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MHD-PIC Simulations of Cosmic-ray Gyro-resonant instabilities

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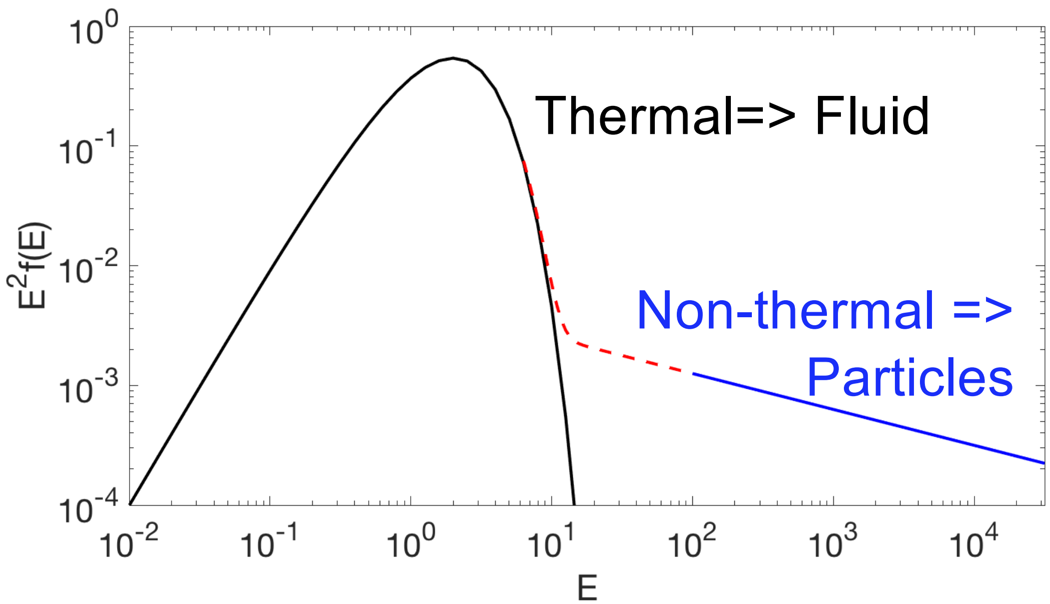
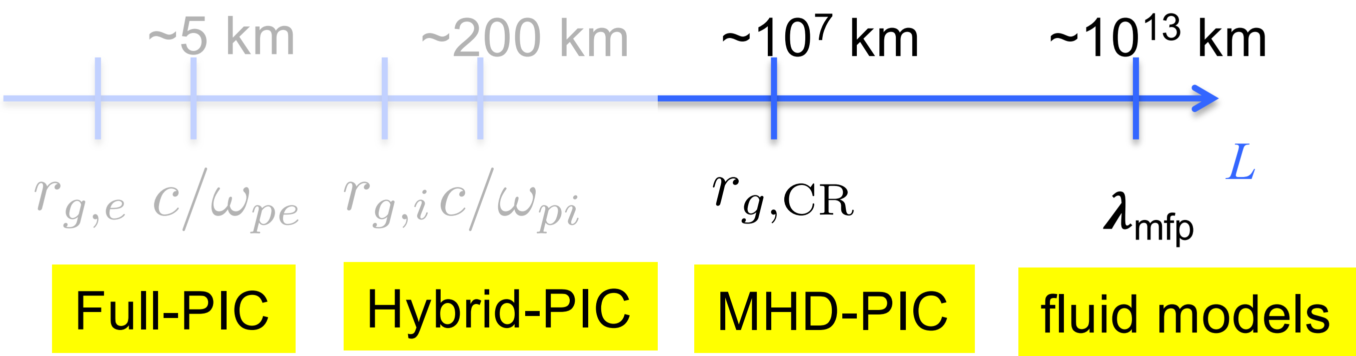
MHD-PIC method: the philosophy



Freshly accelerated, or pre-existing.

Generally required to resolve R_g.



The MHD-PIC method: formulation

Equations for the (relativistic) CR particles:

$$\frac{d(\gamma_j \boldsymbol{u}_j)}{dt} = \frac{q_j}{m_j}\left(\boldsymbol{E} + \frac{\boldsymbol{u}_j}{c} \times \boldsymbol{B}\right)$$

Specify numerical speed of light c >> any MHD velocity.

Full equations for the gas:

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \mathbf{P}^*) =$$ - Lorentz force on the CRs

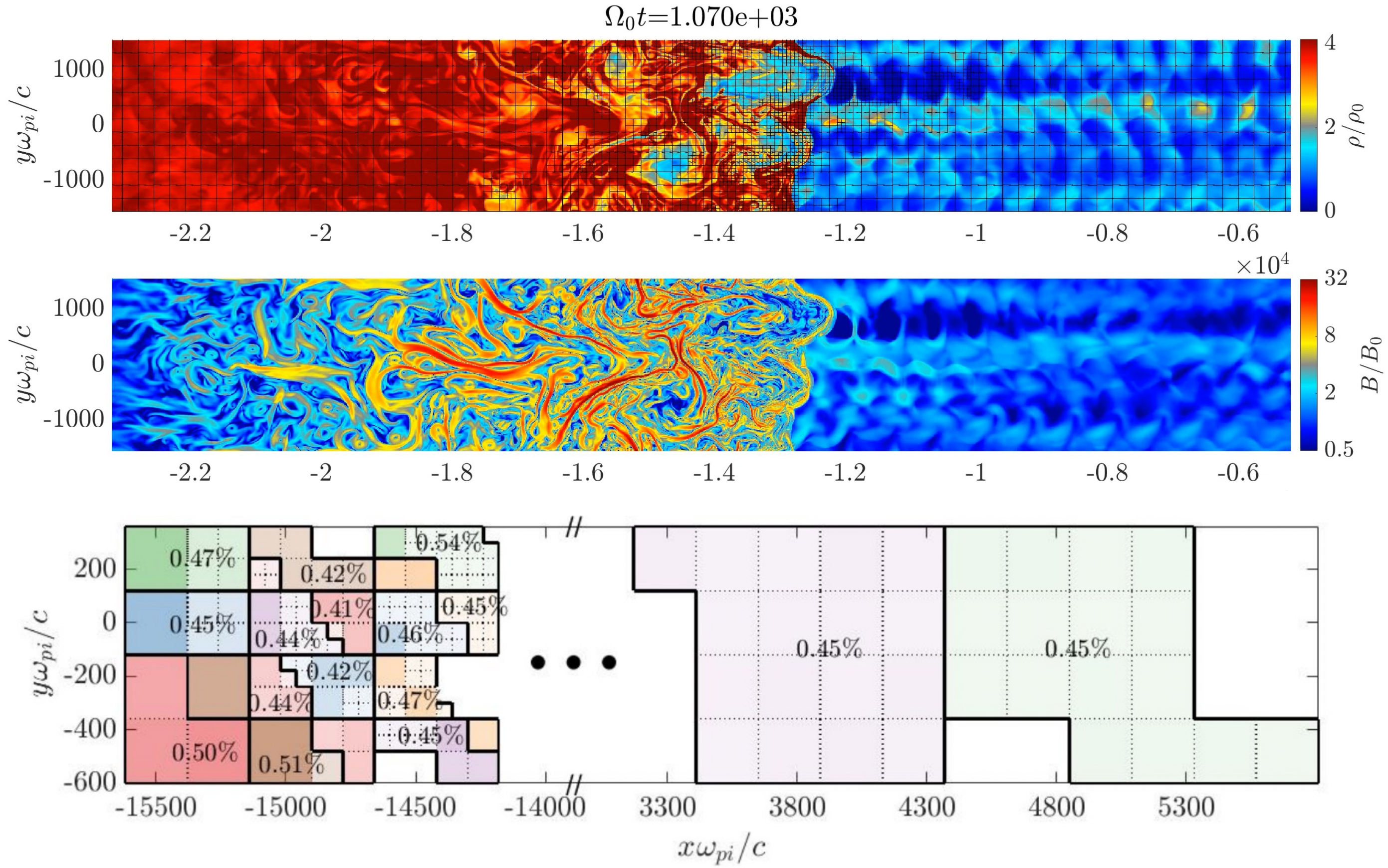
$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*)\boldsymbol{v} - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})] =$$ - energy change rate on the CRs

Known implementations in Athena (Bai, Caprioli, Sironi & Spitkovsky 2015), Pluto (Mignone et al. 2018), MPI-AMRVAC (van Marle et al. 2018), Athena++ (Sun & Bai 2023).

MHD-PIC in Athena++

Xiaochen Sun

2D simulation of a parallel non-relativistic parallel shock:



Trigger the Bell instability, leading to particle acceleration.

(artificial injection at shock front).

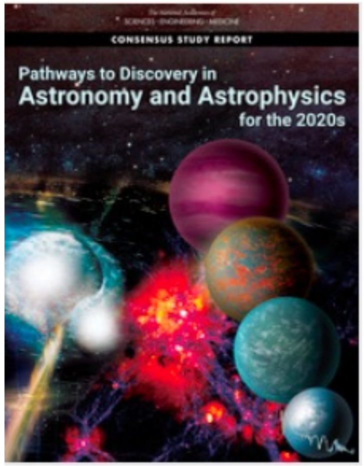
More flexible, efficient (with vectorization), and has AMR.

Sun & Bai (2023)

Cosmic ecosystem: the role of CR feedback

The impact of cosmic rays is one of the largest uncertainties in understanding feedback in galaxy formation.

The primary uncertainty is how CRs are scattered by small-scale fluctuations in the magnetic field, which sets whether CRs can escape a region or whether their pressure builds up to the point where it can drive an outflow.



Rate of pitch angle scattering:

$$\nu_s \sim \Omega \left(\frac{\delta B}{B} \right)^2$$

At resonant wavelength

For small δB, known as *quasi-linear diffusion*

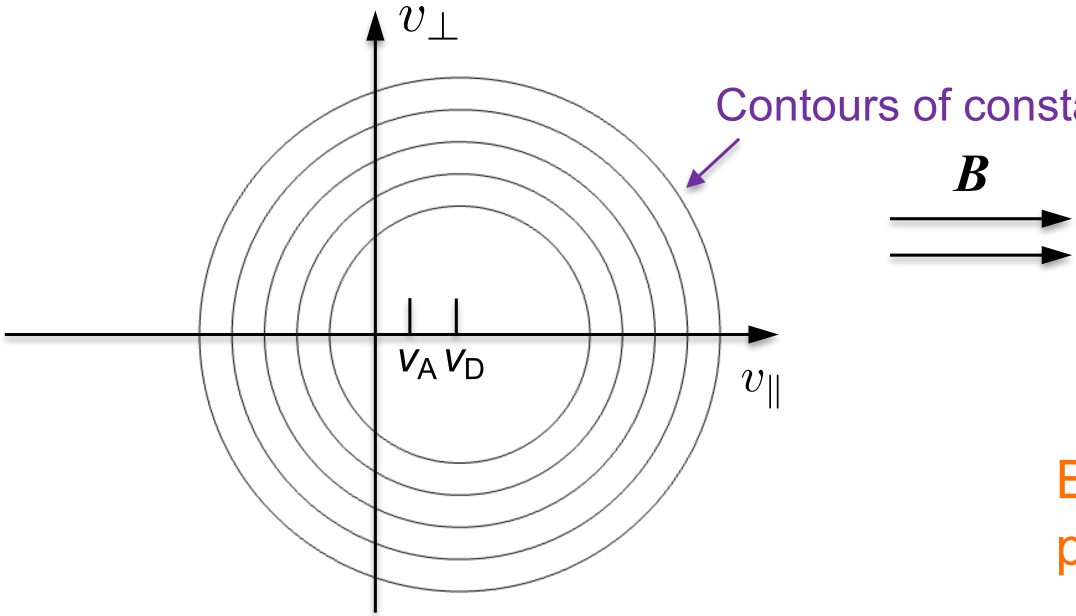
CR mean free path: $l_{\rm mfp} \sim c/\nu_{\rm s}$, diffusion coefficient: $\kappa \sim c^2/\nu_{\rm s}$

CR gyro-resonant instabilities

When the level of CR anisotropy exceeds ~v_A/c, CRs resonantly excite Alfvén waves. (see book by Kulsrud 2005)

Classical case: CR streaming instability (Kulsrud & Pearce 1969)

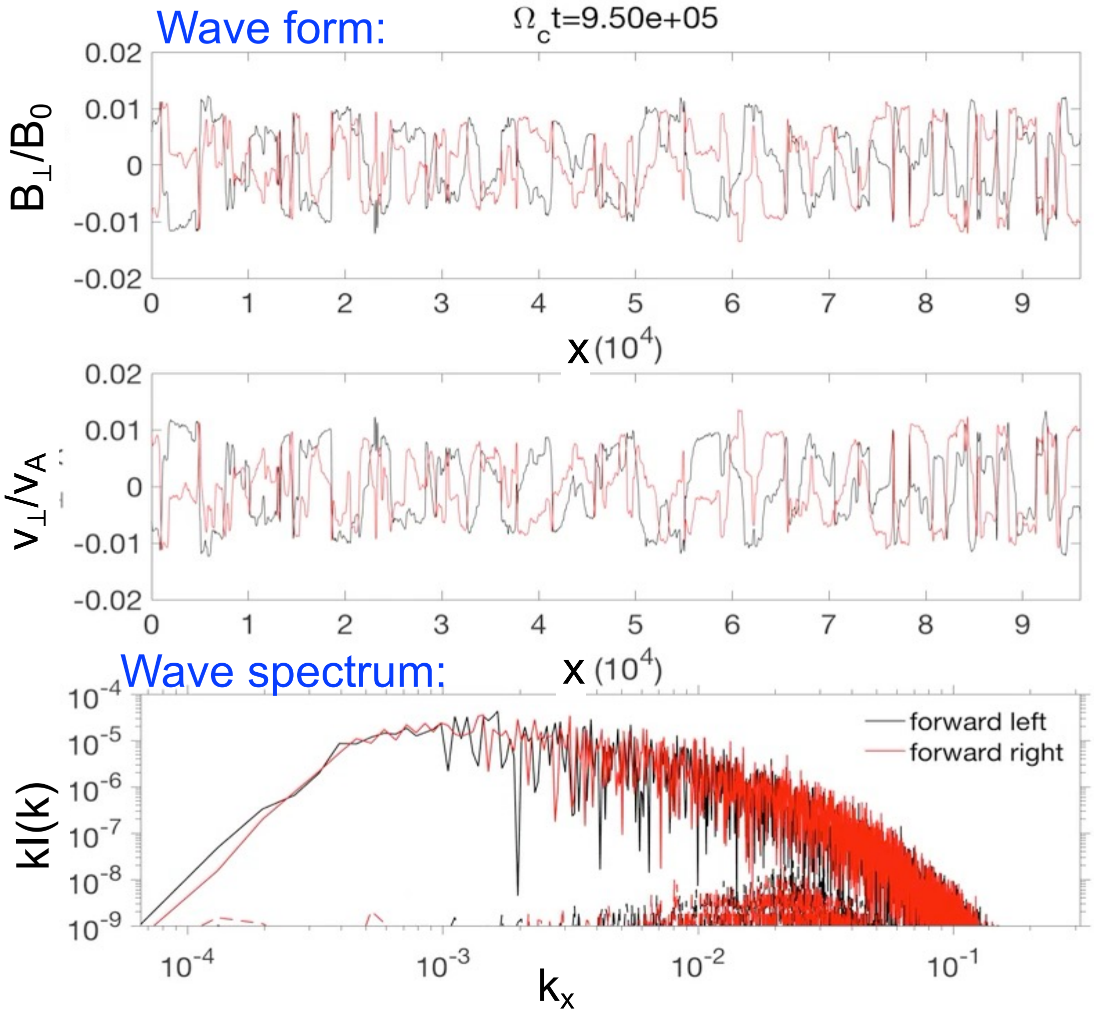
Triggered when the bulk CRs drift speed relative to gas is faster than v_A



CR streaming naturally arises when CRs escape from the source.

Excite forward-traveling waves (two polarizations) down the CR gradient.

1D simulation: growth and saturation



Periodic BC in the rest frame of the CRs.

Gas travels to the left at v_D.

A κ-distribution of CRs peaking at $p=p_0$.

Fiducial parameters:

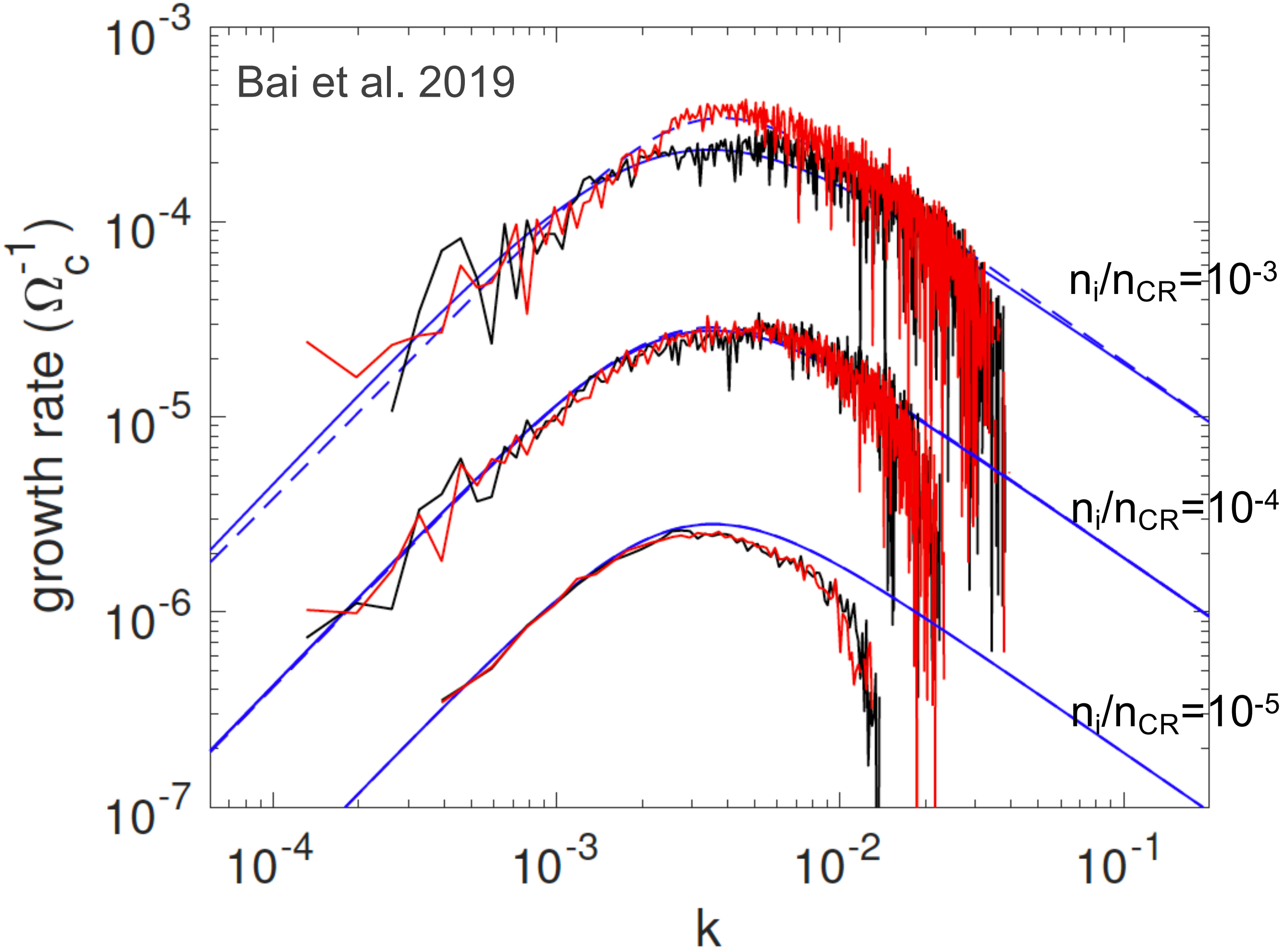
$v_D=2v_A$

$N_{CR}/n_0=10^{-4}$

$p_0/m=300v_A$

Bai, Ostriker, Plotnikov & Stone, 2019

Matching analytical dispersion relation



Accurately reproduce the linear growth rate over broad spectrum.

Need δf method to reduce noise.

PIC simulations captures maximum growth rate, but hardly over broad range of k (Holcomb & Spitkovsky 19)

Towards saturation: quasi-linear diffusion

$$\frac{\partial f_w}{\partial t} = \frac{\partial}{\partial \mu_w}\left[\frac{1-\mu_w^2}{2}\nu(\mu_w)\frac{\partial f_w}{\partial \mu_w}\right]$$ + reflection

Scattering frequency:

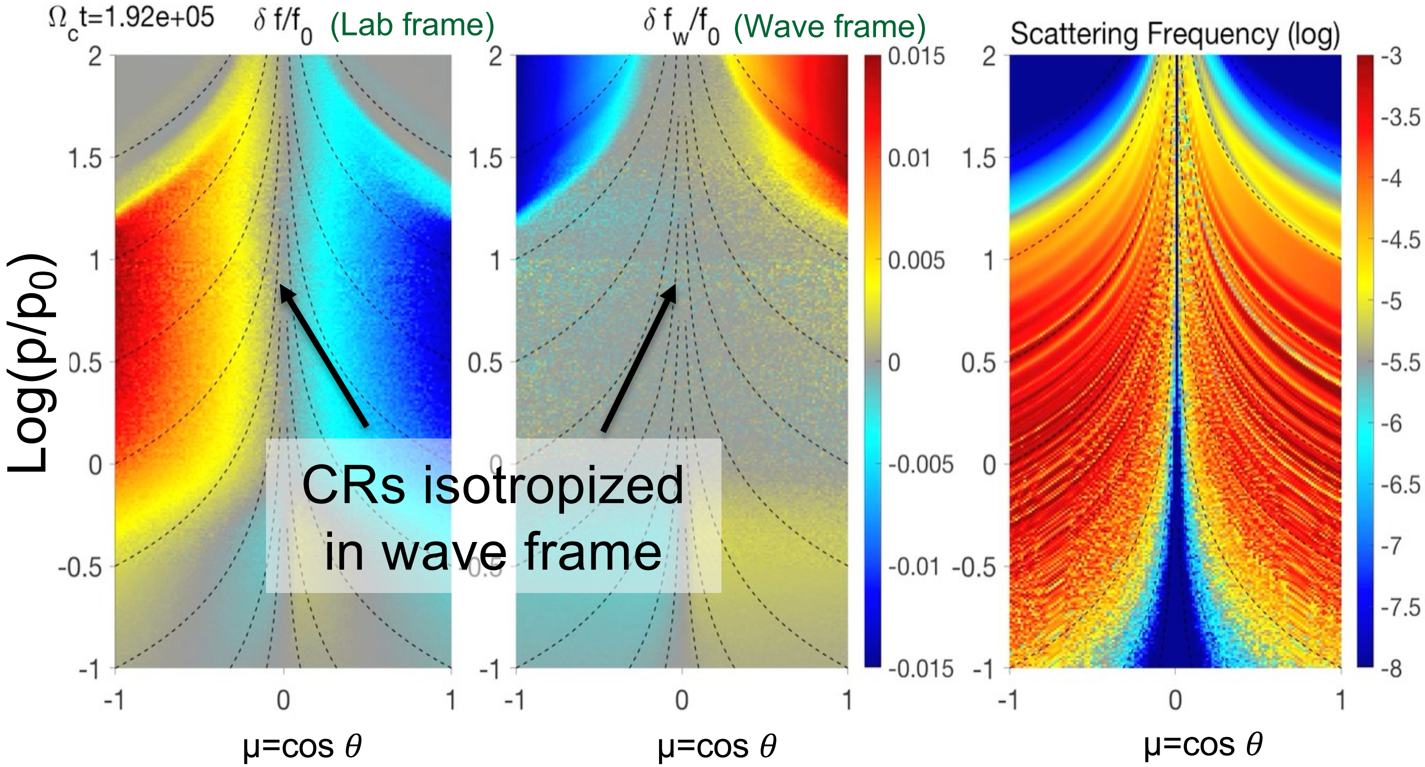
$$\nu(\mu_w) = \pi\Omega k_{\rm res} I(k_{\rm res})$$

wave intensity

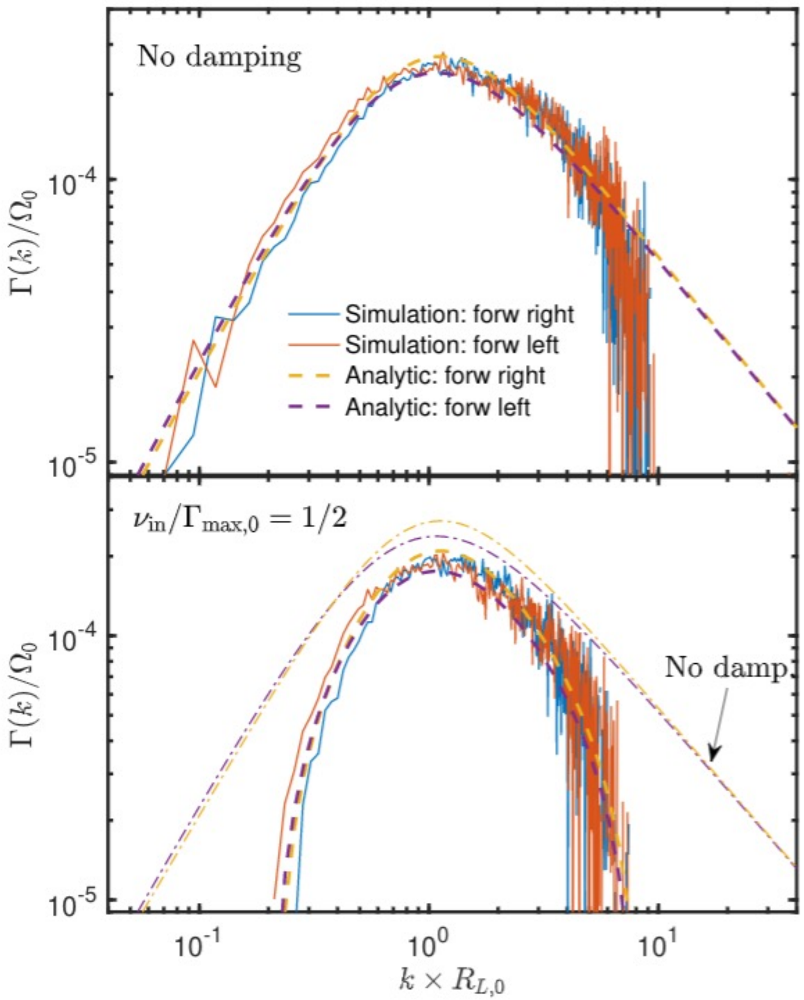
Parameters:

$v_D=2v_A$

$N_{CR}/n_0=10^{-3}$



Effect of ion-neutral damping: linear growth



Ion-neutral damping dominates in the neutral phase of ISM & molecular clouds.

In the high-frequency regime:

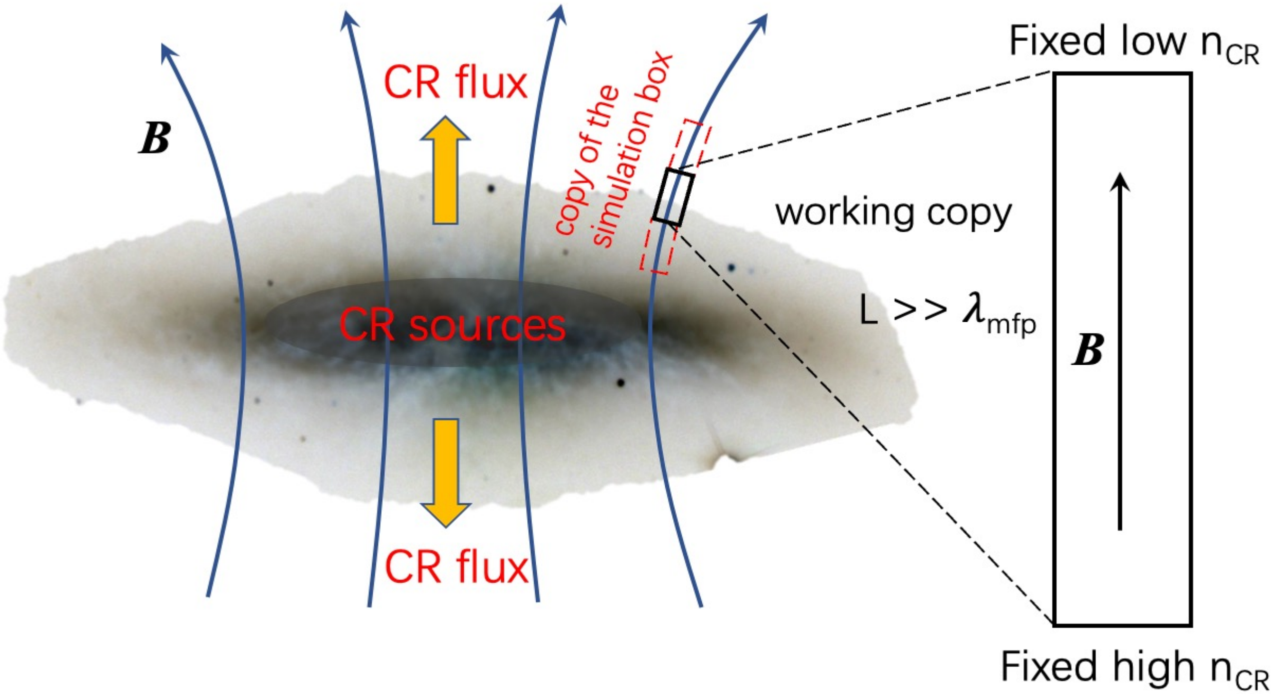
$$\Gamma_{\rm tot}(k) = \Gamma_{\rm CR}(k) - \frac{\nu_{\rm in}}{2}$$

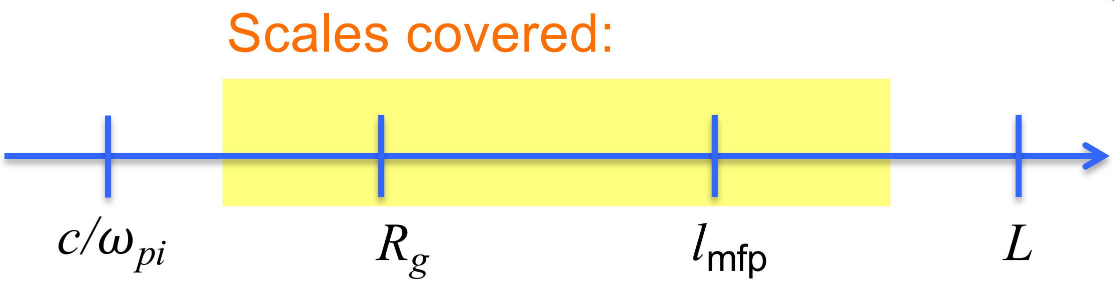
Reduced growth rate & instability bandwidth.

Caveat: other damping mechanisms need the kinetic physics of background MHD.

Plotnikov, Ostriker & Bai, 2021

Streaming box: a multi-scale sim. framework





Sustained CR pressure gradient to drive instability.

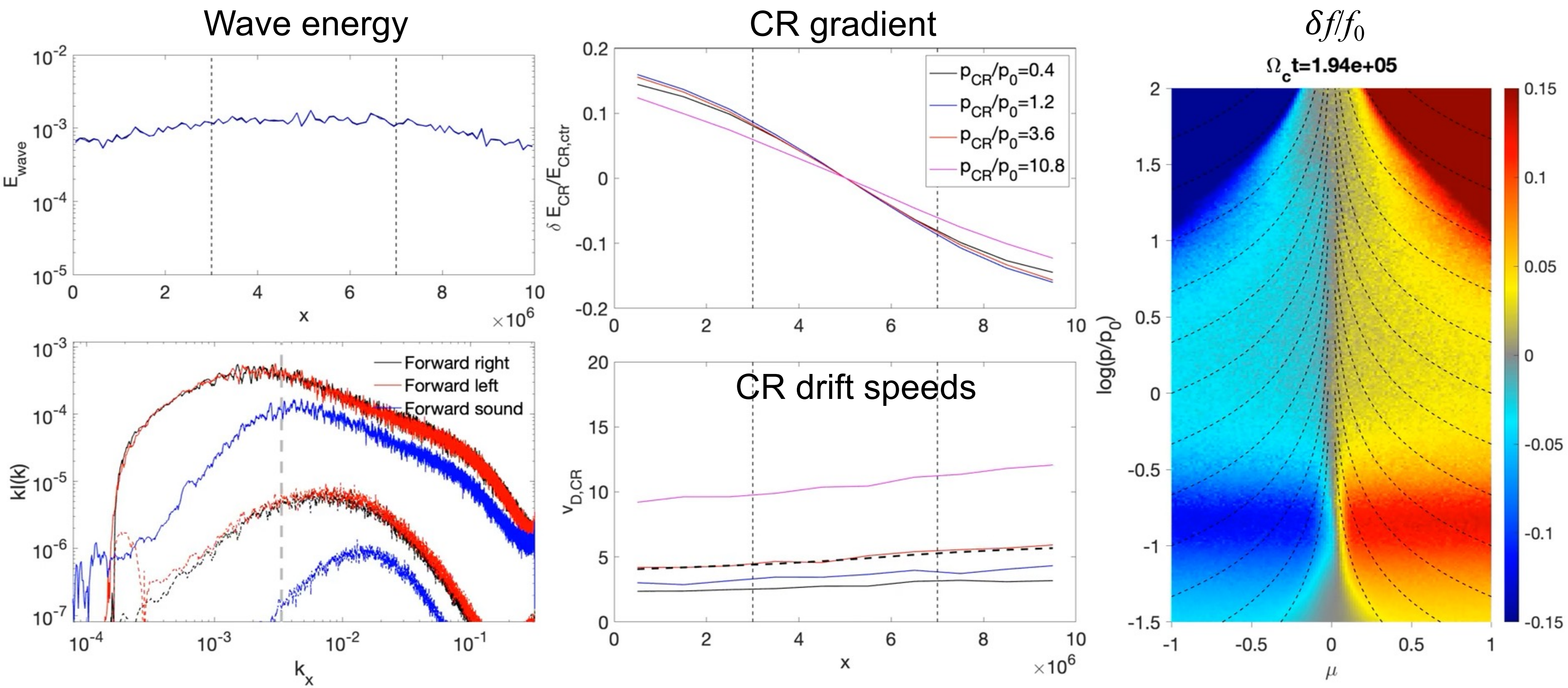
(See also Bambic+2021 where a transient CR gradient is realized)

Saturation amplitude achieved by balancing driving and damping.

Bai, 2022

Saturation towards a steady state

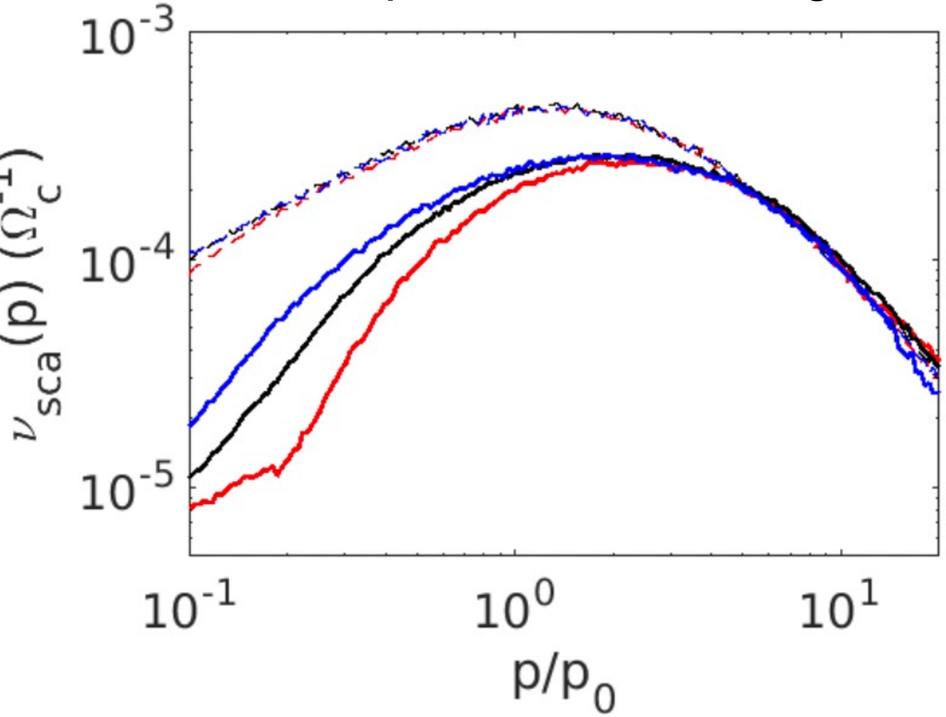
Bai, 2022



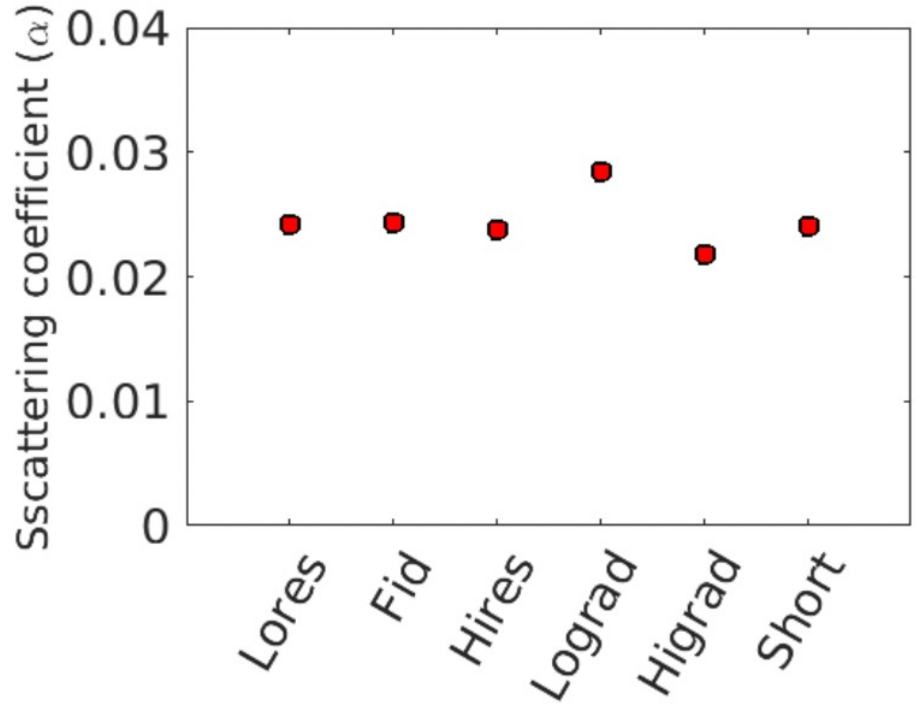
Long box ($10^7 d_i$), $N_{CR}/n_0=10^{-4}$, $p_0/m=300v_A$, with IN damping ($10^{-4}\Omega_c$).

Measuring scattering rates

Momentum-dependent scattering rate



Momentum-integrated (fluid) scattering rate



Bai, 2022

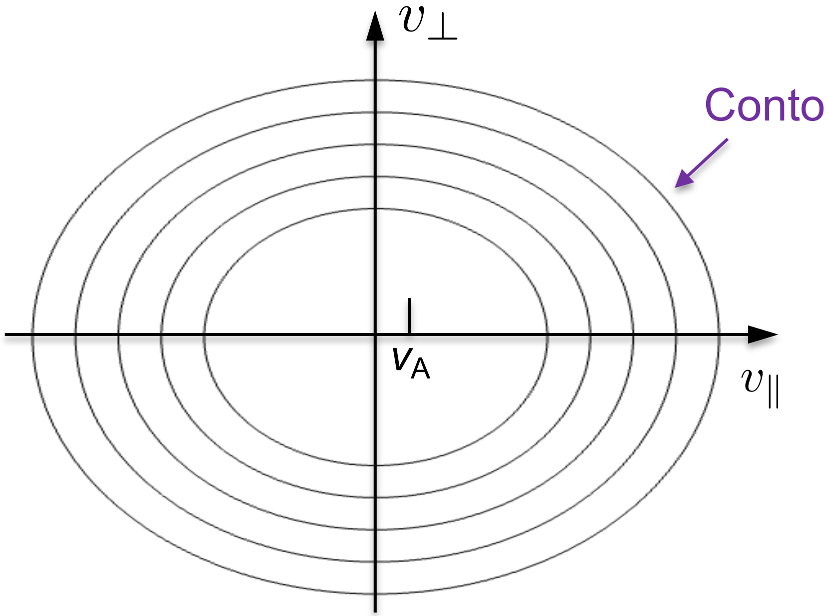
$$\nu_{\rm sca}^{\rm eff} = \alpha_{\rm sca}\Omega_c \frac{\mathbb{C}^2}{v_A \nu_{\rm in} L_{\rm CR}} \left(\frac{n_{\rm CR}^{\rm ctr}}{n_i}\right)$$

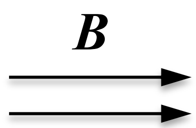
Gyro-resonant instability by CR anisotropy

When the level of CR anisotropy exceeds ~v_A/c, CRs resonantly excite Alfvén waves. (see book by Kulsrud 2005)

Less studied case: (Lazarian & Beresnyak 2006, Lebiga+2018, Zweibel 2020)

Gyro-resonant instability by CR pressure anisotropy

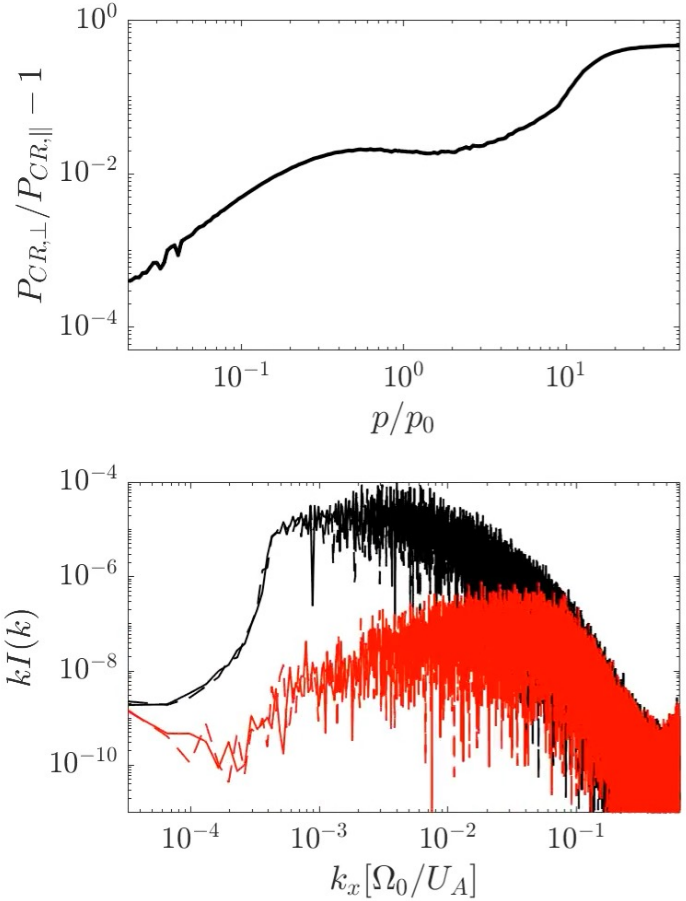


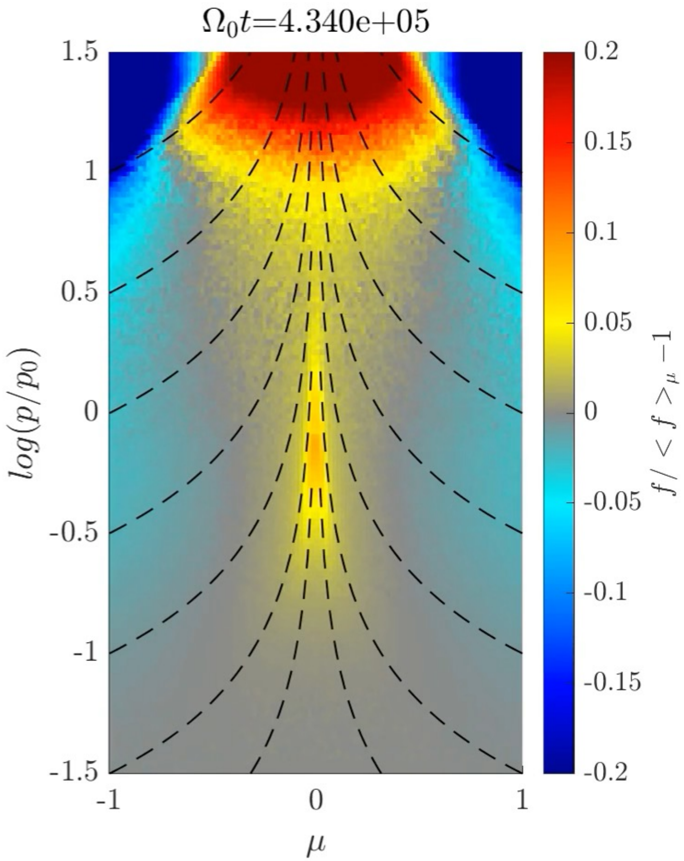


CR anisotropy naturally arises when background gas expands/compresses.

More generally, where there is $\dot{B}$. Excite both forward and backward traveling waves.

Driving CR anisotropy in an expanding box



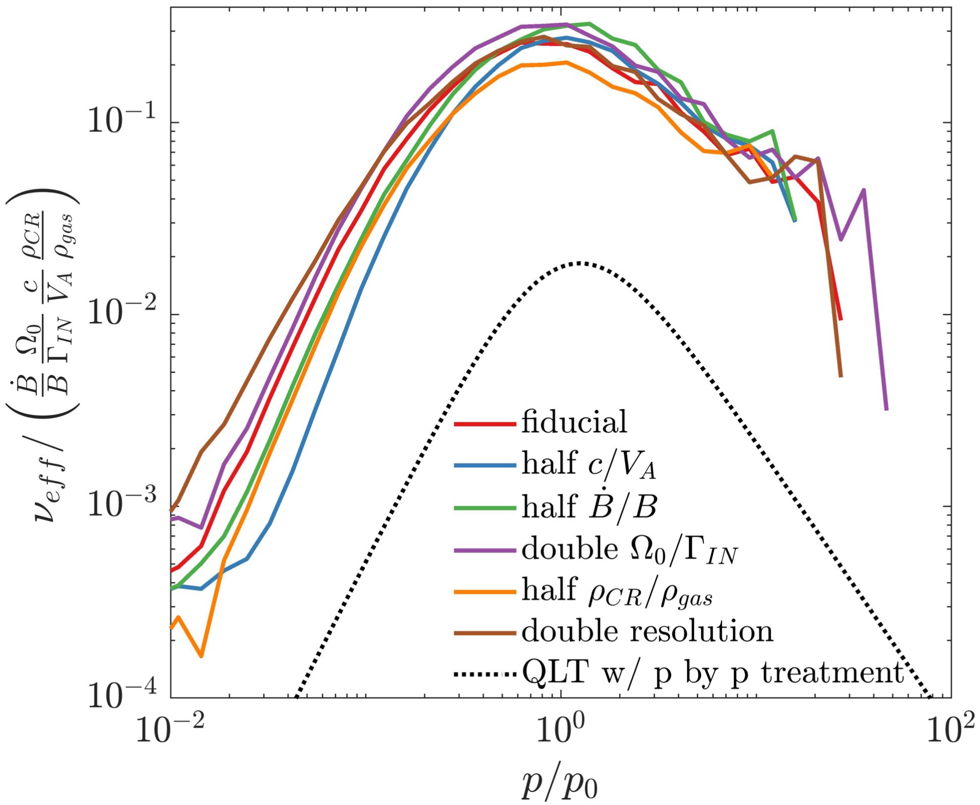


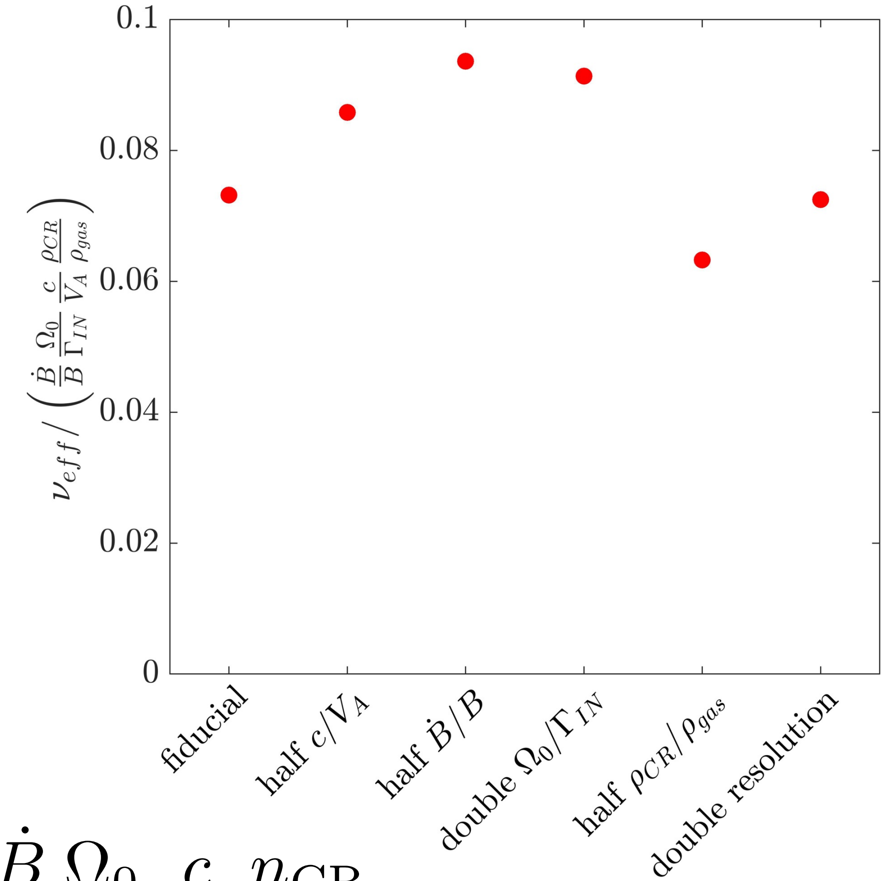
(Sun & Bai, in prep)

Using Athena++, with expanding box, driving the CR pressure anisotropy instability.

Xiaochen Sun

Measuring scattering rates





(Sun & Bai, in prep)

$$\nu = \alpha \frac{\dot{B}}{B} \frac{\Omega_0}{\nu_{\rm in}} \frac{c}{v_A} \frac{n_{\rm CR}}{n_i}$$

Piecing together

Goal: provide microphysical prescription of CR scattering rates.

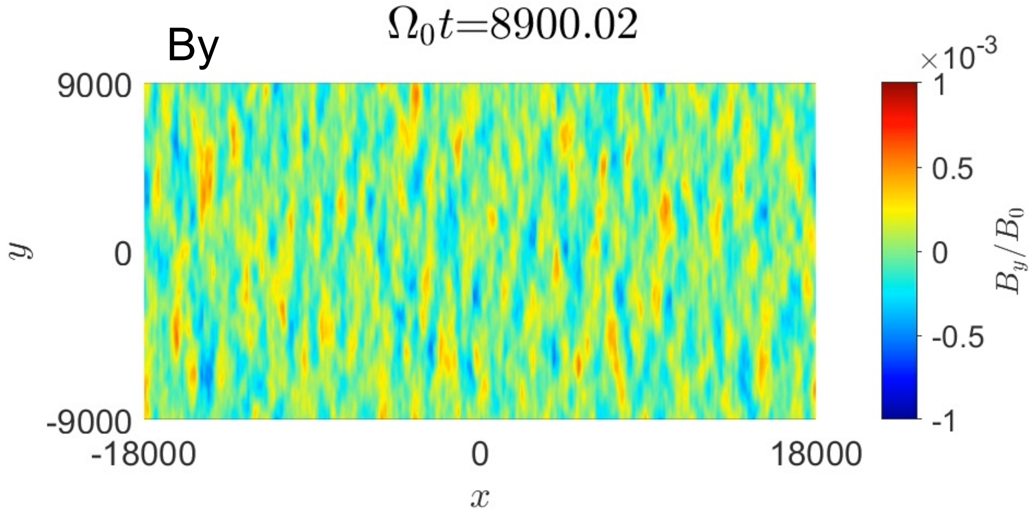
Currently, we can provide the scattering rates under the following cases:

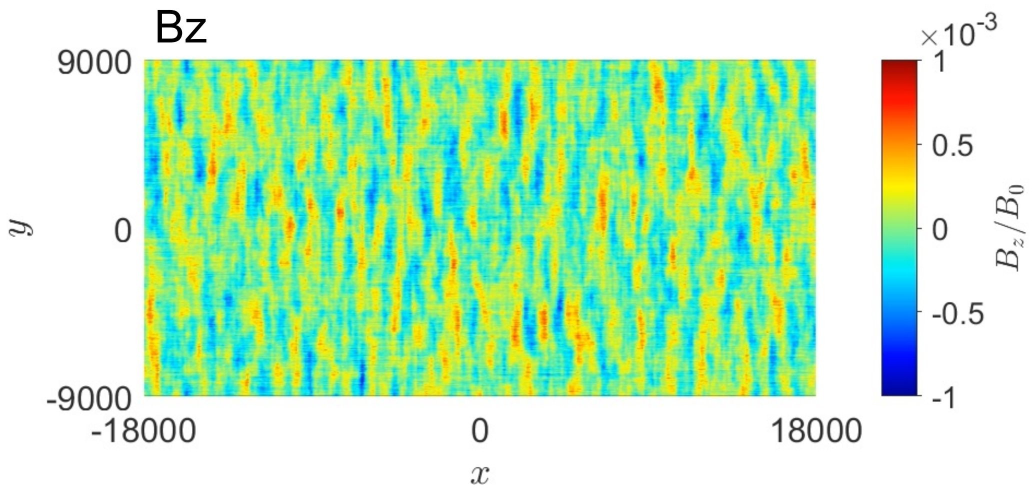
- Pure streaming (as a function of CR pressure gradient)
- Pure expansion/compression (as a function of $\dot{B}/B$)

Missing:

- The general mixed cases (there are both CR gradient and $\dot{B}$).
  - MHD-PIC + streaming box + expanding/compressing box.
- Damping dominated by other mechanisms (e.g., nonlinear Landau).
  - Replace background MHD by multi-fluid with closure?

Extension to multi(2)-D: linear growth





The dispersion relation gets very complex…

$$\frac{\Gamma_i}{\Omega_c} = \left(\frac{k_\parallel v_d}{\omega} - 1\right)\frac{n_{\rm CR}}{n_i}\frac{2\sqrt{\pi}}{\kappa^{3/2}}\frac{\kappa+1}{\kappa}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\cdot$$

$$\sum_{n=-\infty}^{+\infty}\frac{1}{|\tilde{k}_\parallel|}\int_{|n|\tilde{p}_{res}}^{+\infty} \mathrm{d}\tilde{p}\,\tilde{p}\left[1+\frac{\tilde{p}^2}{\kappa}\right]^{-(\kappa+2)}\cdot$$

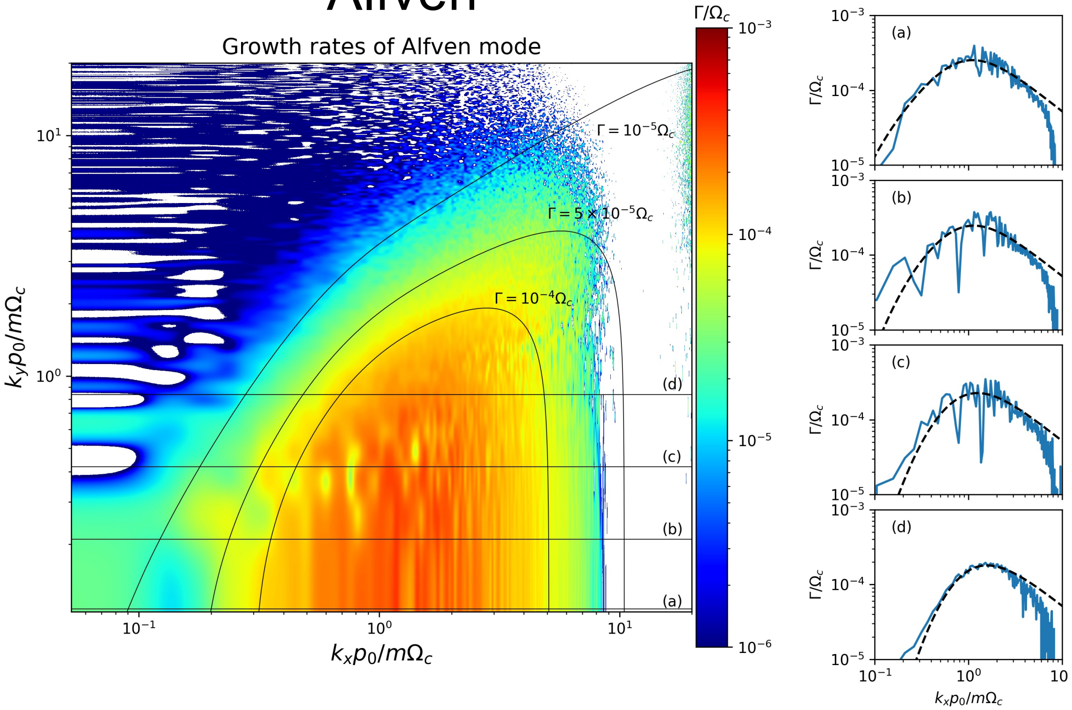
$$\begin{array}{ll} \text{Alfvén} & \left\{\begin{array}{c} \frac{n^2}{\tilde{k}_\perp^2}J_n^2\left(\tilde{k}_\perp\sqrt{\tilde{p}^2-n^2\tilde{p}_{res}^2}\right) \\ (\tilde{p}^2-n^2\tilde{p}_{res}^2)\left[J_n'\left(\tilde{k}_\perp\sqrt{\tilde{p}^2-n^2\tilde{p}_{res}^2}\right)\right]^2\cos^2\alpha \\ (\tilde{p}^2-n^2\tilde{p}_{res}^2)\left[J_n'\left(\tilde{k}_\perp\sqrt{\tilde{p}^2-n^2\tilde{p}_{res}^2}\right)\right]^2\sin^2\alpha \end{array}\right\} \\ \text{Fast} & \\ \text{Slow} & \end{array}$$

Shuzhe Zeng

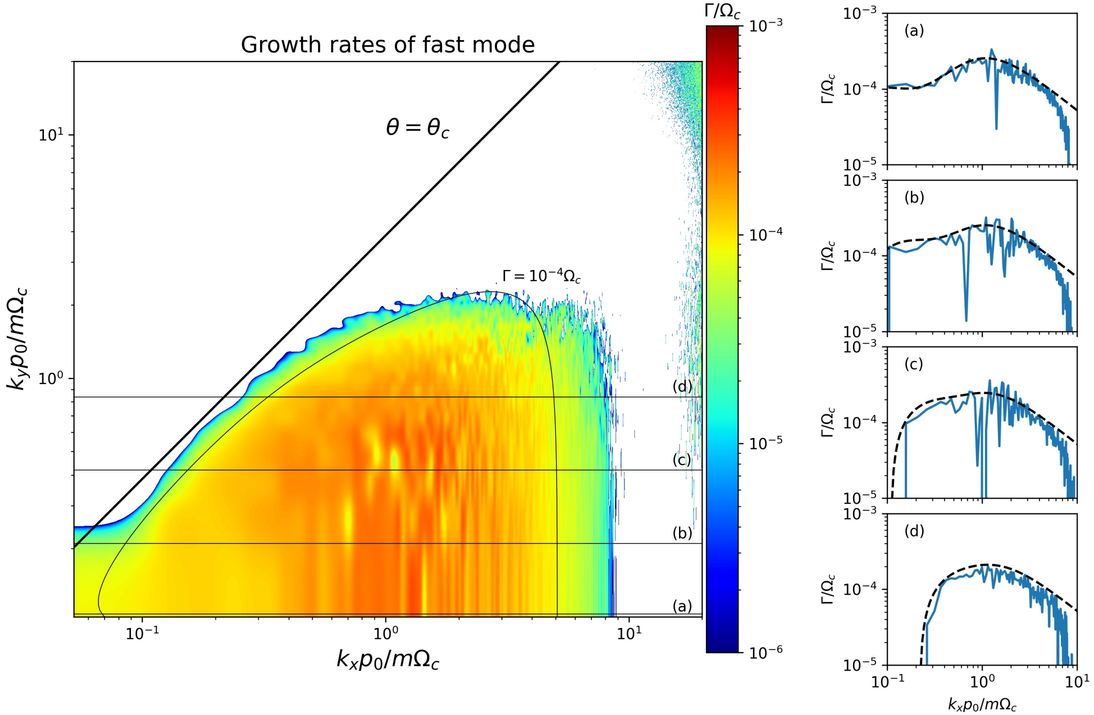
Zeng, Bai & Sun, in prep

Extension to multi(2)-D: linear growth

Alfvén



Fast

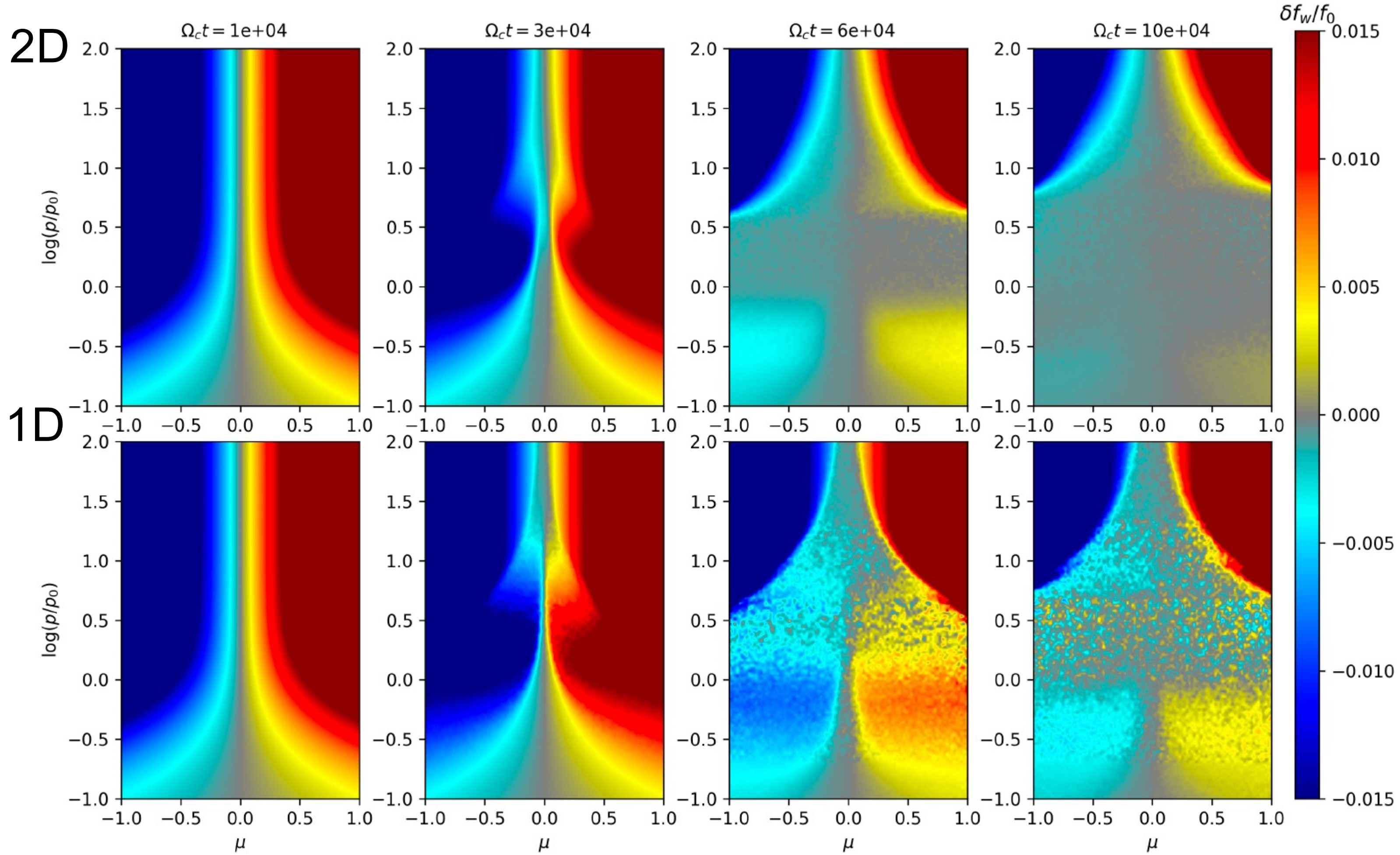


Growth of oblique fast modes dominated by Landau (n=0) resonance.

[likely alleviates the 90° problem]

Zeng, Bai & Sun, in prep

Extension to multi(2)-D: quasilinear evolution



With oblique modes, easier to cross the 90° barrier towards isotropization.

Zeng, Bai & Sun, in prep

Summary

- Motivation and development of MHD-PIC method
  - To study kinetic aspect of CRs interacting with background plasmas
  - PIC for CRs, MHD for background plasmas, valid on scales > ion skin depth.
- The microphysics: CR gyro-resonant instabilities + damping
  - Need δf method to accurately capture waves over broad wavelengths.
- Measuring CR scattering rates from MHD-PIC simulations
  - Streaming box: a general framework to for driving the CR streaming instability.
  - Expanding/compressing box: to drive the CR pressure anisotropy instability.
  - Multi-D effect: easier to cross the 90deg barrier, more isotropization.
  - Next: mixed streaming/pressure anisotropy, and more damping mechanisms.