A minimal SM/LCDM cosmology

Neil Turok

Higgs Centre, University of Edinburgh
and
Perimeter Institute for Theoretical Physics

with Latham Boyle (also @ Higgs Centre + Perimeter)

LCDM provides a remarkably simple description of the large-scale universe: just 5 fundamental physics parameters

the matter/energy content

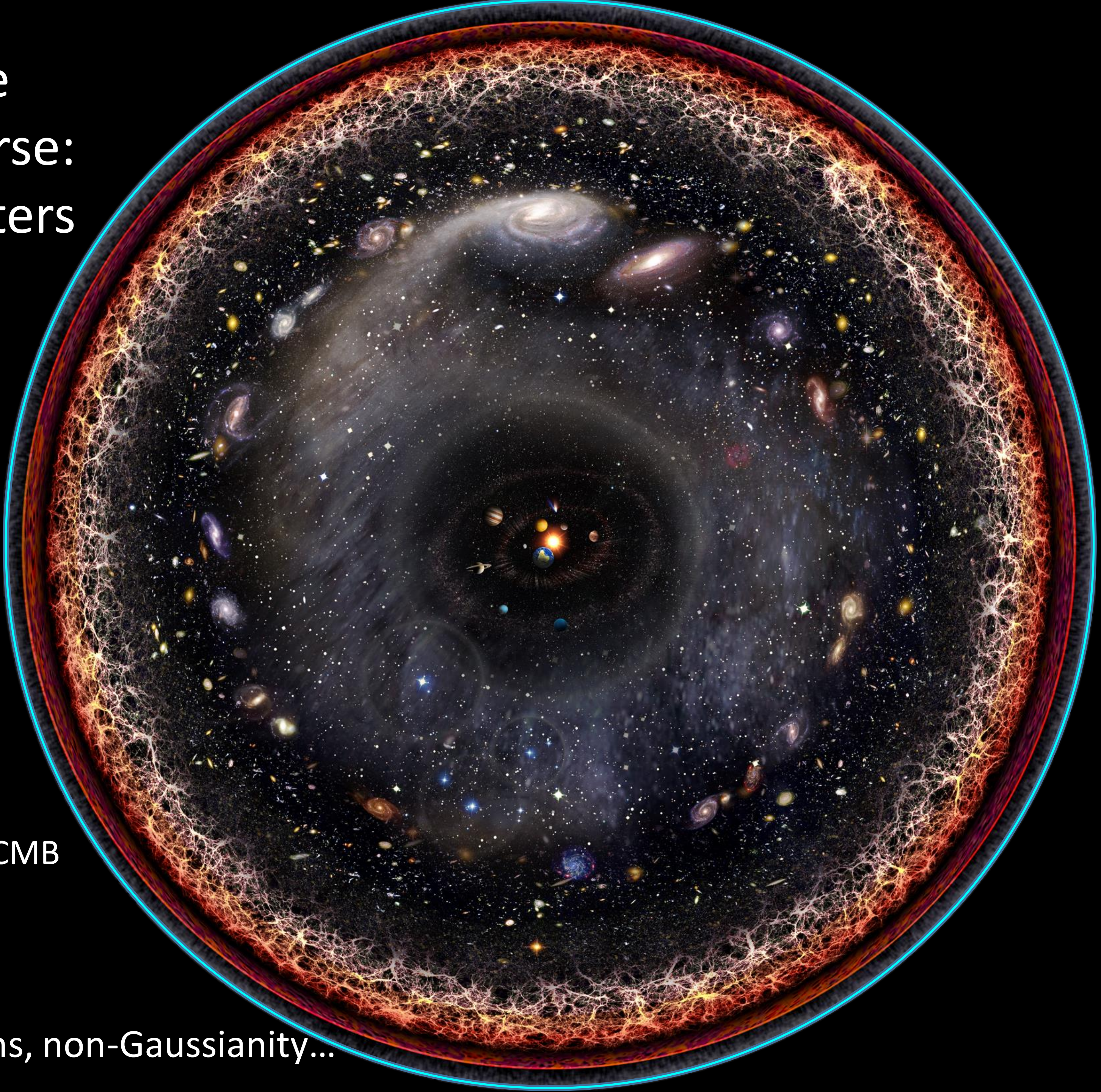
1. ρ_Λ cosmological constant
2. ρ_{DM}/ρ_B DM/baryon density
3. n_B/n_γ baryons per photon

the large-scale geometry

Large scale Newtonian potential

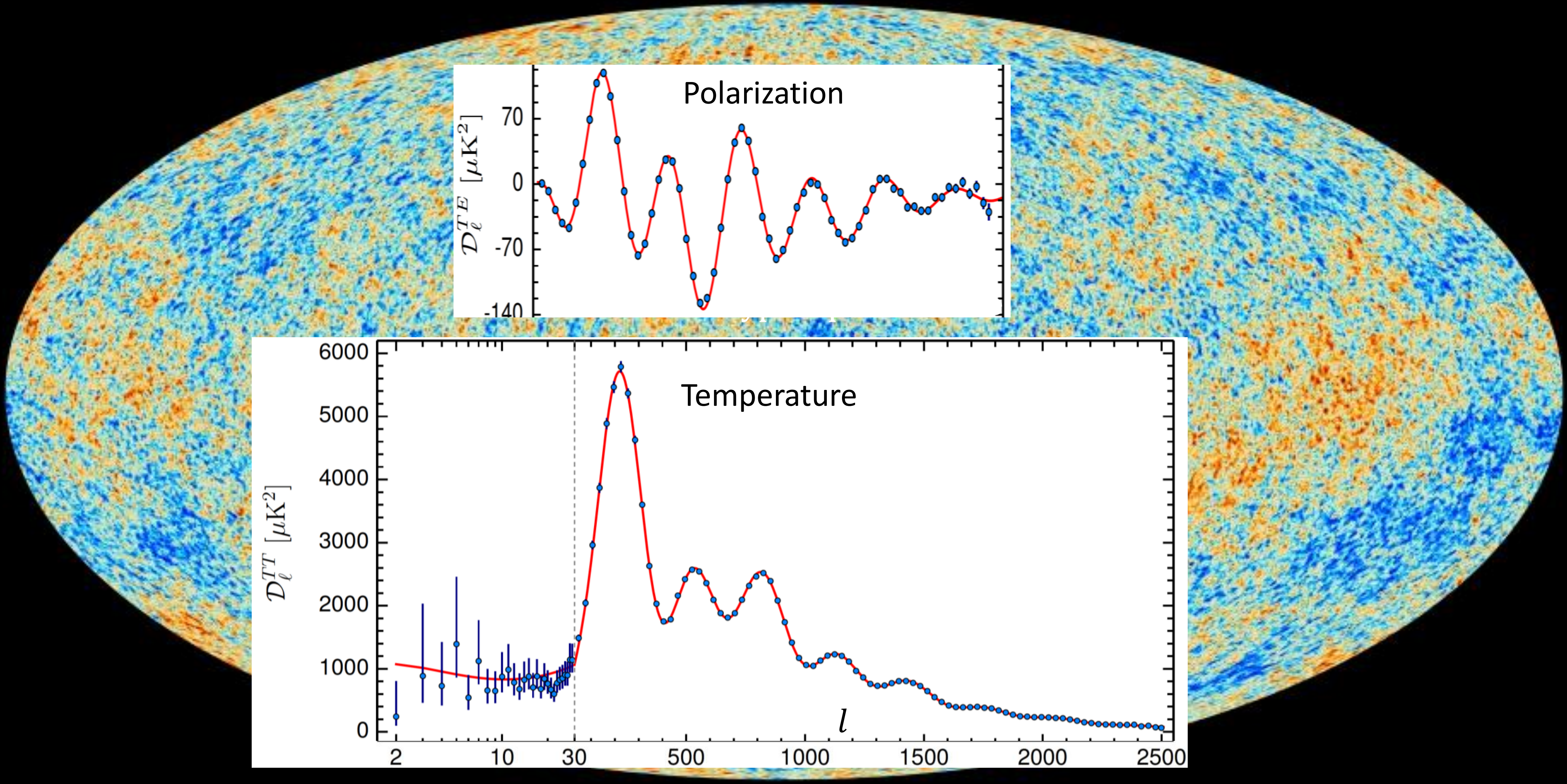
$$\langle \Phi^2 \rangle = \int \frac{dk}{k} A_\Phi \left(\frac{k}{k_*}\right)^{n_s-1} \quad (k_* \equiv 0.05\mathrm{Mpc}^{-1})$$

$$\left.\begin{array}{l} 4.\ A_\Phi = (7.6 \pm 0.1) \times 10^{-10} \\ 5.\ n_s - 1 = -0.04 \pm .006 \end{array}\right\} \text{Planck CMB}$$



many quantities are so far consistent with zero:
space curvature, tensor/isocurvature perturbations, non-Gaussianity…

Large scale perturbations



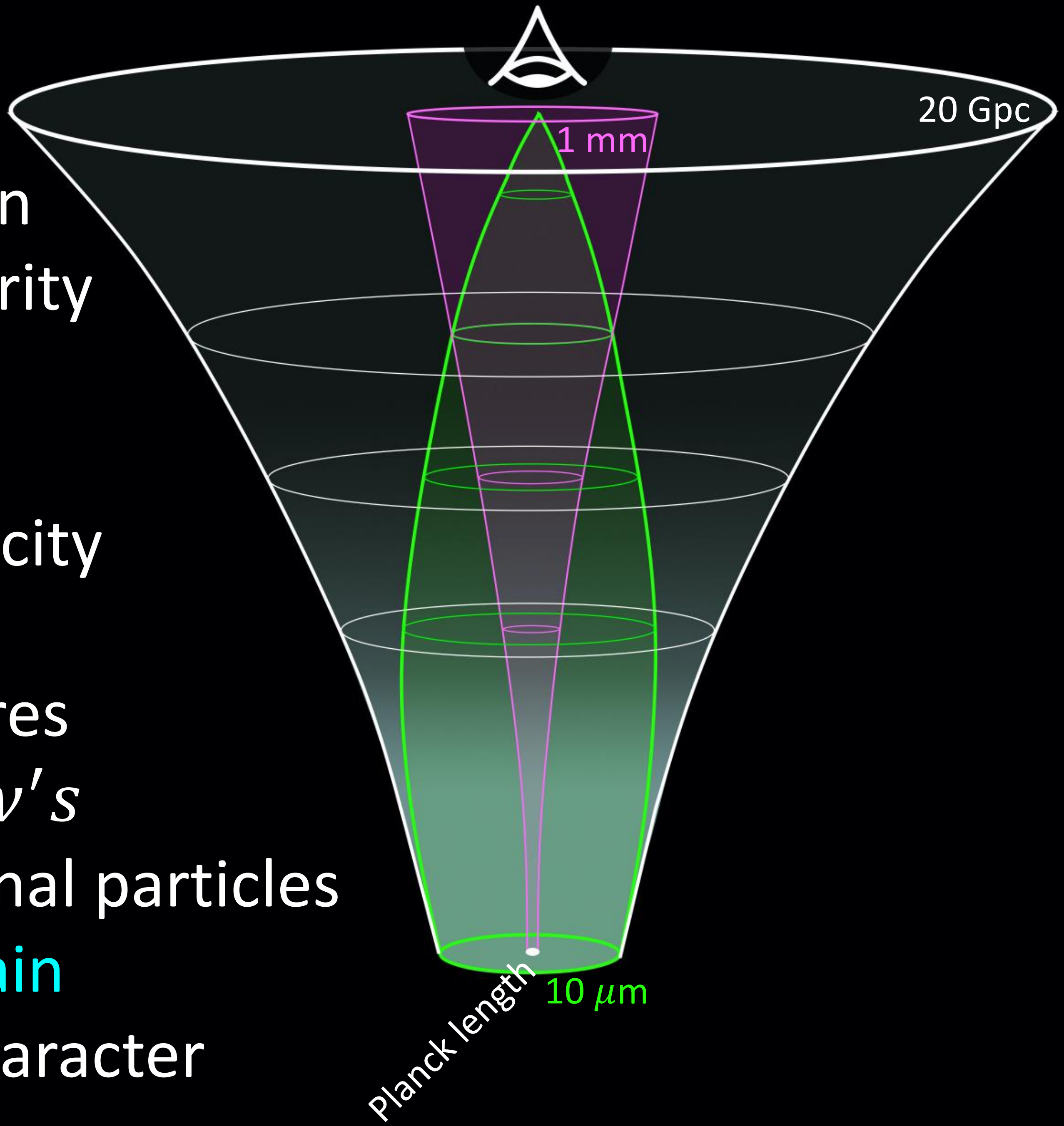
ESA Planck satellite

The fact that the phenomena are consistent with such a simple description suggests the possibility of a highly economical theoretical explanation

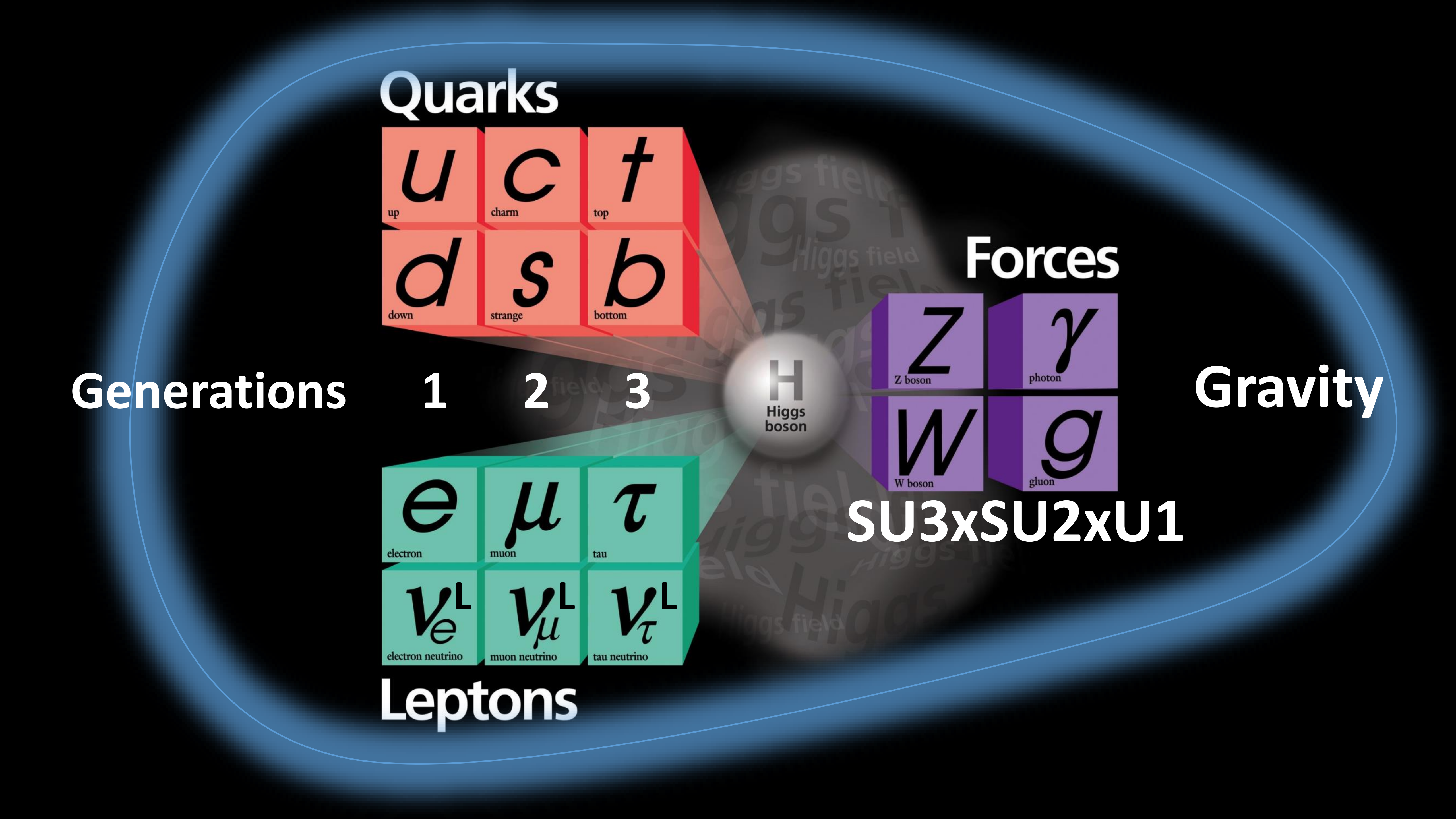
This talk:
a minimal, unified framework
based on extrapolating the radiation
epoch (and SM) back to the singularity

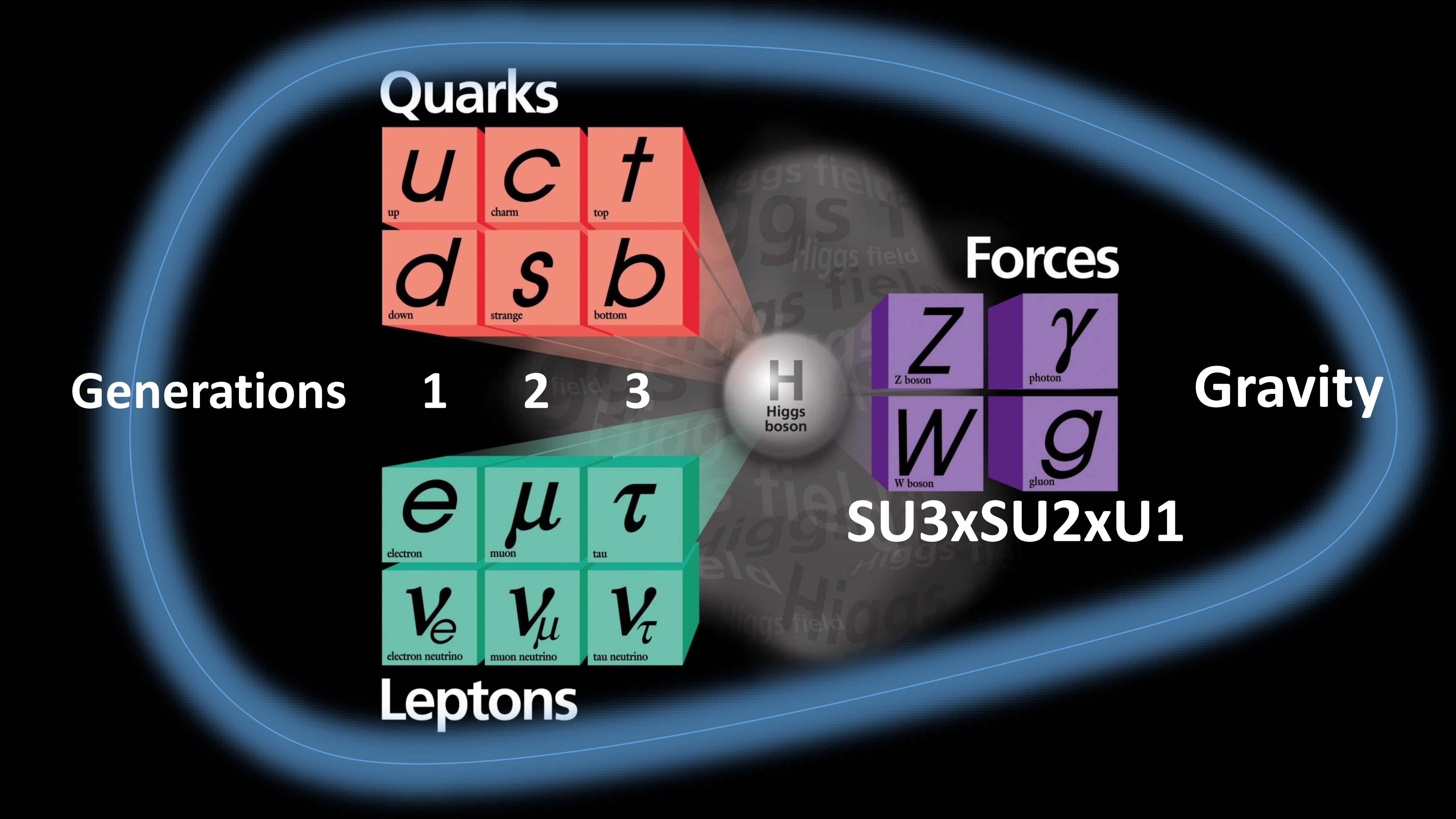
Guiding principles:
CPT & conformal symmetry; analyticity

new explanations for LCDM's features
predict 3 generations including RH $\nu's$
no need for inflation or any additional particles
with minimal assumptions we explain
the amplitude, tilt and statistical character
of the primordial fluctuations

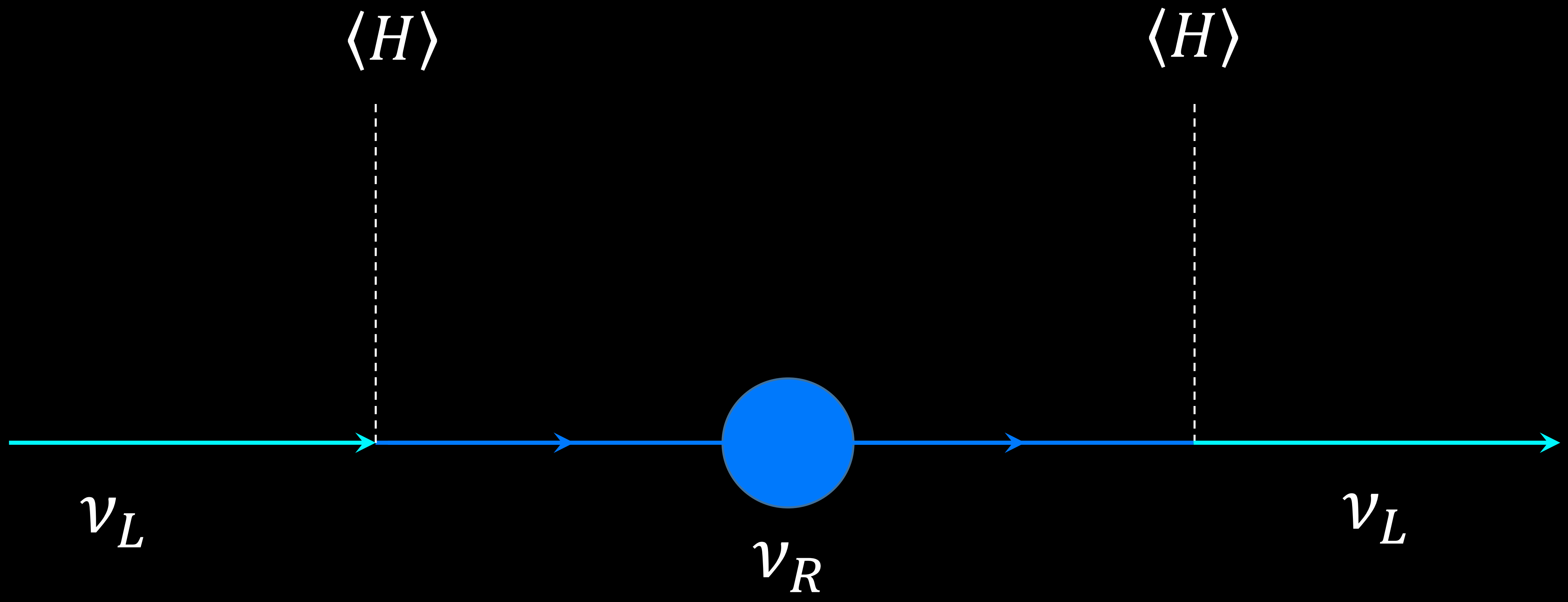


a minimal explanation of the dark matter


Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
Generations
1
2
3
H
Higgs boson
Higgs field
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
Gravity
SU3xSU2xU1
e
electron
μ
muon
τ
tau
ν_e^L
electron neutrino
ν_μ^L
muon neutrino
ν_τ^L
tau neutrino
Leptons


Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
Generations
1
2
3
H
Higgs
boson
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
Gravity
SU3xSU2xU1
e
electron
μ
muon
τ
tau
ν_e
electron neutrino
ν_μ
muon neutrino
ν_τ
tau neutrino
Leptons

Right-handed neutrinos:



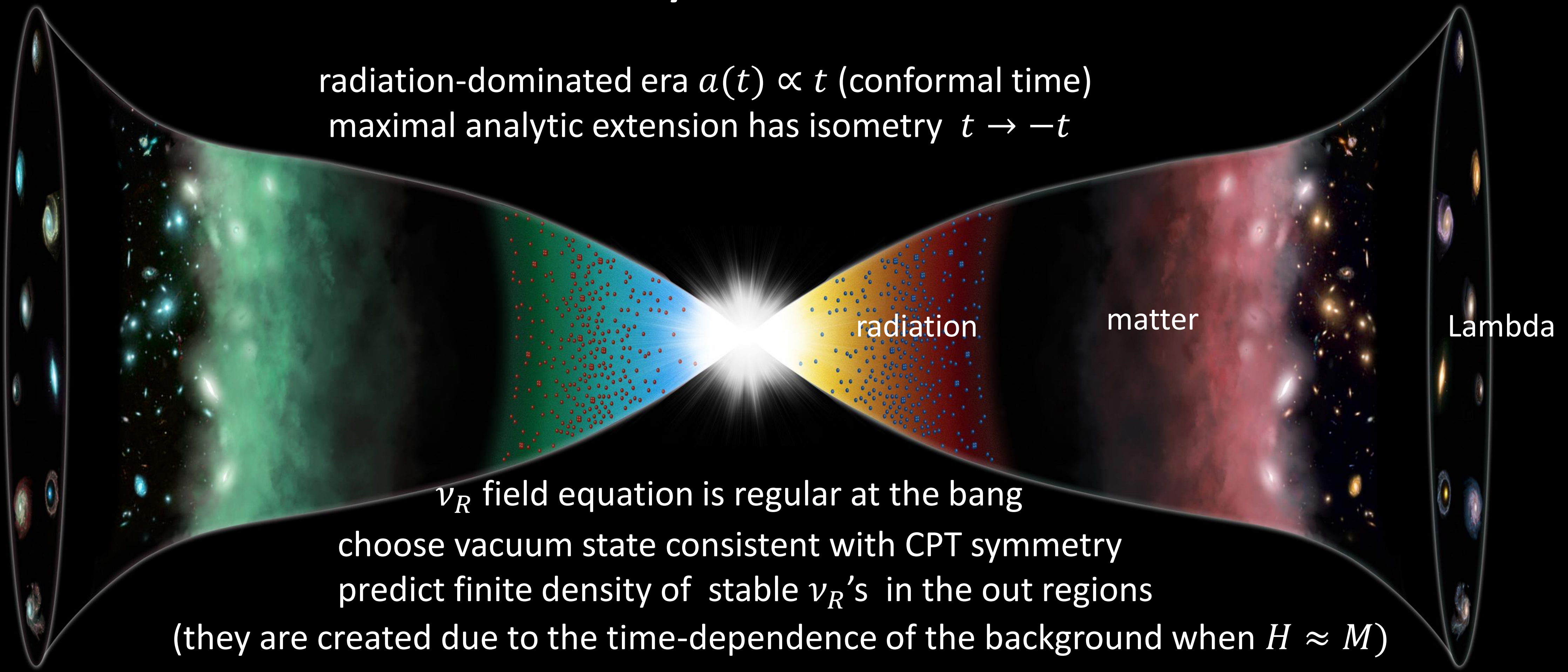
explain observed light neutrino masses
(seesaw mechanism, 1970's)

Later, I'll mention a new explanation for why ν_R's must exist

a right-handed neutrino as the dark matter in a CPT-symmetric universe

radiation-dominated era $a(t) \propto t$ (conformal time)

maximal analytic extension has isometry $t \to -t$



ν_R field equation is regular at the bang

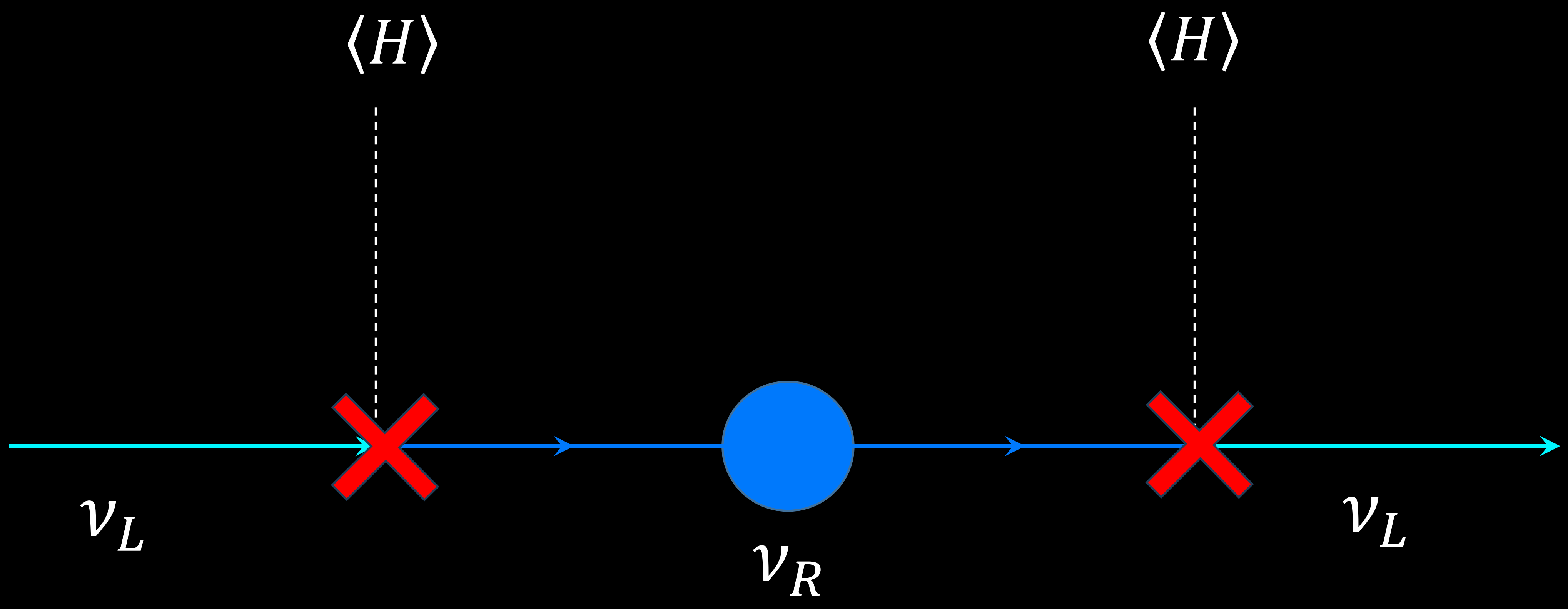
choose vacuum state consistent with CPT symmetry

predict finite density of stable ν_R's in the out regions

(they are created due to the time-dependence of the background when $H \approx M$)

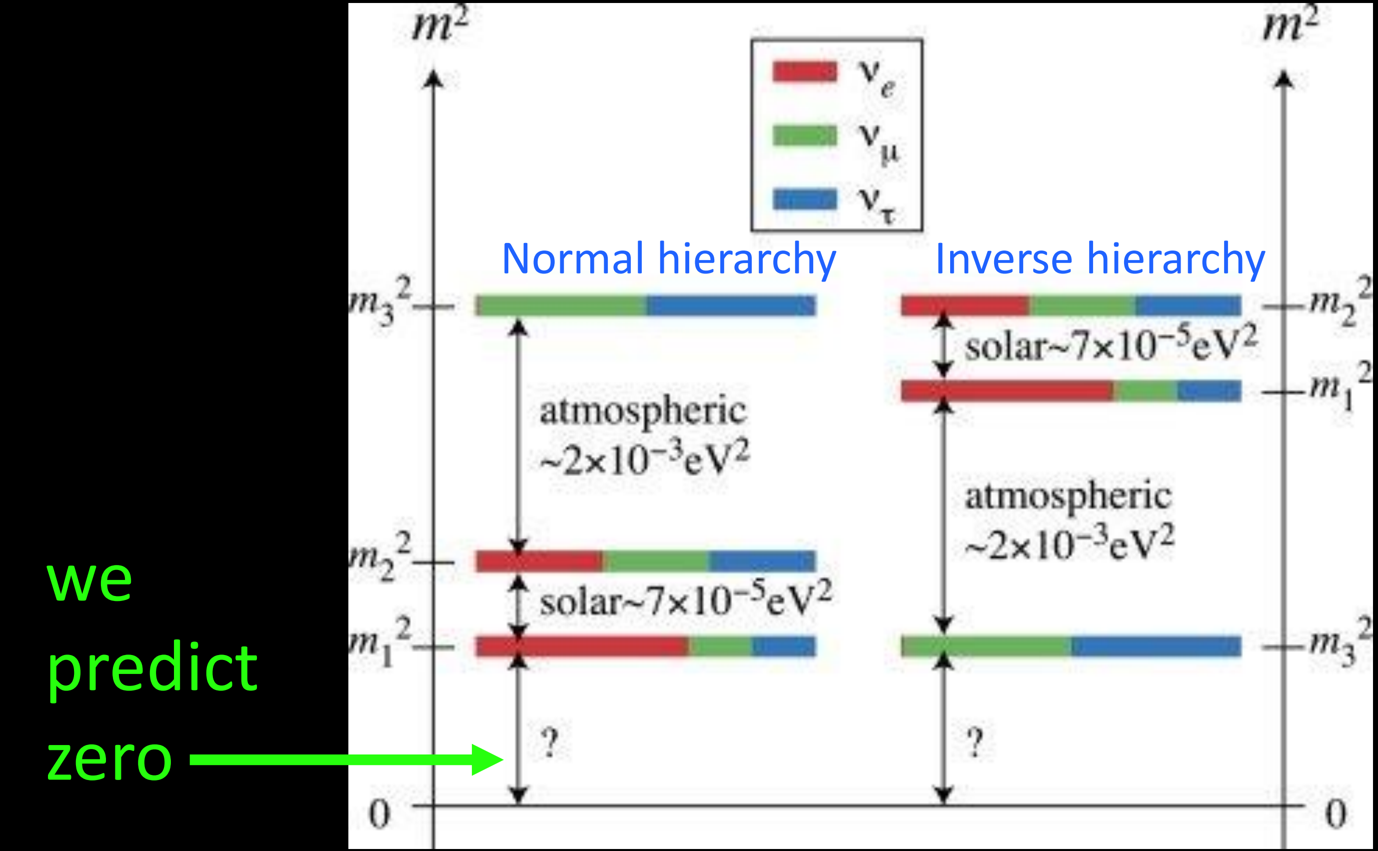
if one ν_R is stable, its density matches Ω_{DM} if its mass $M \approx 5 \times 10^8 GeV$

Stability of one RH neutrino $\Rightarrow$ $\mathbb{Z}_2$ symm $\Rightarrow$ lightest ν massless



will be tested using EUCLID, LSST and S4

Light neutrinos: observations



Normal hierarchy: $M_\nu \equiv \sum m_\nu \approx 0.06\ eV$

Inverted hierarchy: $M_\nu \approx 0.1\ eV$

current data

eBOSS 2007.08991

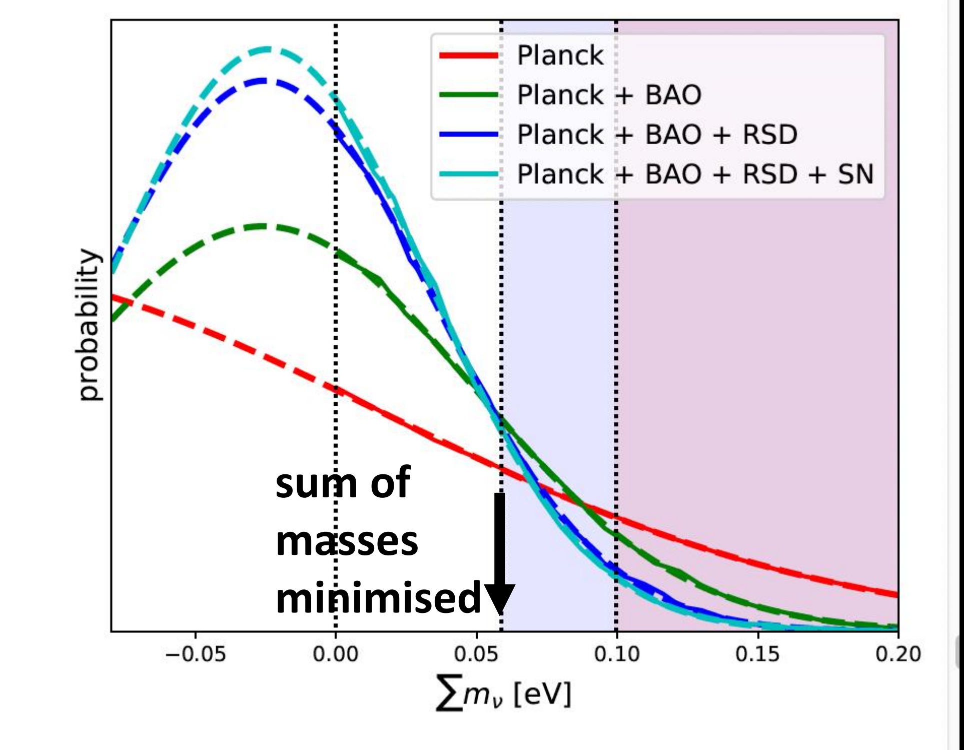
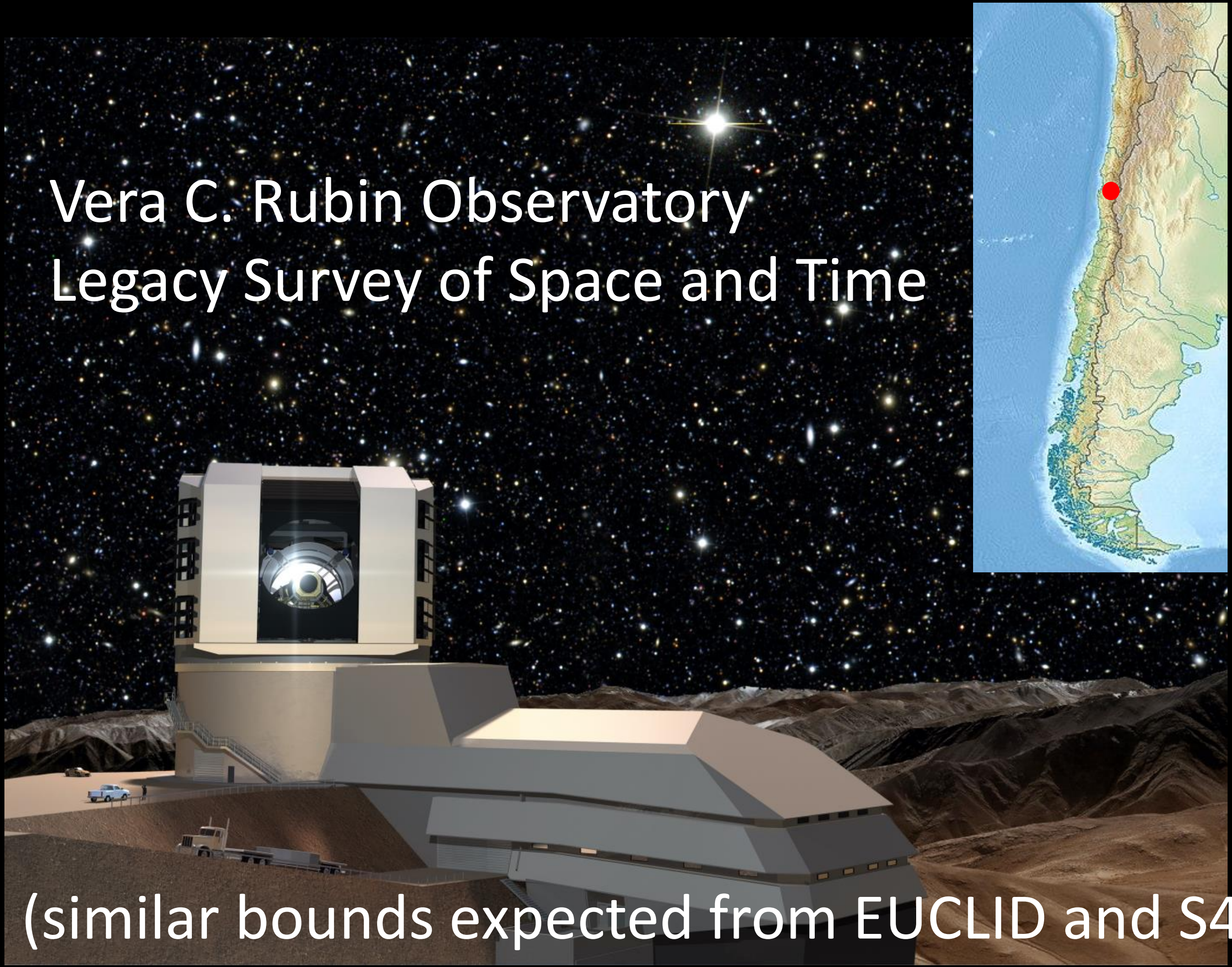
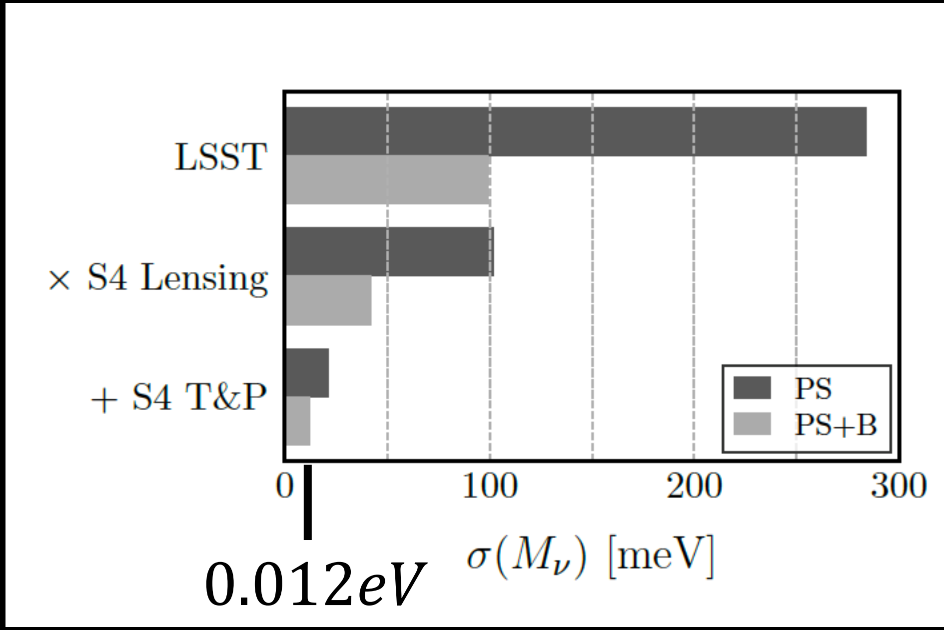


FIG. 13.— Posterior for sum of neutrino masses for selected con binations of data with a $\nu\Lambda$CDM cosmology. Dashed curves sho the implied Gaussian fits. Shaded regions correspond to lower lin its on normal and inverted hiearchies. Likelihood curves are no malized to have the same area under the curve for $\sum m_\nu > 0$.

Chen et al. 2103.01229

Vera C. Rubin Observatory
Legacy Survey of Space and Time





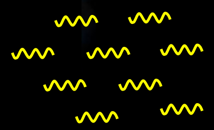
(similar bounds expected from EUCLID and S4)

a minimal explanation of the large-scale geometry

Penrose

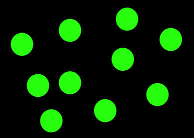
Path integrals and gravity

gauge fields



Higgs

$$\int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G}-\frac{1}{4}F^2+\bar{\psi}i\not{D}\psi-\lambda H\bar{\psi}\psi+|DH|^2-V(H)\right)}$$



particles

gravity

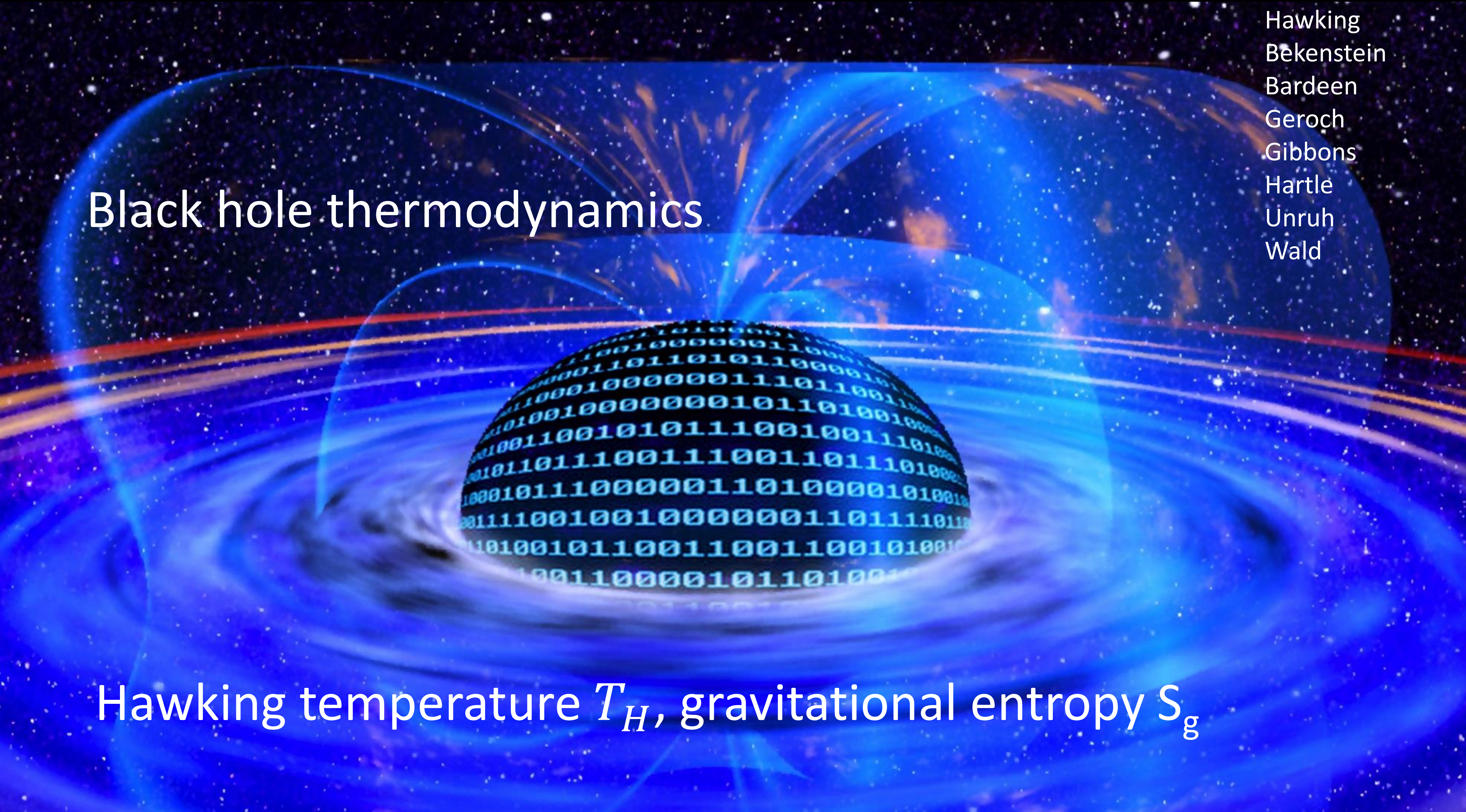
gravitational entropy

With pbcs in imaginary time, $Z = e^{S_g}$

partition
function

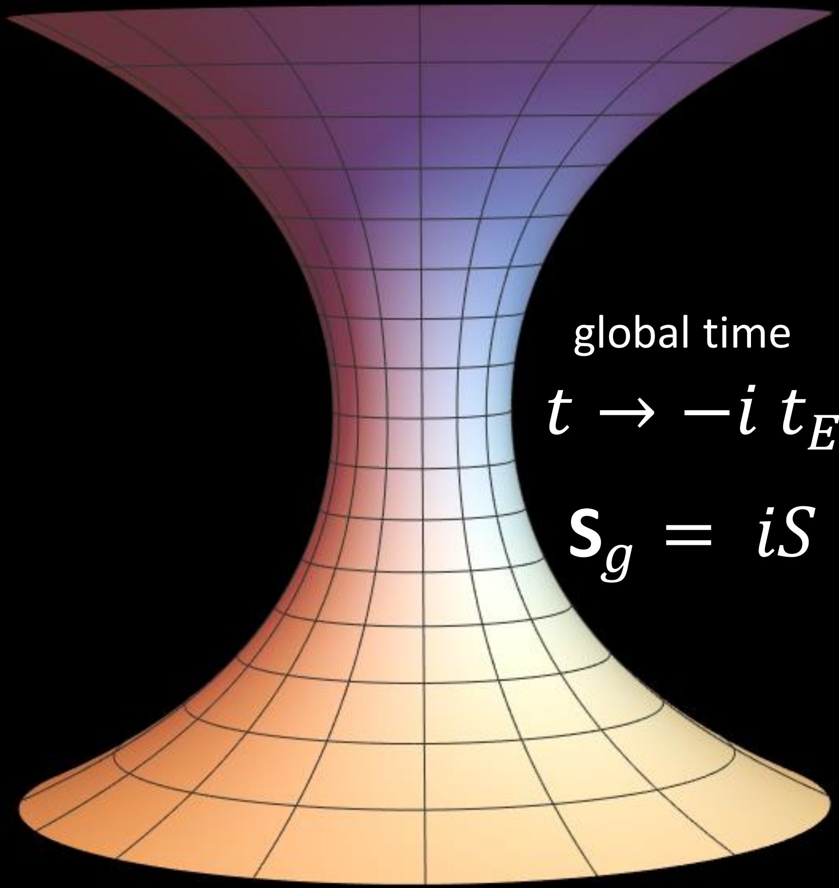
Black hole thermodynamics

Hawking
Bekenstein
Bardeen
Geroch
Gibbons
Hartle
Unruh
Wald



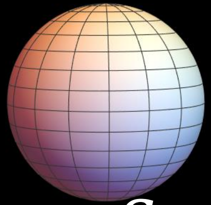
Hawking temperature T_H, gravitational entropy S_g

de Sitter



gravitational entropy from the Euclidean path integral

S^4



global time

$t \to -i\, t_E$

trace of Einstein

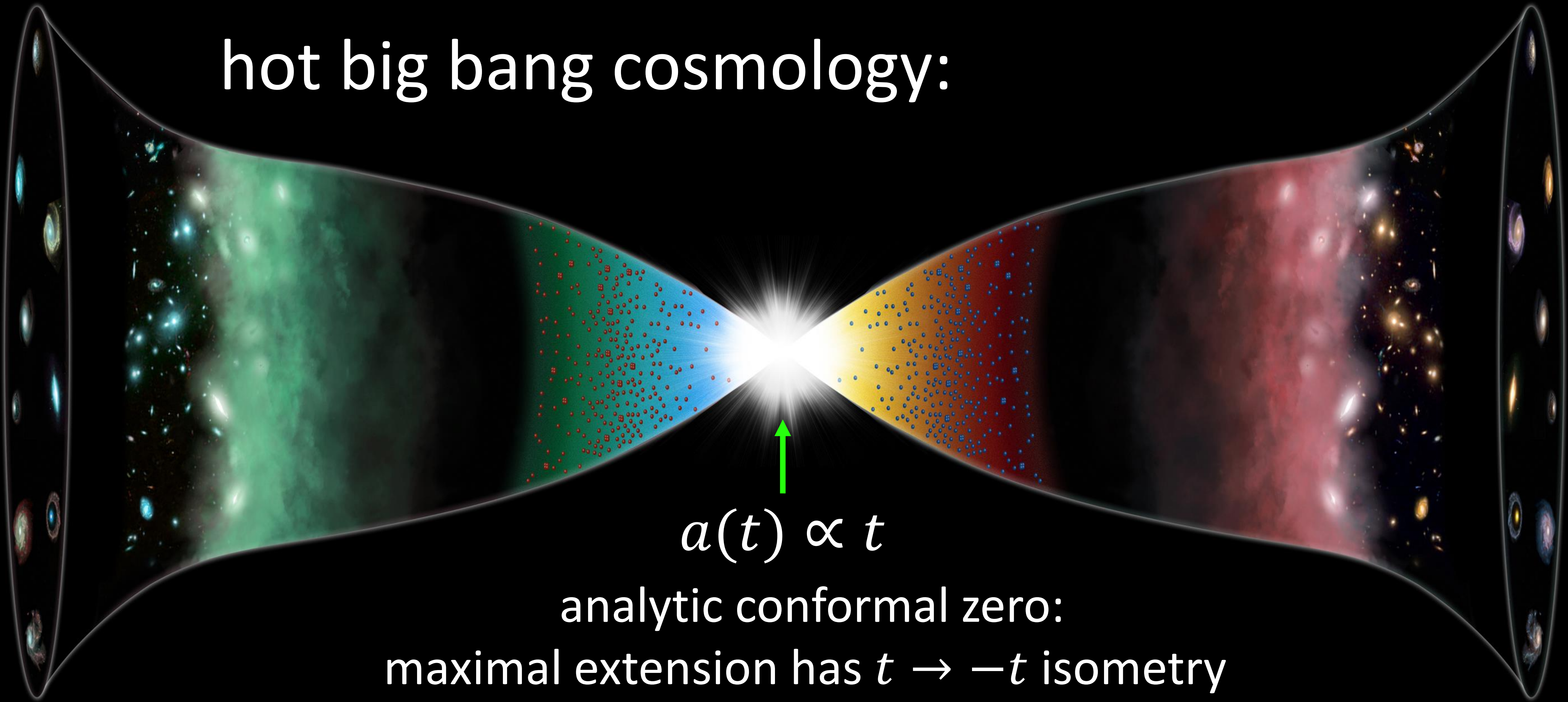
$R = \frac{4\rho_\Lambda}{M_P^2}$

$$\mathbf{S}_g = iS = -S_E = \int\left(\tfrac{1}{2}M_P^2 R - \rho_\Lambda\right) = \rho_\Lambda \,\mathrm{Vol} = \frac{24\pi^2 M_P^4}{\rho_\Lambda}$$

$\equiv \mathbf{S}_\lambda \approx 3.26 \times 10^{122}$ for measured ρ_Λ

de Sitter Entropy

hot big bang cosmology:



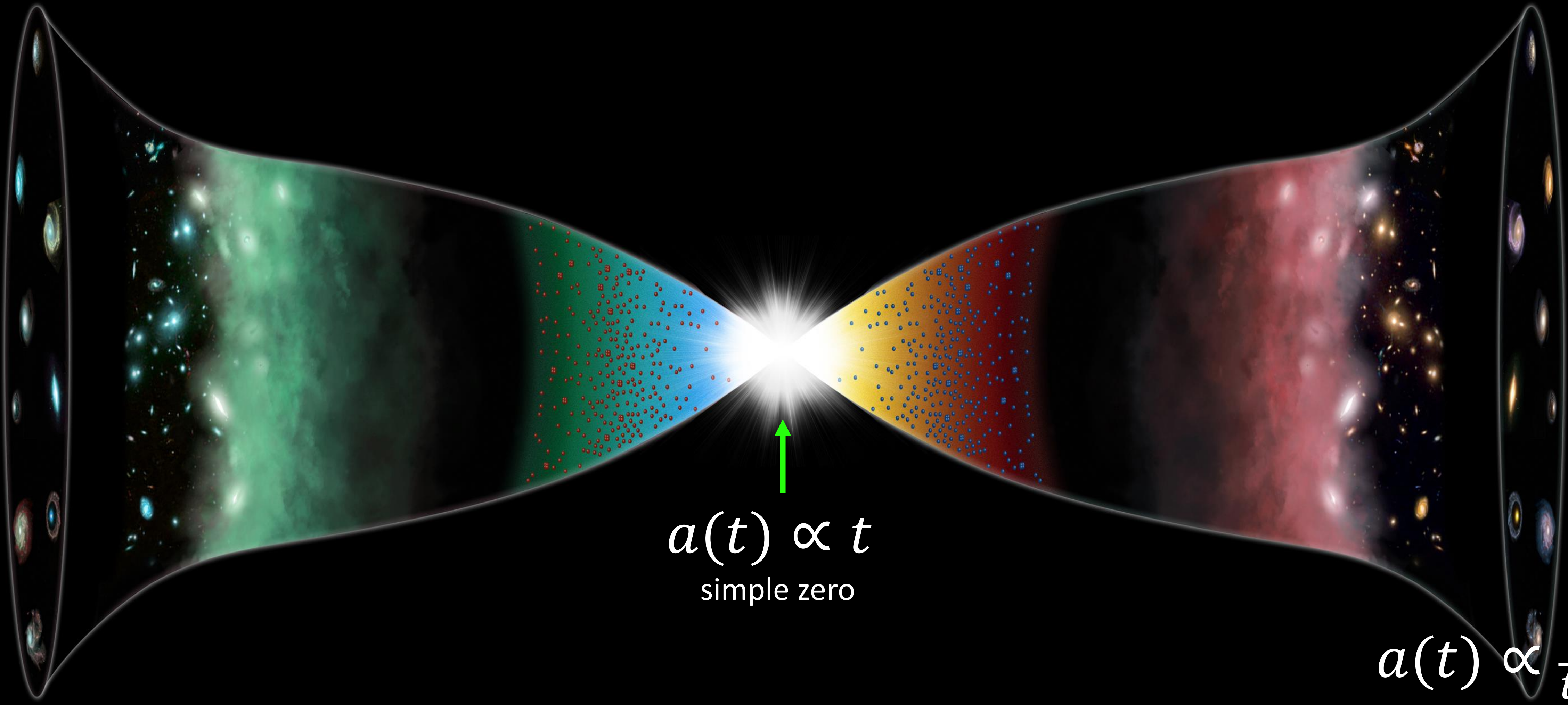
$a(t) \propto t$

analytic conformal zero:

maximal extension has $t \rightarrow -t$ isometry

can impose CPT symmetry via the “method of images”

the big bang singularity is then a mirror!



$$a(t) \propto t$$

simple zero

$$a(t) \propto \frac{1}{t_* - t}$$

simple pole

realistic cosmology:

scale
factor

$$ds^2 = a(t)^2\left(-dt^2 + \gamma_{ij}dx^i dx^j\right)$$

conformal
time

comoving symmetric space
(assume compact)

$R^{(3)} = 6\kappa$

In Planck units

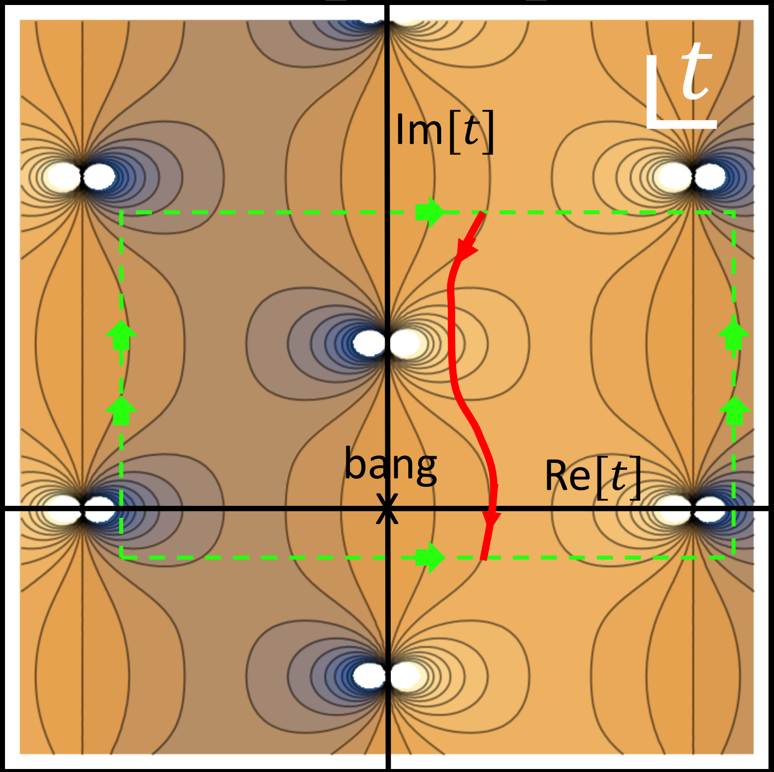
radiation matter space curvature Lambda

Friedmann $$3\dot{a}^2 = r + \mu a - 3\kappa a^2 + \lambda a^4$$

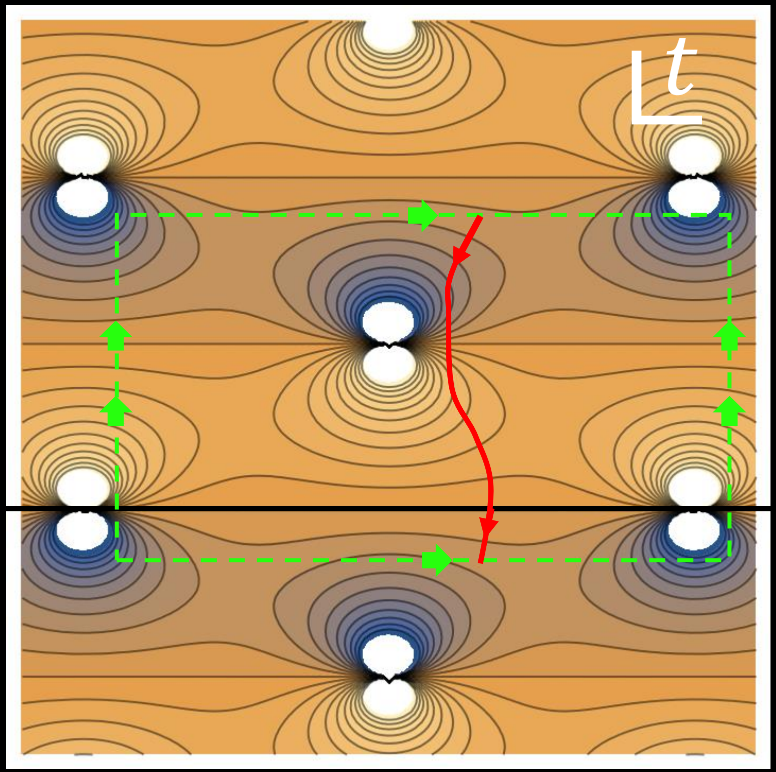
general solution (Jacobi elliptic function) is periodic in imaginary time

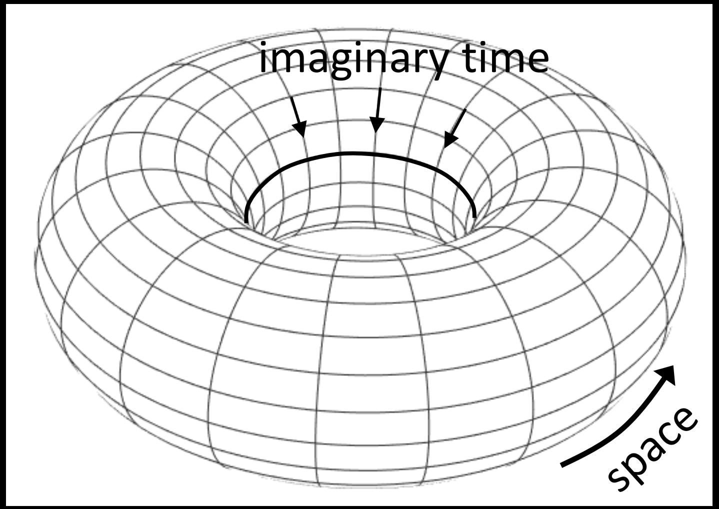
$a(t)$ is single-valued and doubly periodic in the complex t-plane: its only singularities are simple poles. The imaginary time period and the action computed over a period determine T_H and the gravitational entropy S_g

$\mathrm{Re}[a(t)]$



$\mathrm{Im}[a(t)]$





Euclidean instanton for a universe w/radiation, matter, curvature, Lambda

S_g can be calculated analytically for a general cosmology with radiation, matter, space curvature and a cosmological constant (*i.e.*, all globally conserved quantities).

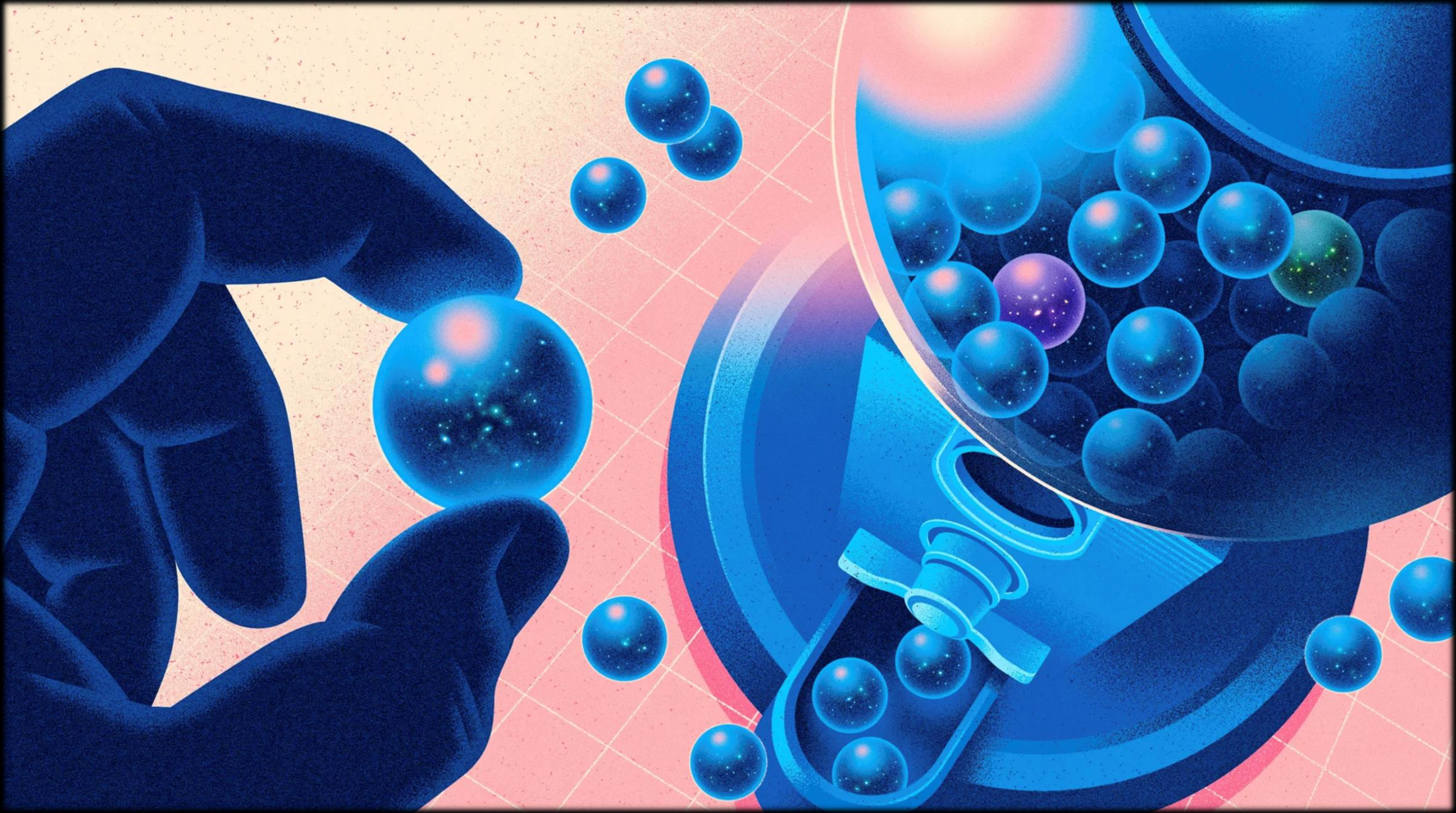
Inhomogeneities and anisotropies treated in cosmological perturbation theory.

S_g gives the total number of states associated with a cosmology. It favours

1. homogeneous, isotropic, spatially flat universes
2. a small, positive cosmological constant (echoing earlier arguments of Baum, Hawking, Coleman…)

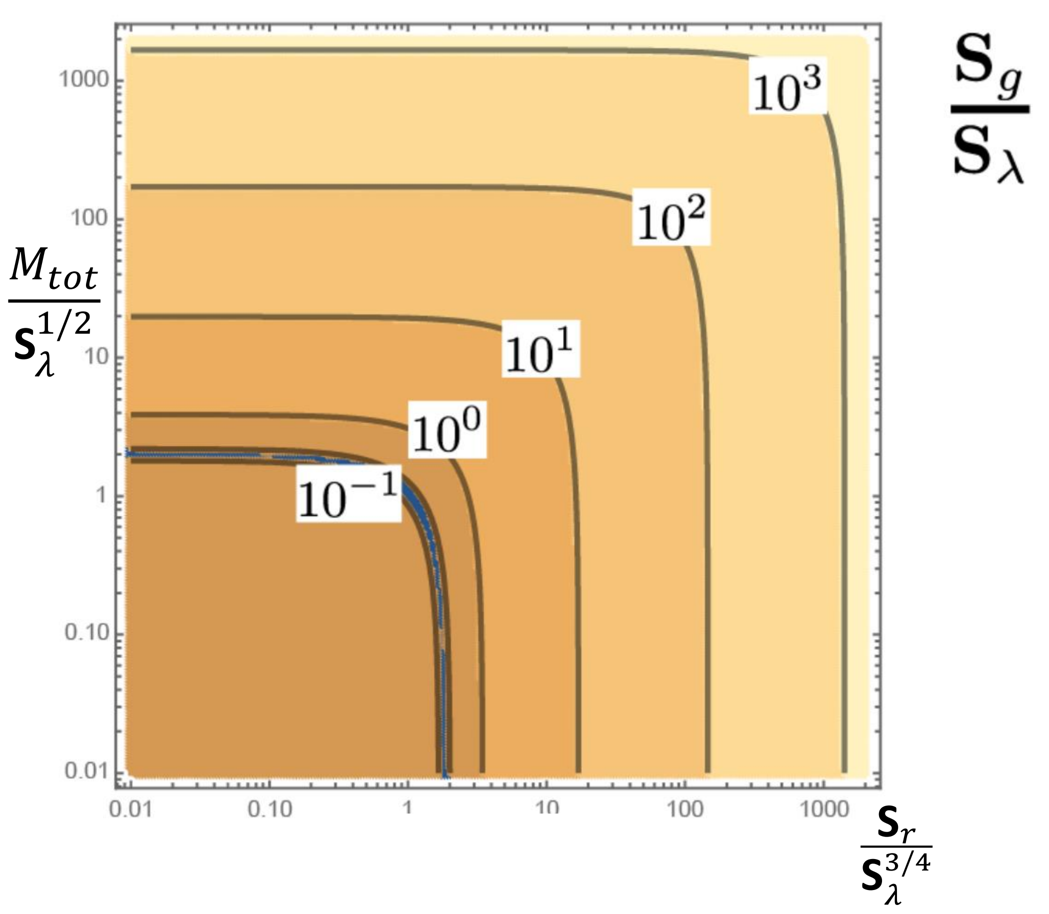
This is a thermodynamic explanation of the large-scale geometry of the cosmos. No smoothing or flattening mechanism is required.

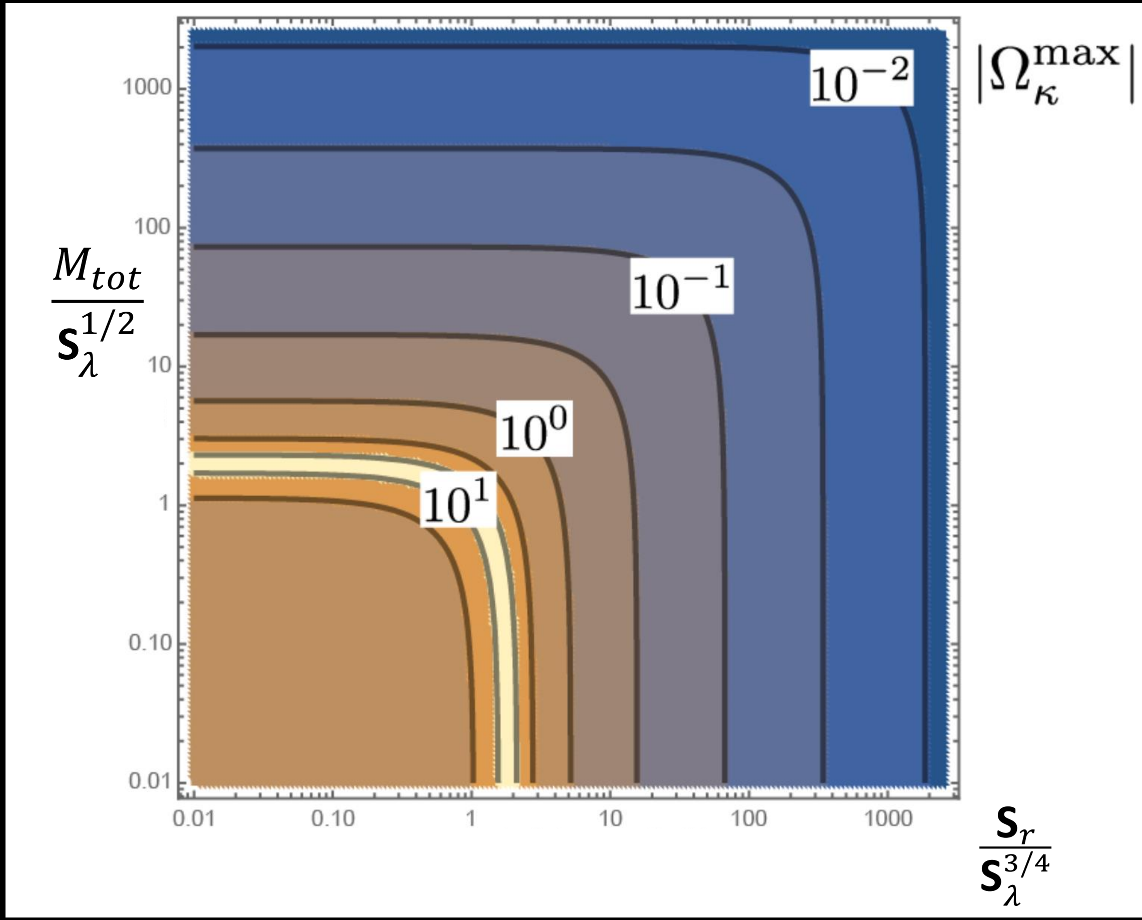
Note: S_g is the *global* entropy for the entire 4d spacetime. It is independent of real time via Cauchy's theorem. It provides a measure on the set of possible cosmologies.



Quanta Magazine, Nov 17, 2022; *WIRED*, Jan 22, 2023

Including matter





Suggests an explanation of the matter/radiation and Lambda/matter coincidence from equipartition (must also include gravitational entropy due to black holes)

For a perfect radiation fluid, $T^{\mu}{}_{\mu} = 0 \;\; (P = \frac{1}{3}\rho)$, *i.e.*, local conformal symmetry, $\exists\ \infty^3$ solutions to Einstein-fluid equations which are analytic at $t = 0$.

$$ds^2 = t^2(-dt^2 + h_{ij}(t,\boldsymbol{x})dx^i\, dx^j);\; h_{ij}(t,\boldsymbol{x}) = h^0_{ij}(\boldsymbol{x}) + t^2\, h^2_{ij}(\boldsymbol{x}) + \ldots,$$

regular 4-metric

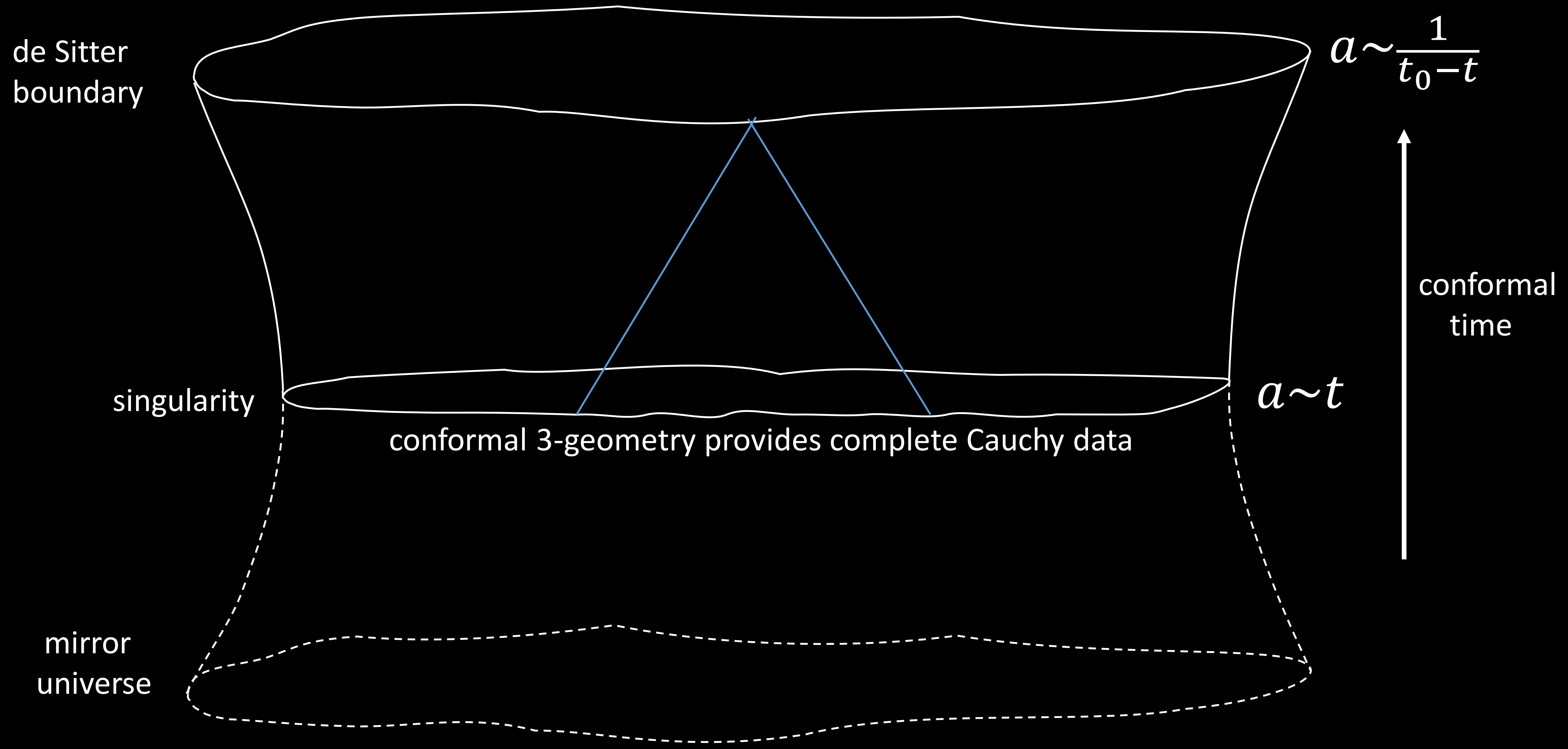
regular 3-metric

determined by Einstein eqns

They all have a global isometry $t \leftrightarrow -t$. They are saddles of the real-time path integral for gravity with CPT-symmetric boundary conditions.

The singularity is purely *conformal* and invisible to conformally invariant matter

BKL or Mixmaster excluded because they are singular hence not genuine saddles

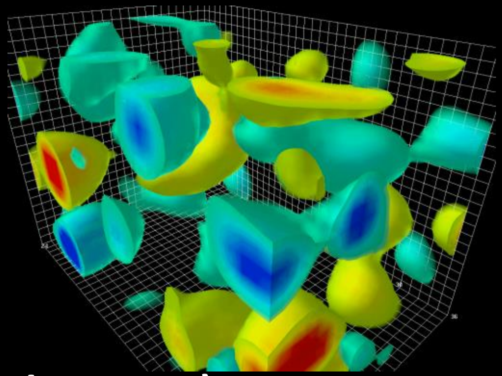


The full nonlinear Einstein's equations for inhomogeneous cosmology can be solved in a covariant gradient expansion by matching the two asymptotic series

(NT, in prep. 2023)

a minimal explanation of the perturbations

Quantum fields and gravity



vacuum energy and pressure are divergent,
physical regularizations give (*e.g.*, Maxwell, point-splitting):

B.S.DeWitt, Phys. Rep. 19 (1975) 295

$$\Longrightarrow \langle T^{\mu\nu}\rangle \sim \frac{3}{\pi^2 \Delta t^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}, \quad \text{where } \Delta t^2 = \text{invart time-like separation}$$

Can be renormalized away but leaves us with little physical understanding

Similarly, quantum divergences spoil the local scale (Weyl) invariance of Maxwell and Dirac fields in curved backgrounds: two independent conformal anomalies.

$$\langle T^{\mu}{}_{\mu}\rangle = a\,E + c\,C^2; \;\; E = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2; C^2 = C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

These cannot be renormalized away.

Dimension zero scalars: "fields without particles"

Described by a four-derivative, Weyl-invariant (*i.e.,* locally scale-invariant) action

$$S_4 = -\frac{1}{2}\int d^4x\sqrt{-g}(\Box\varphi)^2 + ..$$

Bogoliubov *et al.* (1987) recognized this as a specially simple kind of gauge theory

Infinite-dimensional symmetry: $\varphi(x) \longrightarrow \varphi(x) + \alpha(x)$ with $\Box\alpha = 0$

allows one to project out negative norm (ghost) states*

The only physical state is the vacuum: it possesses scale-invariant fluctuations

$$\langle\varphi(0,\boldsymbol{x})\varphi(0,\boldsymbol{y})\rangle = \int \frac{d^3\boldsymbol{k}}{(2\,\pi)^3}\frac{e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{y})}}{4\,k^3}$$

cf. Newtonian potential Φ in cosmology

* for a general analysis, see S. Bateman + NT in prep. (2023)

These fields can cancel the above 3 anomalies in coupling the SM to gravity

L. Boyle+NT, arXiv:2110.06258

Dim 1 scalars, fermions, gauge bosons, Dim 0 scalars

1. Vacuum energy $\propto n_{s,1} - 2n_F + 2n_A + 2n_{s,0}$
2. Conformal anomaly (Euler) $\propto n_{s,1} + \frac{11}{2} n_F + 62\, n_A - 28\, n_{s,0}$
3. Conformal anomaly (Weyl^2) $\propto n_{s,1} + 3\, n_F + 12\, n_A - 8\, n_{s,0}$

1) All three vanish iff $n_{s,1} = 0 \Longrightarrow$ no fundamental dimension one scalars (the Higgs must be composite)

2) Any two equations then give $n_F = 4n_A$ and $n_{s,0} = 3n_A$

3) For gauge group $SU3 \times SU2 \times U1$, *predict* n_F =48, *i.e.*, 3 fermion generations, each with a RH ν

primordial perturbations from dimension 0 fields

Boyle+NT
arXiv: 2302.00344

Running couplings violate scale symmetry: at high temperature,

$$T_\beta^{SM} \equiv \left\langle T^{SM\mu}{}_{\mu} \right\rangle_\beta = 3P - \rho \approx \sum c_i \alpha_i^2 T^4 \equiv c_\beta^{SM} T^4; \text{ in SM, } c_\beta^{SM} \equiv \tfrac{125}{108}\alpha_Y^2 - \tfrac{95}{72}\alpha_2^2 - \tfrac{49}{6}\alpha_3^2$$

This anomalous trace can be cancelled by introducing a linear coupling in the effective action,

$$\Gamma^\varphi = \sum_{j=1}^{n_{s,0}} \frac{1}{2}\int -a\varphi_j \Delta_4 \varphi_j + \left[a\left(E - \tfrac{2}{3}\Box R\right) + cC^2 - n_{s,0}^{-1} T_\beta^{SM}\right]\varphi_j$$

(generalises a trick used in string theory)

Note: there is still a ``gauge” symmetry a la Bogoliubov.

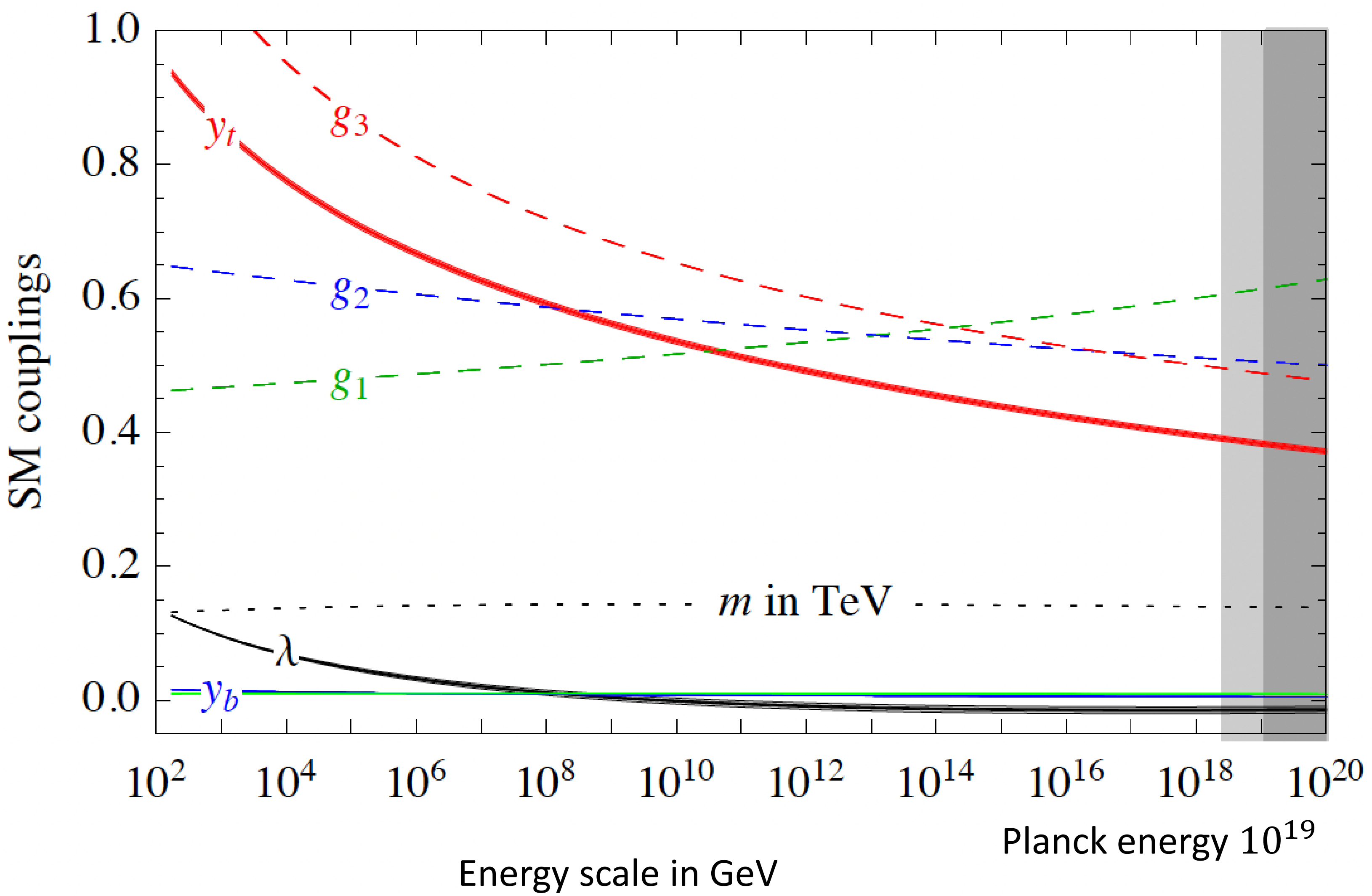
The final term corrects the Friedmann-fluid equations, converting quantum correlations in the dim-0 fields into large scale curvature fluctuations:

$\dot{a}^2 = \frac{8\pi G}{3}\rho_r a^4 (1 + c_\varphi \bar{\varphi}(x))$ with $\bar{\varphi}(x) = n_{s,0}^{-1} \sum \varphi_j(x)$, $c_\varphi = c_\beta^{SM} / \left(\frac{\pi^2}{30}\mathcal{N}_{eff}\right)$, $\mathcal{N}_{eff} \approx 106\frac{1}{4}$

This creates “comoving curvature perturbation” $\mathcal{R}(x) = \frac{1}{4} c_\varphi \bar{\varphi}(x)$

(adiabatic, Gaussian, scalar; no primordial long-wavelength gravitational waves)

Buttazzo et al
1307.3536
[hep-ph]



Spectral tilt

Dominated by QCD coupling α_3: asymptotic freedom $\Longrightarrow$ red tilt!

We argue that $\mathcal{P}_{\mathcal{R}}(k)$ runs with k as α_3^2, as $k \to 0$

This leads to the prediction $n_s - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{\mathrm{d} \ln k} = 2\frac{\beta_\alpha}{\alpha} = -\frac{7}{\pi}\alpha_{QCD}(M_P)$

Since this is a critical exponent we may extrapolate all the way from the Planck length to today's Hubble radius traced comoving back to the Planck time.

The amplitude and tilt agree with Planck's observations!

Prediction for primordial perturbations

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{3^2 5^2}{7(2\,\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2\left(\frac{k}{k_P}\right)^{-\frac{7\alpha_3}{\pi}};$$ k_P= comoving Planck wavenumber

with $c_\beta^{SM} \equiv \frac{125}{108}\alpha_Y^2 - \frac{95}{72}\alpha_2^2 - \frac{49}{6}\alpha_3^2$ and $\mathcal{N}_{eff} = 106\frac{1}{4}$

Now use $(k_P/k_*)^{1-n_s} = 14.8 \pm 5.1,$ $k_* \equiv$0.05 Mpc^{-1}

Thus, we predict $\mathcal{P}_{\mathcal{R}} = A\left(\frac{k}{k_*}\right)^{n_s-1},\ A = (13 \pm 5) \times 10^{-10};\ n_s = 0.958$

cf. Planck satellite: $A = (21 \pm 0.3) \times 10^{-10};\ n_s = 0.959 \pm 0.006$

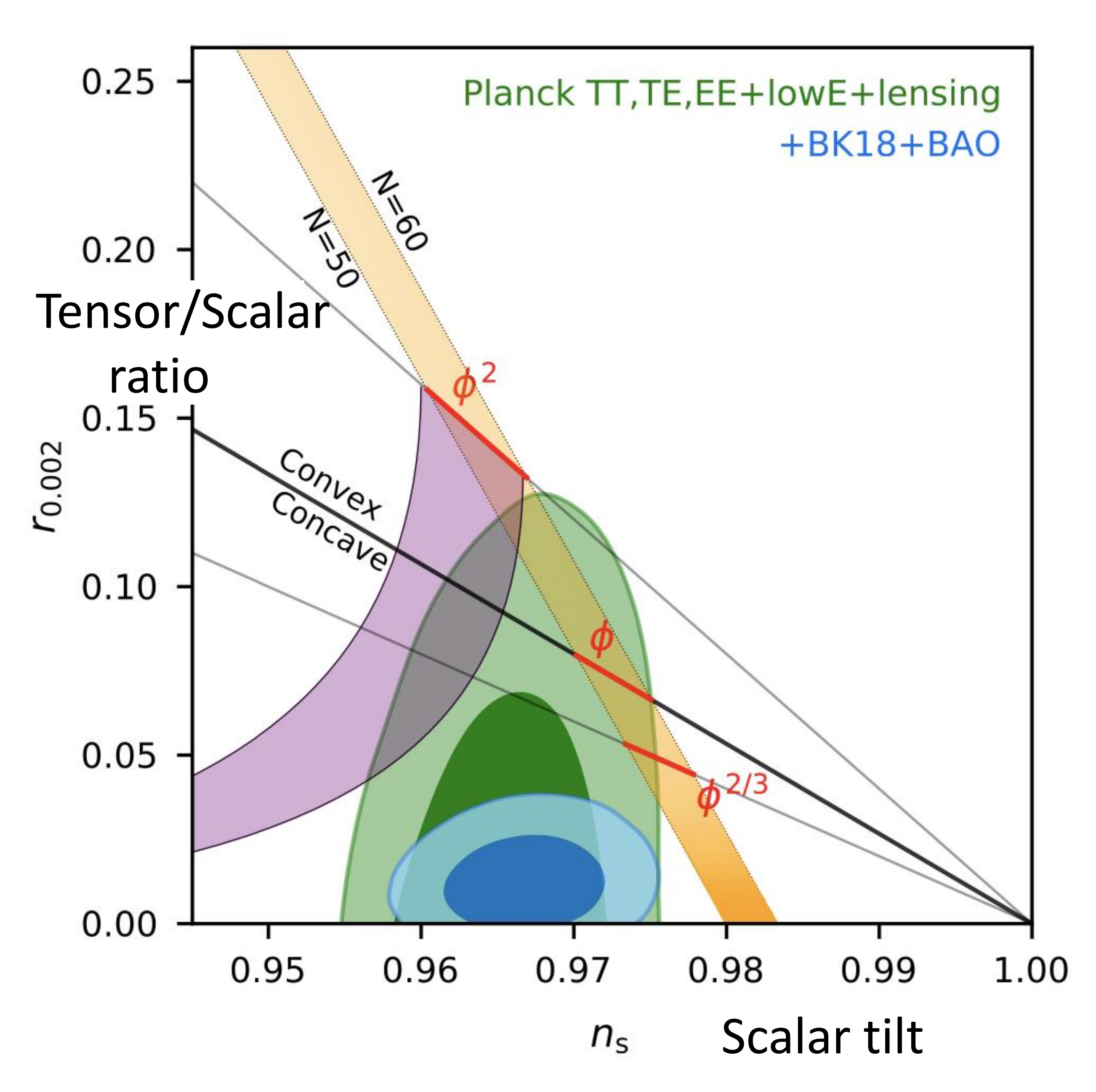
Thank you for listening!

Boyle, Finn, NT Phys. Rev. Lett. 121 (2018) 251301; Annals of Physics 438 (2022) 168767
Boyle & NT arXiv: 2201.07279 (Phys. Lett. B in press) , 2110.06258 (Phys. Lett. B in press)
arXiv: 2302.00344 and references therein

no sign of
inflationary
gravitational waves
(tensor modes)

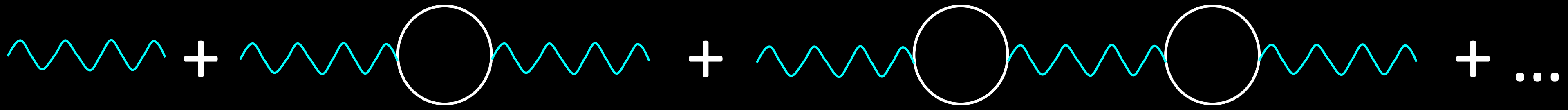
BICEP/Keck
Collaboration
2203.16556 [astro-ph]
PRL **127**, 151301 (2021)
r< 0.036 at 95% confidence

anticipated limit
r<.003
using SPT for
"delensing"
(2027)

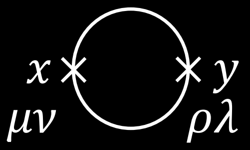


Tomboulis 70's
Han Willenbrook
Donoghue Menezes

Graviton propagator with 1 loop SM corrections



Loop is given by the Fourier transform of the stress-energy correlator: for a CFT,



$$= \left\langle T^{\mu\nu}(x) T^{\rho\lambda}(y) \right\rangle = C^T \frac{1}{4\,\pi^4 x^8}\, I^{\mu\nu,\rho\lambda}(x-y)$$

where $I^{\mu\nu,\rho\lambda}(x) = \frac{1}{2}\big(I^{\mu\rho}(x) I^{\nu\lambda}(x) + I^{\mu\lambda}(x) I^{\rho\nu}(x)\big) - \frac{1}{4}\eta^{\mu\nu}\eta^{\rho\lambda}$ and $I^{\mu\nu}(x) = \eta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}$

$C^T = \frac{4}{3}[n_{s,1} + 3\, n_F + 12\, n_A - 8\, n_{s,0}] \equiv \frac{4}{3} n_{eff}$ ($\propto$ coefft c of Weyl squared trace anomaly)

Projector onto spin 2 component
- gauge invariant

Dim reg and min sub $\Rightarrow$ $D^{\alpha\beta,\mu\nu}(k) = \dfrac{P^{\alpha\beta,\mu\nu}(k)}{k^2\Big((1 - \frac{n_{eff}}{240\pi} G k^2 \ln(-\frac{k^2}{\mu^2})\Big)}$

SM corrections to the graviton propagator:

1. Inconsistent with Källén-Lehmann repn. $D(k) = \int_0^\infty dm^2\, \rho(m^2) \frac{1}{k^2 - m^2 + i\varepsilon}$

(follows from Poincare invariance and positivity of the physical Hilbert space)

2. Specifically, resummed $D(k)$ (i) falls off as $|k|^{-4}$ at large $|k|$

(ii) has complex (acausal) poles on physical sheet

Similarly, dim-0 scalar loops alone violate K-L: (i) $|k|^{-4}$ fall off; (ii) a tachyonic pole

BUT:

SM + dim-0 combination is consistent with Poincaré, positivity and microcausality (at one loop in SM gauge+fermion fields: we are now examining higher orders)