Towards a unified picture of energy partition in unmagnetized collisionless shock waves

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Based on
Vanthieghem et al., submitted
Vanthieghem, Lemoine, Gremillet, ApJL, 2022

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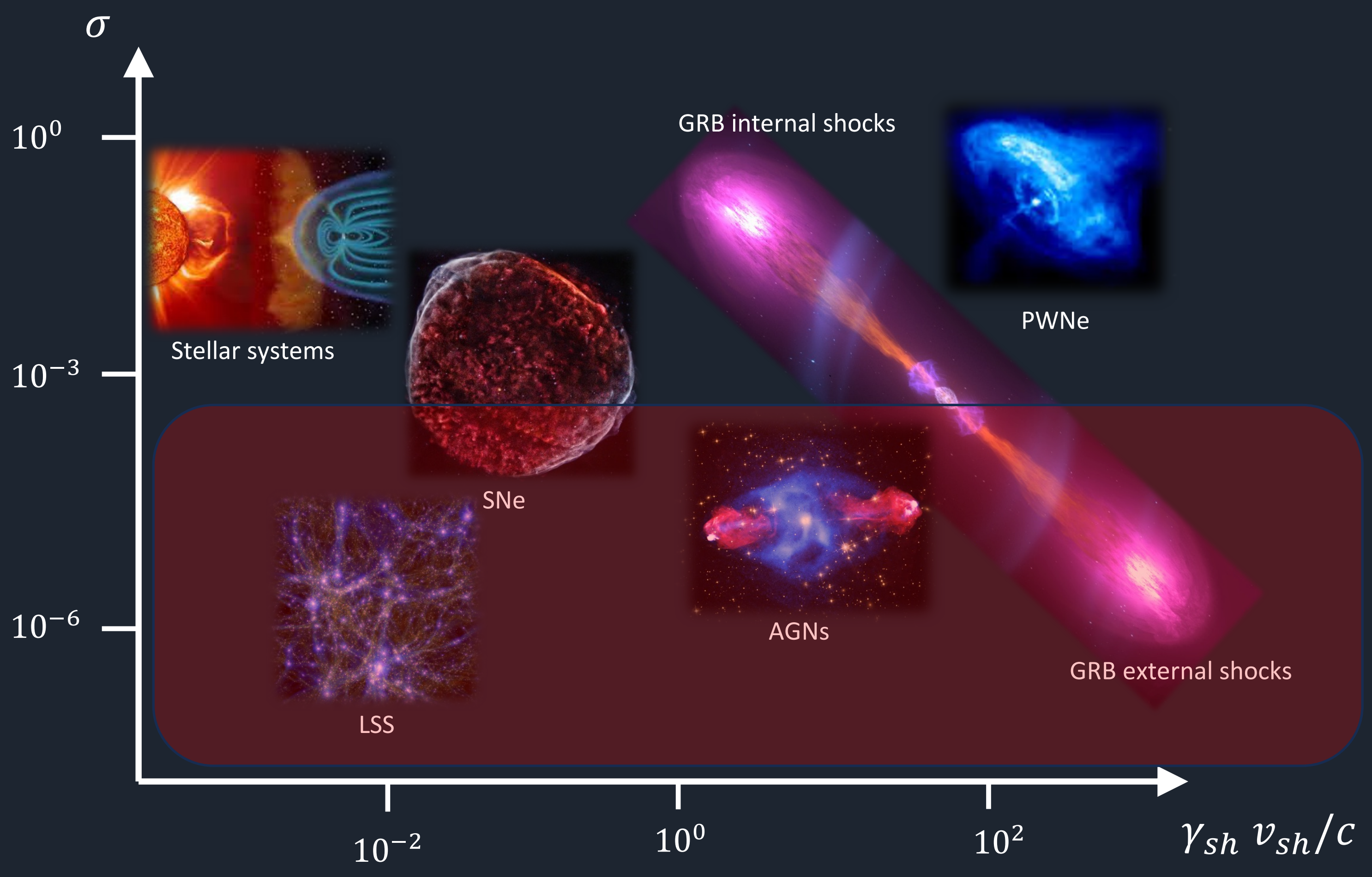
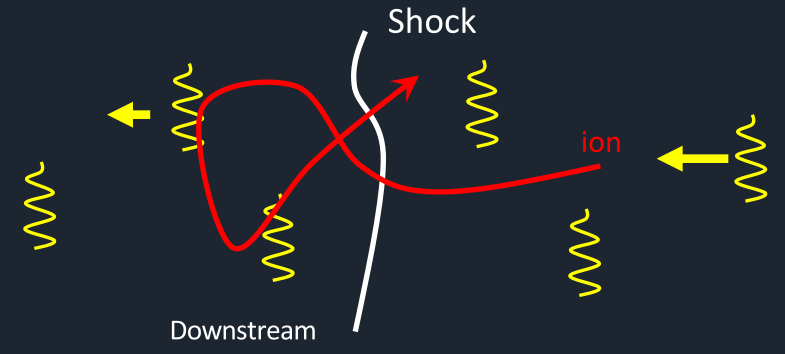
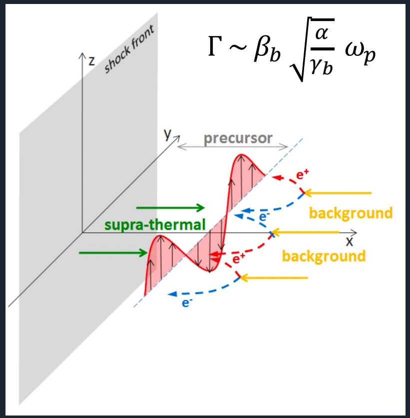
Magnetization and shock speed are key parameters to characterize the shock physics

$$\sigma \equiv \frac{B^2}{4\,\pi(\gamma-1)n\,m\,c^2}$$



Weakly magnetized shocks are dominated by magnetic modes advected towards the shock by the bulk plasma flow $M_A \gg 100$





Dominant instability

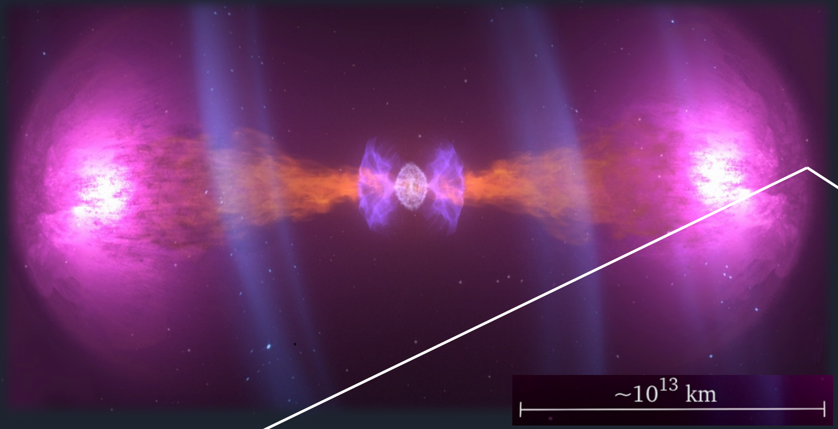
Plasma is coupled through the Weibel instability from the interplay between the background plasma and accelerated particles

Bret et al., 2010, Pelletier et al., 2017

Weibel, 1956, Brendan et al., 1975, Medvedev & Loeb, 1999

Equipartition between electrons and ions is observed in the ultra-relativistic regimes of Weibel-mediated shocks

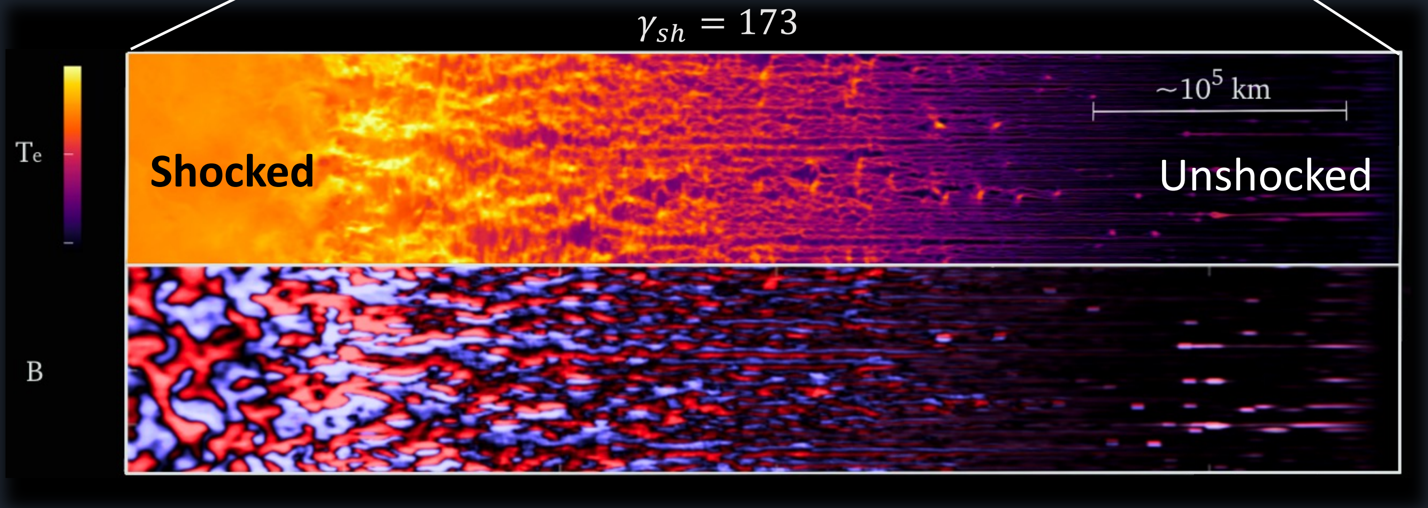
$\gamma_{sh} = 1/\sqrt{1-\beta_{sh}^2}$



Relativistic Weibel-mediated shocks efficiently heat the electrons up to close to equipartion, leading to efficient electron injection

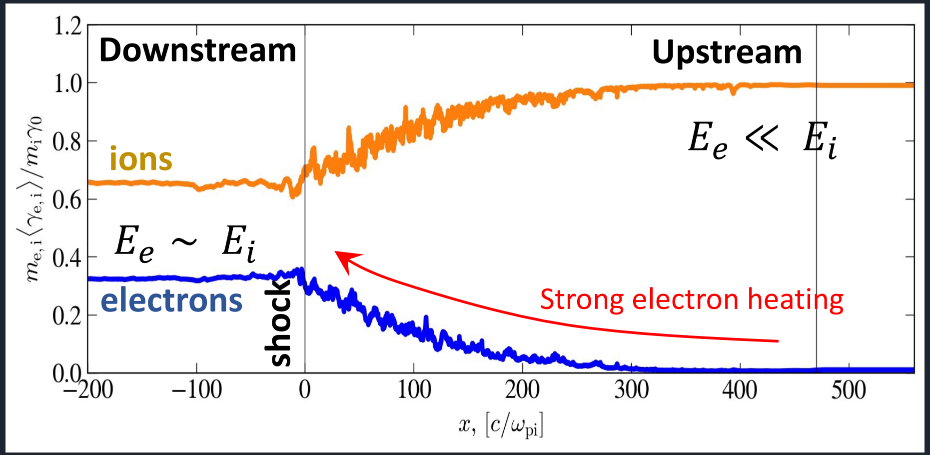
$$E_e \sim E_i \Rightarrow \langle \gamma_e \rangle \sim \frac{m_i}{m_e} \langle \gamma_i \rangle \sim 10 \text{ GeV}$$

- Modeling of gamma-ray burst emission[1]
- In kinetic simulations[2,3,4]



Particle-In-Cell simulation – (top) Temperature; (bot) B-field

Mean kinetic energy per particle



$\frac{m_e}{m_i} E_i$

[1]D. Freedman, E. Waxman ApJ 547, 192 (2001)

[2]A. Spitkovsky, ApJL 673, L39 (2008)

[3]S. Martins et al., ApJL 695, L189 (2009)

[4]T. Haugbolle, ApJL 739, L42 (2011)

High Alfvén Mach number collisionless blast waves efficiently heat electrons

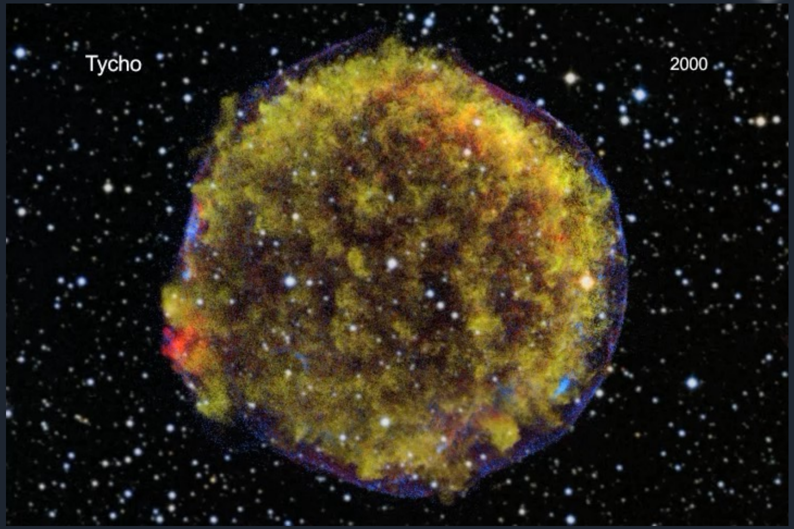
A temperature ratio of the order of unity is observed over a large range of shock velocities in high-Mach number shock waves

- From SNR shock waves observations[1]
- Via in-situ measurements[2]
- In kinetic simulations[3,4]

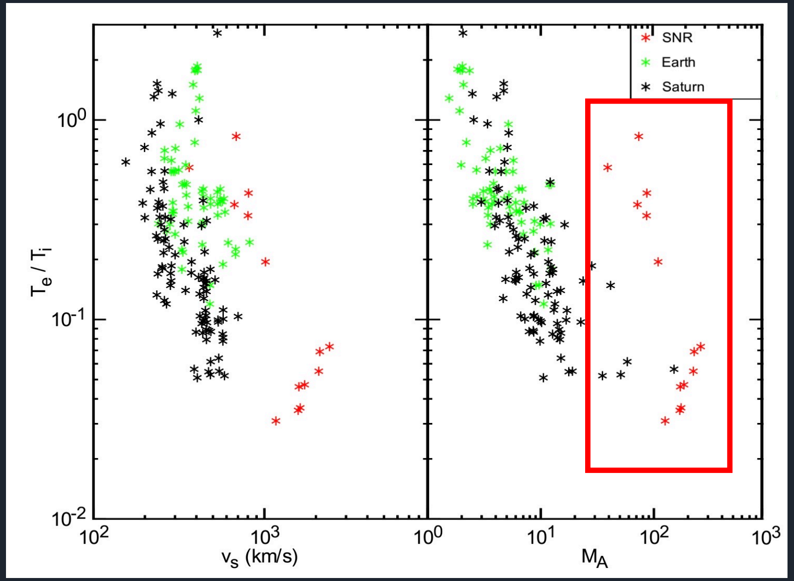
In all cases, different sources of electron heating have been identified:

- Inductive electric field[4,5]
- Break up of filaments[6]
- Secondary modes[7]
- Etc.

⇒ Need to extract a dominant contribution.



X-ray: NASA/CXC/GSFC/B.Williams et al; Optical: DSS

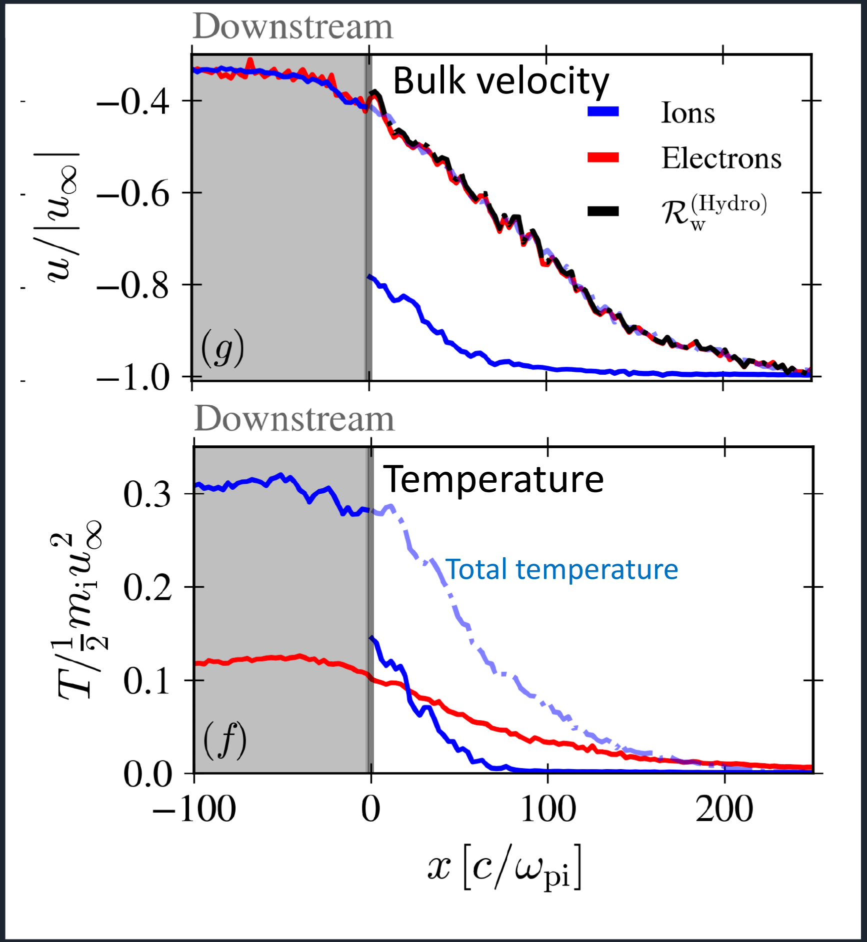


P. Ghavamian et al Space Sci. Rev. 178, 633 (2013)

[1] P. Ghavamian et al Space Sci. Rev. 178, 633 (2013)
[2] A. Johlander et al., GRL 50 (2023)
[3] T. Amano et al., ApJ 690, 244 (2009)
[4] A. Bohdan et al., PRL 126 (2021)
[6] M. Gedalin et al EPL 97, 35002 (2012)
[7] R. Kumar et al Astroph. J. 806, 2 (2015)
[6] I.Plotnikov et al., MNRAS 430, 1280 (2013)
[7] M. Milosavljevic Astroph. J. 641, 978-983 (2006)

The turbulence is magnetically dominated, and drifts close to the electron bulk velocity

Background plasma parameters



A natural scattering center frame: the Weibel frame

- The precursor of the shock is dominated by the Weibel instability

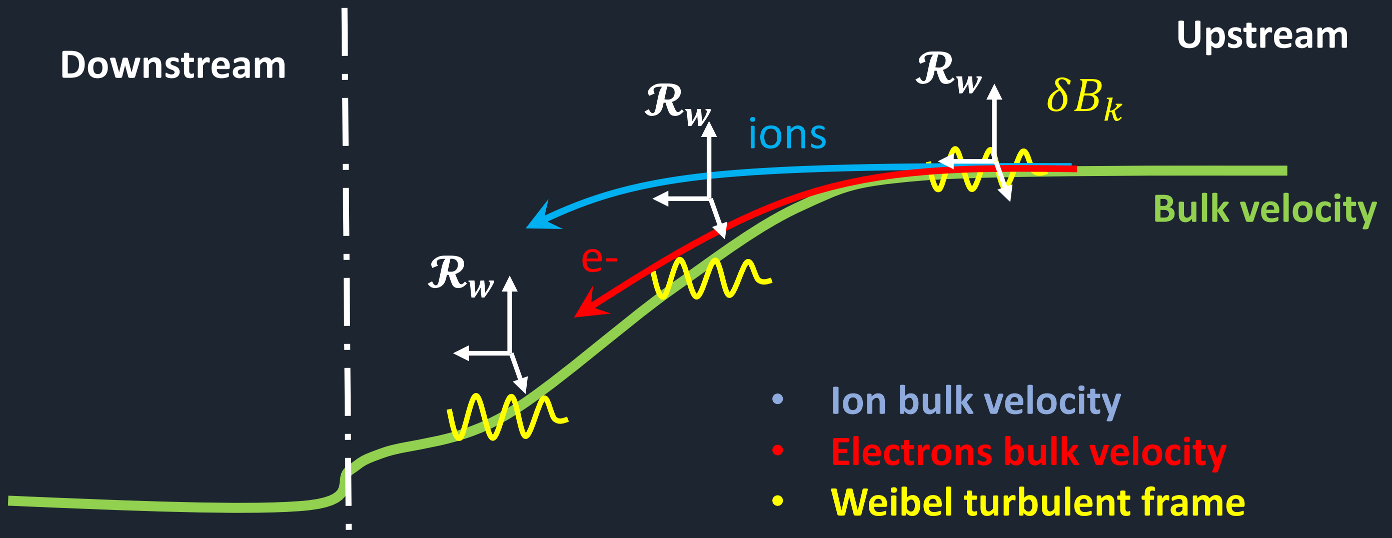
$$\Rightarrow \boldsymbol{E}^2 - \boldsymbol{B}^2 < 0$$

- At each point, one can define a local quasi-magnetostatic reference frame $\mathcal{R}_w$[1,2]

$$\Rightarrow u_w \sim \frac{E\times B}{B^2} \sim \frac{\omega}{k}\frac{\epsilon_{xy}}{\epsilon_{yy}} \sim u_e$$

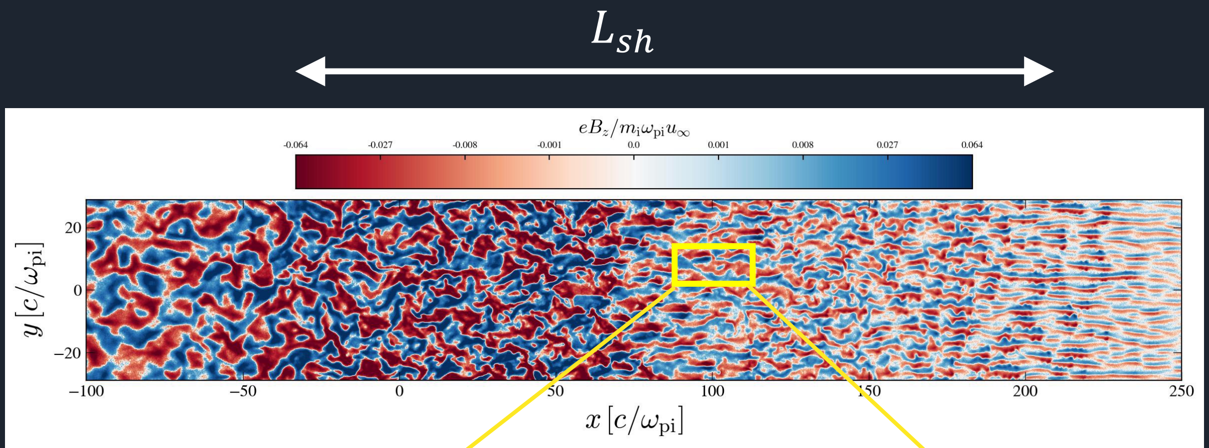
- Electrons drift close to the Weibel frame in the shock precursor

[1]C. Ruyer et al PRL 117, 065001 (2016) [2]G. Pelletier et al PRE 100, 013205 (2019)



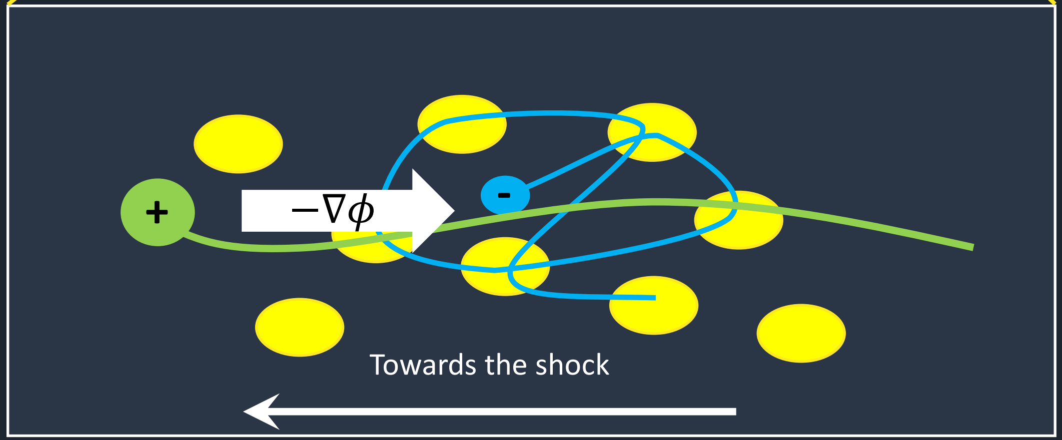
AV, V. Tsiolis et al., submitted

Energy partition through ambipolar heating in a decelerating microturbulence



$u_{sh} = 0.075; \Delta x = 0.1\frac{c}{\omega_{pe}}; c\Delta t = 0.45\,\Delta x$; 32 ppc; $m_i = 49\,m_e$ (Tristan-mp)

Joule effect/ambipolar heating

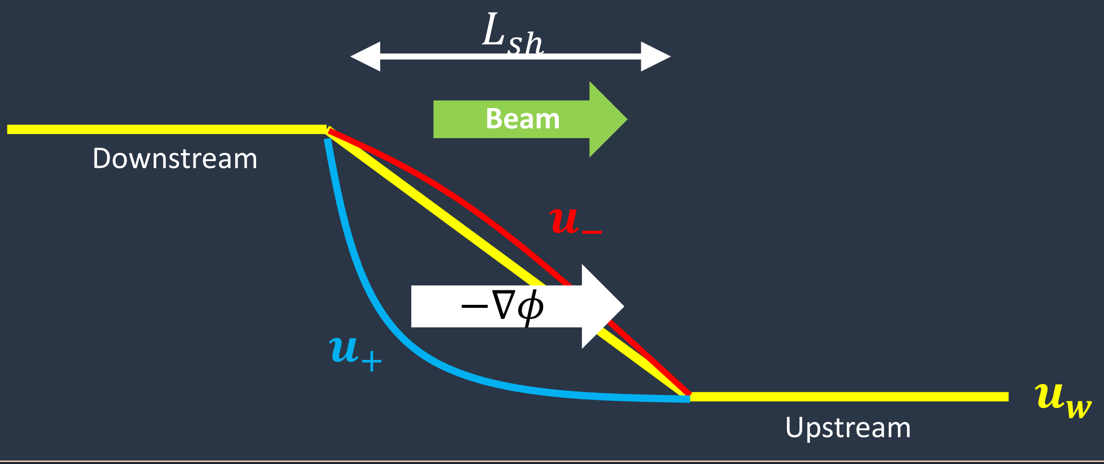


Model

- Collisionless friction with a magnetized microturbulence
- Electrostatic field from charge separation in a decelerated plasma

Equation of motion in the turbulence frame (Weibel frame $\boldsymbol{u_w}$)

$$\dot{\boldsymbol{p}} = \boldsymbol{p}\cdot\boldsymbol{\delta\widehat{\Omega}_t} + q\,\boldsymbol{E} - m\,\dot{\boldsymbol{u}}_w$$



AV, V. Tsiolis et al., submitted

A Monte Carlo-Poisson approach to solve the coupled transport of electrons and ions in a decelerated microturbulence

Semi-dynamical approach to electron-ion transport

For a white noise with isotropic scattering, the transport equation reduces to

$$\dot{\boldsymbol{p}} = \boldsymbol{p}\cdot\underbrace{\boldsymbol{\delta\widehat{\Omega}_t}}_{1.} + \underbrace{q\,\boldsymbol{E}}_{2.} - \underbrace{m\,\dot{\boldsymbol{u}}_w}_{3.}$$

1. **Pitch-angle scattering:** Gaussian white-noise process
Scattering center frame

$$<\delta\widehat{\Omega}_t> = 0 \qquad\qquad <\delta\widehat{\Omega}_t\,\delta\widehat{\Omega}_{t'}> = 2\,\delta(t'-t)$$

2. **Poisson solver**: self-consistent solution to the electrostatic field
Shock front frame

$$\nabla^2\phi = -4\,\pi\,\rho$$

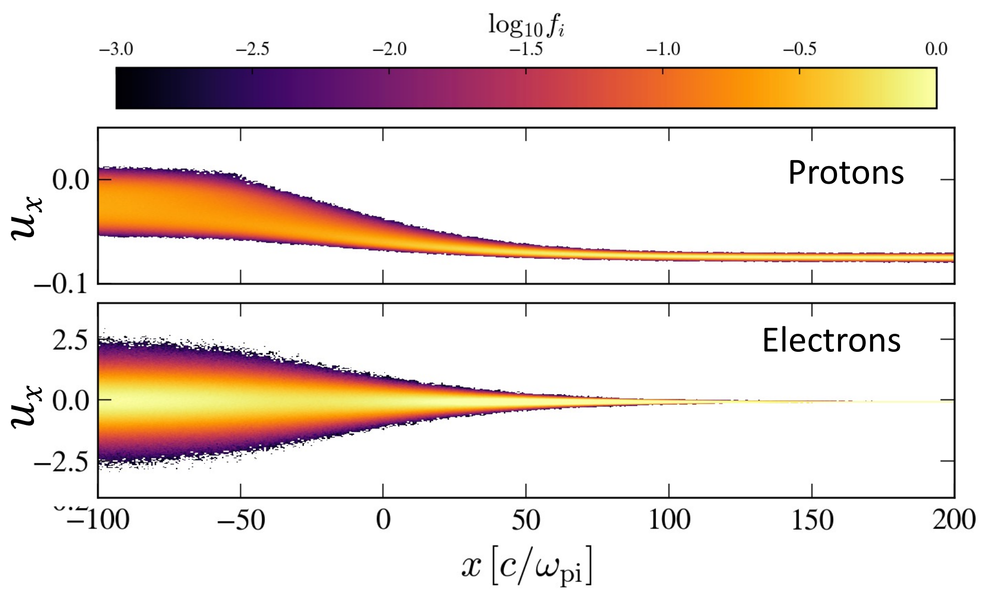
3. Stationary scattering center frame deceleration // effective gravity

Free parameters: ν, L_{sh}

Automatically decompose the work from electric field:

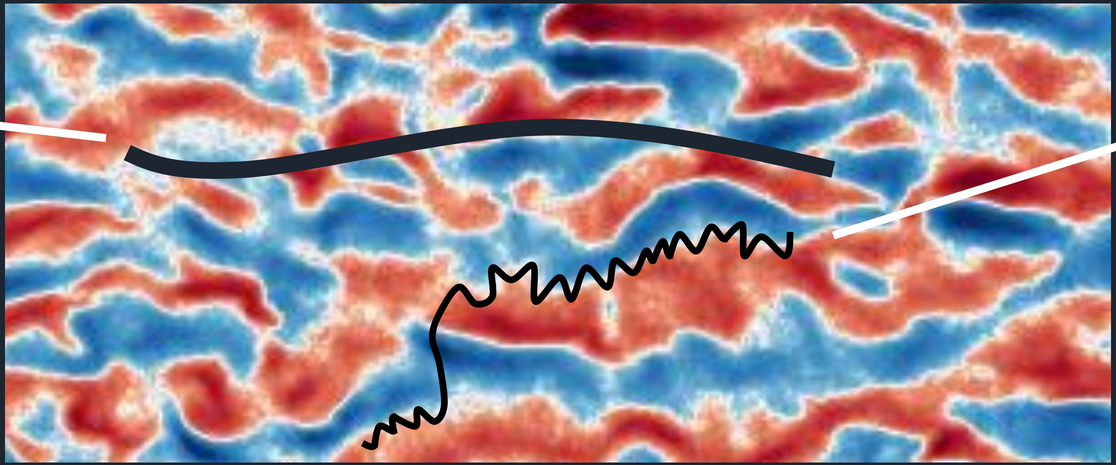
- Monte Carlo ⇒ Motional electric field ($\sim E_\perp$)
- Poisson ⇒ cross shock potential ($E_\parallel$)

Coupled dynamics of electrons and ions:



$$m_i = 1836\,m_i;\ \nu_i = \frac{|u_\infty|}{L_{sh}};\ \nu_e = \frac{m_i\nu_i}{m_e};\ L_{sh} = 150\,c/\omega_{pi}$$

Strongly magnetized electrons scatter through decoherence of the betatron motion in the finite filaments



Magnetic field profile

Ion scattering frequency

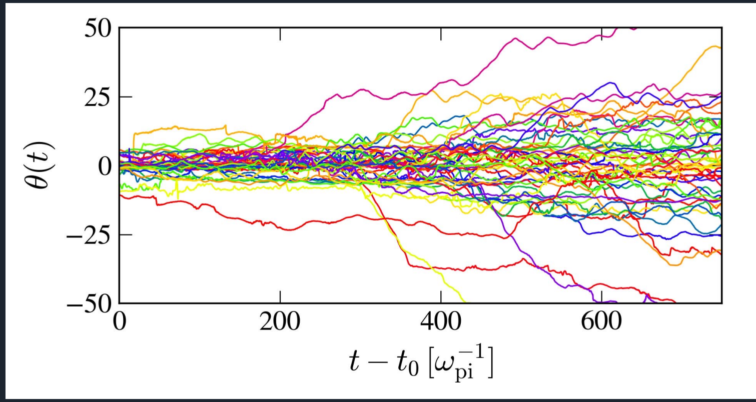
Small angle pitch-angle scattering captures the essential dynamics

$\nu_i^{\mathrm{PIC}} \sim \nu_i^{\mathrm{Th}}$

$$\nu_i^{\mathrm{PIC}} \sim \lim_{t\to\infty} \frac{<\alpha^2>}{2\,t} \sim \left\langle \frac{1}{2}\frac{d}{dt}\alpha^2 \right\rangle$$

$$\nu_i^{\mathrm{Th}} \sim r_\perp \frac{|u_\infty|}{r_{g,i}^2}$$

Ion pitch-angle



Electron scattering frequency

Decorrelation of the betatron oscillation at filament transition

$$\nu \sim \Delta\alpha^2/\Delta t$$

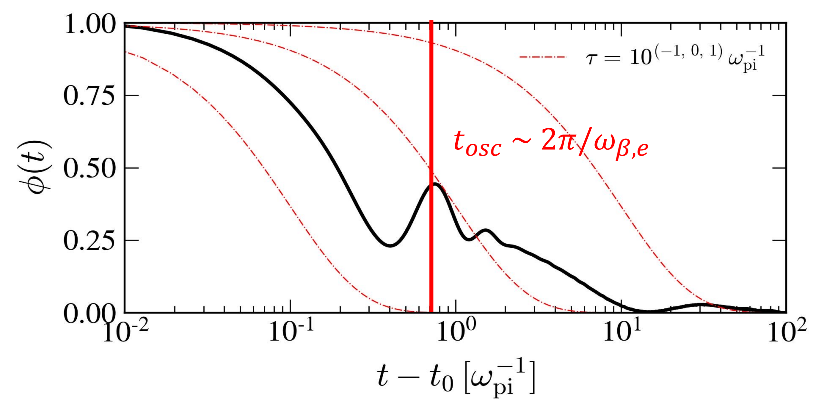
Betatron deflection angle

$$\Delta\alpha^2 = \omega_\beta^2\, r_\perp^2/u_{th}^2$$

Crossing time of a filament

$$\Delta t \sim r_\parallel/|u_{th}|$$

Electron self-correlation function

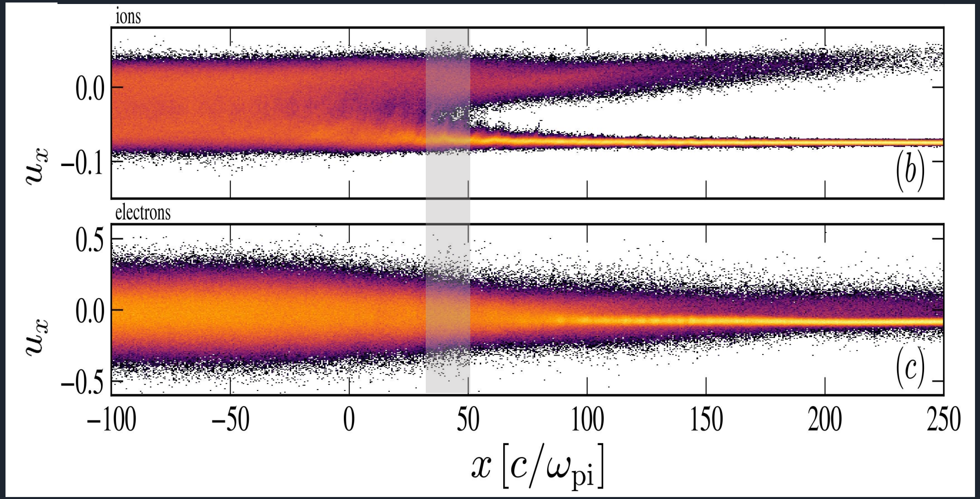


$$\nu_e^{\mathrm{Th}} \sim 2\pi \frac{k_\parallel}{k_\perp}\frac{m_i}{m_e}\frac{|u_{sh}|}{r_{g,i}}$$

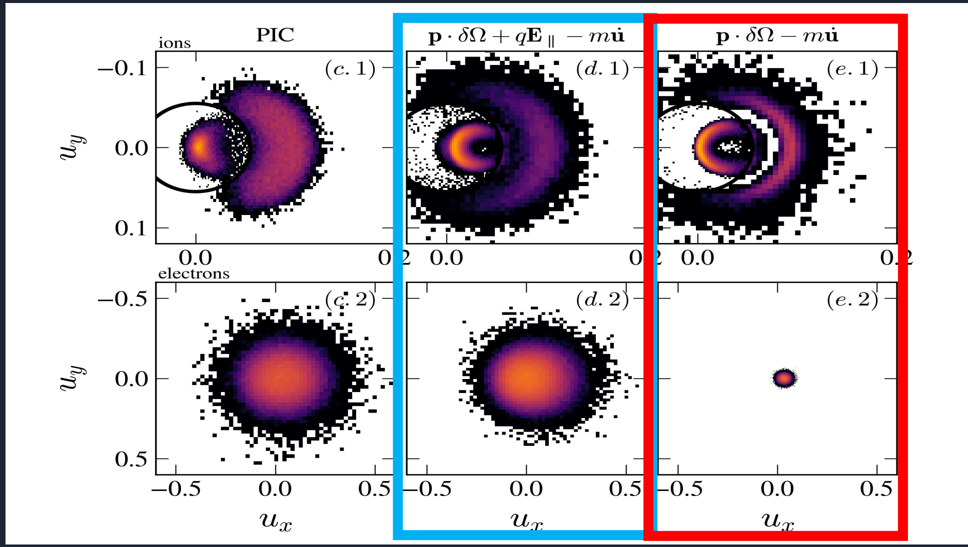
AV, V. Tsiolis et al., submitted

The transport equation captures ambipolar electron heating and non-adiabatic ion heating in the decelerating turbulence

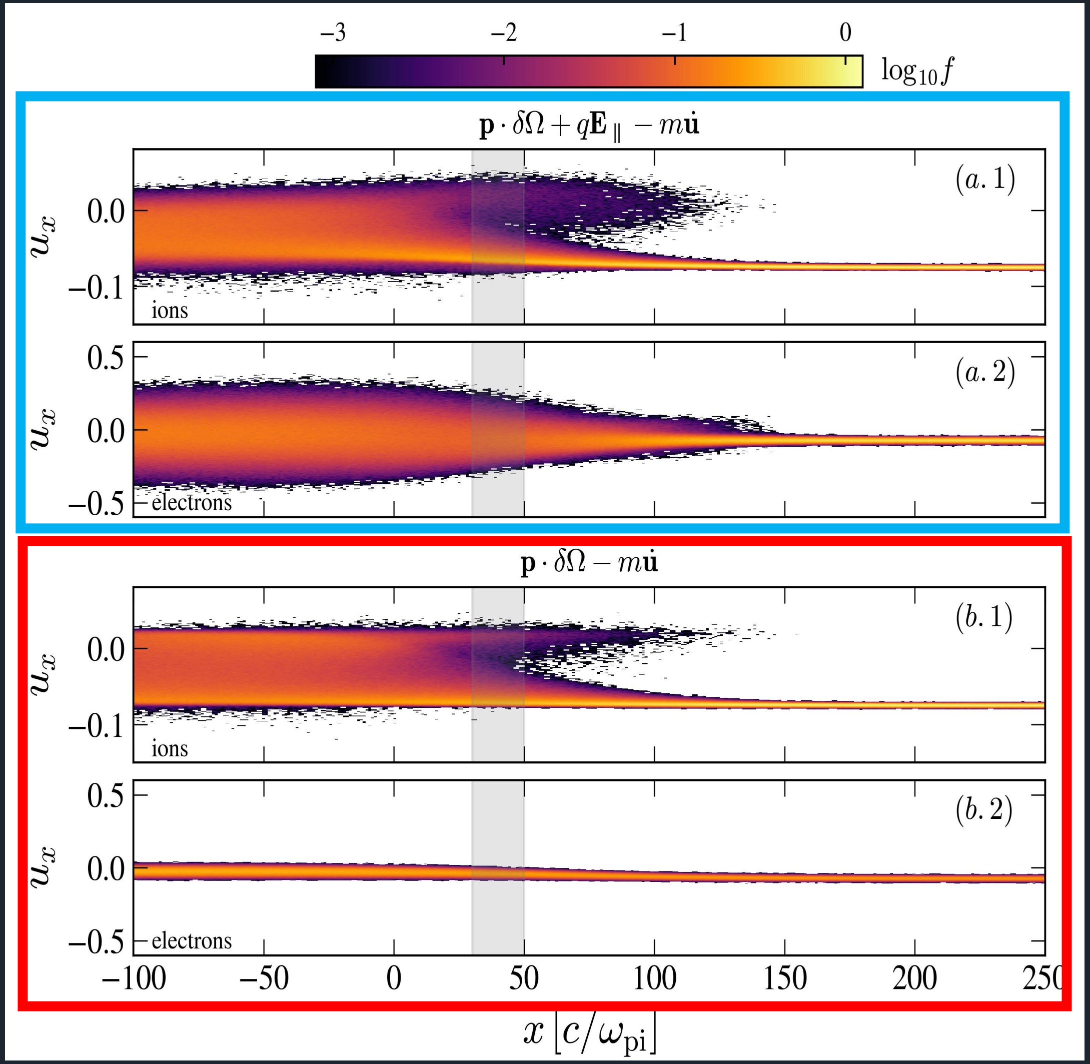
Particle-In-Cell



$u_{sh} = 0.075$; $\mathrm{m_i} = 49\ \mathrm{m_e}$ (**Tristan-mp**)



Theory



$m_i = 49\, m_i$; $v_i = \dfrac{|u_\infty|}{L_{sh}}$; $v_e = \dfrac{m_i v_i}{m_e}$; $L_{sh} = 150\, \dfrac{c}{\omega_{pi}}$

AV, V. Tsiolis et al., submitted

Ambipolar heating in Weibel-mediated shock waves naturally leads to the expected energy partition

Heating rate

$$L_{sh}|\partial_x \tau^{xx}| = \phi_\infty \begin{cases} \frac{3}{16}\xi & \text{if } \xi \lesssim 1 \\ \frac{3}{4}\xi^{-1} & \text{if } \xi \gg 1 \end{cases}$$

$\phi_\infty = m_i\, n_\infty\, u_\infty^2$

Free parameter: $\xi = \frac{L_{sh}\, m_e \nu_e}{m_i |u_\infty|}$

Ambipolar parameter

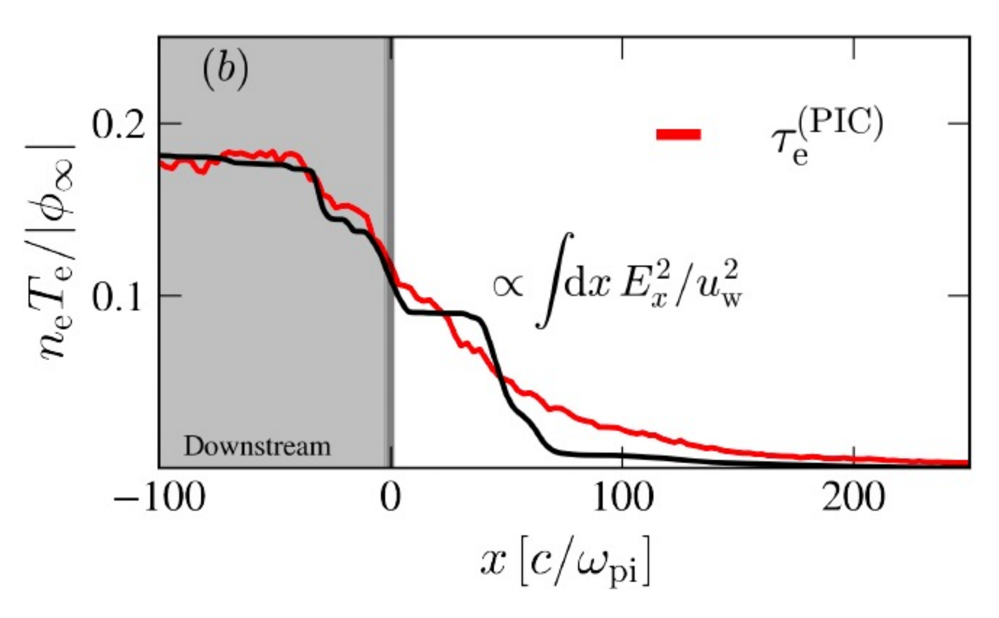
$$\xi \sim \frac{m_e \nu_e}{m_i \nu_i} \sim k_\parallel\, r_{g,i} \sim 1$$

for a typical kink-driven filament disruption

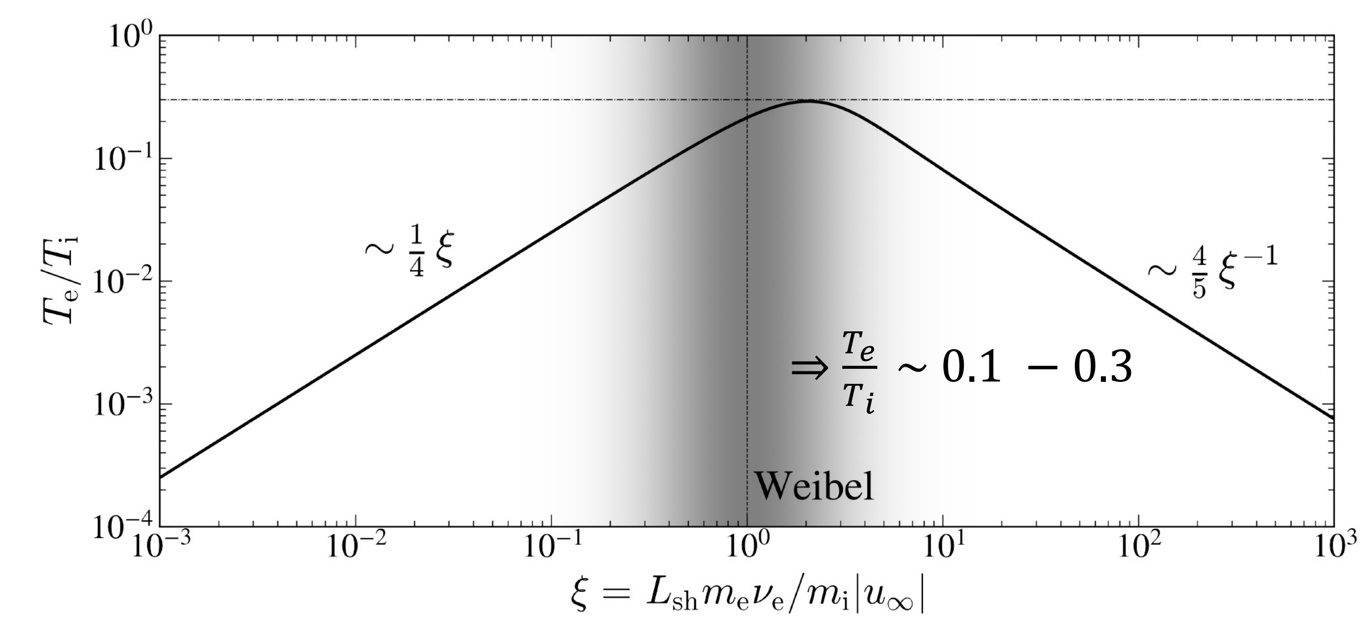
$\nu_i \sim |u_\infty|/L_{sh}$

Weibel saturation

$\omega_{\beta,i} \sim \frac{|u_\infty|}{c}\, \omega_{pi}$



Downstream temperature ratio



AV, V. Tsiolis et al., submitted

In the relativistic regime, the energy dependence of the scattering frequency is important to account for equipartition

The model extends to the relativistic regime of turbulent deceleration

$$\dot{\boldsymbol{p}}^i = \left(\boldsymbol{p} \cdot \delta\widehat{\boldsymbol{\Omega}}_t \right)^i - \Gamma^i_{ab}\, p^a\, \beta^b + q\, E_\parallel$$

⇒ Linear Fokker-Planck equation

$$\sim \partial_x f + \cdots \frac{\boldsymbol{du_w}}{\boldsymbol{dx}}\, \boldsymbol{\partial_p f} + \cdots \boldsymbol{\partial_p} \left(\boldsymbol{D_{pp}\, \partial_p f} \right) = \boldsymbol{0}$$

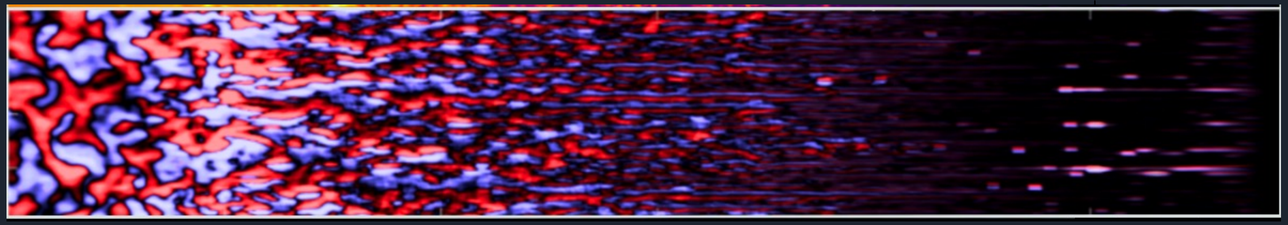
$$\boldsymbol{D^e_{pp}} \propto \frac{\boldsymbol{1}}{\boldsymbol{v}} \left(\frac{\boldsymbol{2}}{\boldsymbol{3}} \frac{\boldsymbol{du_w}}{\boldsymbol{dx}} + \frac{\boldsymbol{q\, E_\parallel}}{\boldsymbol{p}} \right)^2 \sim \frac{\boldsymbol{1}}{\boldsymbol{v}} \left(\frac{\boldsymbol{q\, E_\parallel}}{\boldsymbol{p}} \right)^2$$

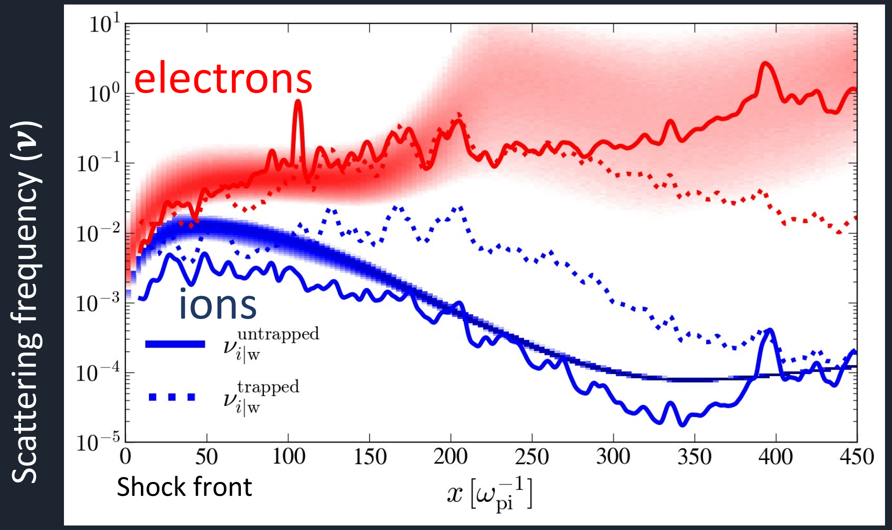
For E ~ 0, heating is analogous to shearing acceleration

$$\boldsymbol{D_{pp}} \propto \frac{\boldsymbol{1}}{\boldsymbol{v}} \left(\frac{\boldsymbol{du_w}}{\boldsymbol{dx}} \right)^2$$

The scattering frequency of the electron varies by order of magnitude from the far upstream to the shock transition

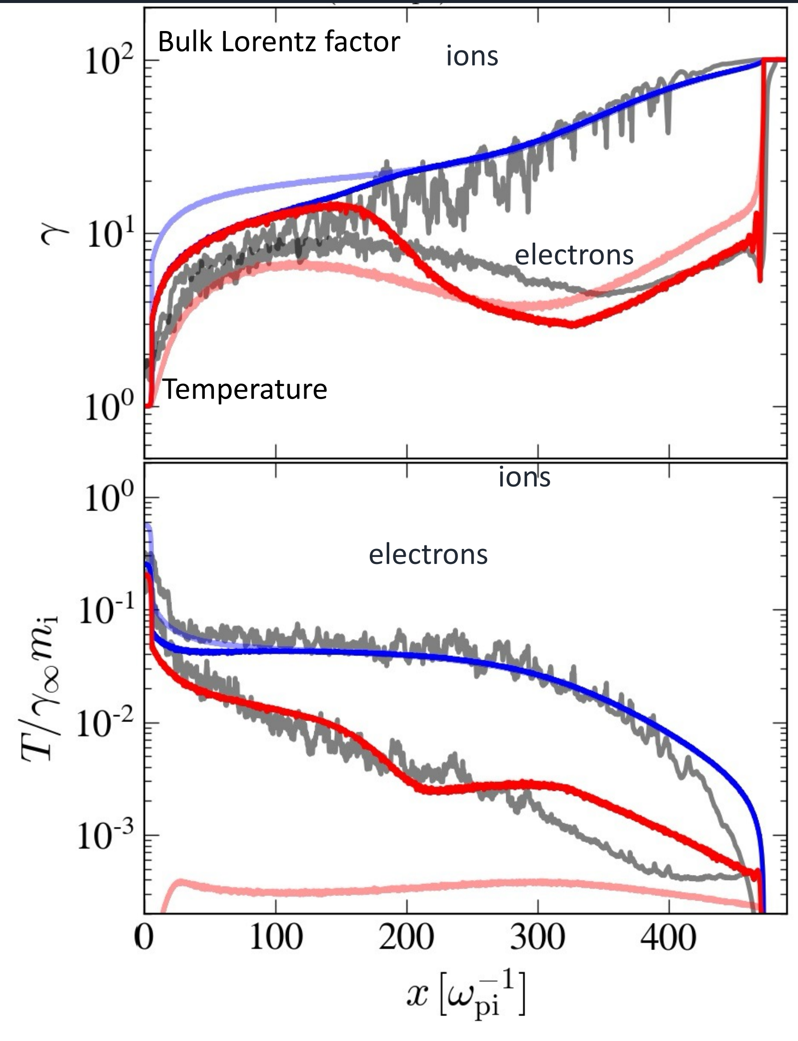
$\lambda \sim d_e \sim d_i$ ← $\lambda \sim d_e$





AV, Lemoine, Gremillet, ApJL 2022

The ambipolar electric field also accounts for the electron dynamics in the relativistic regime



Blue/**Red** : Reconstructed trajectories of ions and electrons from the transport equation

PIC
Theoretical model
- **ions**
- **electrons**

Light red: T_e from theoretical model in **absence of longitudinal electrostatic field**

⇒ Pure pitch-angle scattering cannot explain equipartition.

Red : T_e from theoretical model **with longitudinal electrostatic field**

⇒ Overall, satisfactory reconstruction of velocity and temperature, with electron heating up to equipartition

AV, Lemoine, Gremillet, ApJL 2022

To conclude…

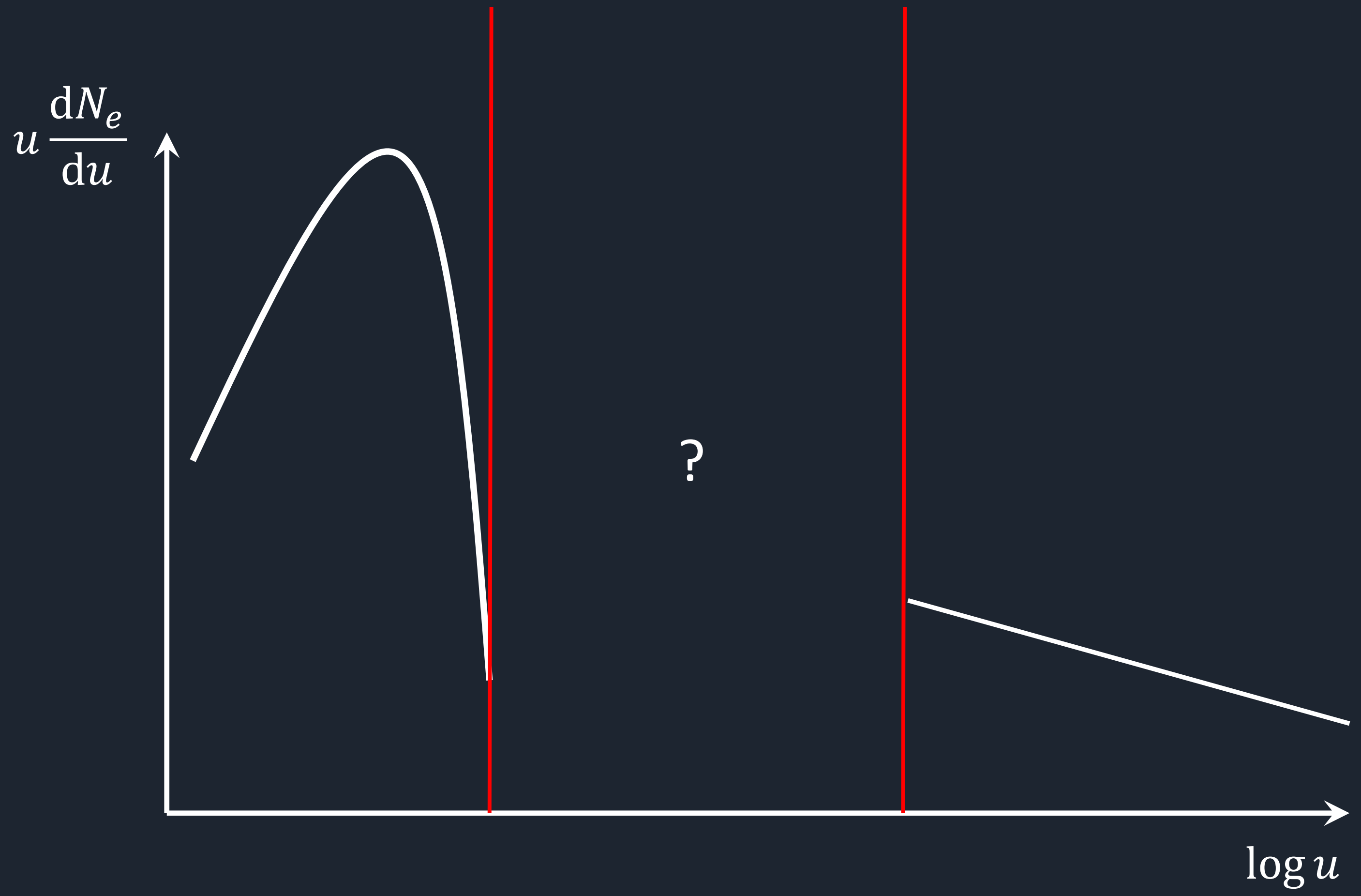
In short

- An electrostatic ambipolar potential develops in the precursor of Weibel-mediated shocks
- Energy partition between electrons and ions can be modeled as ambipolar heating in a decelerating magnetically dominated microturbulence
- A simple Gaussian white-noise description of isotropic pitch-angle scattering is sufficient to capture the dominant electron dynamics
- The model captures electron thermalization (and acceleration) for natural parameters of Weibel-mediated shocks

Prospects

- Scaling of the pitch-angle scattering frequency in energy and effect on the spectrum of electrons
- Magnetically dominated shock precursors are observed in various shock conditions with $\delta B/B \gg 1$ (parallel, perpendicular) in which the model should remain valid
- Electron acceleration efficiency?

Particle acceleration and formation of a nonthermal tail?



The full Fokker Planck equation for electron transport in a Weibel-mediated shock with coherent electrostatic potential

Dimensionless Fokker-Planck

To leading order, the full Fokker-Planck equation can be rewritten as

$$\partial_{\tilde{t}} f + \tilde{u}_w \, \partial_{\tilde{x}} f \; - \frac{1}{3} \, \tilde{u} \, \partial_{\tilde{x}} \tilde{u}_w \, \partial_{\tilde{u}} f = \frac{1}{\tilde{u}^2} \partial_{\tilde{u}} (D_{\tilde{u}\tilde{u}} \, \tilde{u}^2 \, \partial_{\tilde{u}} \, f) + \frac{2}{3} \, \tilde{u} \frac{q}{e} \, \tilde{E} \, \partial_{\tilde{x}\,\tilde{u}} \, f + \partial_x D_{\tilde{x}\tilde{x}} \, \partial_x \, f$$

Bulk heating

Dominant for $\tilde{u}^2 \ll 1$ ($\tilde{E} \sim 1$)

Fermi acceleration

Dominant for $\tilde{u}^2 \gg 1$ ($\tilde{E} \sim 1$)

$$D_{\tilde{u}\tilde{u}} = \frac{1}{3} \, \tilde{E}^2$$

$$D_{\tilde{x}\tilde{u}} = \frac{2}{3} \frac{q}{e} \tilde{E} \, \tilde{u}$$

$$D_{\tilde{x}\tilde{x}} = \frac{1}{3} \tilde{u}^2$$

Dimensionless variables:

$$\tilde{E} = \frac{eE}{m_e \, v_e \, |u_\infty|} \qquad \xi = \frac{v_e \, m_e \, L_{sh}}{m_i |u_\infty|} \qquad r = \frac{m_e}{m_i} \qquad \tilde{u} = \sqrt{r/\xi} \, \frac{u}{|u_\infty|} \qquad \tilde{x} = \frac{x}{L_{sh}} \qquad \tilde{t} = \frac{|u_\infty| ct}{L_{sh}}$$

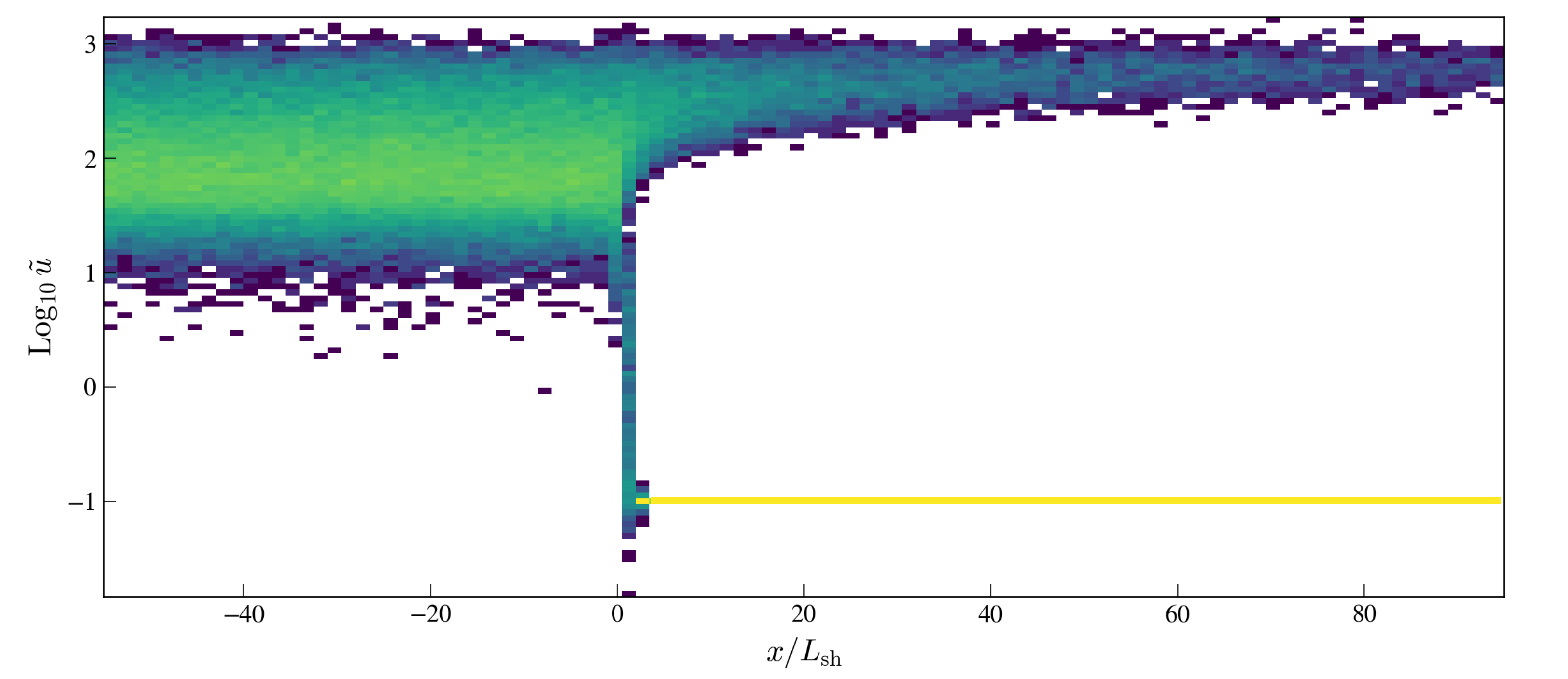
The transport equation leads to thermalization and Fermi acceleration

Monte Carlo solution – Langevin equation

No unique mapping of the FP equation to a Langevin equation but the simplest formulation reads

$$\mathrm{d}U_t = -\frac{1}{3}\partial_{\tilde{x}}\tilde{u}_w\tilde{u}\,\mathrm{d}t + \sqrt{D_{\tilde{u}\tilde{u}}}\,\mathrm{d}W_t^U + \frac{2}{U_t}D_{\tilde{u}\tilde{u}}\,\mathrm{d}t \quad \& \quad D_{\tilde{x}\tilde{u}} \text{ imposes } \mathrm{dW_t^u} = \mathrm{dW_t^x}$$

$$\mathrm{d}X_t = \tilde{u}_w\,\mathrm{d}t + \sqrt{D_{\tilde{x}\tilde{x}}}\,\mathrm{dW_t^X}$$



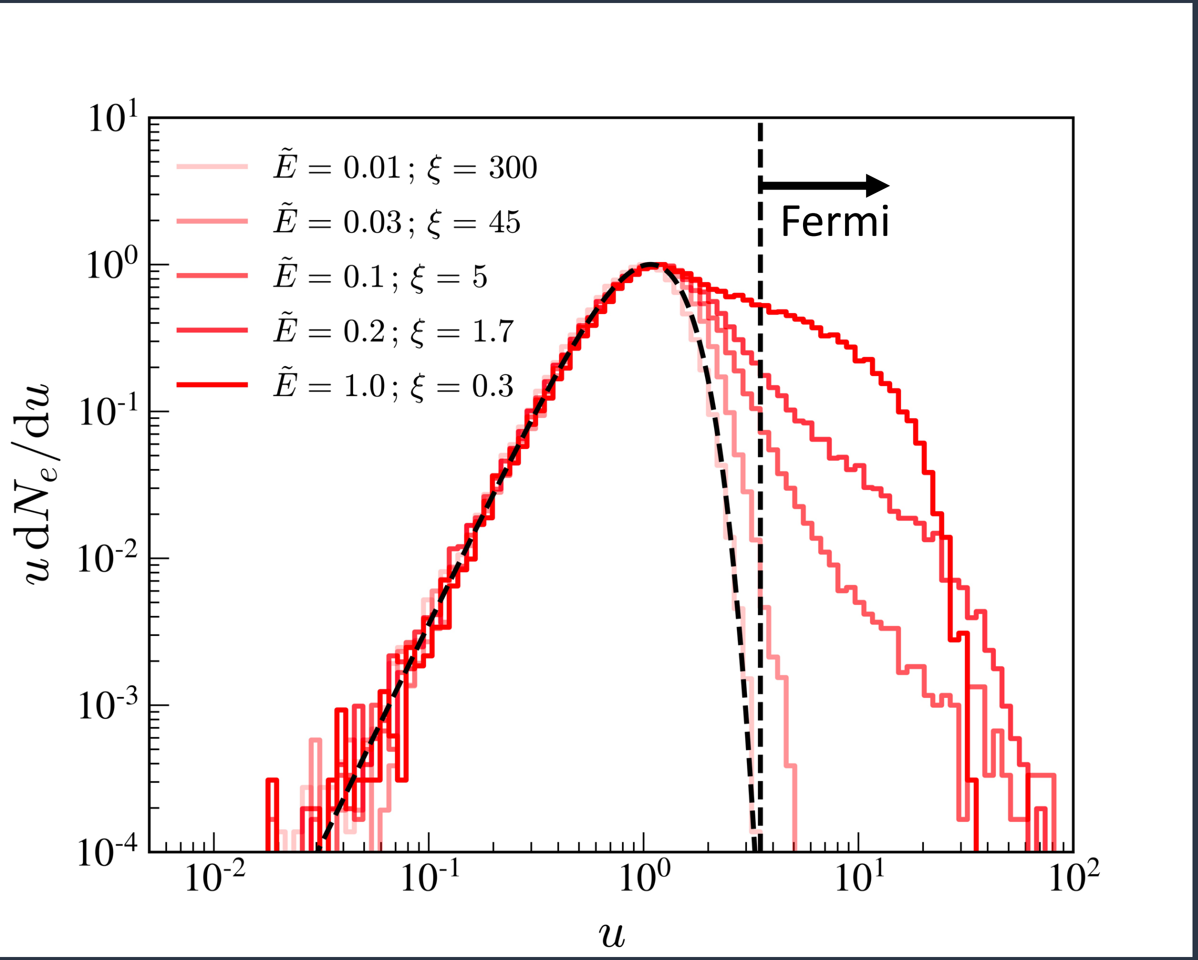
$\xi \sim 1$

$< |\tilde{E}| > \sim 1$

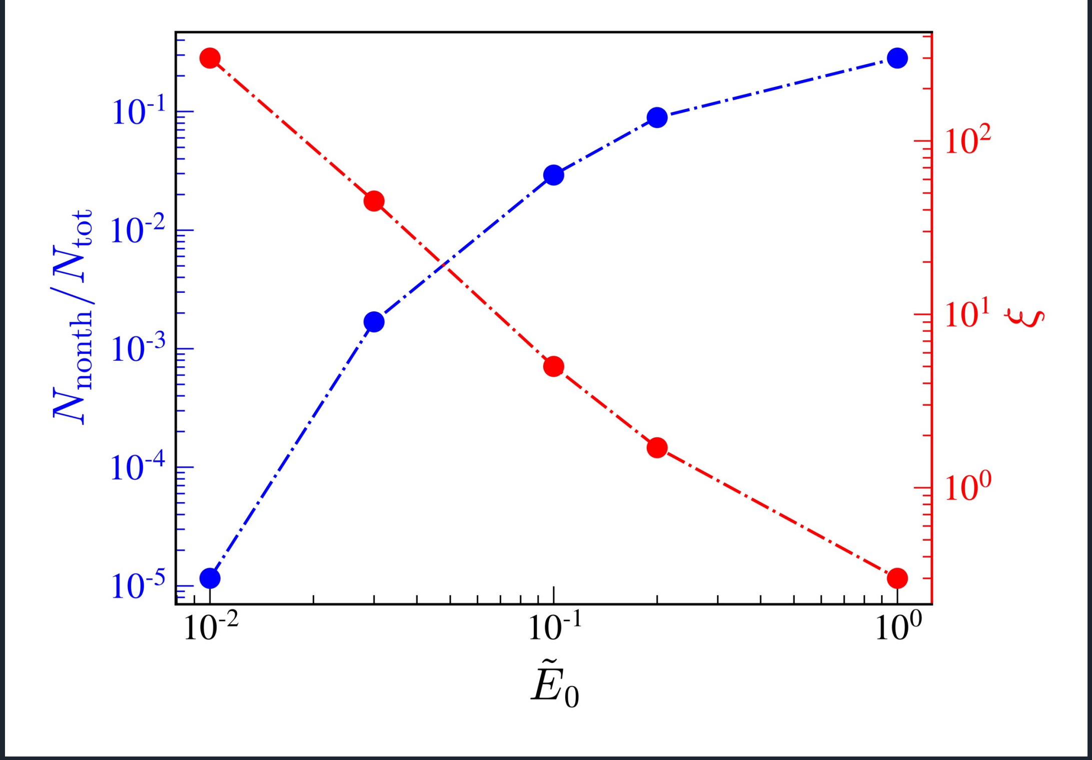
$\frac{m_i}{m_e} = 1836$

Reasonable values of the Weibel-mediate coherent electrostatic field are sufficient to accelerate electrons via Fermi process

Thermalization and acceleration



Parametric dependence



Injection into Fermi process requires $D_{xx} \gtrsim D_{pp}$

$$\frac{u^2}{u_\infty^2}\frac{m_e}{m_i} \gtrsim \tilde{E}^2\,\xi$$