

Spin Up in Neutron Stars : The Future of the Vela Pulsar

[GORDON BAYM](#), [CHRISTOPHER PETHICK](#), [DAVID PINES](#) & [MALVIN RUDERMAN](#)

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Assuming that the speed up of the Vela pulsar was caused by a “starquake”, a number of predictions can be made, and the event can be taken as evidence that the interior of the pulsar is a superfluid.

[Published: 03 July 1975](#)

Pulsar glitches and restlessness as a hard superfluidity phenomenon

[P. W. ANDERSON](#) & [N. ITOH](#)

[Nature](#) **256**, 25–27 (1975) | [Cite this article](#)

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“the macroglitches, especially those in the Vela pulsar, may be caused by the catastrophic release of pinned vorticity.”

Dynamics of a superfluid in a neutron star crust

Yuri Levin, Columbia (<https://yurilevincolumbia.github.io/>)
in collaboration with Bennett Link, Montana State

Link & Levin (2022), ApJ, 941, 148

Levin & Link (2023), ApJ, 959, 84

1. Strong hysteresis in pinning/unpinning of a vortex. Bistability of pinned and unpinned states.
2. Non-trivial background configurations with strong localized currents - superfluid rivers.
3. Great observational progress, lots of data. Pushback against observers (“give me the theory of glitches today”). We don’t understand basic things.

Vortex motion.

2d - view from above.

$\vec{V}_s - \vec{V}_v$ ↑
superfluid
velocity
relative
to the vortex.

$\otimes \xrightarrow{\vec{F}_{\text{Magnus}}}$

$$\vec{F}_{\text{Magnus}} = \vec{\kappa} \times (\vec{V}_s - \vec{V}_v)$$

↑
vector
along the
vortex.

↑
relative
velocity.

Equation of motion.

$$\vec{F}_{\text{Magnus}} - \nabla V - \alpha \vec{V}_v = 0$$

↑
pinning
potential.

↑
friction.

Particular case:

$$V_s = 0$$

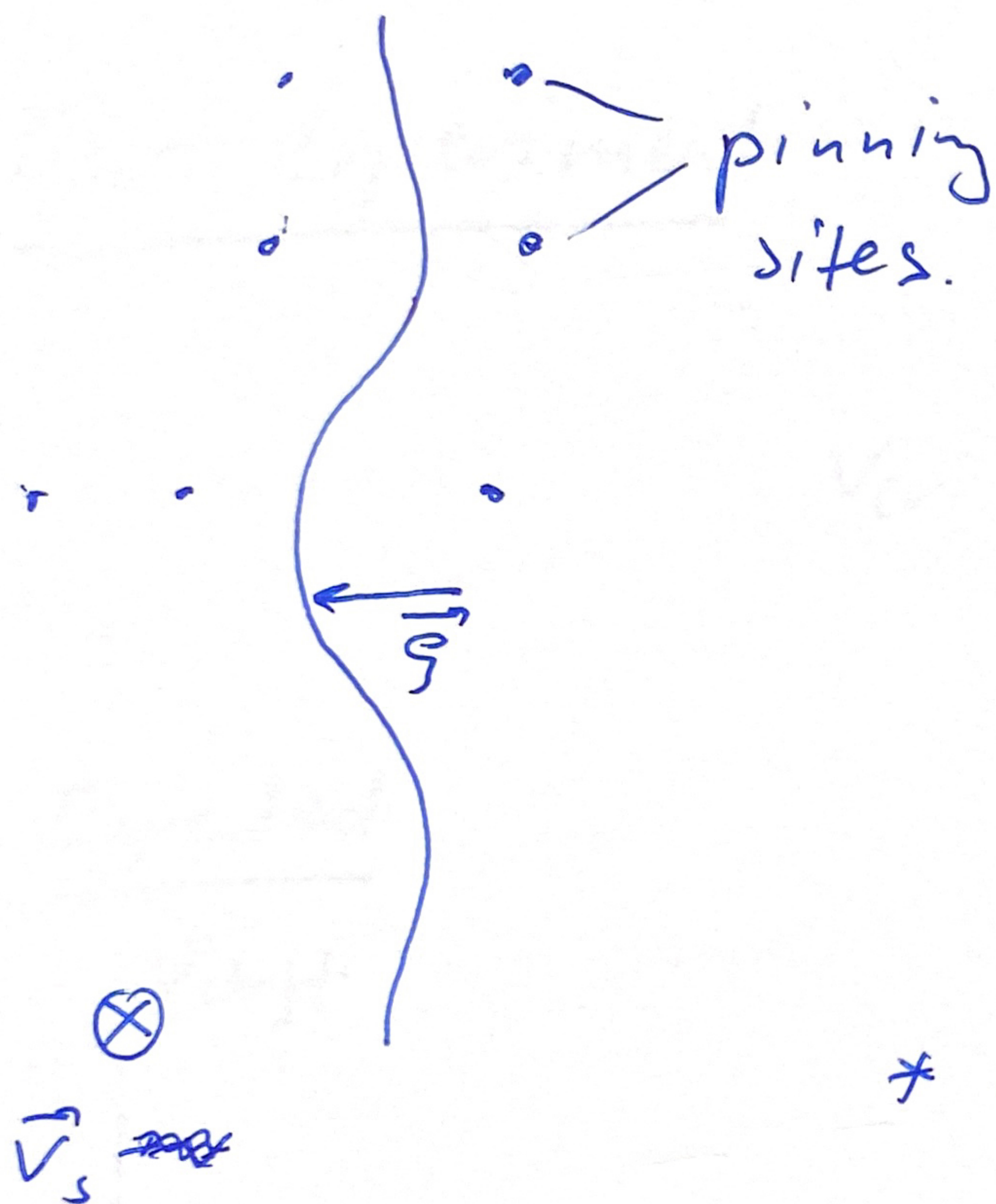
$$\alpha = 0 - \text{no friction.}$$

vortex circles
around maxima
or minima of

$$V(\vec{r})$$

Vortex motion

3d - view from the side.



Equations of motion:

$$\vec{F}_{\text{Magnus}} - \nabla V + T \frac{\partial^2 \xi}{\partial z^2} + 2\vec{V} = 0$$

Magnus

pinning

tension

friction

* One low-resolution simulation of
Link 2005. Saw unpinning.

* Innovation: spectral method, very fast.
analytical form of pinning

Link & Levin 2012

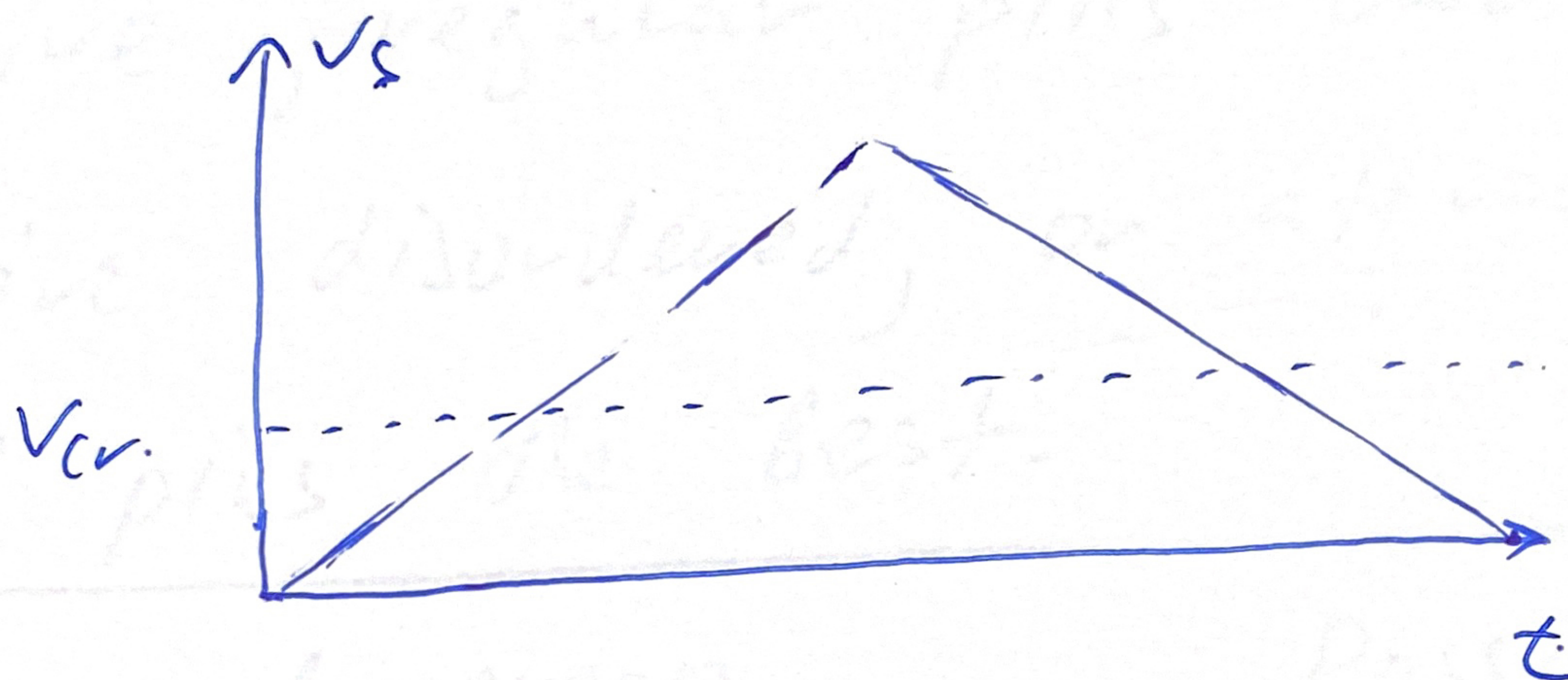
potentials, varied controlled disorder

Standard Core.

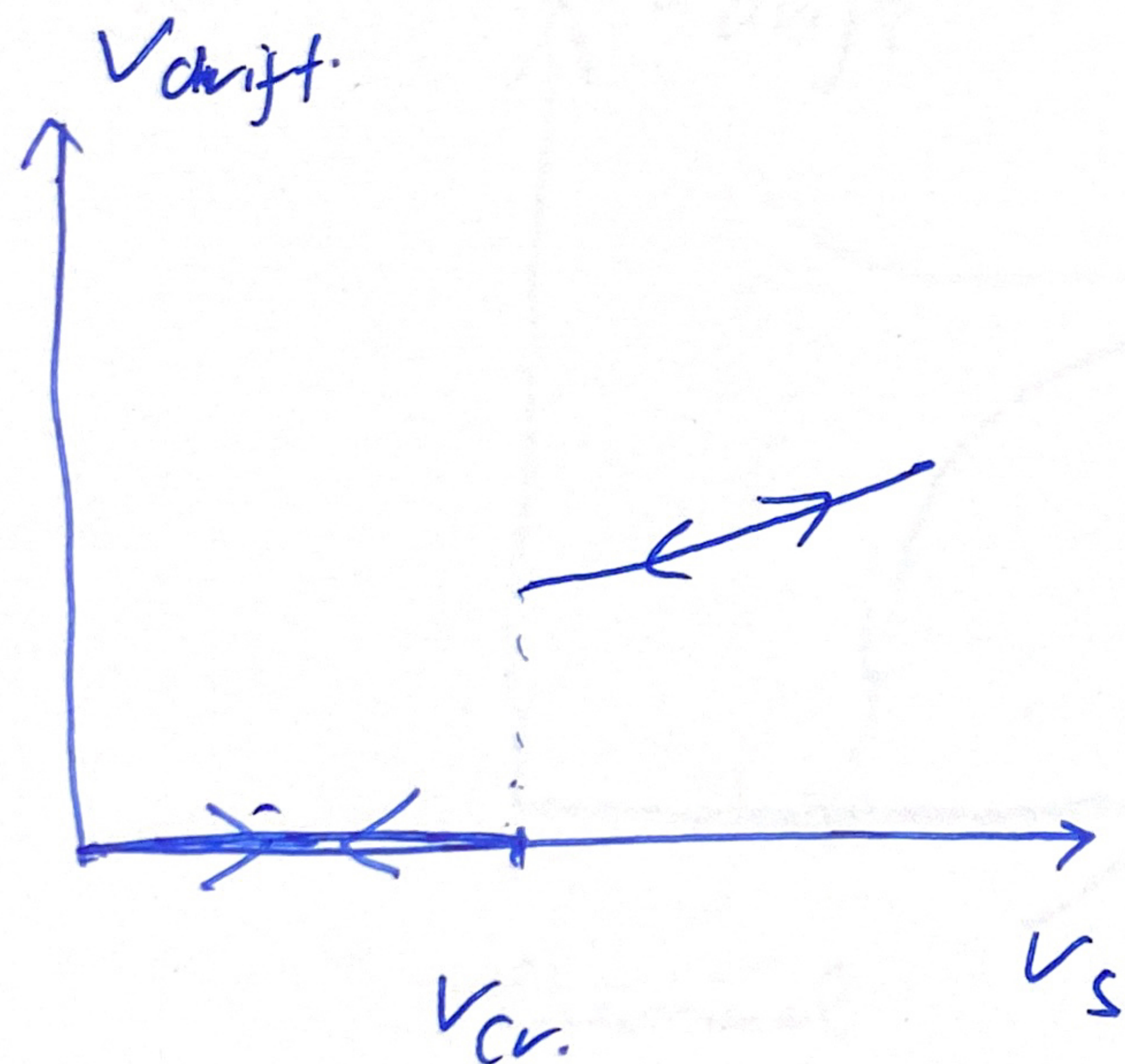
$$|\vec{V}_s - \vec{V}_v| < V_{cr.} - \text{pinned.}$$

$$|\vec{V}_s - \vec{V}_v| > V_{cr.} - \text{unpinned.}$$

Our experiment

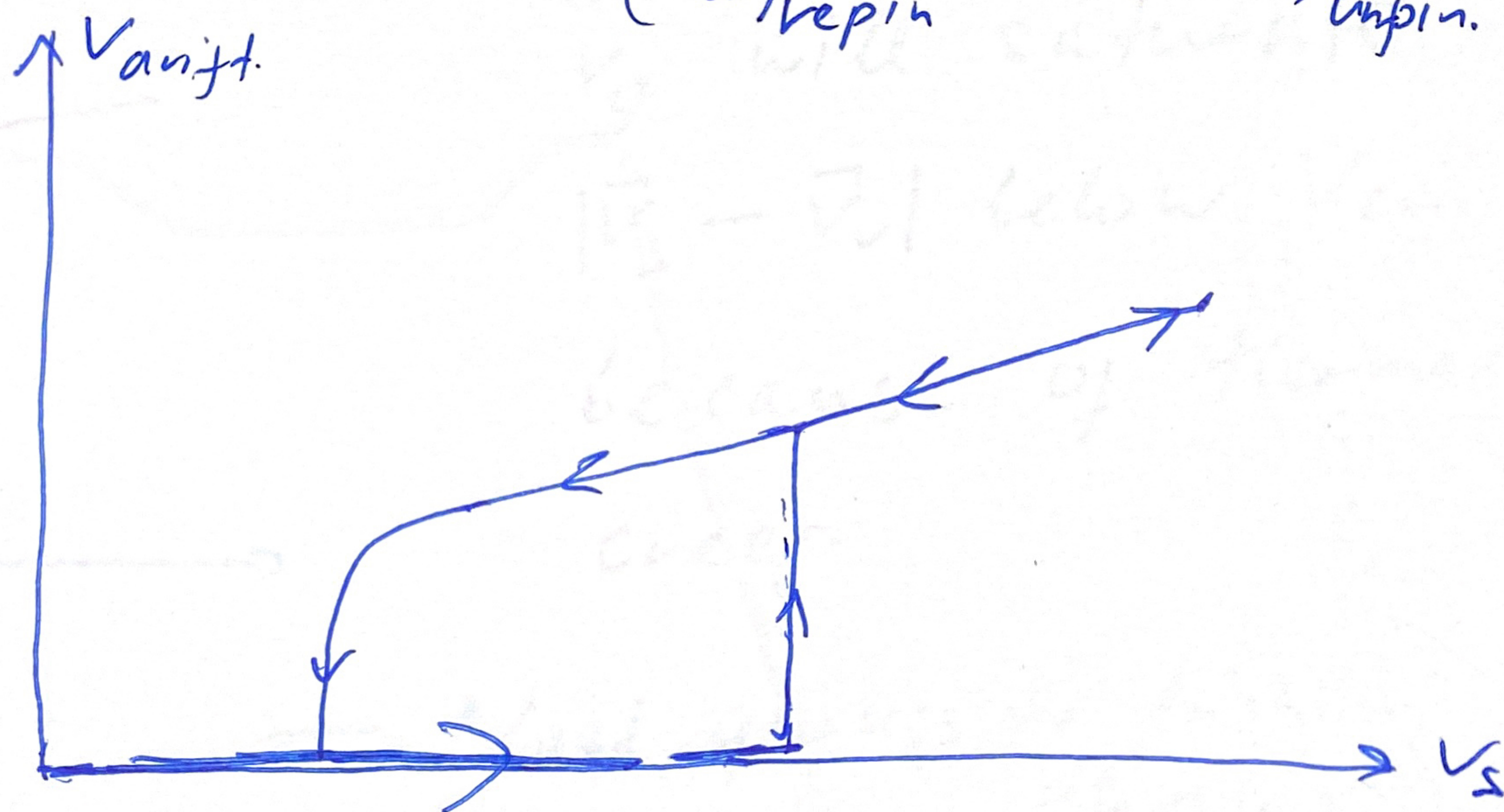


Expected



Results

$$(V_{cr.})_{repin} < (V_{cr.})_{unpin.}$$



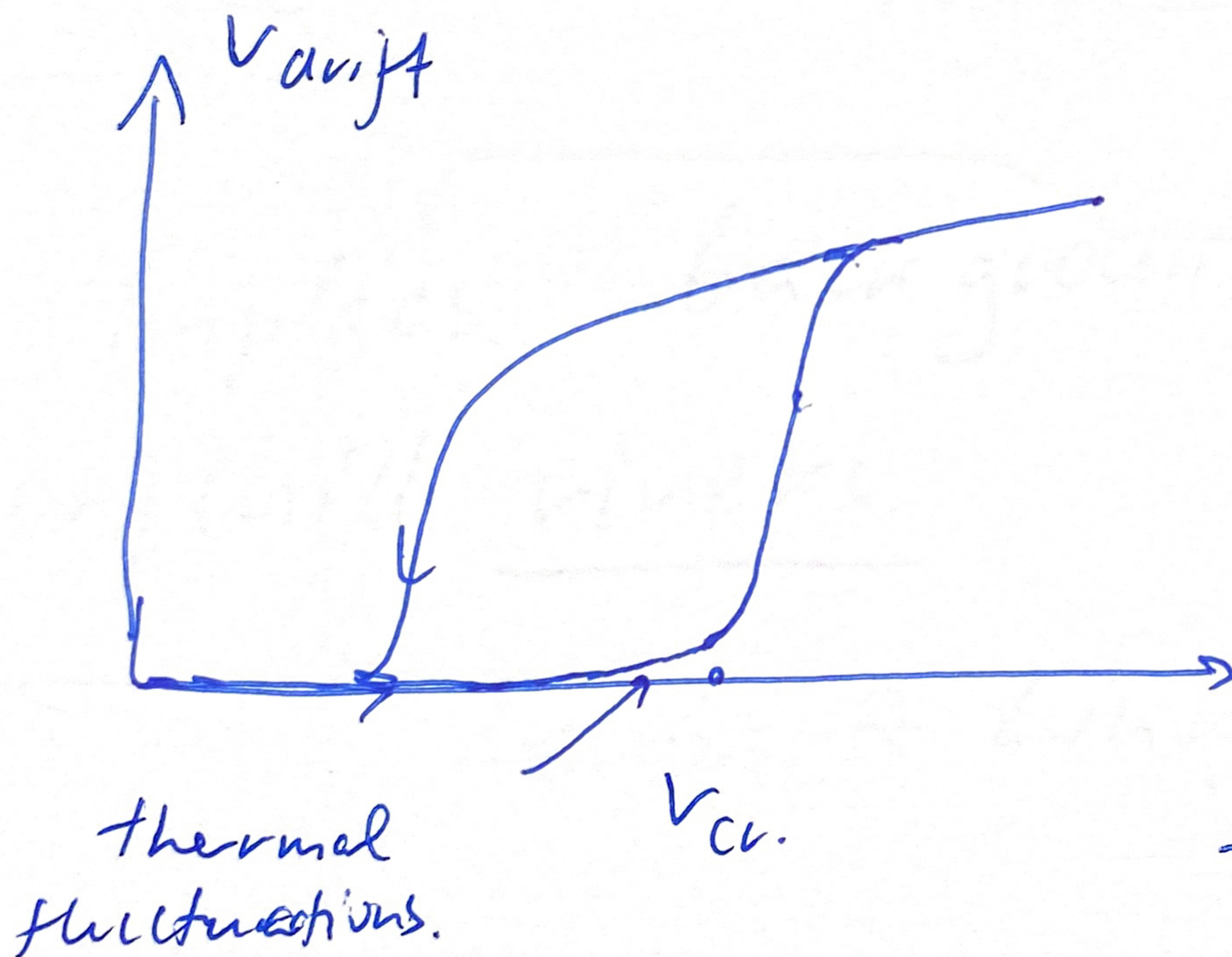
* Hysteresis, bistability.

* Extreme dependence on lattice structure.

— repulsive, regular pins badly.

— attractive disordered, or attractive aligned pins the best.

Caveat: thermal creep.



Possibility that gradual increase of V_d will saturate $|\vec{V}_s - \vec{V}_d|$ below V_{cr} because of thermal creep.

— Need quakes to trigger glitches?

Hypothesis

In most of NS

$|\vec{V}_s - \vec{V}_v|$ saturates below

$(V_{cr})_{unpin}$ as

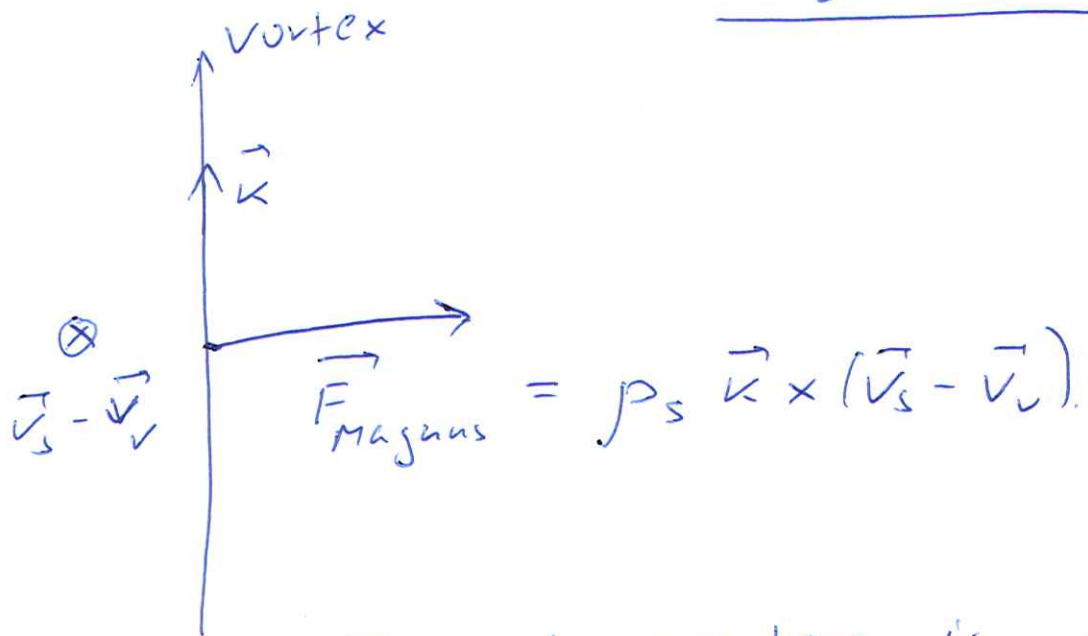
the NS spins down.

This implies background state with

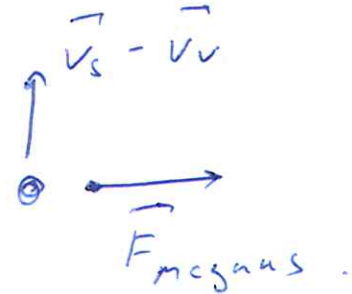
superfluid rivers.

Levin & Lin 2023

Magnus force



From above.



If a vortex is pinned, as the pulsar is spun down, it feels Magnus force pushing it.

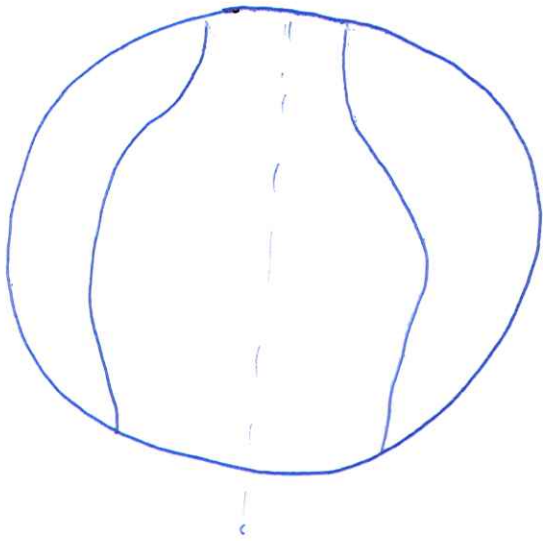
This force builds up.

$$\vec{F}_{\text{magnus}} = -\vec{F}_{\text{other}}$$

Vortex has no inertia.

Evolution in axisymmetry.

Levin & Lin 03
accepted.



* Lemma vortex is a source of circulation.

\Rightarrow vortex in axisymmetry traces
constant circulation surface.

* Corollary Stationary vortex \Rightarrow
 \Rightarrow no change in V_s
at that location.

* Assumption vortex creeps in the direction of Magnus force.

when $|\vec{V}_s - \vec{V}_{star}| > V_{cr}(r)$

$V_{cr}(r)$ - critical velocity above which
the vortex becomes mobile. $V_{cr} \sim 10^7 \text{ cm/sec}$
 $\sim 10 \text{ rad/sec}$.

* Theorem vortex motion in the direction
of Magnus force leads to
local reduction of $|\vec{V}_s - \vec{V}_{star}|$

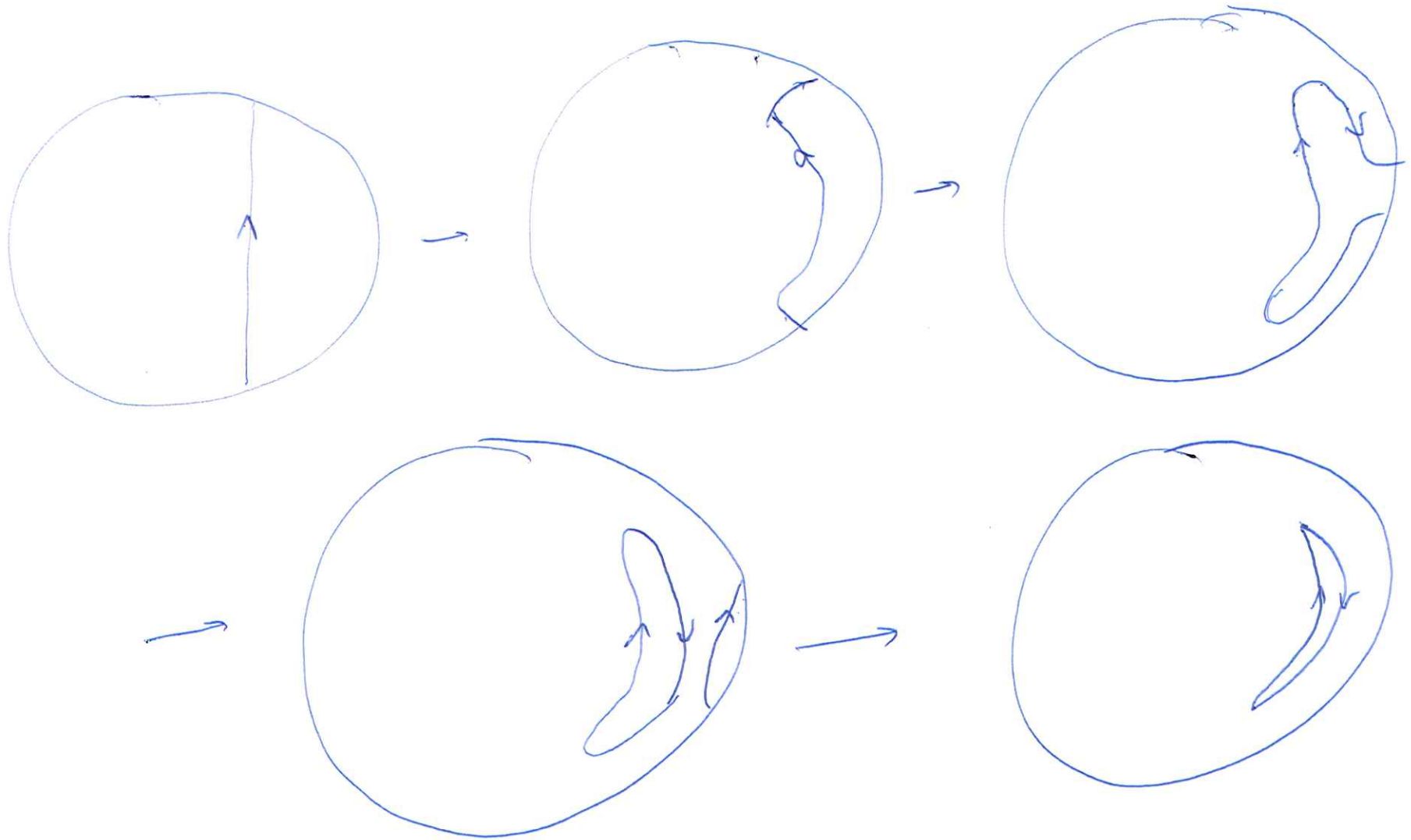
* Corollary as $|\vec{V}_s - \vec{V}_{star}|$ increases above V_{cr} ,
vortex mobility will bring it down to V_{cr} .
at that location. So once $|\vec{V}_s - \vec{V}_{star}|$ reaches
 V_{cr} , it stays at that value.

* Corollary. fully prescriptive scenario for vortex
motion, since each vortex is specified uniquely
by the value of \vec{V}_s - circulation.

* Fact most models predict large variations
in $V_{cr}(r)$: crust-core interface, pasta-normal
lettice transition, etc.

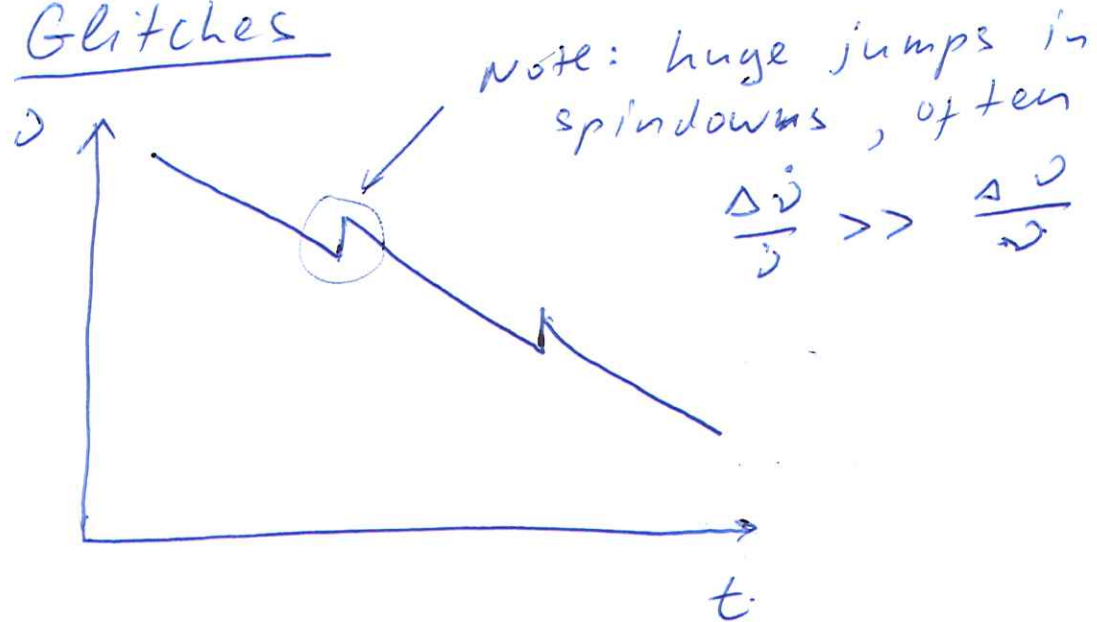
Large $\left| \frac{\partial V_{cr}}{\partial r} \right| \Rightarrow$ strong vortex concentration

Typical evolution.



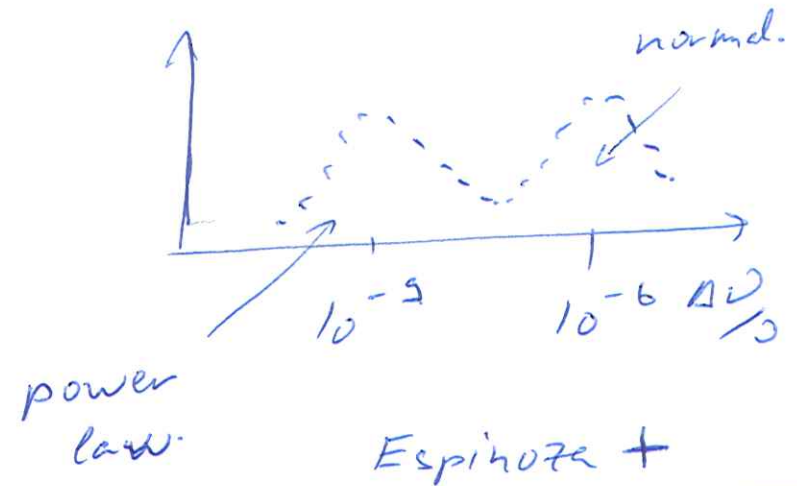
Formation of vortex loops around regions of high V_{cr} . Fast flow inside — "river".

Glitches



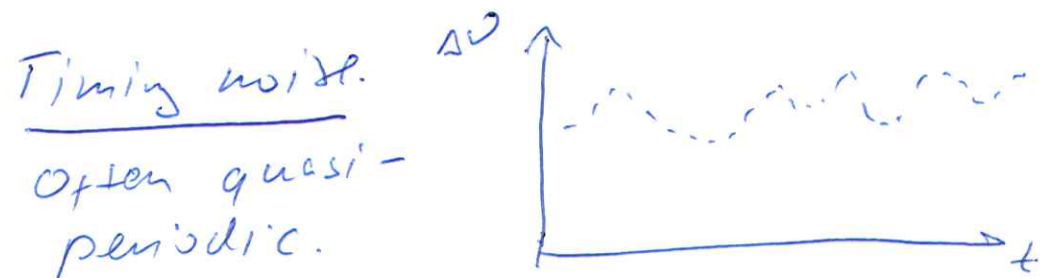
* Only young and middle-aged pulsars

* Distribution bimodal



* Suspected quake in Vela.
Paltreyman + 2016. Null
Need more data.

Bransgrove + theory.
- incomplete. Need more detail on discharge & Alfvén wave damping.



Magnetar glitches & antiglitches

Radiative change \Rightarrow timing features

timing features \nRightarrow radiative change.

Dib & Kaspi 2014.

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Lessons

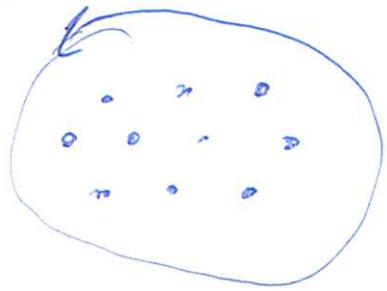
In axisymmetry, with simplest prescriptions for vortex motion.

- * Vortices are strongly bent.
- * Form loops with rivers inside
- * Strongly concentrated in regions of $\left| \frac{\partial v_{\phi}}{\partial r} \right|$ that are high, by factors $\sim 100 - 1000$ relative to expectations of steady rotation.

Caveats

- * Lith & Levin (2022) show that unpinning - repinning is complex, stochastic process with large hysteresis.
- * Thermal creep can destroy rivers. We are simulating it now.

View from above.



$$\oint \vec{V}_s \cdot d\vec{r} = \frac{2\pi\hbar}{m} N$$

If SF rotates
homogeneously,
effective angular
velocity.

$$\Omega = \frac{\oint \vec{V}_s \cdot d\vec{r}}{2(\text{Area})} = \frac{\pi\hbar}{m} n$$

where $n \equiv \frac{N}{\text{Area}}$ is

the density of vortices.

SF vortices are physical
objects

- * They feel attraction or repulsion to lattice nuclei (people still argue which one)
- * They might interact with fluxtubes in the wire.
- * If vortices are fixed in space by interactions — "pinned" then the circulation pattern of \vec{V}_s is fixed.
- * Corollary: pinned superfluid cannot be spun down.
- * Important other force:
Magnus force.

Superfluid Rivers.

Yuri Serin.

(Columbia.
Flatiron)

Normal Fluid.

Vorticity: $\vec{\omega} \equiv \nabla \times \vec{v}$

Taylor - Proudman

Theorem:

unstratified fluid
in steady state
rotates "on cylinders"

$$v_{\phi} = v_{\phi}(\tilde{\rho})$$

cylindrical
radius

$$v_{\tilde{\rho}} = v_{\theta} = 0$$

Superfluid.

$$\vec{v}_s = \frac{\hbar}{m} \nabla \phi \quad \phi - \text{condensate phase.}$$

$$\Rightarrow \nabla \times \vec{v}_s = 0$$

$$\Rightarrow \text{circulation } \oint \vec{v}_s \cdot d\vec{r} = 0$$

Right?

Wrong. Superfluid rotates

by forming a dense array
of vortices, inside of which
superfluidity breaks down.

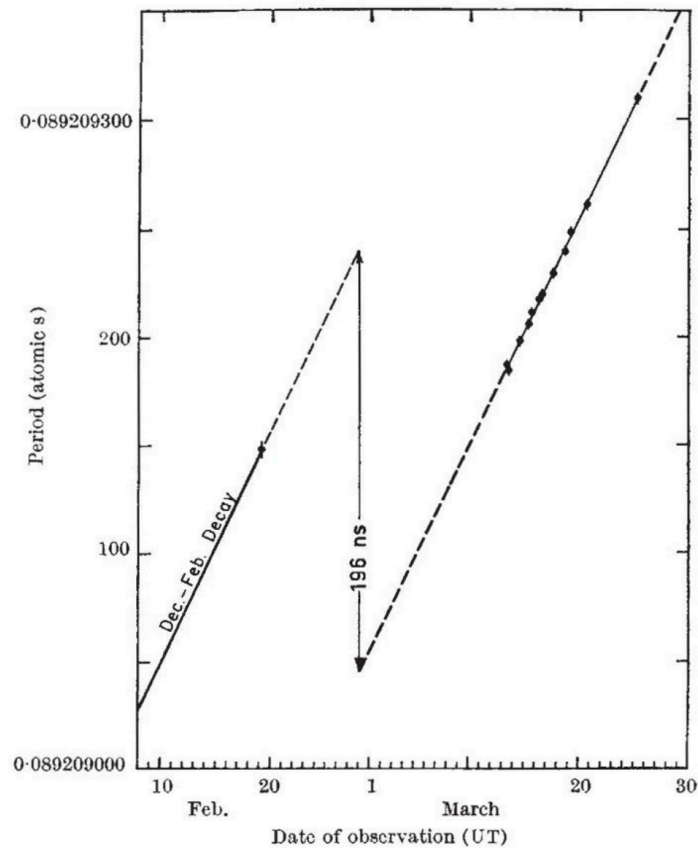
$$\oint_{\text{vortex}} \vec{v}_s \cdot d\vec{r} = \frac{2\pi\hbar}{m} \quad \text{around a single vortex}$$



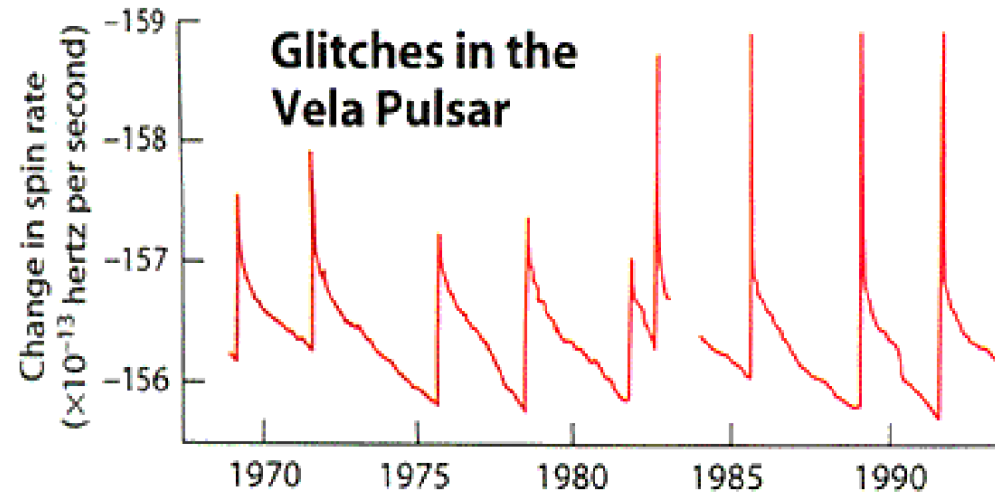
Ensures that phase
changes by 2π

2. Unpinning and repinning superfluid vortices in a neutron star crust

The first glitch in Vela



Radhakrishnan & Manchester 1969



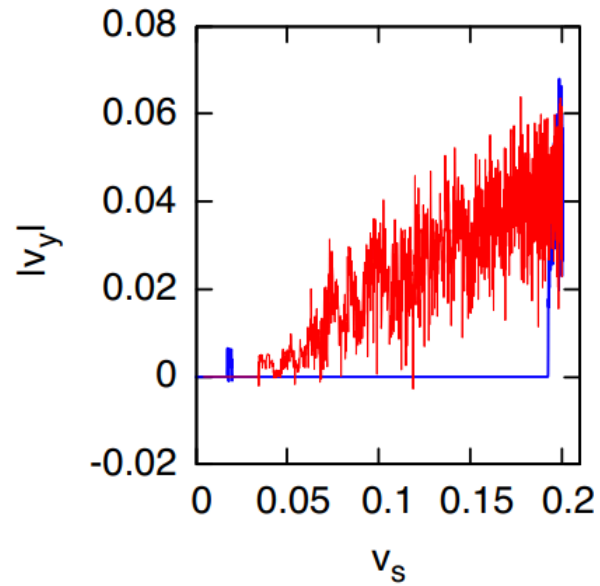
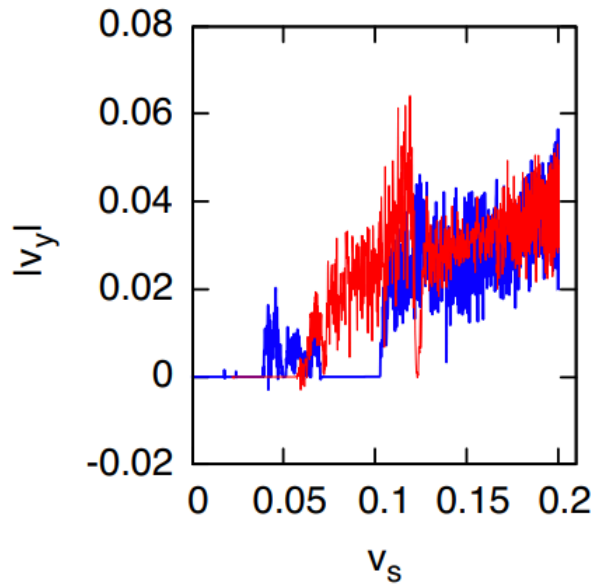
A.G. Lyne, F. Graham-Smith

- Sudden change in spin frequency and/or frequency derivative
- Probably due to the sudden unpinning of many ($\sim 10^{11}$) superfluid vortices inside the neutron star

Vortex Pinning in Neutron Stars, Slip-stick Dynamics, and the Origin of Spin Glitches

BENNETT LINK¹ AND YURI LEVIN^{2,3,4}

Main results



- Hysteresis
- Repulsive regular lattice the worst for pinning
- Attractive irregular or glass the best
- Vortex tangle
- Some ideas for large-scale avalanche of unpinning

