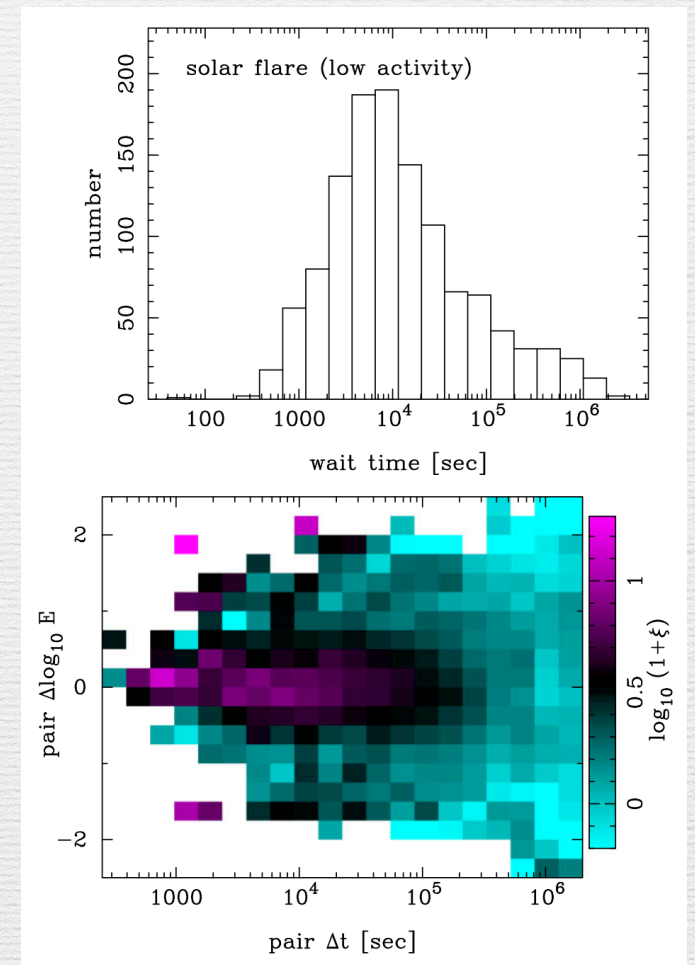
## FRBs



## Earthquakes



## solar flares



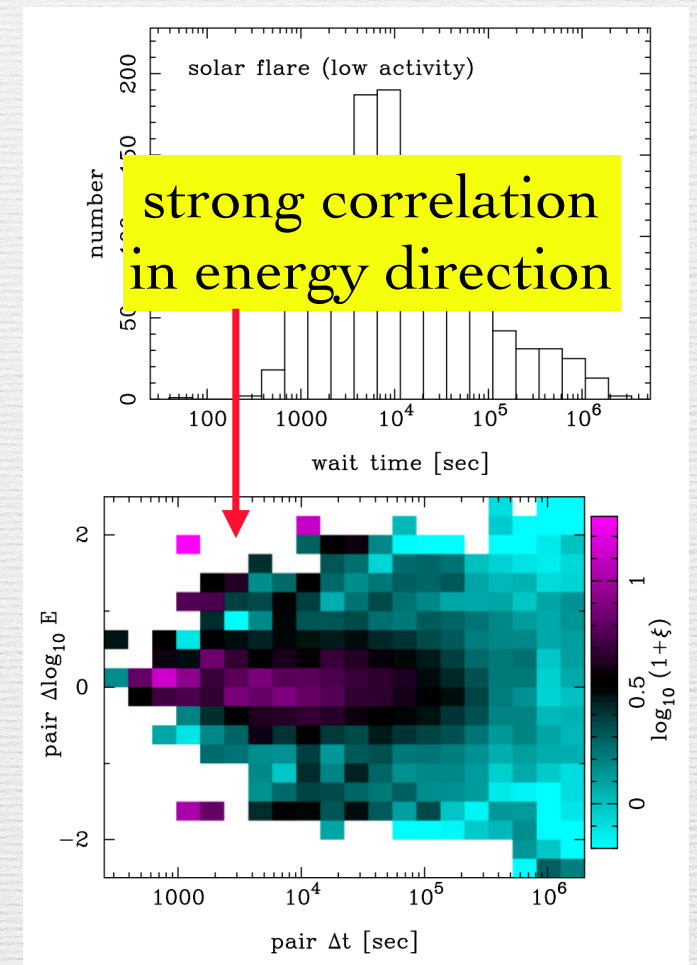
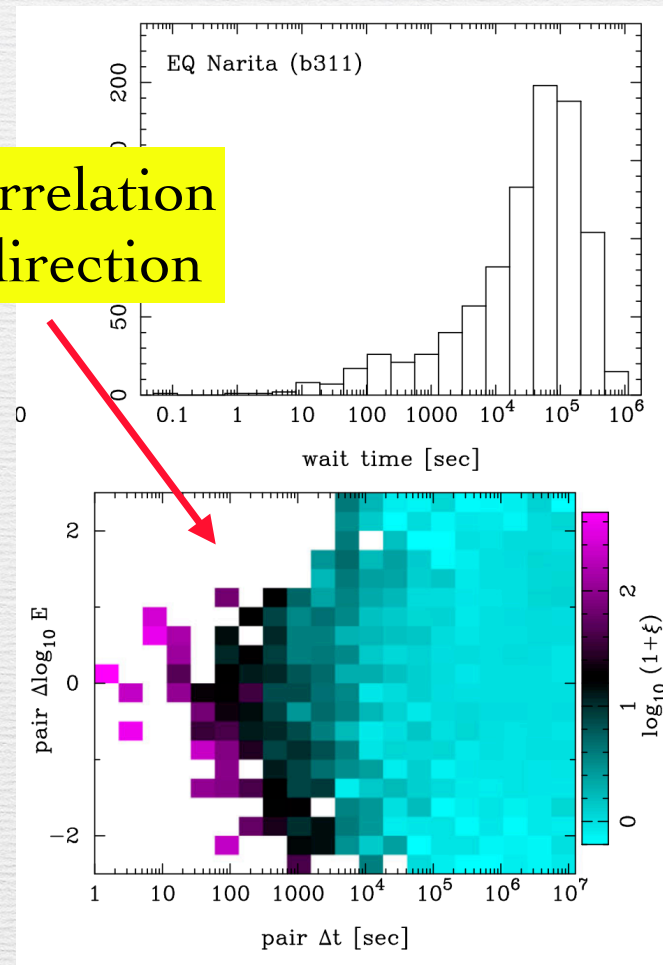
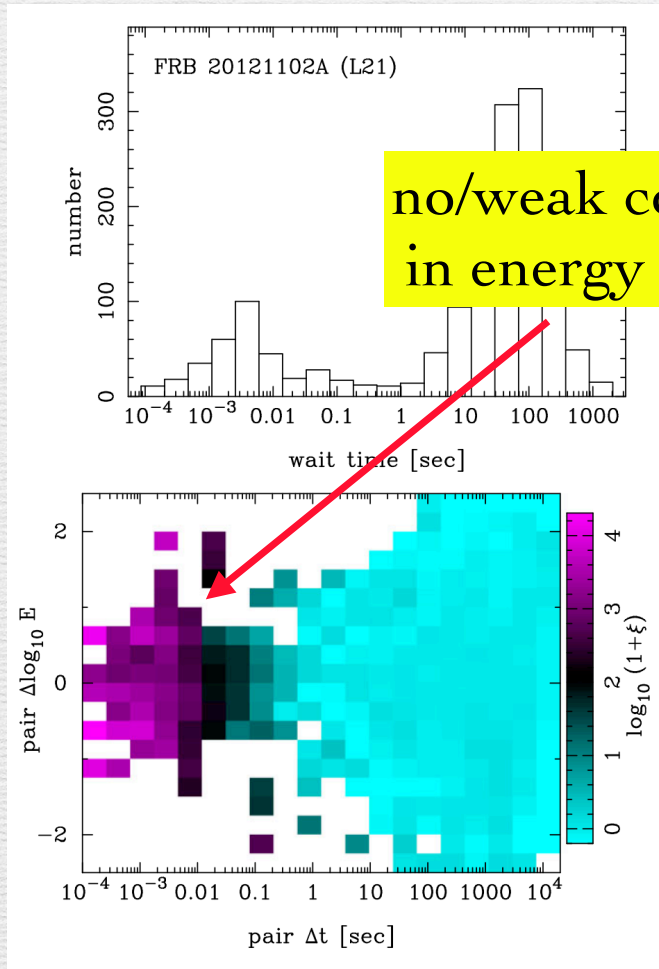


# time-energy correlation: FRBs vs. earthquakes vs. solar flares

## FRBs

## Earthquakes

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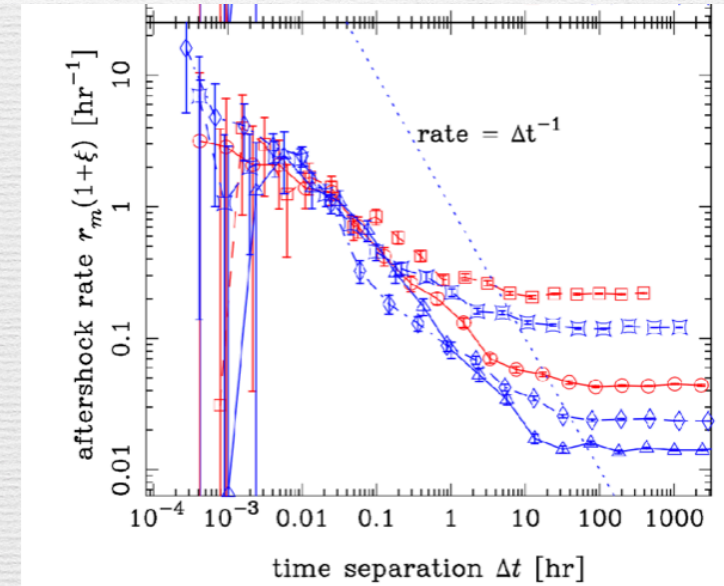
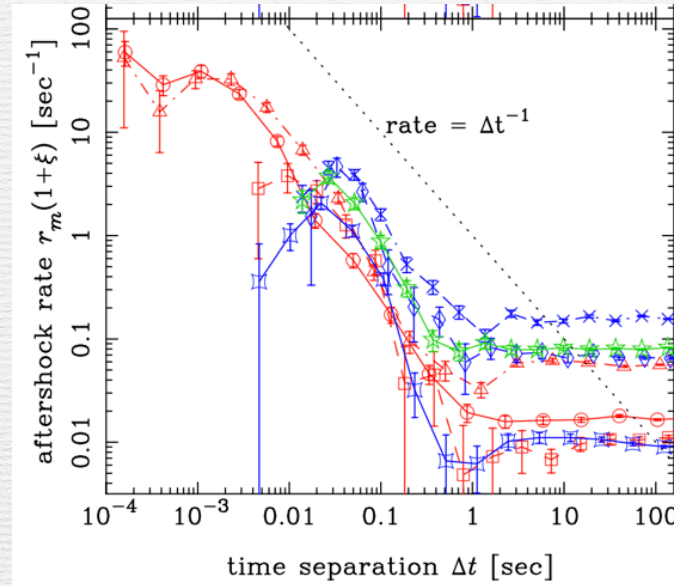




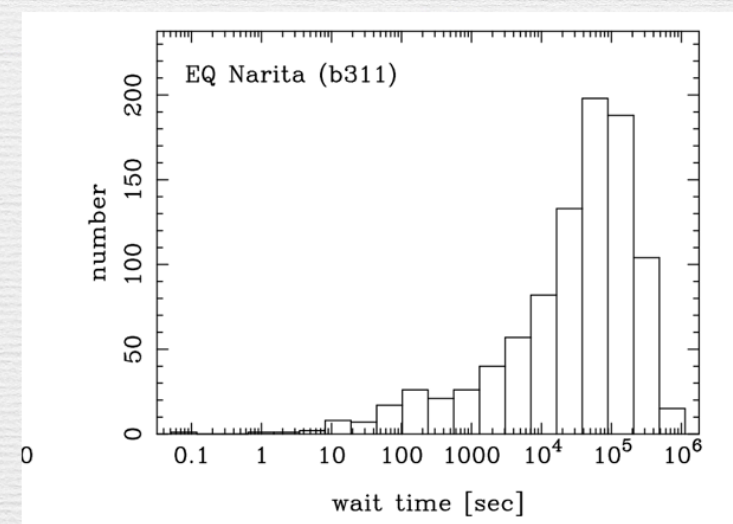
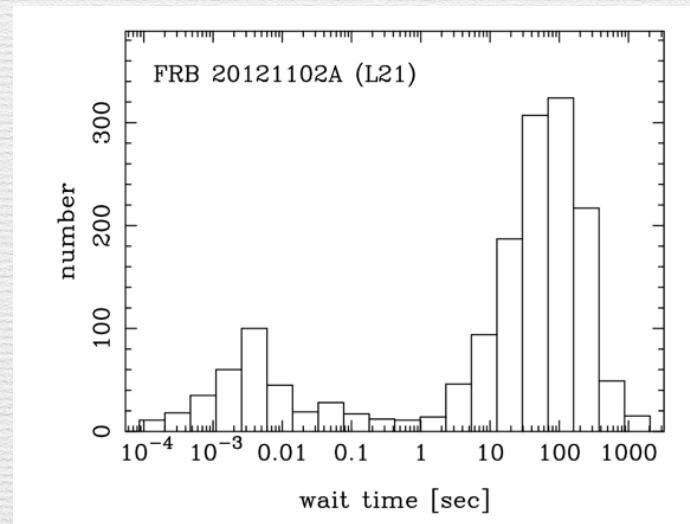
# Difference between FRBs & earthquakes?

- index  $p$  of Omori-Utsu law

- $\propto (\Delta t + \tau)^{-p}$



- wait-time distribution

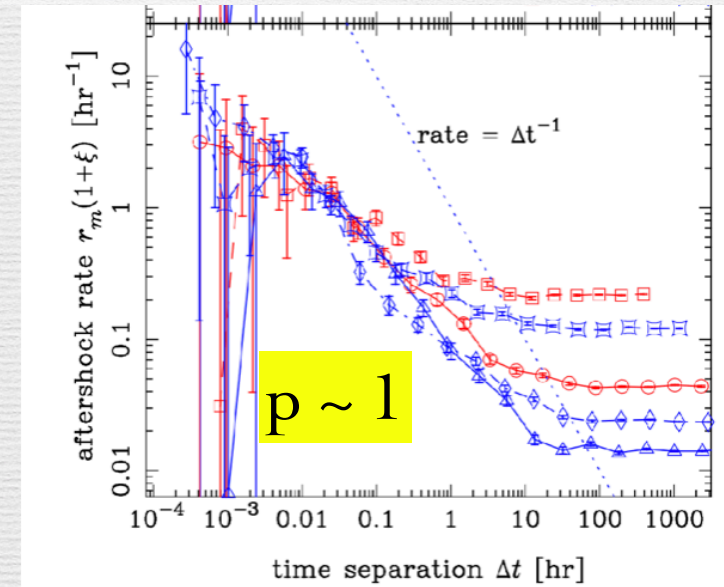
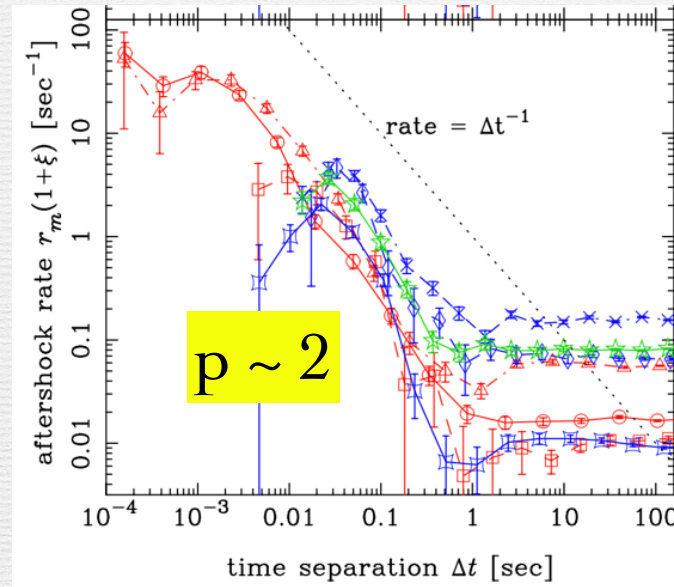




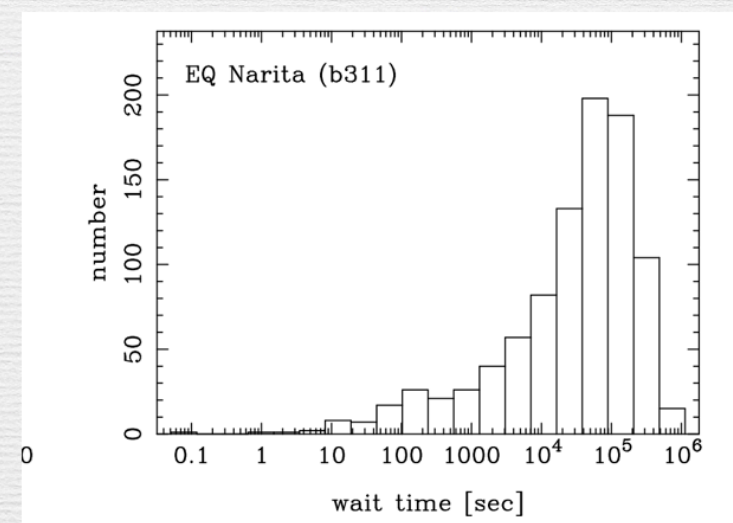
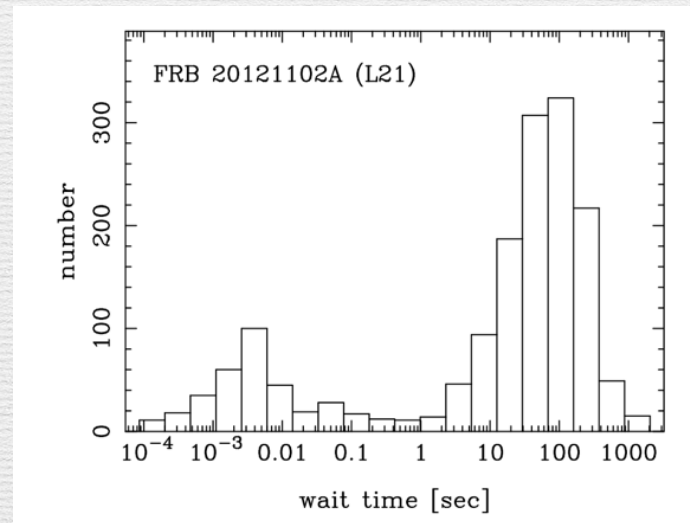
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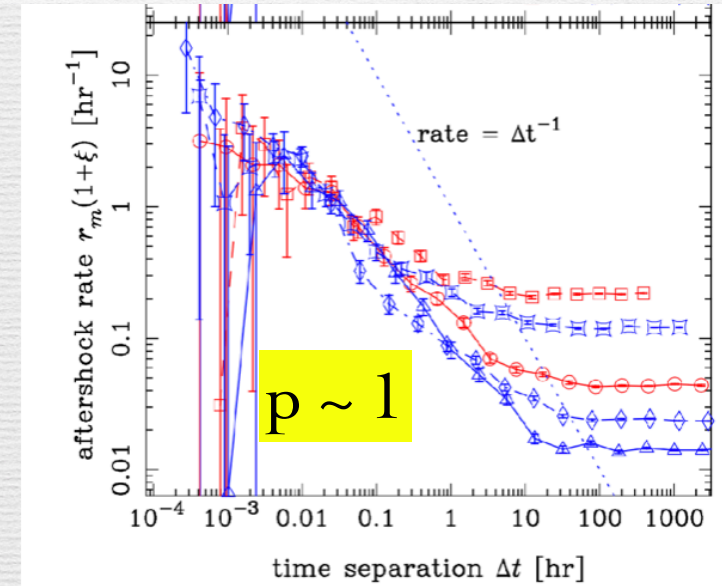
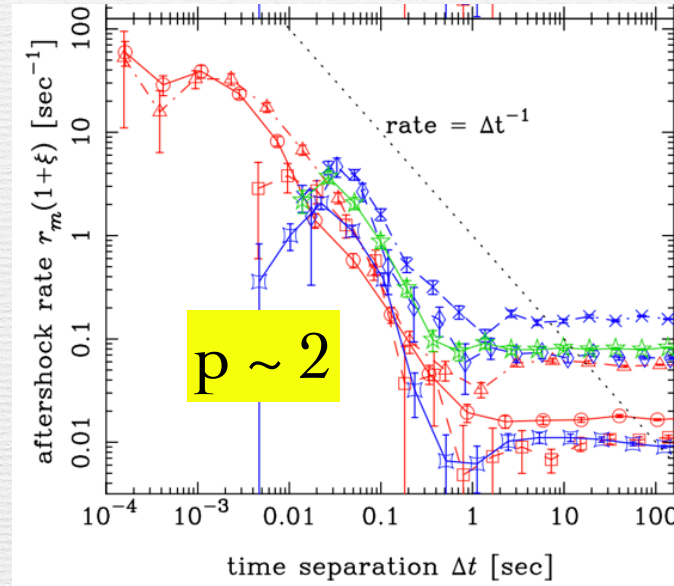




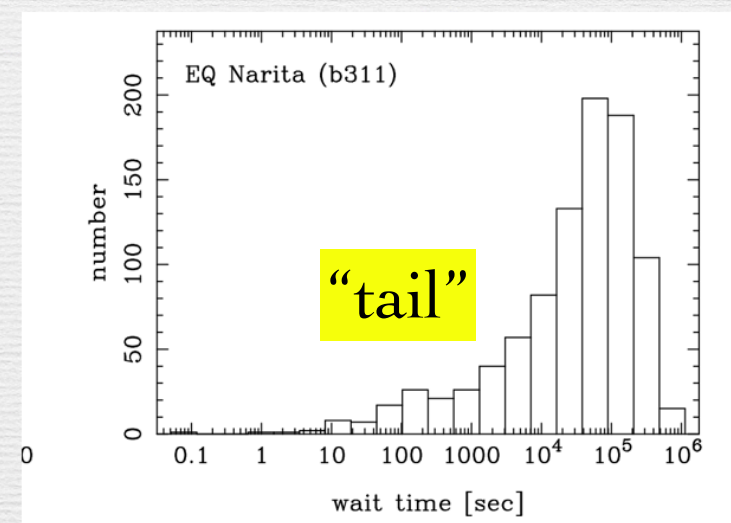
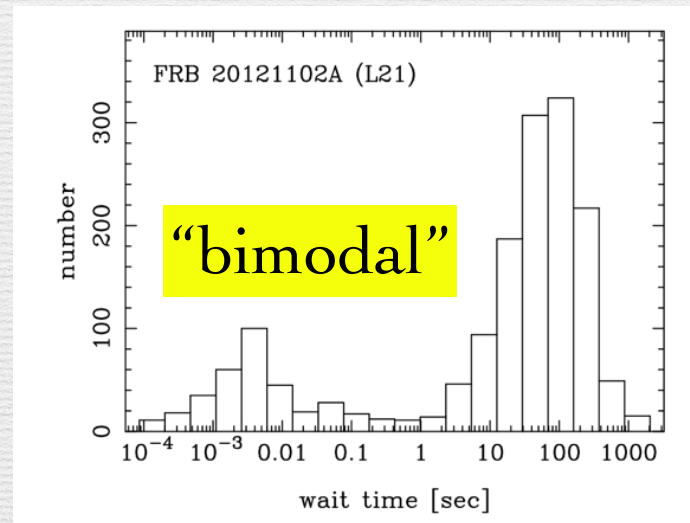
# Difference between FRBs & earthquakes?

- index  $p$  of Omori-Utsu law

- $\propto (\Delta t + \tau)^{-p}$



- wait-time distribution





## Conclusions (1)

- FRBs are remarkably similar to earthquakes in time-energy correlation, with the universal laws on the aftershock statistics
  1. each event induces 0.1-0.5 aftershocks
  2. aftershock rate obeys the Omori-Utsu law  $\propto (\Delta t + \tau)^{-p}$
  3.  $\tau$  is close to the event duration (10 msec for FRBs, 1 min for earthquakes)
  4. even if the source activity changes, the aftershock rate remains stable
  5. almost no correlation between time and energy
- These features have been known for earthquakes as the ETAS (epidemic-type aftershock sequence) model
- the only difference is  $p \sim 2$  (FRB) or  $\sim 1$  (earthquakes)
- In contrast, solar flares are NOT similar to FRBs/earthquakes
  - perhaps related to fluid surface of the Sun, compared with solid crusts at FRB/Earth surfaces?



## Conclusions (2)

- A natural interpretation: repeating FRBs are produced when the energy stored in solid neutron star crust is liberated by seismic activity
- Other FRB mechanism may not be excluded, but these aftershock properties must be explained in any FRB theory, putting strong constraints
- Future theoretical studies on FRB aftershock properties may give us new information about the neutron star crust / dense nuclear matter