

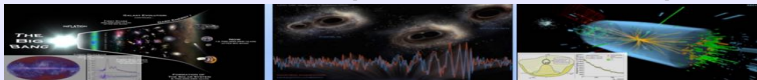
Pulsar Timing Residuals Induced by Wideband Ultralight Dark Matter

Speaker: Yun-Long Zhang (NAOC)

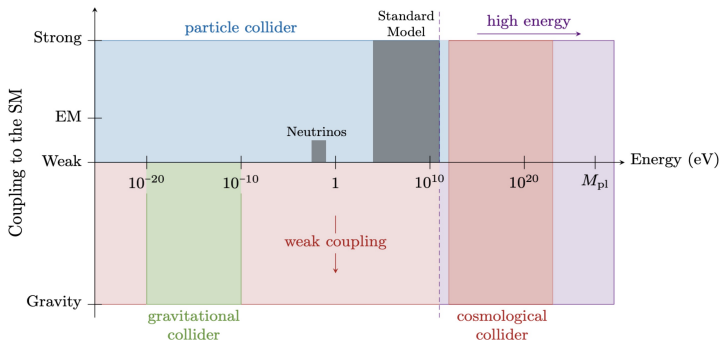
National Astronomical Observatories,
Chinese Academy of Sciences

Phys.Rev.D 106, 066006(2022) with S. Sun, Xing-Yu Yang(ITP-CAS)
Phys.Rev.D 104, 103009(2021) with Sichun Sun(Beijing Inst.Tech.)

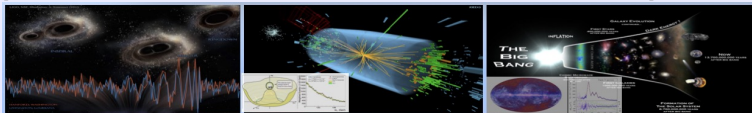
Dec.11@Texas in Shanghai [email: zhangyunlong@nao.cas.cn]



Motivation: new physics in ultra-low energy



[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



Ultra Light Dark Matter

GW & ULDM

Ultralight DM

Axion-like DM

BH Superradiance

Resonance

Radio Burst

GW Burst

PTA and DM

spin-0 DM

spin-2 DM

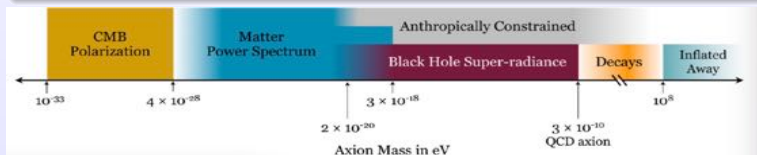
Summary

QCD axion

- $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \alpha_\theta \theta F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + \bar{\psi} \left(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5} \right) \psi$
- Strong CP problem: no CP violation in measurement
- Peccei–Quinn (1977): introduce new pseudoscalar
- Wilczek–Weinberg: relax the CP-violation parameter

Axion-like: ultra light dark matter

- e.g. Fuzzy Dark Matter, Dark photon dark matter





Ultralight dark matter and gravitational wave

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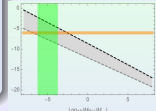
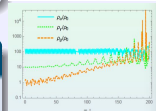
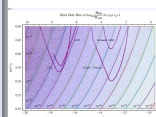
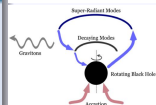
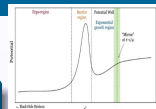
Summary

Black Hole Superradiance & GW signal

- Axion annihilation $\vartheta + \vartheta \rightarrow h$ (Stochastic GW)
Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$ (Monochromatic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_{\vartheta}} \simeq \left(\frac{M_{BH}}{M_{\odot}} \right) \left(\frac{m_{\vartheta}}{10^{-10} \text{eV}} \right)$
- Fast Radio Burst from Axion $\sim \vartheta F \tilde{F}$ ($\vartheta \rightarrow \gamma\gamma$)
- GW burst from Axion $\sim \vartheta R \tilde{R}$ ($\vartheta \rightarrow hh$)

Ultra-light DM and multi band GW detection

- Tabletop exp: Axion star & GW burst ($\sim \text{GHz}$)
- LISA & LVK: Superradiance ($\sim \text{mHz} - \text{kHz}$)
- FAST & SKA : Ultra-light dark matter ($\sim \text{nHz}$)



Axion annihilation and Stochastic GWs

GW & ULDM

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Radio Burst

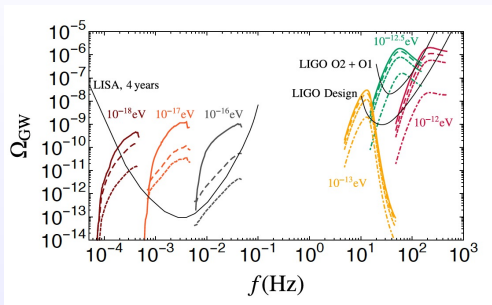
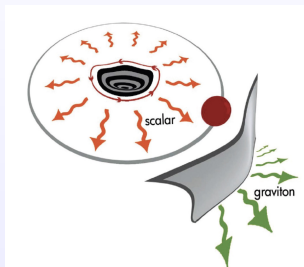
GW Burst

PTA and DM

spin-0 DM

spin-2 DM

Summary



- Axion annihilation $\vartheta + \vartheta \rightarrow hh$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Energy level transition and Monochromatic GW

GW & ULDM

Ultralight DM

Axion-like DM

BH Superradiance

Resonance

Radio Burst

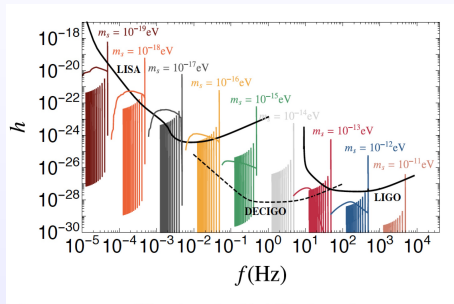
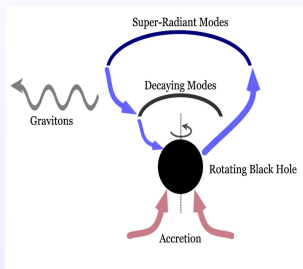
GW Burst

PTA and DM

spin-0 DM

spin-2 DM

Summary



- Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromatic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

i. Axion-photon coupling ($\vartheta \rightarrow \gamma\gamma$)

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spin-2 DM

Summary

Interaction

- The interaction

$$S_{EM} = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F^2 + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta F \tilde{F}} \right).$$

- Axion term $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - m_\vartheta^2 f_\vartheta^2 (1 - \cos \frac{\vartheta}{f_\vartheta})$.

- Interaction $\mathcal{L}_{\vartheta F \tilde{F}} = -\frac{\alpha_\gamma}{4} \vartheta F_{\mu\nu} \tilde{F}^{\mu\nu}$, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}$.

Equation of motion

- $(\partial_\mu \partial^\mu - m_\vartheta^2) \vartheta = \frac{\alpha_\gamma}{4} F \tilde{F}$, $\partial_\mu F^{\mu\nu} = -\alpha_\gamma \tilde{F}^{\mu\nu} \partial_\mu \vartheta$
- Axion like field $\langle \vartheta \rangle = \bar{\vartheta}(t) = \vartheta_0 \sin(m_\vartheta t + \phi_0)$,
- Electromagnetic field $A_\pm(t, z) \equiv [A_x(t, z) \pm i A_y(t, z)] / \sqrt{2}$

Mathieu equation and parametric resonance

GW & ULDM

Ultralight DM

Axion-like DM
BH Superradiance

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Radio Burst
GW Burst

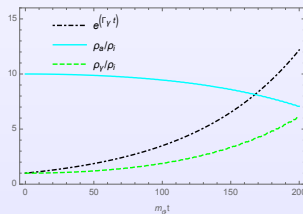
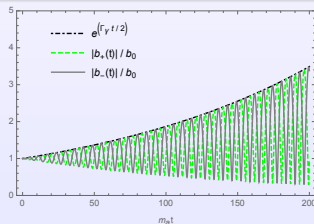
PTA and DM

spin-0 DM
spin-2 DM

Summary

$$\left[-\partial_t^2 + \partial_z^2 \mp i\alpha_\gamma \dot{\vartheta}(t) \partial_z \right] A_\pm(t, z) = 0, \quad A_\pm(t, z) = b_\pm(t) e^{ikz}.$$

- In momentum space $\ddot{b}_\pm(t) + \left[k^2 \mp \alpha_\gamma k \dot{\vartheta} \right] b_\pm(t) = 0.$
- Amplification factor $e^{\Gamma_\gamma t_\gamma}$, $\Gamma_\gamma = \alpha_\gamma \vartheta_0 \frac{m_\vartheta}{2}$, $t_\gamma \simeq \frac{1}{m_\vartheta v_\vartheta c}.$
- Time evolution of energy density: photon ρ_γ & axion ρ_ϑ .



ii. Axion gravitation coupling ($\vartheta \rightarrow hh$)

GW & ULDM

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Summary

Gravitational interaction

- Action $S_{GW} = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa_4} R + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta R \tilde{R}} \right).$
- Axion like field $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2}m_\vartheta^2\vartheta^2.$
- Chern-Simons term
$$\mathcal{L}_{\vartheta R \tilde{R}} = \frac{\alpha_g}{4} \vartheta R^\beta_{\alpha\gamma\delta} \tilde{R}^\alpha{}_\beta{}^{\gamma\delta}, \quad \tilde{R}^\alpha{}_\beta{}^{\gamma\delta} \equiv \frac{1}{2} \epsilon^{\gamma\delta\mu\nu} R^\alpha{}_{\beta\mu\nu}.$$

Equation of motion

- GWs: $\square h_{ij} = \kappa_4 \alpha_g \tilde{\epsilon}^{pk}{}_{(i} [\dot{\vartheta}(\partial_p \square h_{j)k}) - \ddot{\vartheta}(\partial_p \partial_t h_{j)k})],$
- Axion like field: $(\square - m_\vartheta^2)\vartheta = -\frac{\alpha_g}{4} R \tilde{R},$
- Polarization: $h_{ij}(t, z) = [h_R(t) \epsilon_{0ij}^R + h_L(t) \epsilon_{0ij}^L] e^{ikz} + \text{h.c.}$

Parametric Resonance and GW Amplification

GW & ULDM

Ultralight DM

Axion-like DM

BH Superradiance

Resonance

Radio Burst

GW Burst

PTA and DM

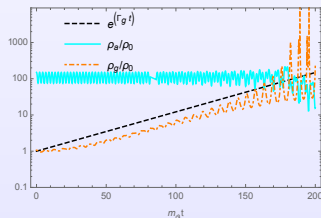
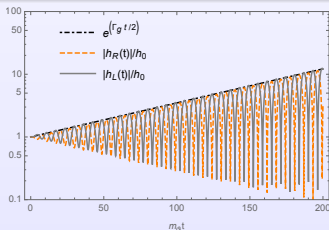
spin-0 DM

spin-2 DM

Summary

E.O.M. of gravitational wave ($l = L, R$)

- $[\ddot{h}_l(t) + k^2 h_l(t)] [1 - \varepsilon_l \kappa_4 \alpha_g k \dot{\vartheta}(t)] = \varepsilon_l \kappa_4 \alpha_g k \ddot{\vartheta}(t) \dot{h}_l(t).$
- Factor $e^{\Gamma_g t_g}$, & $\frac{\Gamma_g}{m_{\vartheta}} \sim \left(\frac{\kappa_4 \alpha_g}{1 \text{eV}^{-3}} \right) \left(\frac{m_{\vartheta}}{10^{-9} \text{eV}} \right)^2 \left(\frac{\dot{\vartheta}_0}{10^9 \text{GeV}} \right).$
- Time evolution of gravitational field ρ_g & axion field $\rho_{\vartheta}.$



[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

Pseudoscalar or Axial current to Two-graviton

GW & ULDM

Ultralight DM

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BH Superradiance

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spin-0 DM
spin-2 DM

Summary

Volume 40B, number 3

PHYSICS LETTERS

10 July 1972

THE GRAVITATIONAL CORRECTION TO PCAC

R. DELBOURGO and A. SALAM

Physics Department, Imperial College, London S.W. 7 2B7, U.K.

$$\partial_\alpha A_\alpha = 2m P + e^2 \epsilon_{\kappa\lambda\mu\nu} F_{\kappa\lambda} F_{\mu\nu} / 16\pi^2 + \epsilon_{\kappa\lambda\mu\nu} R_{\kappa\lambda\rho\sigma} R_{\mu\nu\rho\sigma} / 768\pi^2.$$

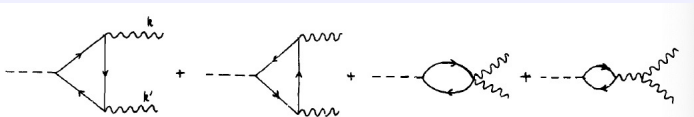


Fig. 1. One-fermion loop contributions to the two-graviton mode.

cf. Partially Conserved Axial vector Current theory (PCAC)

Joint analysis & Branch ratio ($\vartheta \rightarrow \gamma\gamma$ & $\vartheta \rightarrow hh$)

GW & ULDM

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spin-0 DM

spin-2 DM

Summary

The interaction

- $S_{total} = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa_4} R - \frac{1}{4} F^2 + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta F\tilde{F}} + \mathcal{L}_{\vartheta R\tilde{R}} \right)$
- EM field $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2}m_\vartheta^2\vartheta^2$.
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}$, $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^\beta_{\alpha\gamma\delta}\tilde{R}^{\alpha\gamma\delta}_\beta$.

Equation of motion

- $\square h_{ij} = \kappa_4 \alpha_g \tilde{\epsilon}^{pk}_{(i} [\dot{\vartheta}(\partial_p \square h_{j)k}) - \ddot{\vartheta}(\partial_p \partial_t h_{j)k})] - 2\kappa_4 (T_{ij}^{(\gamma)} + T_{ij}^{(\vartheta)})$
- axion like field $(\square - m_\vartheta^2)\vartheta = \frac{\alpha_\gamma}{4} F\tilde{F} - \frac{\alpha_g}{4} R\tilde{R}$
- Polarization $\nabla_\mu F^{\mu\nu} = -\alpha_\gamma \partial_\mu \vartheta \tilde{F}^{\mu\nu}$

Branch Ratio of GWs and EMs

GW & ULDM

Ultralight DM

Axion-like DM
BH Superradiance

Resonance

Radio Burst
GW Burst

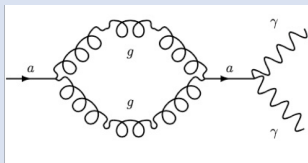
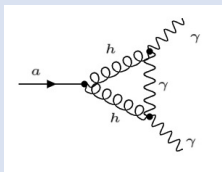
PTA and DM

spin-0 DM
spin-2 DM

Summary

- The triangle Feynman diagram: where the axion-photon coupling is generated from Chern-Simon gravity coupling.

$$\mathcal{L}_{\vartheta F \tilde{F}} = -\frac{\alpha_\gamma}{4} \vartheta F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\vartheta R \tilde{R}} = \frac{\alpha_g}{4} \vartheta R^\beta_{\alpha\gamma\delta} \tilde{R}^\alpha_{\beta}{}^{\gamma\delta}.$$

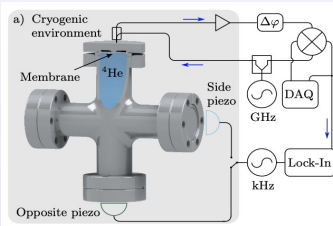
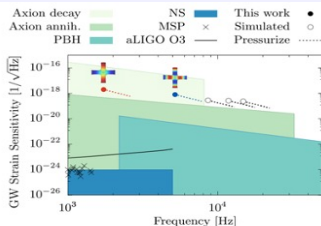


- The triangle diagram is divergent as $\alpha_\gamma \sim \alpha_g (\Lambda_{CS}/M_{pl})^4$, where Λ_{CS} is the cut-off for Chern-Simons theory.
- Two powers of M_{pl} from $h_{\mu\nu} T^{\mu\nu}$ coupling.

Parametric resonance and GW amplification

Branch Ratio and GWs

- $\frac{\text{Br}(\vartheta \rightarrow gg)}{\text{Br}(\vartheta \rightarrow \gamma\gamma)} \simeq \frac{\alpha_g^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{pl}}{\Lambda_{cs}}\right)^8$, (Power of FRB $P_{(\gamma)} \sim 10^{42} \text{ ergs/s}$).
- High frequency $h_{(g)} \sim 10^{-26} \left(\frac{1\text{GHz}}{\nu}\right) \left(\frac{P_{(g)}}{P_{(\gamma)}}\right)^{1/2} \left(\frac{1\text{kpc}}{L}\right)$
- Low freq. $h_{(g)} \sim 10^{-21} \left(\frac{10^{-2}\text{Hz}}{\nu}\right)^{1/2} \left(\frac{M_{BH}}{10^7 M_\odot}\right)^{1/2} \left(\frac{1\text{kpc}}{L}\right)$



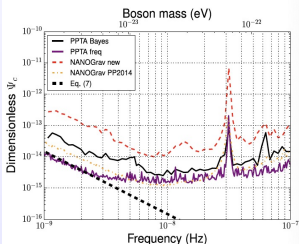
cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.

PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.

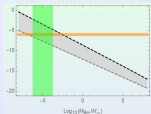
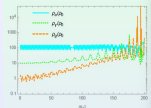
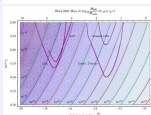
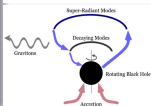
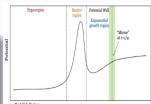
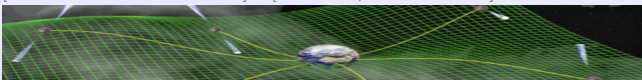
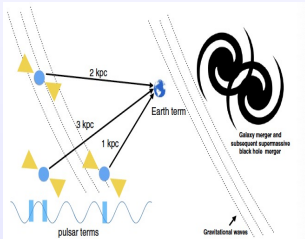
Ultra light dark matter and pulsar timing

Ultra light dark matter

- PTA & SKA: ultra light DM ($\sim$ nHz)
- $\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} \simeq 4\text{kpc} \left(\frac{10^{-23}\text{eV}}{m} \right) \left(\frac{10^{-3}}{v} \right)$
- $f_c = \frac{m}{\pi} \simeq 4.8\text{ nHz} \left(\frac{m}{10^{-23}\text{eV}} \right)$



[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



The astrophysics of nanohertz gravitational waves

GW & ULDM

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Axion-like DM
BH Superradiance

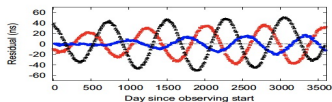
Resonance

Radio Burst
GW Burst

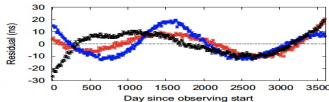
PTA and DM

spin-0 DM
spin-2 DM

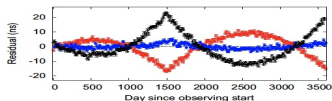
Summary



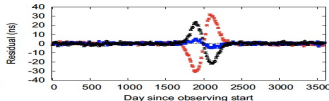
(a) Continuous wave



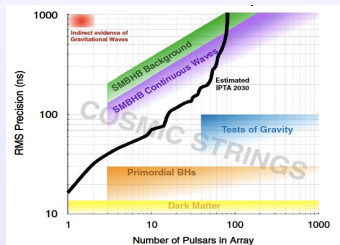
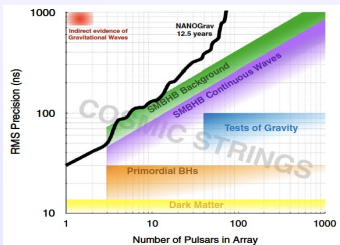
(b) Background



(c) Burst with Memory



(d) Burst



[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]

DM oscillation induced time residual

- Metric: $ds^2 = -(1 + 2\Phi) dt^2 + [(1 - 2\Psi) \delta_{ij} + h_{ij}] dx^i dx^j$.
- e.g. the scalar field $\phi(x, t) = \phi(x) \cos[mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t') dt'$
- Strain $h_\phi = 2\sqrt{3} \Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2} \frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17} \alpha_0 \left(\frac{f_{yr}}{f}\right)^2$,
- GW Timing residual: $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left(\frac{f_s}{f}\right)^{1/2}$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]

Pulsar timing residual and ultralight dark matter

GW & ULDM

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Axion-like DM
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GW Burst

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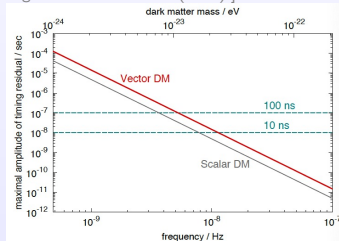
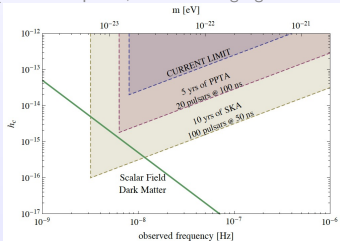
spin-0 DM
spin-2 DM

Summary

DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



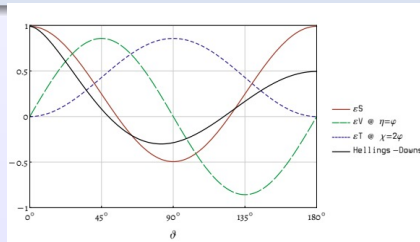
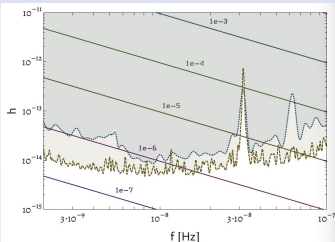
[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

Pulsar timing constraints on spin-2 ULDM

spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,

$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 (M_{\mu\nu} M^{\mu\nu} - M^2)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos[mt + \theta_2(x)] \varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t) - \omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$



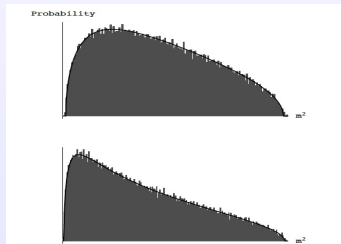
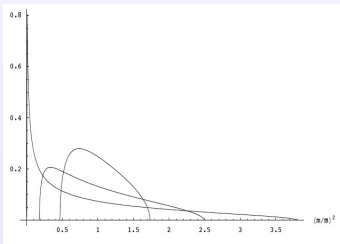
[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM" JCAP(2020)]

Multi fields: Marcenko-Pastur distribution

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m)$, $\int dm P(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices," (1967)]



[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)

Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009)]

Our phenomenological fitting results

GW & ULDM

Ultralight DM

Axion-like DM
BH Superradiance

Resonance

Radio Burst
GW Burst

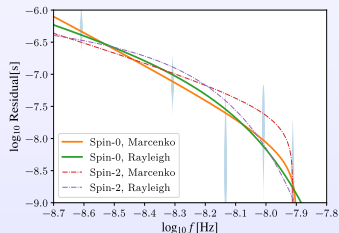
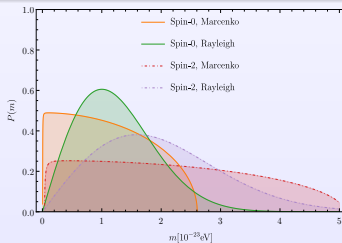
PTA and DM

spin-0 DM
spin-2 DM

Summary

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.



[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Corner Figures of Bayesian Fitting

GW & ULDM

Ultralight DM

Axion-like DM
BH Superradiance

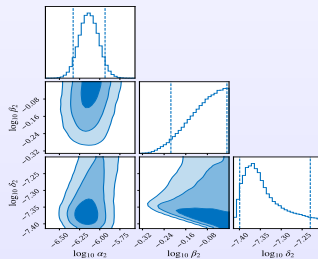
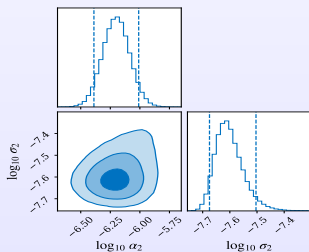
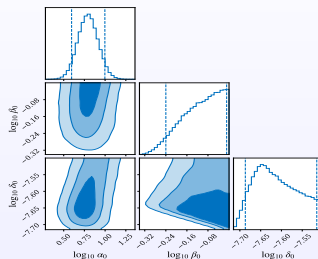
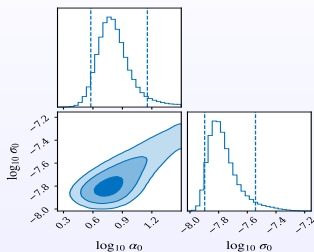
Resonance

Radio Burst
GW Burst

PTA and DM

spin-0 DM
spin-2 DM

Summary



Our phenomenological fitting results

The effective strain

- $h_c^\phi(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$
- $h_c^M(f) = \frac{\alpha_2}{M_{pl}} \frac{mMP(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$

	Parameters	spin-0	spin-1	spin-2
Marcenko	α_i	$5.9^{+1.9}_{-1.3}$	$\sim 3\alpha_0$	$7.6^{+2.2}_{-1.7} \times 10^{-7}$
	$m_-^i/(10^{-23}\text{eV})$	$2.9^{+3.6}_{-0.3} \times 10^{-3}$	$\sim \delta_0(1 - \sqrt{\beta_0})$	$6.3^{+6.0}_{-1.7} \times 10^{-3}$
	$m_+^i/(10^{-23}\text{eV})$	$2.61^{+0.21}_{-0.01}$	$\sim \delta_0(1 + \sqrt{\beta_0})$	$5.08^{+0.02}_{-0.01}$
Rayleigh	α_i	$5.6^{+3.8}_{-1.0}$	$\sim 3\alpha_0$	$6.1^{+2.1}_{-1.3} \times 10^{-7}$
	$\sigma_i/(10^{-23}\text{eV})$	$1.0^{+0.4}_{-0.1}$	$\sim \sigma_0$	$1.6^{+0.3}_{-0.1}$

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Summary and Outlook

Black Hole Superradiance & Gravitational waves

- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda \vartheta} \simeq \left(\frac{M_{BH}}{M_{\odot}} \right) \left(\frac{m_{\vartheta}}{10^{-10} \text{eV}} \right)$
- Axion annihilation $\vartheta + \vartheta \rightarrow h$ (stochastic GW)
Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$ (monochromatic)
- Fast Radio Burst from Axion $\sim \vartheta F \tilde{F}$ ($\vartheta \rightarrow \gamma\gamma$)
- Fast GW burst from Axion $\sim \vartheta R \tilde{R}$ ($\vartheta \rightarrow hh$)

Ultra-light DM and multi band GW detection

- Tabletop exp: GW burst & Axion clump ($\sim \text{GHz}$)
- LVK & LISA-Taiji: Superradiance ($\sim \text{mHz} - \text{kHz}$)
- PTA & SKA : Ultra-light dark matter ($\sim \text{nHz}$)

Thanks a lot for your attention!

