Detection and parameter estimation of ringdown signals using a pulsar timing array

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Introduction

Gravitational wave (GW) searches using a pulsar timing array (PTA) are typically limited to the GW frequency $\lesssim 4 \times 10^{-7}$ Hz, due to the average observational cadence of 2 weeks for a single pulsar. By taking advantage of asynchronous observations from multiple pulsars, PTA has the potential to detect GW signals with frequencies higher than the Nyquist frequency of a single pulsar, such as ringdown signals from the merger of supermassive binary black holes (SMBHBs) [1]. In this work, we propose a likelihood-based method for detecting ringdown signals using a PTA. We only consider a single mode, i.e., the (2,2) mode to demonstrate our method. The ringdown waveform is modeled as an exponentially decaying sinusoidal signal, and its parameters are divided into extrinsic parameters and intrinsic parameters. The former ones are determined analytically, and the latter ones are numerically determined using particle swarm optimization (PSO)[2]. We show that for the optimal signal to noise ratio (SNR) $\rho = 10$ scenario, which corresponds, for example, to a SMBHB with chirp mass $M_c = 10^{10} M_\odot$ at a distance D = 300 Mpc , the has a detection probability of 99% if the threshold is set to be the highest detection statistic value obtained with the $\rho = 0$. For the same SNR, the parameter estimation errors are: $\sigma_\alpha \approx 4.8607^\circ$, $\sigma_\delta \approx 4.2476^\circ$ for sky localization, $\sigma_\omega \approx 7.3198$ rad yr^{-1} for angular frequency, $\sigma_\tau \approx 0.009838$ yr and $\sigma_{t_0} \approx 9.54$ h for the signal start time. This is the first attempt to use PTA to detect ringdown signals above the Nyquist frequency of a single pulsar.

Waveform and algorithms

We focus on the $l = |m| = 2$ mode waveforms, and only consider Earth-term timing residuals induced by ringdown waveform. We characterize the ringdown signal in the data with a parameter set consisting of nine parameters $\lambda: \{\alpha, \delta, \omega, \tau, t_0, \psi, \zeta, \iota, \phi_0\}$. The timing residual for the I-th pulsar in a PTA is as follows

$$s_e^I(t) = \sum_{\mu=1}^{4} a_\mu(\zeta, \iota, \phi_0, \psi) A_\mu^I(t; \alpha, \delta, \omega, \tau, t_0)$$

where α, δ represent the right ascension and declination of the gravitational wave source. ω is the frequency of the ringdown signal. τ is signal decay time scale. t_0 is the time at which the ringdown signal arrived at Earth. In addition, the signal also contains parameters such as the polarization angle ψ, overall amplitude ζ, inclination angle ι and initial phase ϕ_0.

We construct the detection statistics and estimate signal parameters by maximizing the likelihood ratio Λ, leading to the F-statistic [3].The intrinsic parameters $\alpha, \delta, \omega, \tau, t_0$ are numerically determined using PSO, and extrinsic parameters $\psi, \zeta, \iota, \phi_0$ are determined by analytically solving $\frac{\partial \ln \Lambda}{\partial a_\mu} = 0$. Given a set of intrinsic parameters, the external parameters that maximize the likelihood ratio are fully determined, so the PSO only needs to search in the five-dimensional intrinsic parameter space to obtain the detection statistics.

We adjust the parameters of the PSO algorithm until the detection statistic values found by PSO are higher than the values at the true parameters. This condition can be used to tune PSO to have a higher probability of success. This ensures a robust capability for signal detection and parameter estimation in real data [2].

Simulations

We use the nearest 100 pulsars simulated for a PTA in the SKA-era [4], and obtain a simulated timing residual dataset . The timing residual for each pulsar is obtained as the sum of the ringdown-induced signal and a realization of white Gaussian noise with a standard deviation of 100ns. The duration of the simulated observation is 5 years, with a uniform biweekly cadence. The performance of the algorithm is characterized statistically using , 500 independent realizations of the timing residual dataset for each of the SNRs in the set 5, 10, 20, 100. The following table lists the remaining parameters of the ringdown signal we injected. We change the amplitude ζ of the injected signal to obtain data with different SNRs.

α/rad	δ/rad	ω/rad yr^{-1}	τ/yr	t_0/yr	ψ/rad	ι/rad	ϕ_0/rad
1.985	0.625	163.92	0.0396	0.3	0.5	0.4949	0.8

When simulating the data, we use the randomized staggered sampling method: the times t_i^I at which the residual s_e^I of the I–th pulsar is sampled is sampled satisfy $t_i^I + c_i^I = (i-1)\Delta + \delta^I$, with $\delta^I = \frac{(I-1)\Delta}{N_p}$, $\Delta = 2$weeks. Here, following [1], c_i^I is a random number drawn from the truncated Cauchy probability density function, which is consistent with that in [1]. In [1], it is shown that staggered sampling can increase the detectable GW signal frequency by a factor of N_p over that given by the single pulsar observing cadence in a PTA with N_p pulsars.

Results

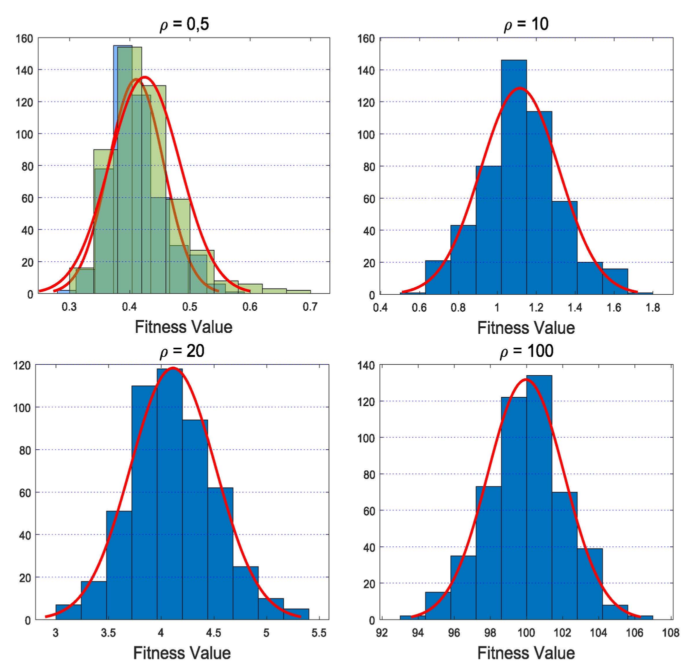

Figure 1. The distribution of detection statistics for ρ= 0, 5, 10, 20, 100. The red curve represents the normal distribution fit to the detection statistics from 500 timing data set realizations.

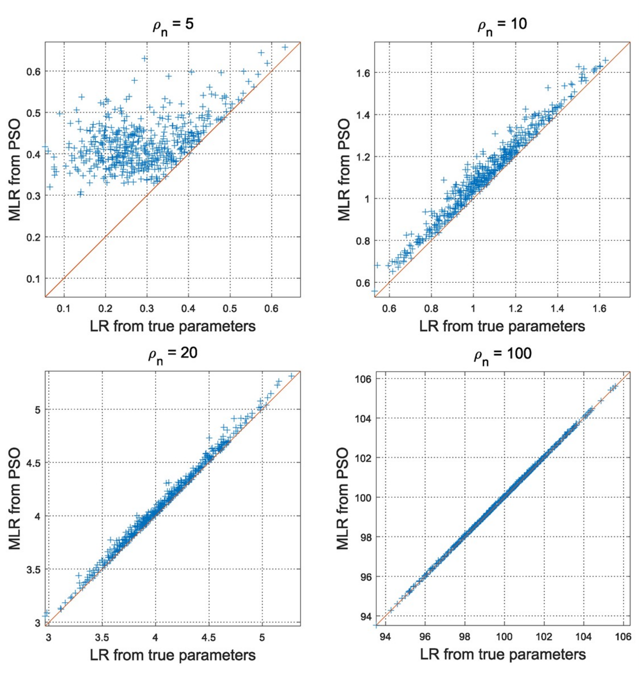

Figure 2. maximized likelihood ratio (MLR) obtained 3 PSO runs vs. likelihood ratio (LR) obtained from the true parameters for the 500 realizations for ρ= 5, 10, 20, 100.

Conclusions

We propose an algorithm based on maximum likelihood to detect and characterize the ringdown signals from mergers of SMBHBs beyond the Nyquist frequency of a single pulsar using PTA. Staggered sampling is used in our simulated observation of an array of 100 pulsars with white noise of 100 ns.

Our algorithm works in the time domain. It separate the nine unknown parameters into intrinsic and external parameters to reduce the dimension of the parameter space searched by PSO. In the simulated data, only the dominating mode of the ringdown signal is considered, and the noise is set to be white. The impact of the high-order mode of the ringdown signal and the red noise in the timing residuals is subject to our future investigations.

Results (cont.)

Figure 1 shows the distributions of detection statistics obtained from the best of three PSO runs for each of the 500 different timing residual datasets corresponding to four different SNRs, i.e., ρ= 0, 5, 10, 20, 100. Figure 2 shows that the detection statistics obtained by our algorithm are higher than those corresponding to the true parameters. As the SNR decreases, the gap between the detection statistic and the value at the true parameter is more obvious. This effect arises from the fact that, as the SNR decreases, the global maximum becomes comparable to the the local peaks in the LR arising from noise alone. Thus, PSO may converge to a local maximum without finding the global maximum and yet find a value that is stronger than that at the true parameters. The detection statistics for the high SNR case exhibit a Gaussian distribution. For $\rho = 10$ the algorithm has a detection probability of 99% if the threshold is set to be the highest detection statistic value obtained with the $\rho = 0$ (i.e., noise-only) datasets. Figure 3 shows the recovered waveform from our algorithm which performs relatively well. Figure 4 shows the distributions of the estimated intrinsic parameters of the signal. Different from other parameters, the estimation of t_0 will biased which usually is advanced by several hours than the true time. However, it may be sufficiently accurate for the follow-up EM observations.

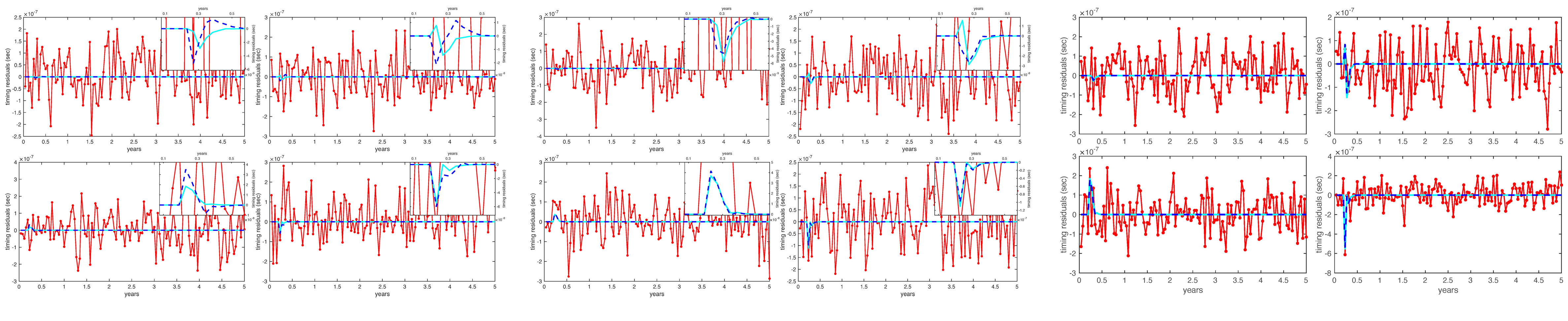

Figure 3. Timing residuals of four pulsars randomly selected from one of timing residual datasets (red solid line). Injected signals are shown by the light blue solid line in each subplot. The reconstructed signals are shown as blue dotted lines. The three sets of figures from left to right represent the case where SNR is 10, 20 and 100. For SNRs of 10 and 20, we show the zoom-in of the waveforms around of the merger in the upper right panels. The reconstructed signals matches well with the injected signals for SNR = 20 and 100.

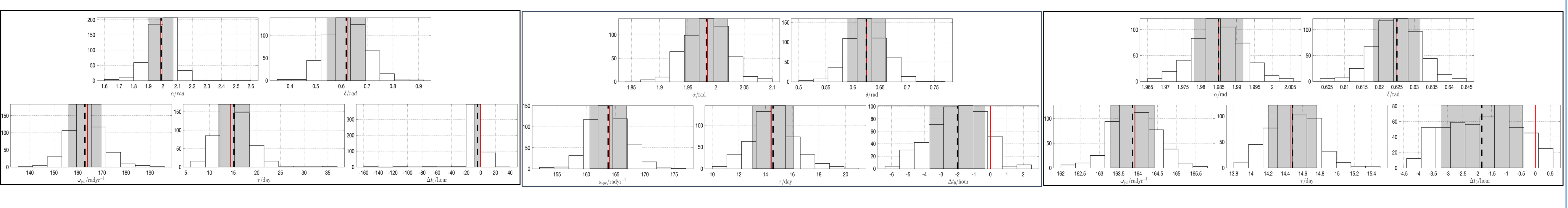

Figure 4. Histograms of estimated intrinsic parameters for SNR of 10, 20, and 100. The red vertical lines mark the true values of the parameter used in the simulations. The total number of realizations for each SNR is 500.

References

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Acknowledgments

Y.W. gratefully acknowledges support from the National Key Research and Development Program of China (Nos. 2022YFC2205201 and 2020YFC2201400), the National Natural Science Foundation of China (NSFC) under Grants No. 11973024, Major Science and Technology Program of Xinjiang Uygur Autonomous Region (No. 2022A03013-4), and Guangdong Major Project of Basic and Applied Basic Research (Grant No.2019B030302001). J. W. Chen acknowledges the support from China Postdoctoral Science Foundation under Grant No. 2021M691146.

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