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An Opacity-Free Method of Testing the Cosmic Distance Duality Relation Using Strongly Lensed Gravitational Wave Signals

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Introduction

The cosmic distance duality relation (CDDR), expressed as $D_L(z) = (1+z)^2 D_A(z)$, plays an important role in modern cosmology. In this study, we propose a new method of testing CDDR using strongly lensed gravitational wave (SLGW) signals. We assess the ability of this novel method in testing CDDR by calculating the parameter estimation based on the simulated SLGW signals from massive binary black holes and the instrument specifications of a space-based GW detector, TianQin. This work considers two types of lens models: point mass (PM) and singular isothermal sphere (SIS).

I. SLGW signals

The observed SLGW signals are given by

$$\begin{aligned}\widetilde{h}^L(f) = &\Big[|\mu_+|^{1/2}\Lambda(t(f))e^{-i(\phi_D+\phi_p)(t(f))} \\ &- i|\mu_-|^{1/2}e^{2\pi i f t_d(f)}\Lambda(t+t_d)(f)e^{-i(\phi_D+\phi_p)(t+t_d)(f)}\Big] \\ &\times\sqrt{\frac{5}{96}}\frac{\pi^{-2/3}\mathcal{M}_z^{5/6}}{D_A^S(z_S)(1+z_S)^2\eta(z_S)}f^{-7/6}e^{i\Psi(f)},\end{aligned} \tag{1}$$

Where the magnification factor and time delay are

$$\mu_\pm = \frac{1}{2}\pm\frac{y^2+2}{2y\sqrt{y^2+4}}, \tag{2}$$

$$t_d = 4M_L(1+z_L)\left[\frac{y\sqrt{y^2+4}}{2}+\ln\left(\frac{\sqrt{y^2+4}+y}{\sqrt{y^2+4}-y}\right)\right]. \tag{3}$$

II. Parameter estimation methods

The Fisher information matrix (FIM) is given by

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial\theta_i}\middle|\frac{\partial h}{\partial\theta_j}\right). \tag{4}$$

The Markov Chain Monte Carlo (MCMC) can obtain a posterior distribution of parameters based on the likelihood function as follows

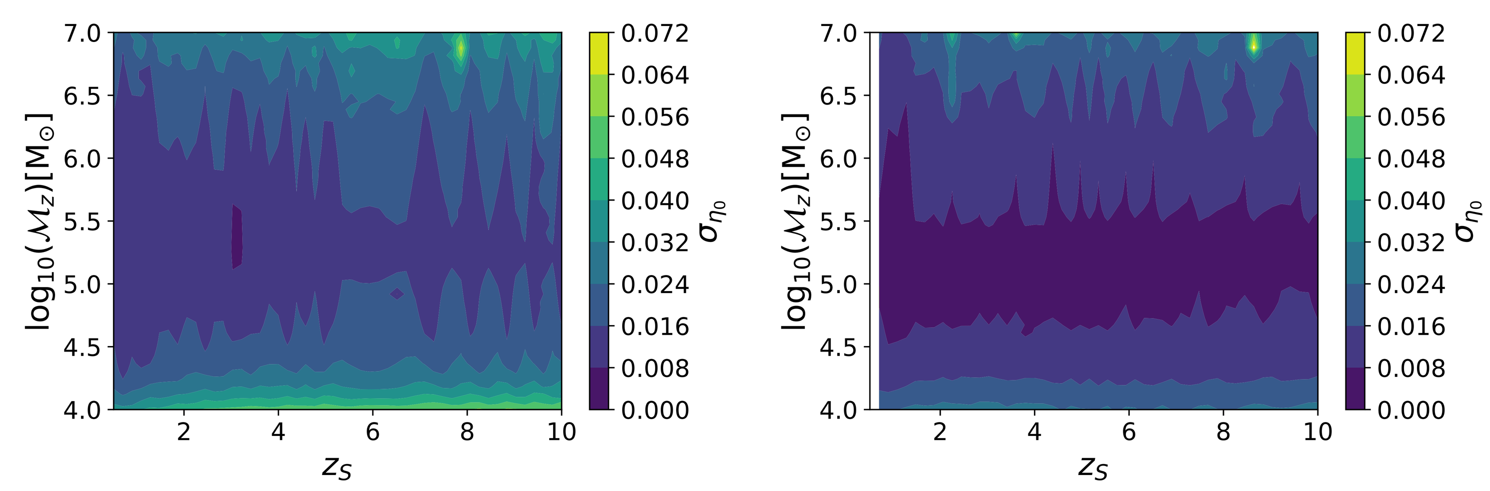
$$p(d|\theta,H)\propto\exp\left[-\frac{1}{2}\left(d-h(\theta)\middle|d-h(\theta)\right)\right]. \tag{5}$$

III. Results

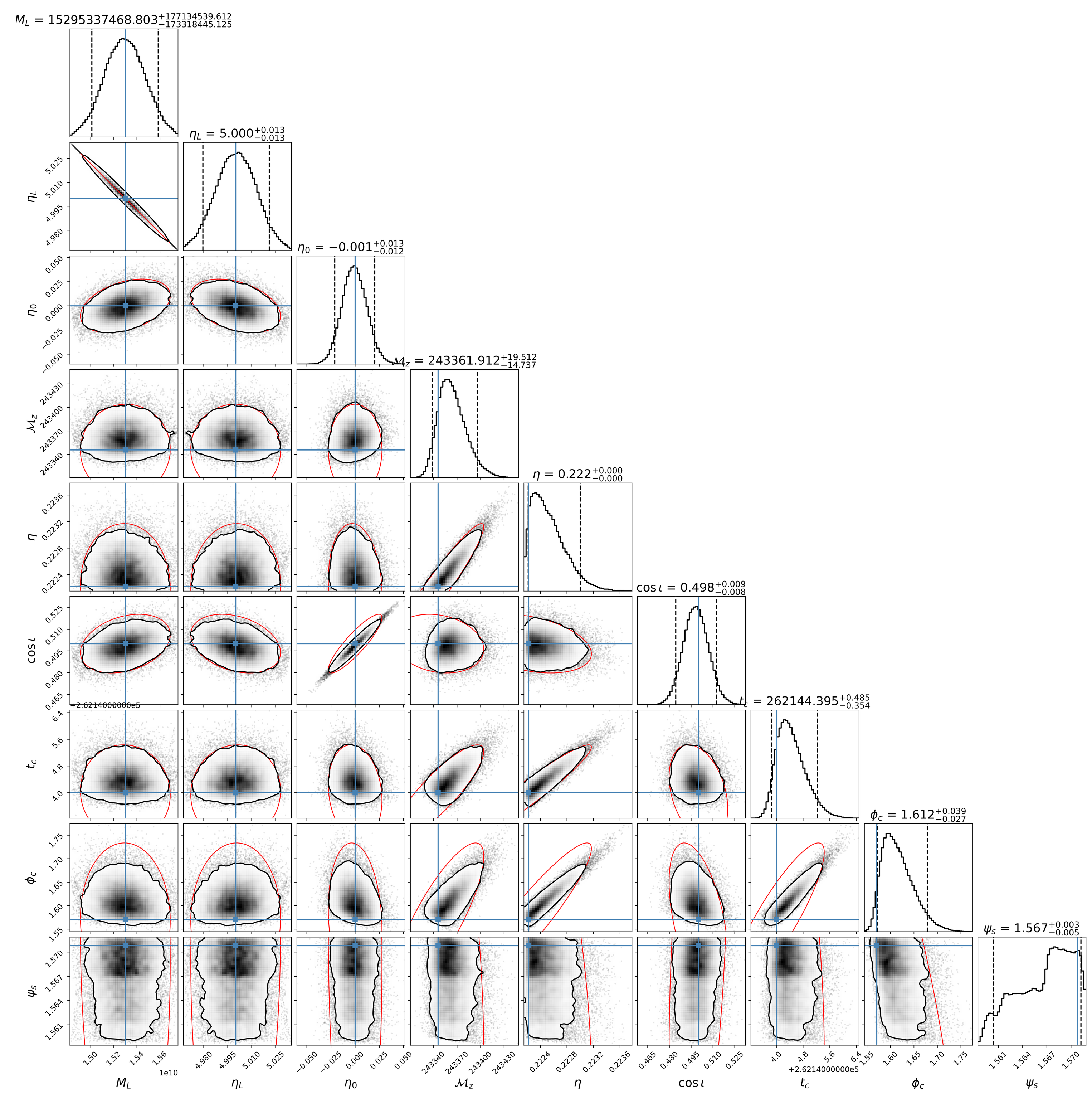
- The measurement precision of η_0 for different models.

Lens model	$\eta_1(z)=1+\eta_0 z$	$\eta_2(z)=1+\eta_0 z/(1+z)$
(a) PM, $y=0.3$	± 0.009	± 0.019
(b) SIS, $y=0.3$	± 0.005	± 0.011
(c) PM, $y=3.0$	± 0.013	± 0.026
(d) SIS, $y=3.0$	-	-

- The contour maps of the measurement precision of η_0. Left panel: PM model; right panel: SIS model.



- The posterior distributions (90% CI)of the parameters in $\eta_1(z)$ for model (c) PM with $y=3.0$. Black line: MCMC; red line: FIM.



IV. Conclusions

- The measurement precision of η_0 can reach a considerable level of 0.5-1.3% for $\eta_1(z)$ and 1.1-2.6% for $\eta_2(z)$.
- High SNR is more inclined to precise measurement of η_0.
- The consistency of parameter estimation methods between FIM and MCMC was established.

V. References

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