Universal relations for rotating and tidally-deformed neutron stars

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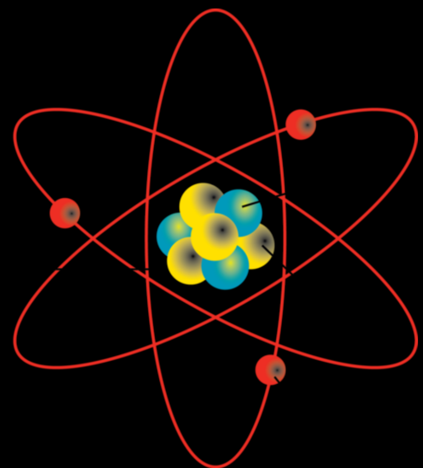
Outline

- What is universal relation for NSs?
- I-Love-Q relation and its applications
- Eccentricity-Moment of inertia relation
- Summary and outlook

The interior of NSs

Normal baryonic matter (Stars)



Compression

$p_F \propto n_e^{1/3}$

$E_e \sim 100\,\mathrm{MeV}$

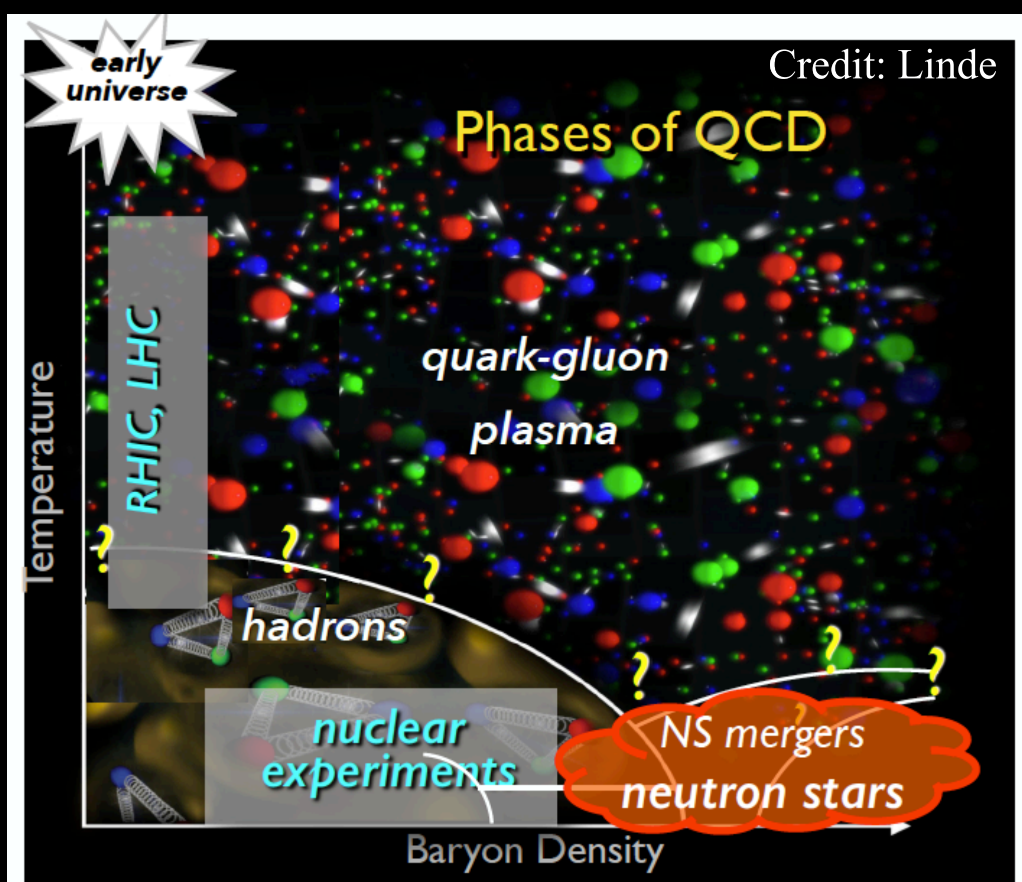
Supranuclear matter

$\sim 20\,\mathrm{km}$

- **Two ways to kill the energetic electrons**

$$e + p \to n + \nu_e$$

$$2f(u,d) \to 3f(u,d,s)$$

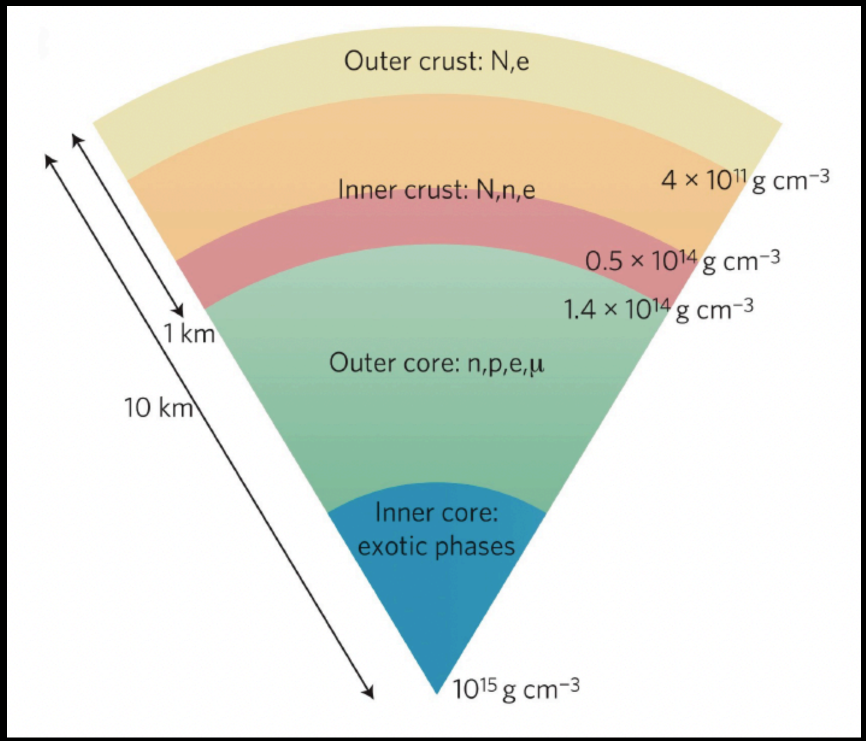


$$\rho_c \sim 10^{15}\,\mathrm{g\,cm^{-3}}$$

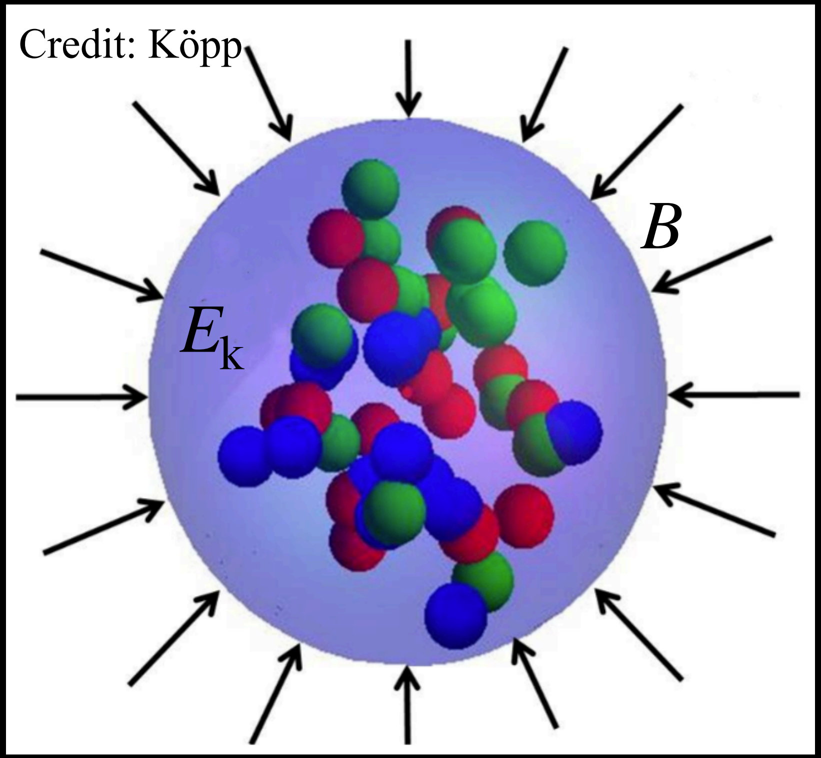
$$T \ll T_F \approx 10^{12}\left(\frac{\rho}{\rho_0}\right)^{2/3}\ \mathrm{K}$$

- Non-perturbative strong interactions of many-body problem

Models of the interior



Lattimer & Prakash 2001; Alford et al. 2005



Witten 1984; Alcock 1986

- Nucleon stars: *npeμ* matter

$$\frac{n_p}{n_n} \approx \left(\frac{E_{F,n}}{2m_u c^2}\right)^{3/2} \sim \left(\frac{100\text{MeV}}{1\text{GeV}}\right)^{3/2}$$

- New freedom in the inner core: hyperons? mesons? quarks?

- Strange stars

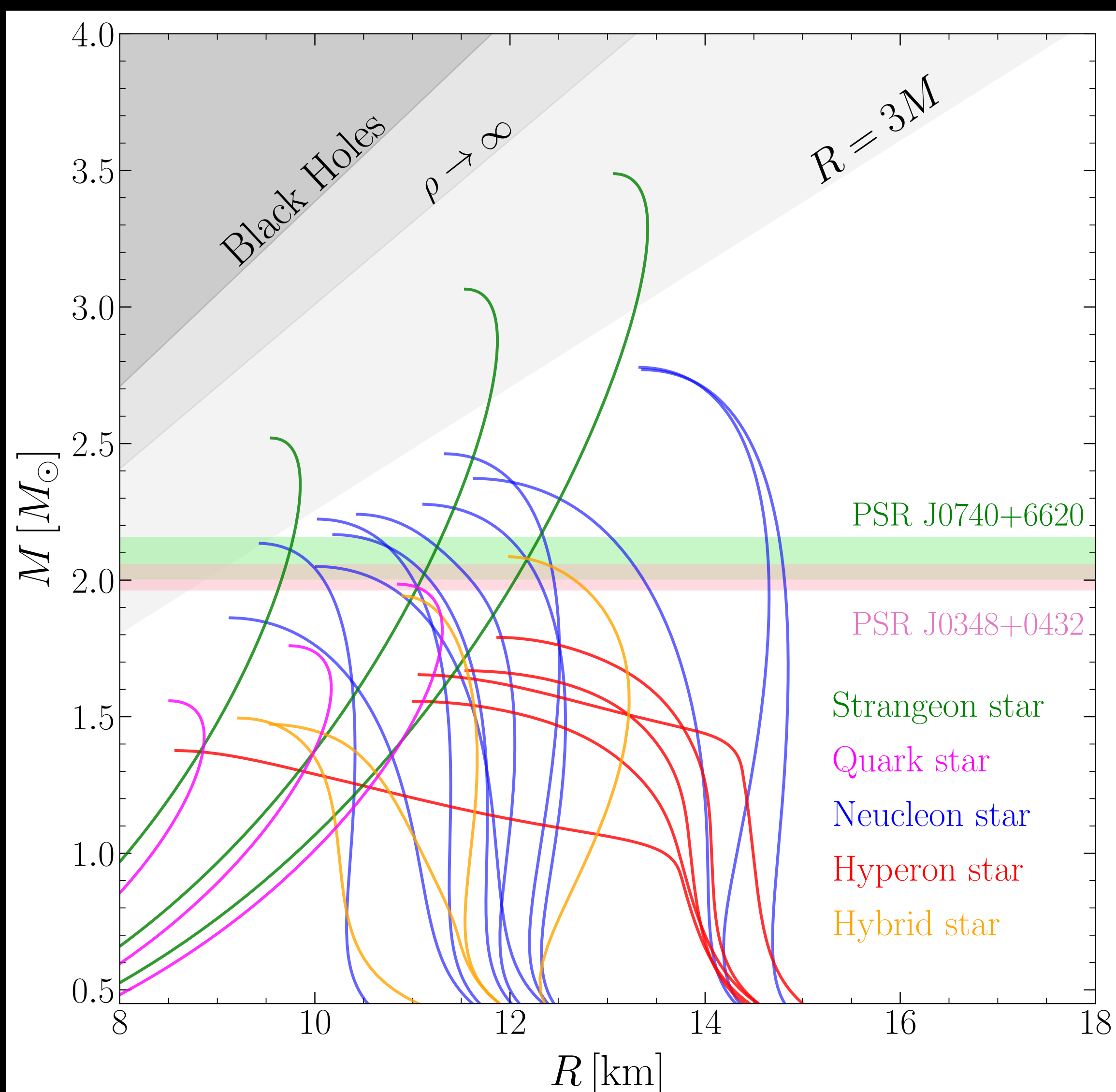
Witten's conjecture: Quark matter composed of nearly equal number of *u*, *d*, *s* is the ground state of strong matter

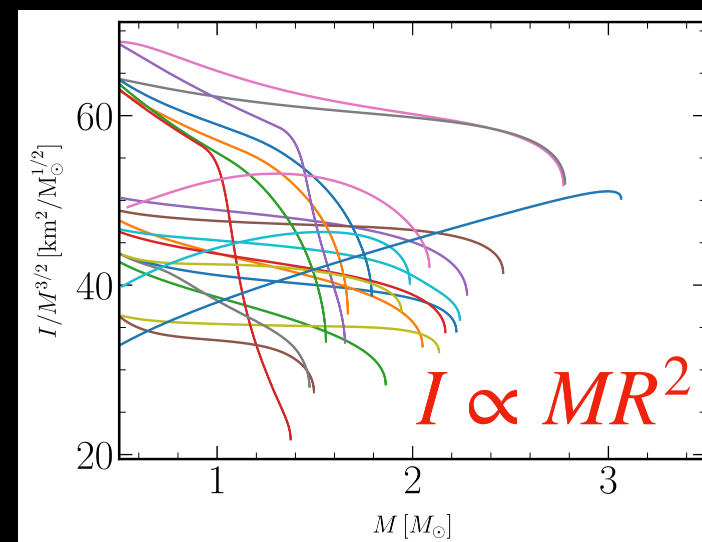
- Color superconducting phase?
- Quark-cluster (solid state)? strangeon star

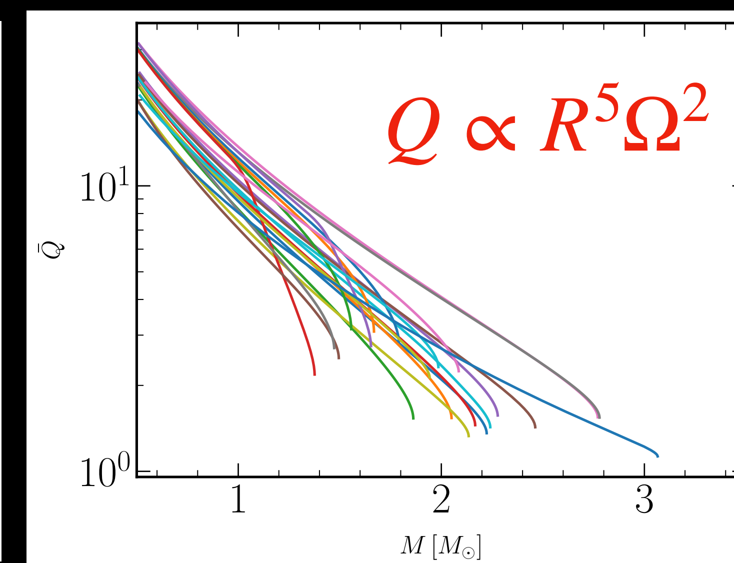
Alford et al. 2007; Xu 2003; Lai & Xu 2009; Gao, Lai, Shao, Xu 2022

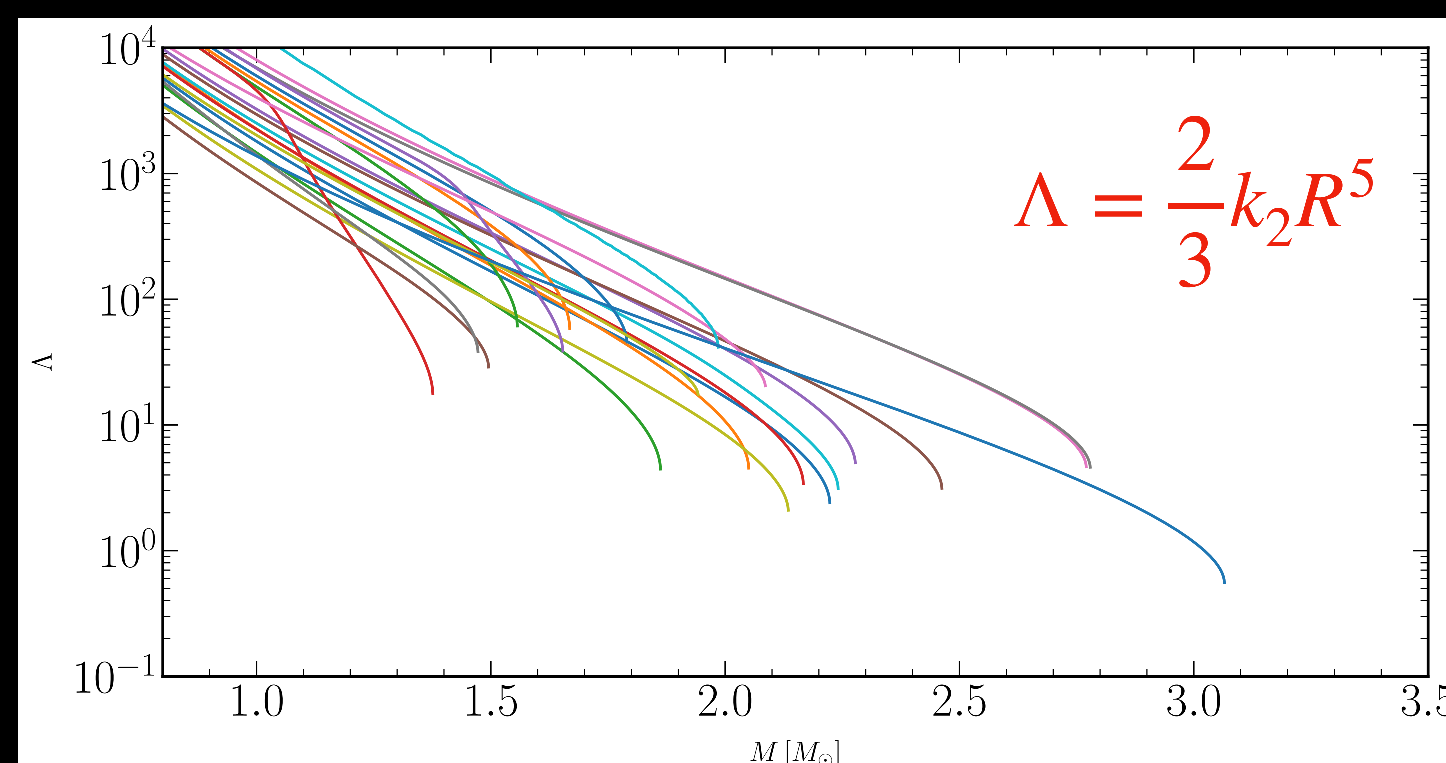
Global properties of NSs

Large uncertainties in EoS → Large scatters of global properties (good for constraining the EoS!)





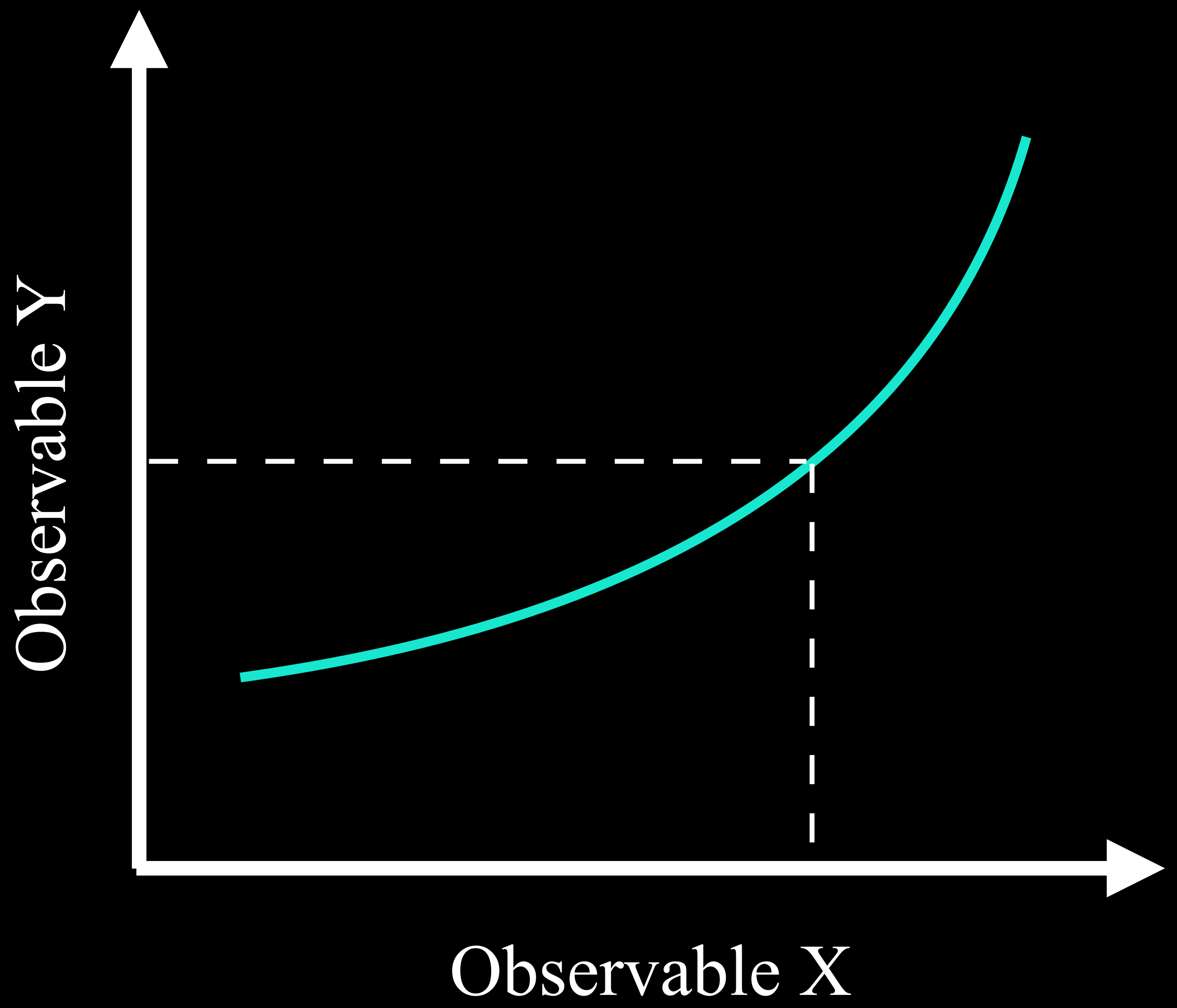




Gao, Lai, Shao, Xu, MNRAS, 2022

What is universal relation?

Relations between NS observables
that are insensitive to the EoS



Applications

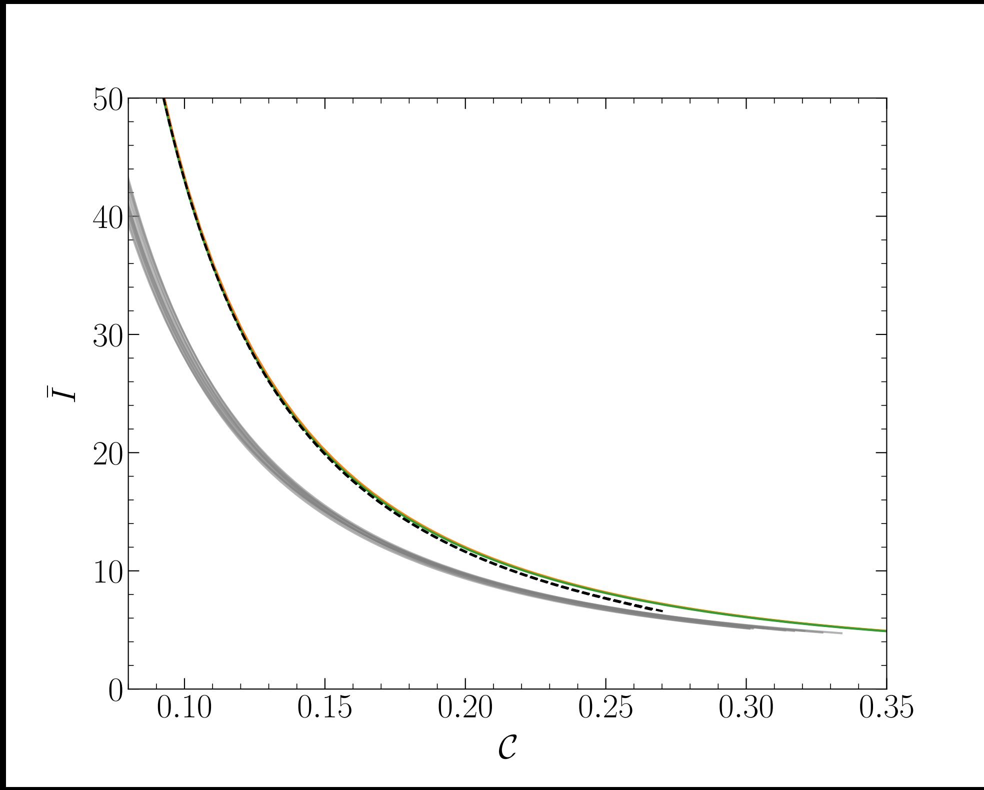
1. Infer one from another: reduce the parameter space

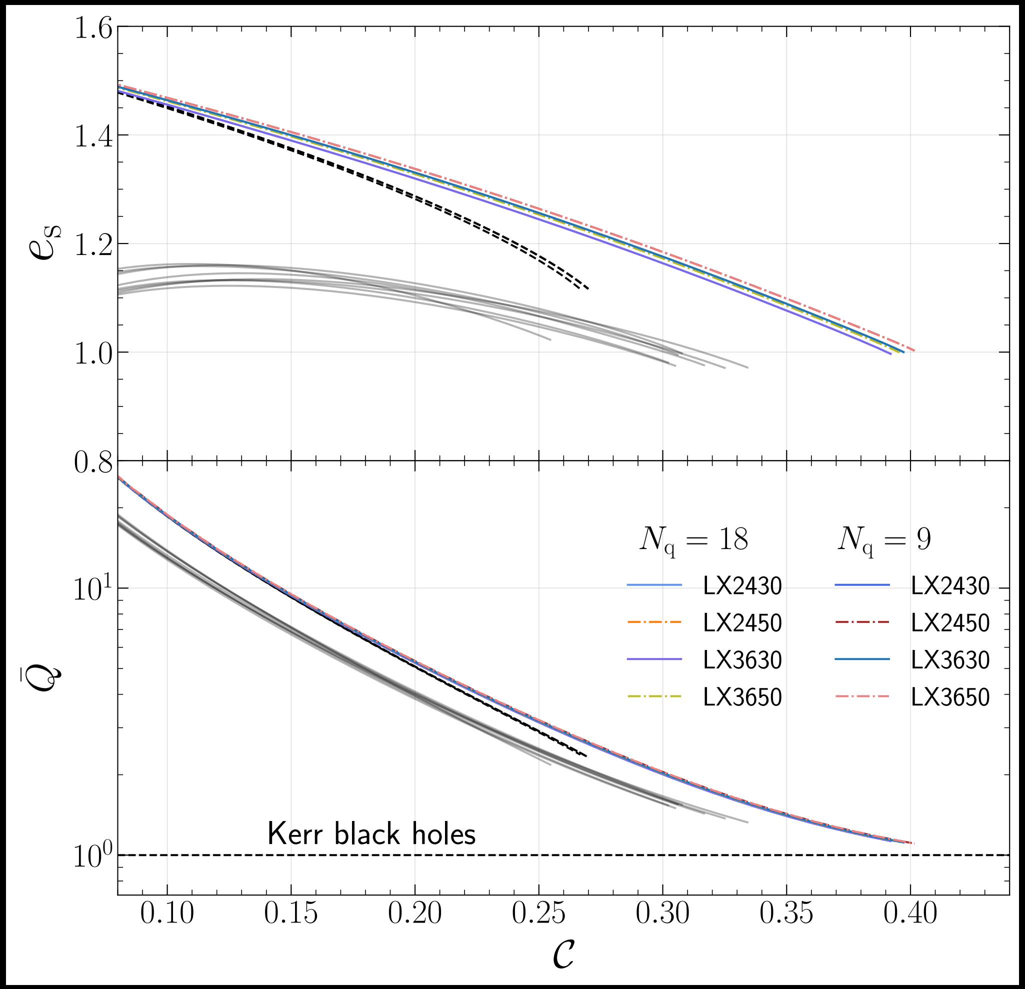
2. Inverse problem of EoS

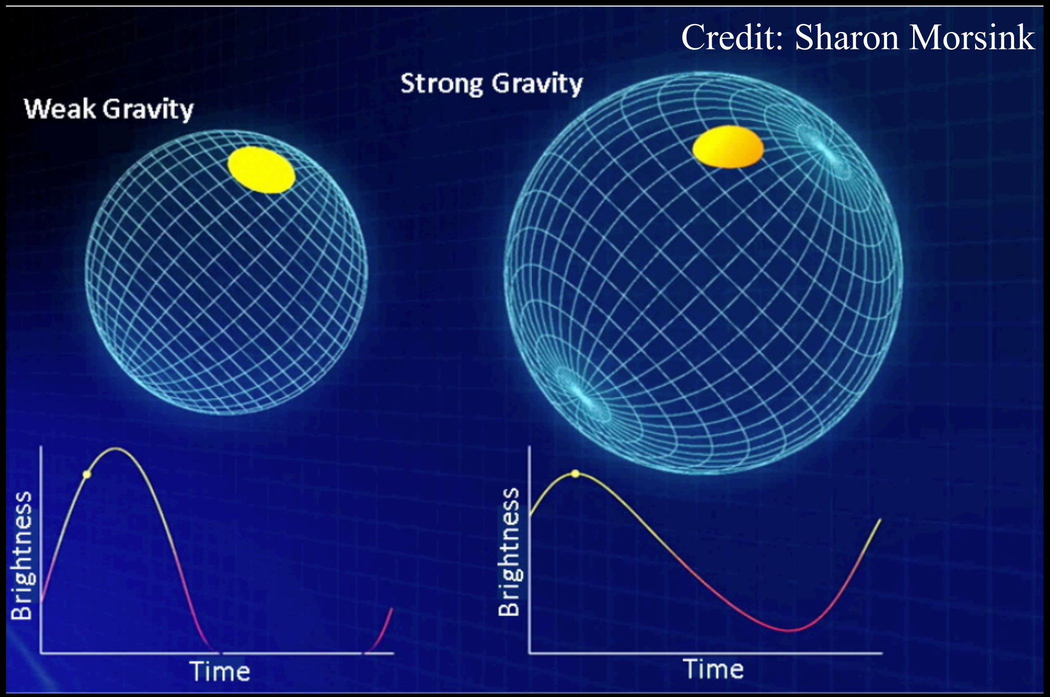
3. Test General Relativity

Universal relations for isolated NSs

Lattimer & Prakash 2001; Urbanec et al. 2013; Gao, Lai, Shao, Xu 2022







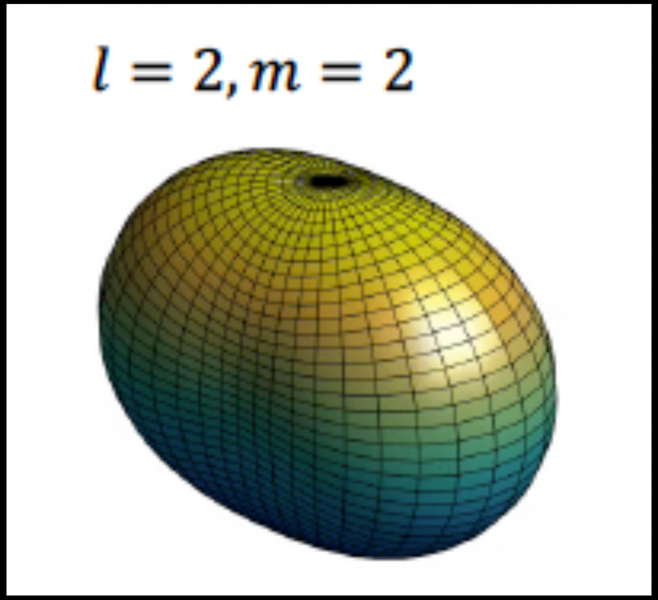
Emission depends on spacetime and geometry of NSs

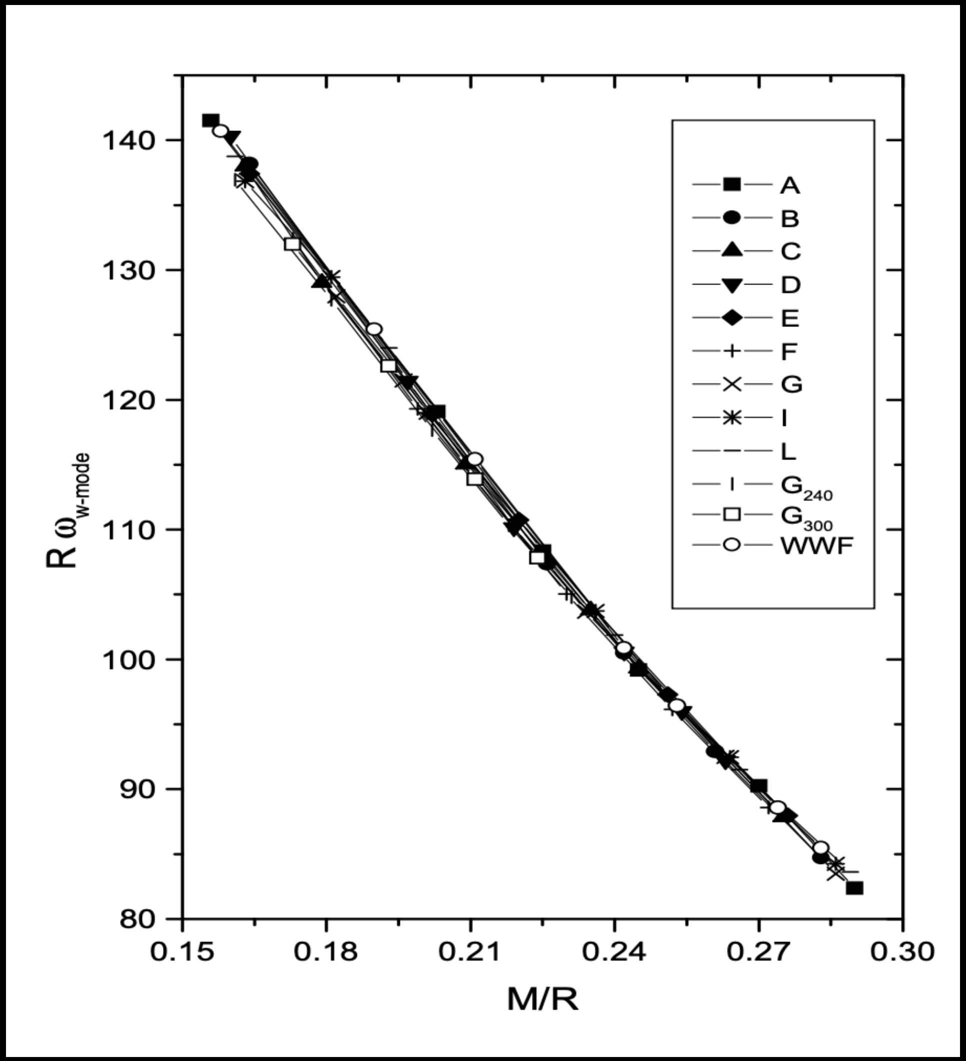
Reduced parameter space for thermal X-ray emission from surface

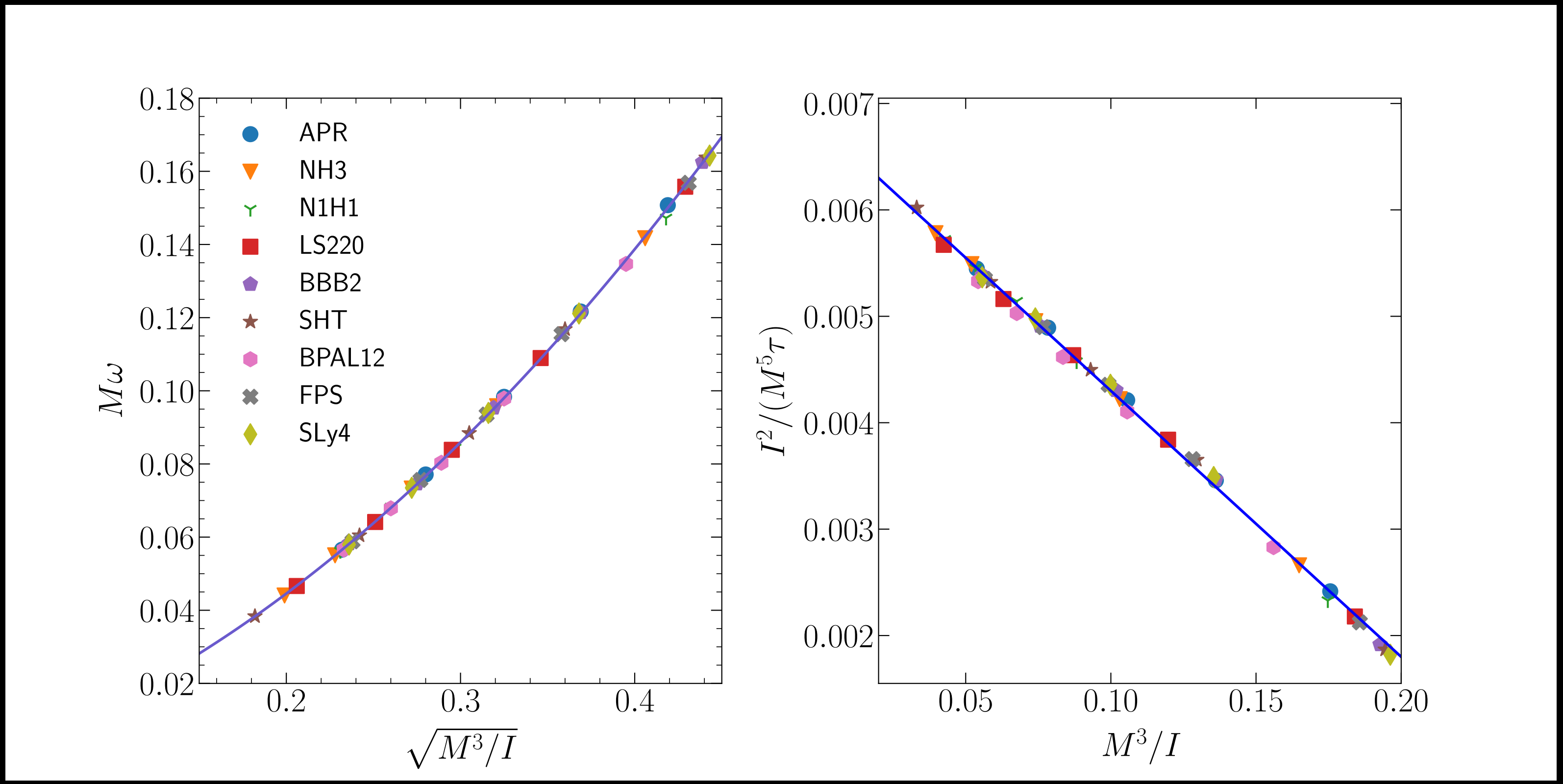
Universal relations for isolated NSs

Gravitational wave asteroseismology

Inverse problem of EoS!



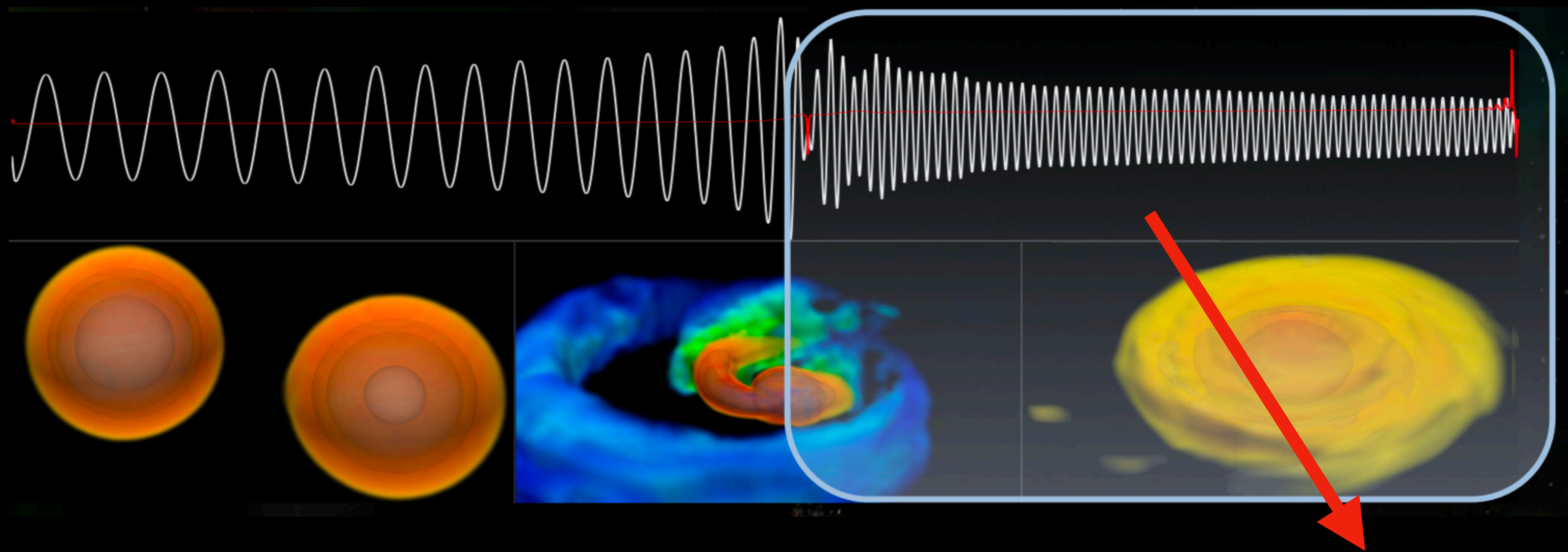


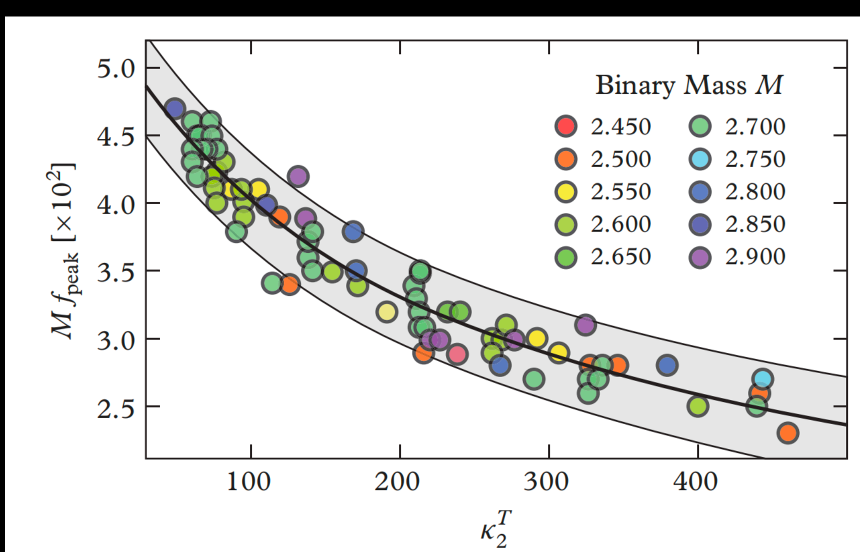


Andersson & Kokkotas 1998

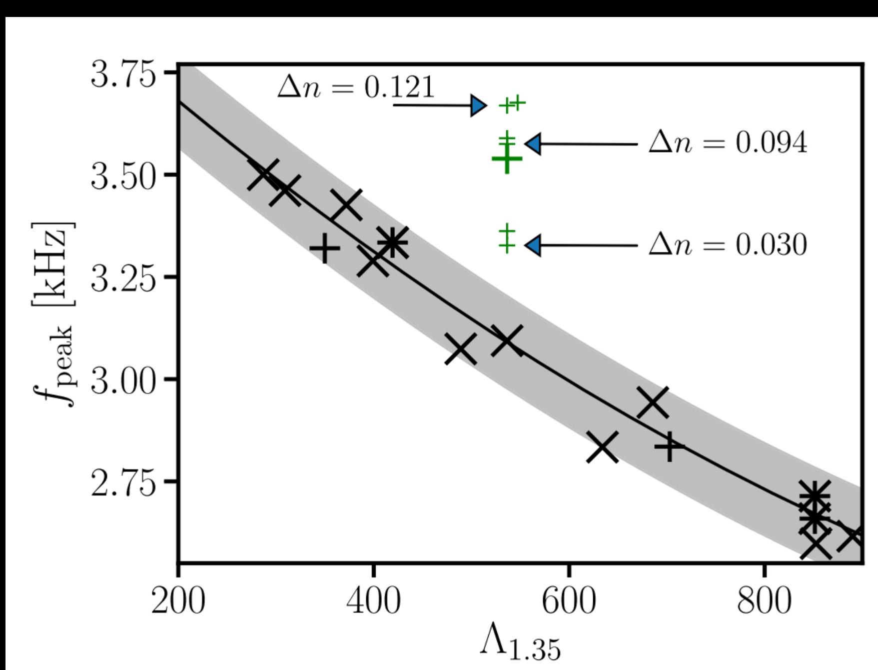
Reproduced from Lau et al. 2011

Universal relations for BNS mergers

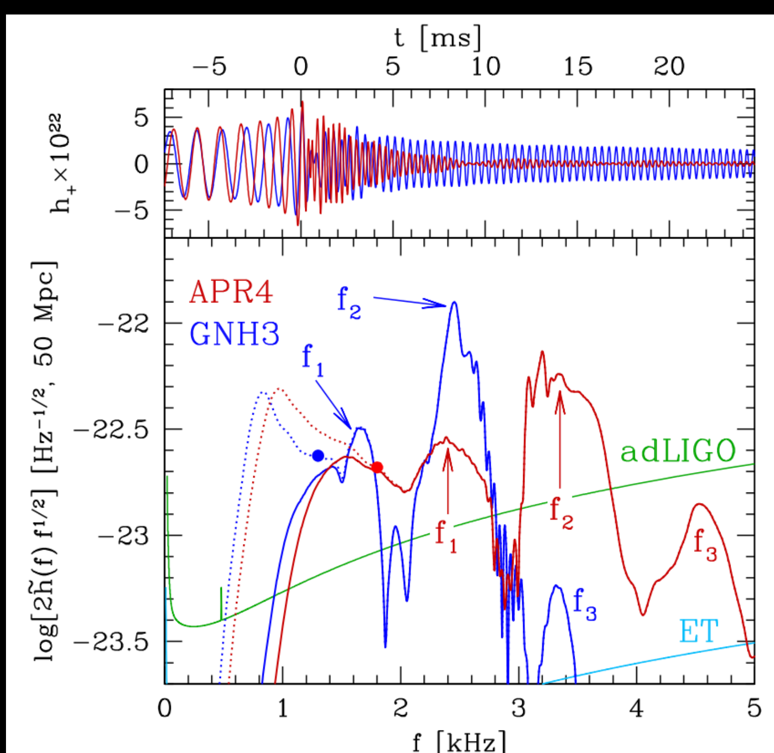




Bernuzzi et al. 2015



Bauswein et al. 2019



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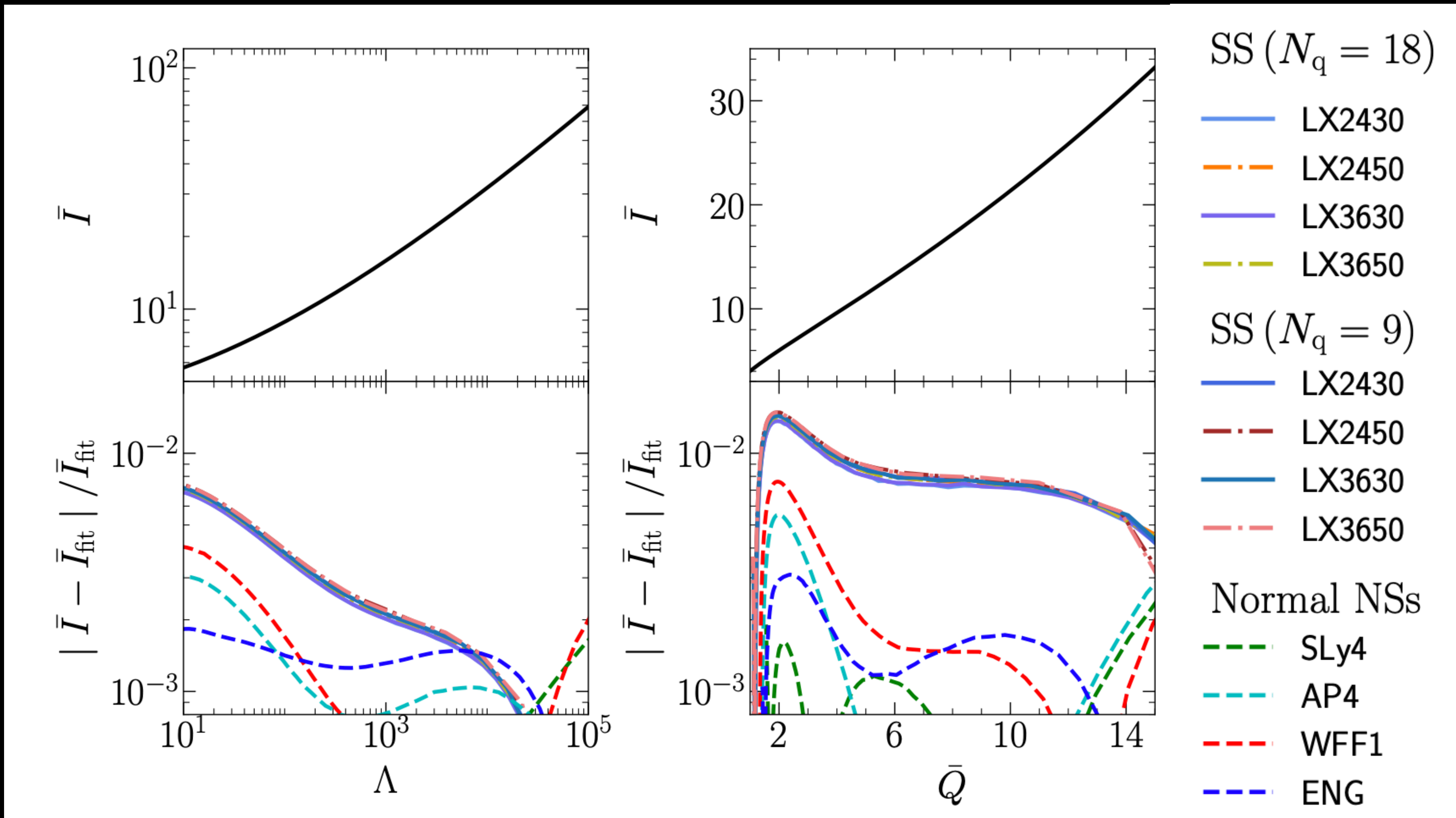
I-Love-Q relation

Yagi & Yunes 2013

$$\bar{I} \equiv \frac{I}{M^3} \quad , \quad \bar{Q} \equiv -\frac{Q}{M^3\chi^2} \quad , \quad \bar{\lambda} \equiv \frac{\lambda}{M^5}$$

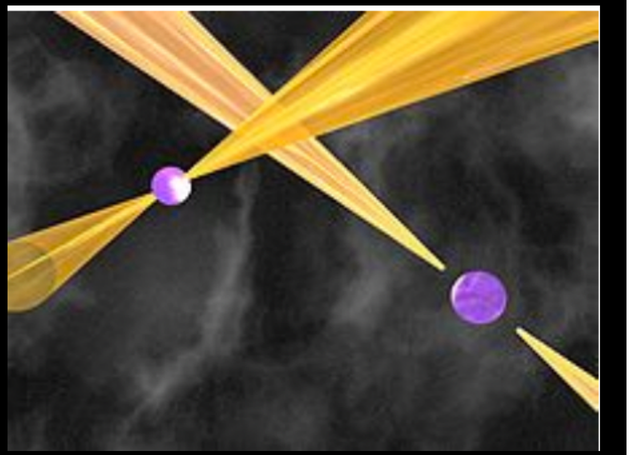


- EoS-insensitive universal relation to 1%
- Still holds for strangeon stars Gao et al. 2022

Test GR with the I-Love-Q relation

Break the degeneracy between EoS and gravity tests

Shao, arXiv:1901.07546

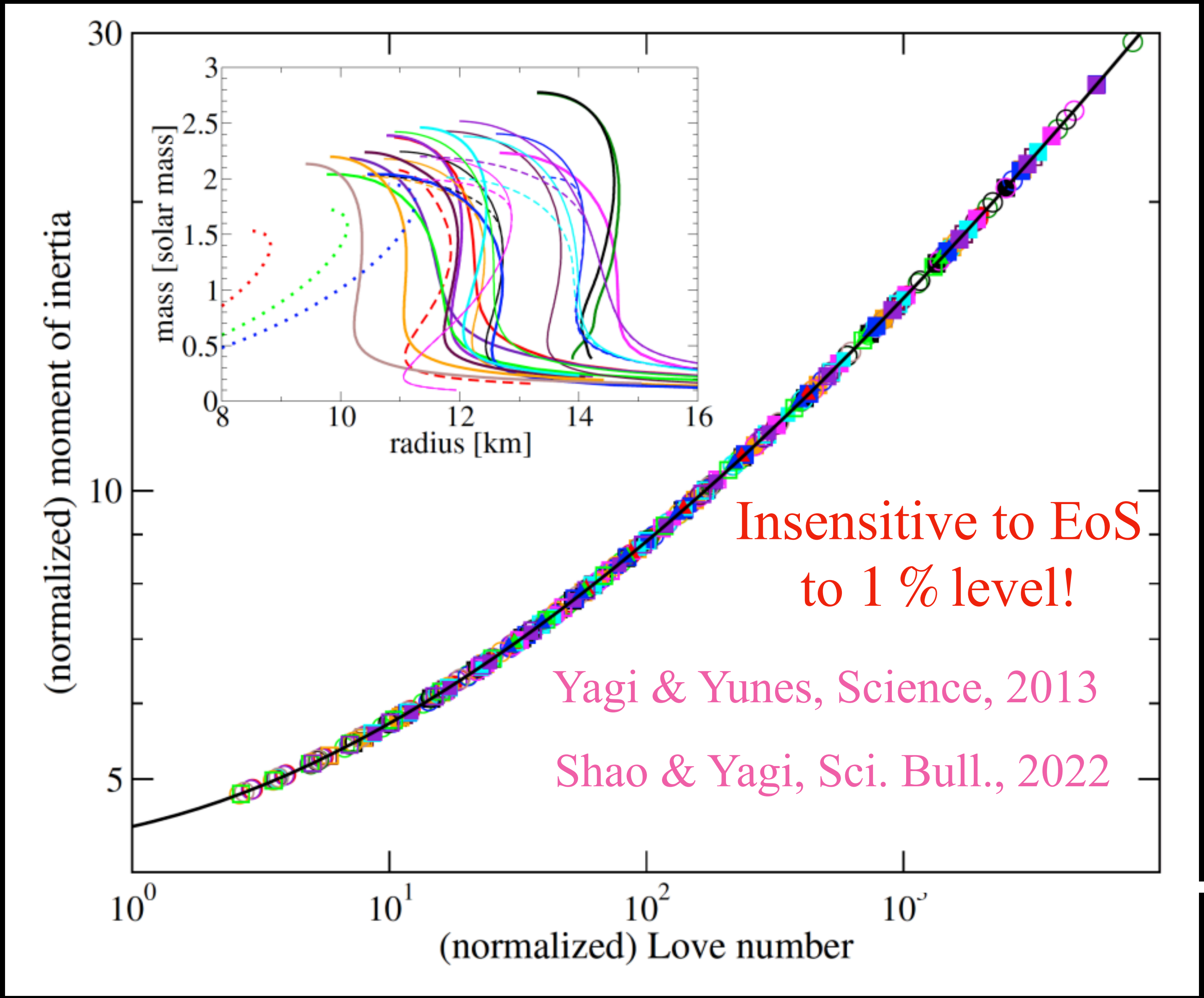


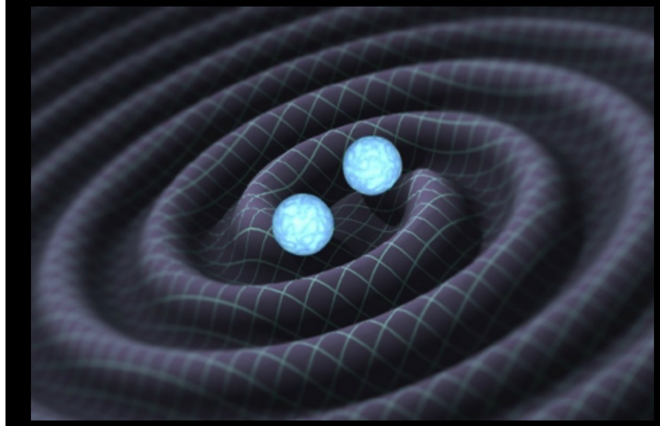
Double pulsar system

Lense-Thirring precession of the orbit

Hu et al. 2020

Kramer et al. 2021





Late stage of binary NS inspiral

GW phase shift due to tidal deformation

- Different theories of gravity may produce different I-Love relation
- Testing gravity without contamination from uncertainties in nuclear physics

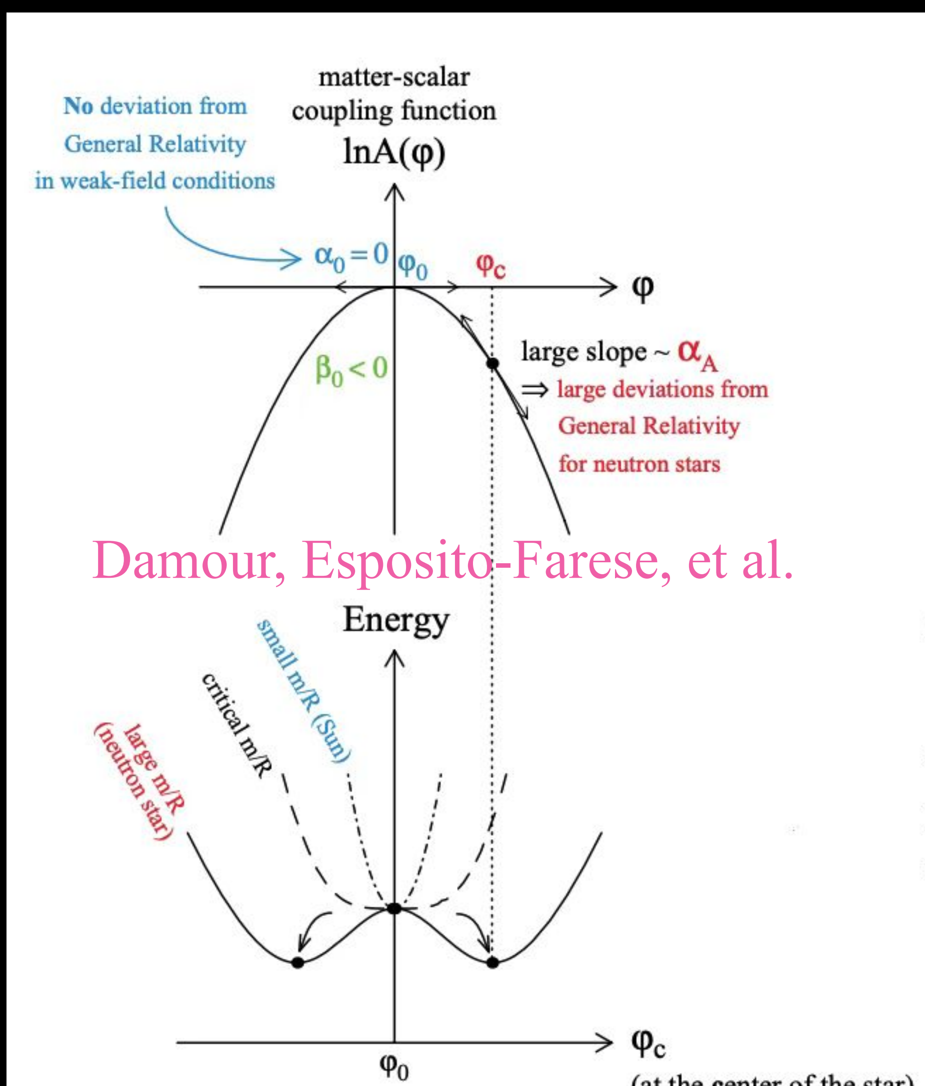
Scalar-tensor gravity and spontaneous scalarization

$$S=\frac{c^4}{16\pi G}\int\frac{d^4x}{c}\sqrt{-g}\left[R-2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V(\varphi)\right]+S_m\left[\Psi_m,\underbrace{A^2(\varphi)g_{\mu\nu}}_{\text{Physical metric}}\right]$$

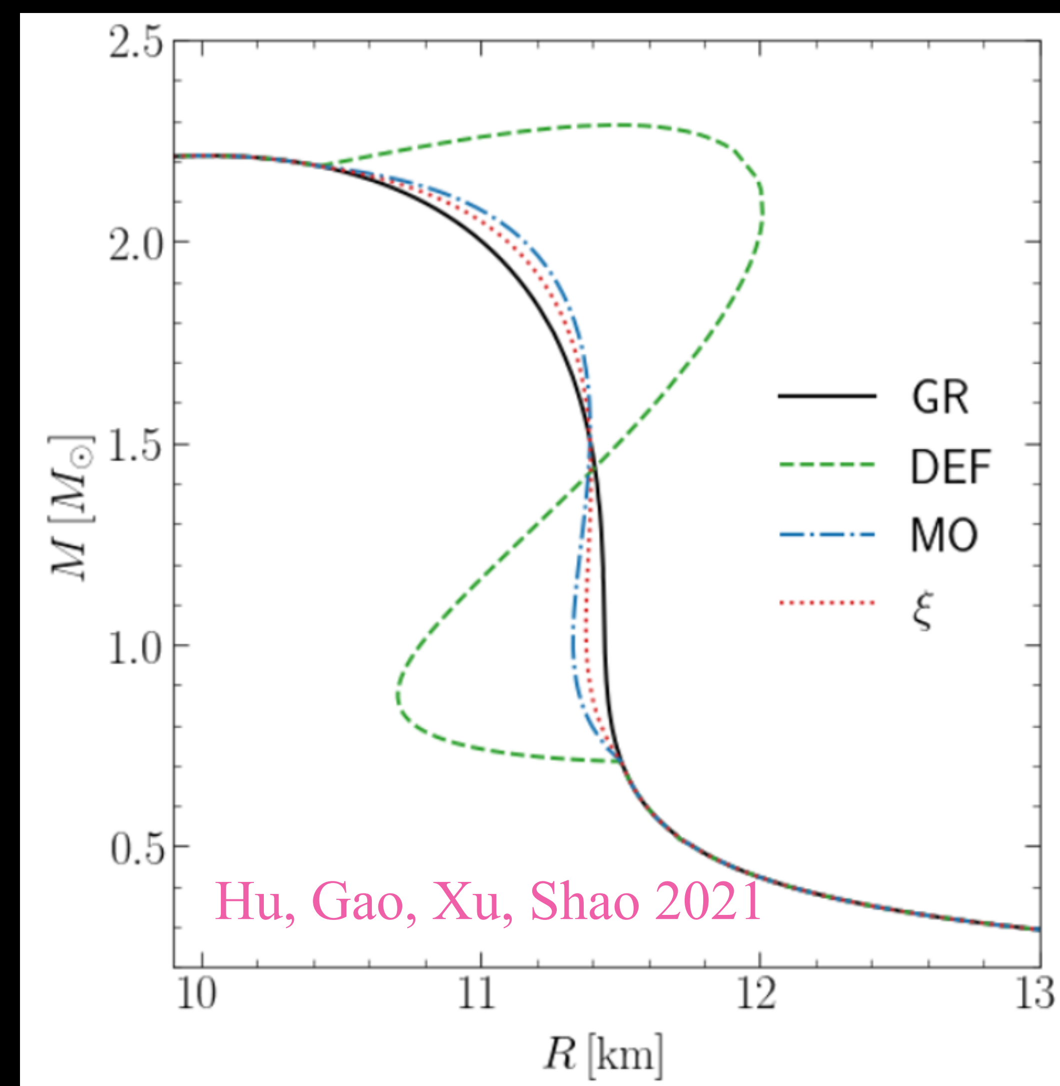
Spin 2

Spin 0

Scalar potential



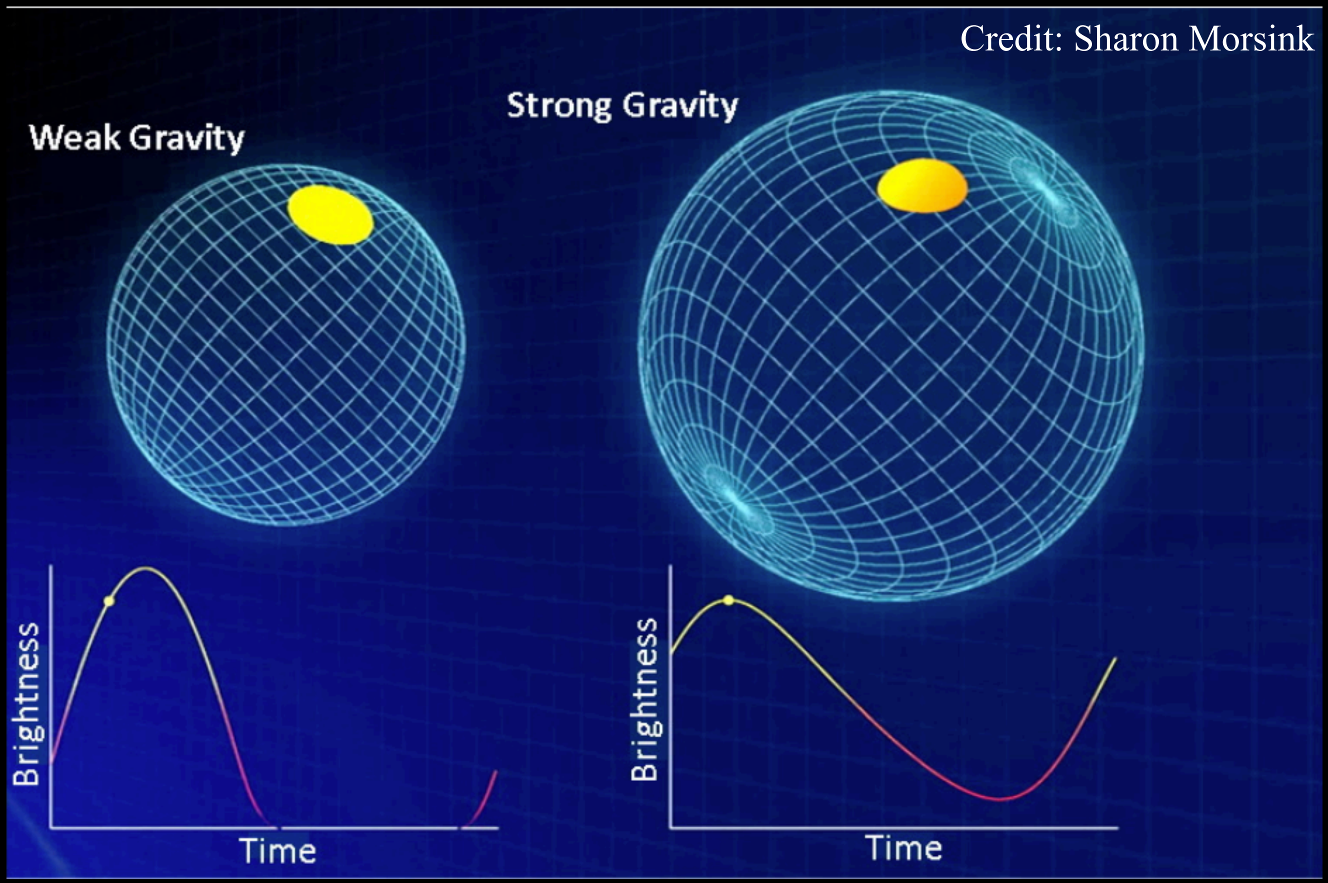
Damour, Esposito-Farese, et al.



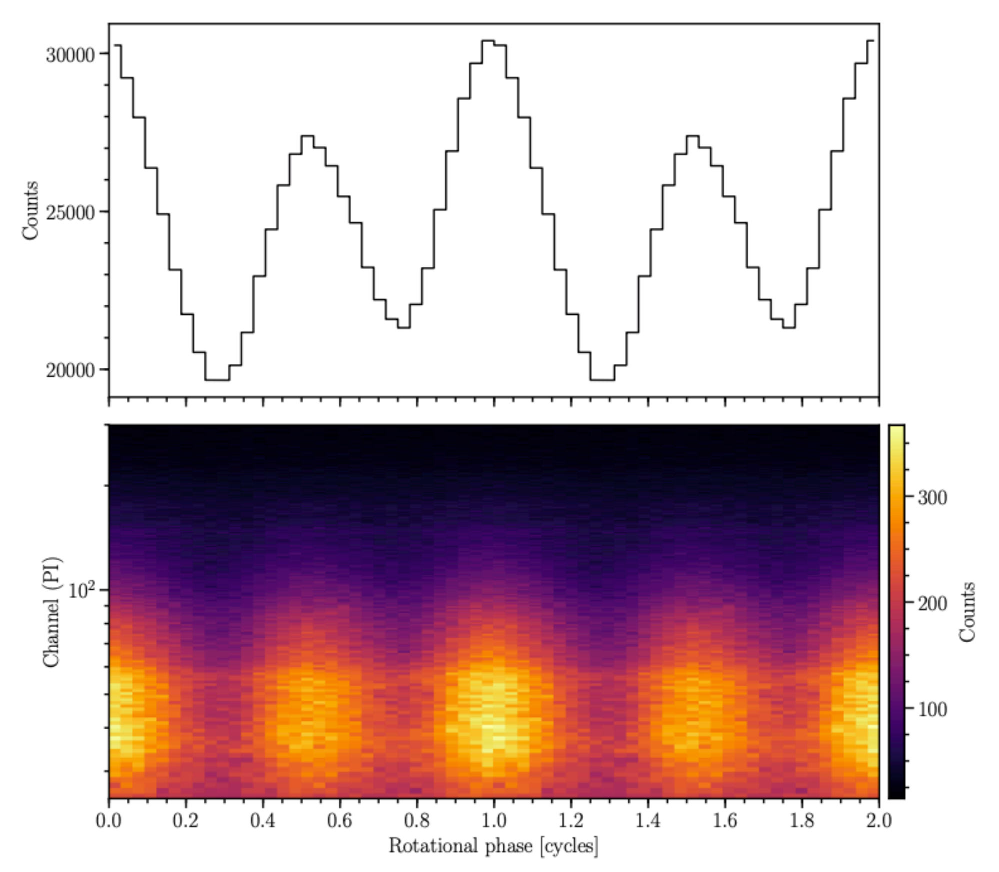
Hu, Gao, Xu, Shao 2021

Scalar-tensor gravity and X-ray pulsars

- Pulsed emission – look for effects of gravitational field (characterized by mass, radius…) on time variations of flux from millisecond period X-ray pulsars



Relativistic effects on pulse profile



PSR J0030+0451 observed by NICER

$M = 1.34^{+0.15}_{-0.16} M_\odot \quad R = 12.71^{+1.14}_{-1.19}$ km

- X-ray pulse profiles is **different** in ST theory because of

different mass-radius relation

different light-like geodesic

Riley et al. 2019

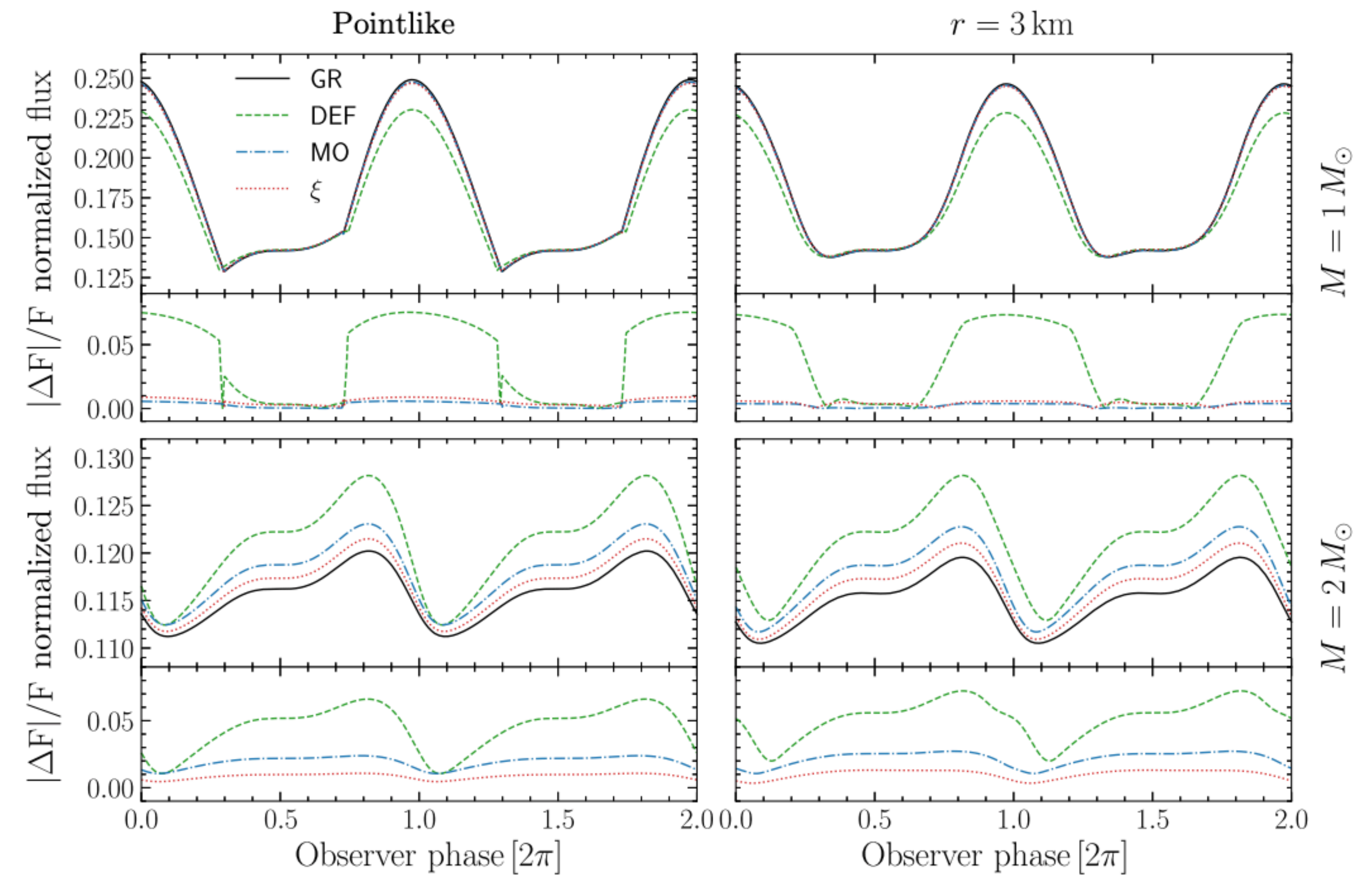
Silva & Yunes 2019

Massive scalar-tensor gravity

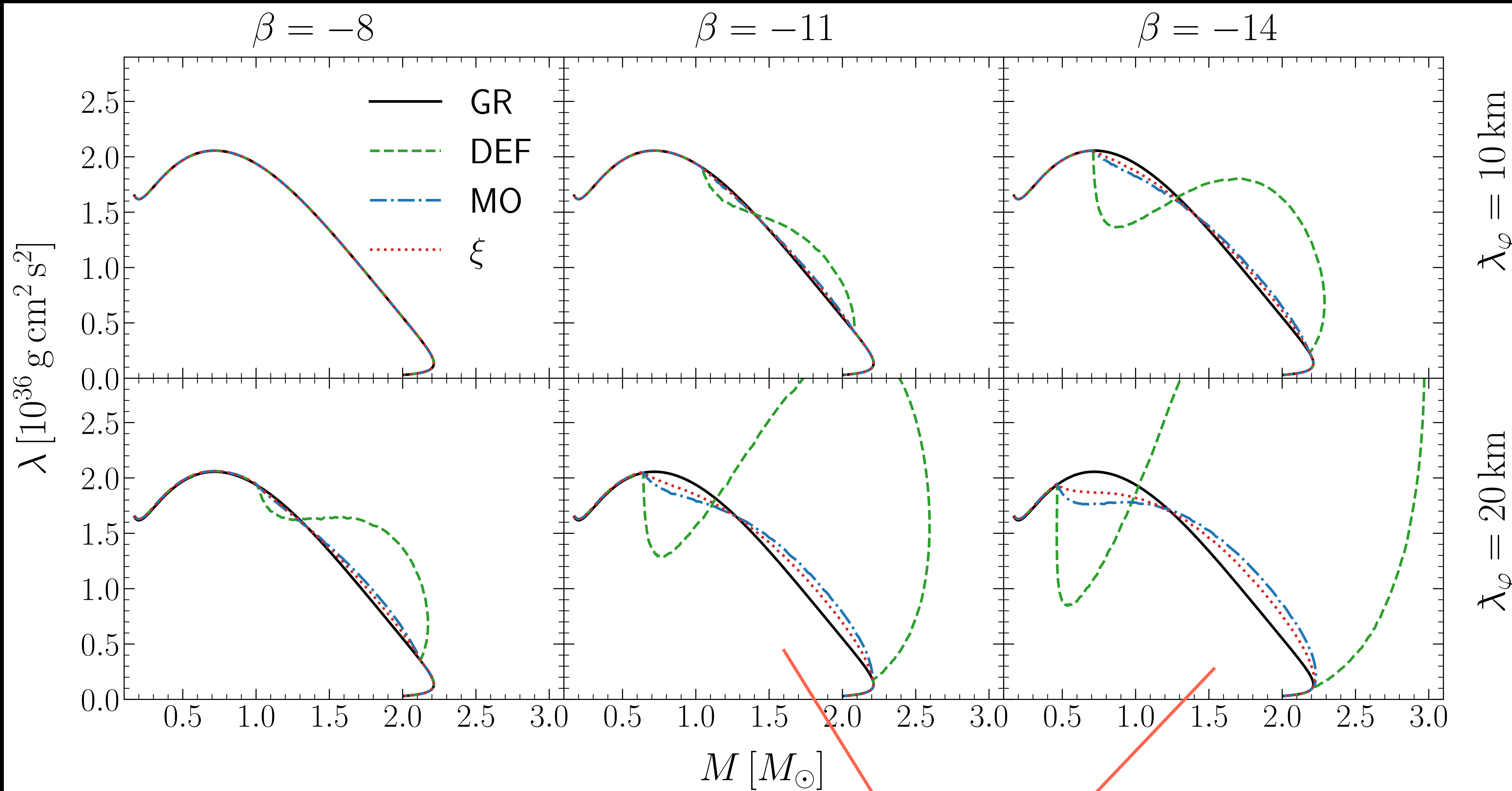
escaping pulsar limits

$\beta = -2\xi = -14, \quad \lambdabar_\varphi = 10$ km

Hu, Gao, Xu, Shao 2021



Massive scalar-tensor gravity



Hu, Gao, Xu, Shao 2021

$$\lambda = \frac{2}{3} k_2 R^5$$

Large deviations for massive DEF theory

Massive scalar Gauss-Bonnet gravity

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi) + F(\Phi)\mathcal{G}\right) + S_m$$

$$\mathcal{G} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2 \qquad U(\Phi) = \left(\frac{2\pi}{\lambda_\Phi}\right)^2\Phi^2 = \frac{1}{\lambda\!\!\!^{-}_\Phi{}^2}\Phi^2 \qquad F(\Phi) = \xi\Phi^2$$

Gauss-Bonnet term | Scalar potential | Coupling

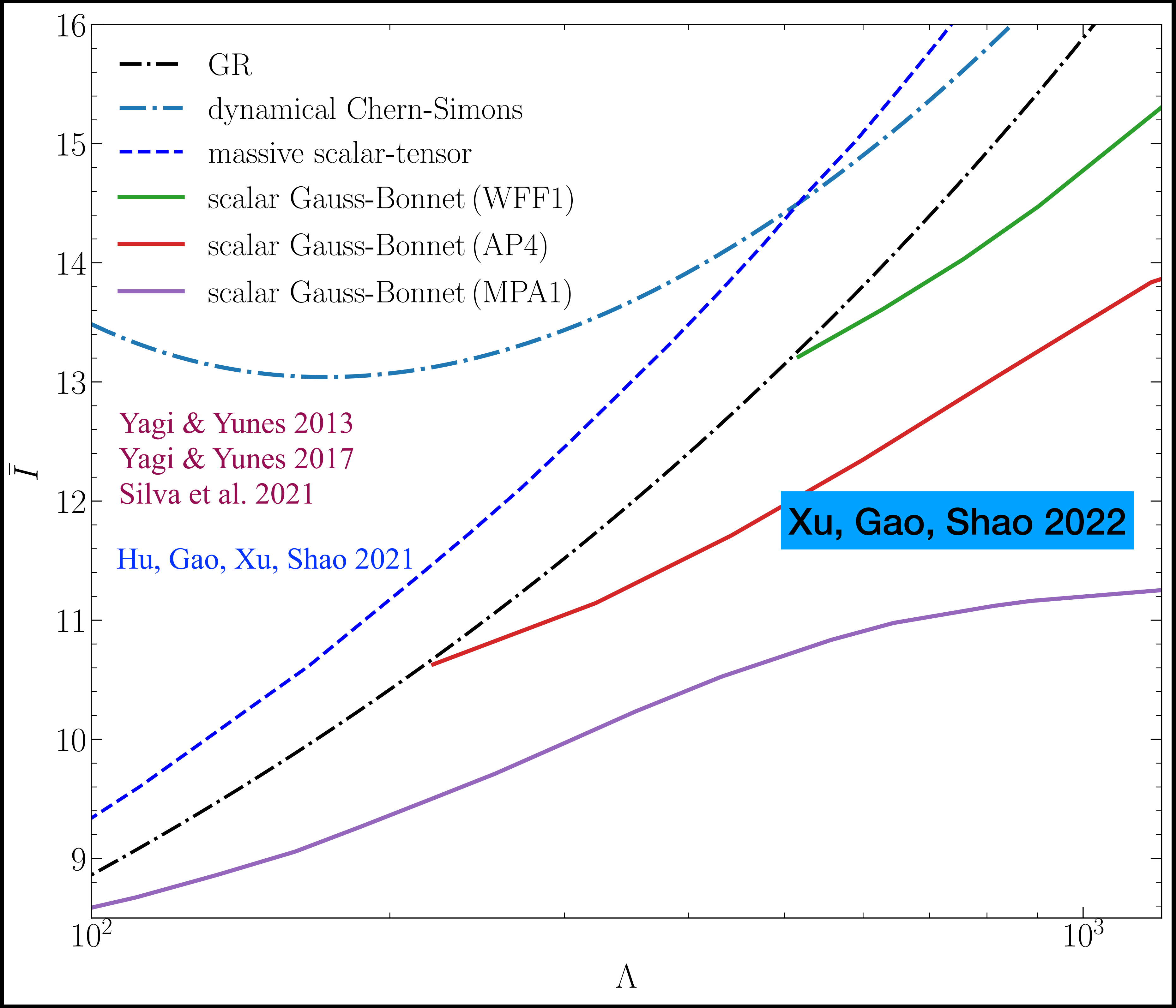
- Has drawn great interests since **solutions of spontaneous scalarization were found for black holes** in these theories

  Antoniou, Bakopoulos & Kanti 2018; Doneva & Yazadjiev 2018

- We investigate **static, slowly-rotating, and tidally-deformed NSs** in massive sGB theory

  Xu, Gao, Shao 2022

I-Love-Q relation in alternative gravities



Outline

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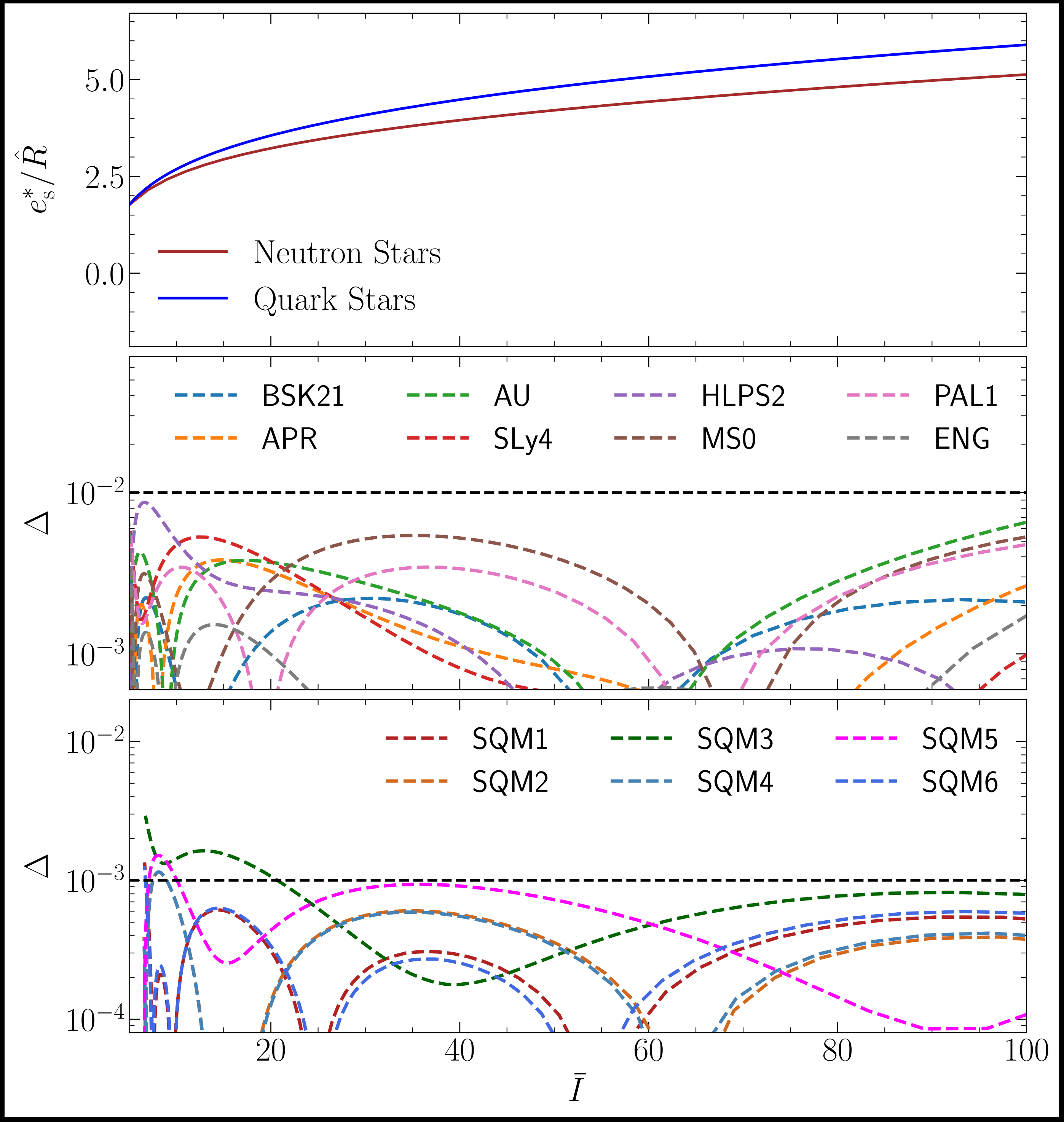
Eccentricity-MoI universal relation

Universal relation for slow rotation



Embedding eccentricity

$$e_s^* = \left\{ -3 \left[v_2(R) - h_2(R) + \xi_2(R)/R \right] \right\}^{1/2}$$

1. Different relations for NSs and QSs
2. For $M \geq 1\, M_\odot$

NSs: $\frac{e_s^*}{\hat{R}} = 0.114 + 1.0412 \ln \bar{I}$

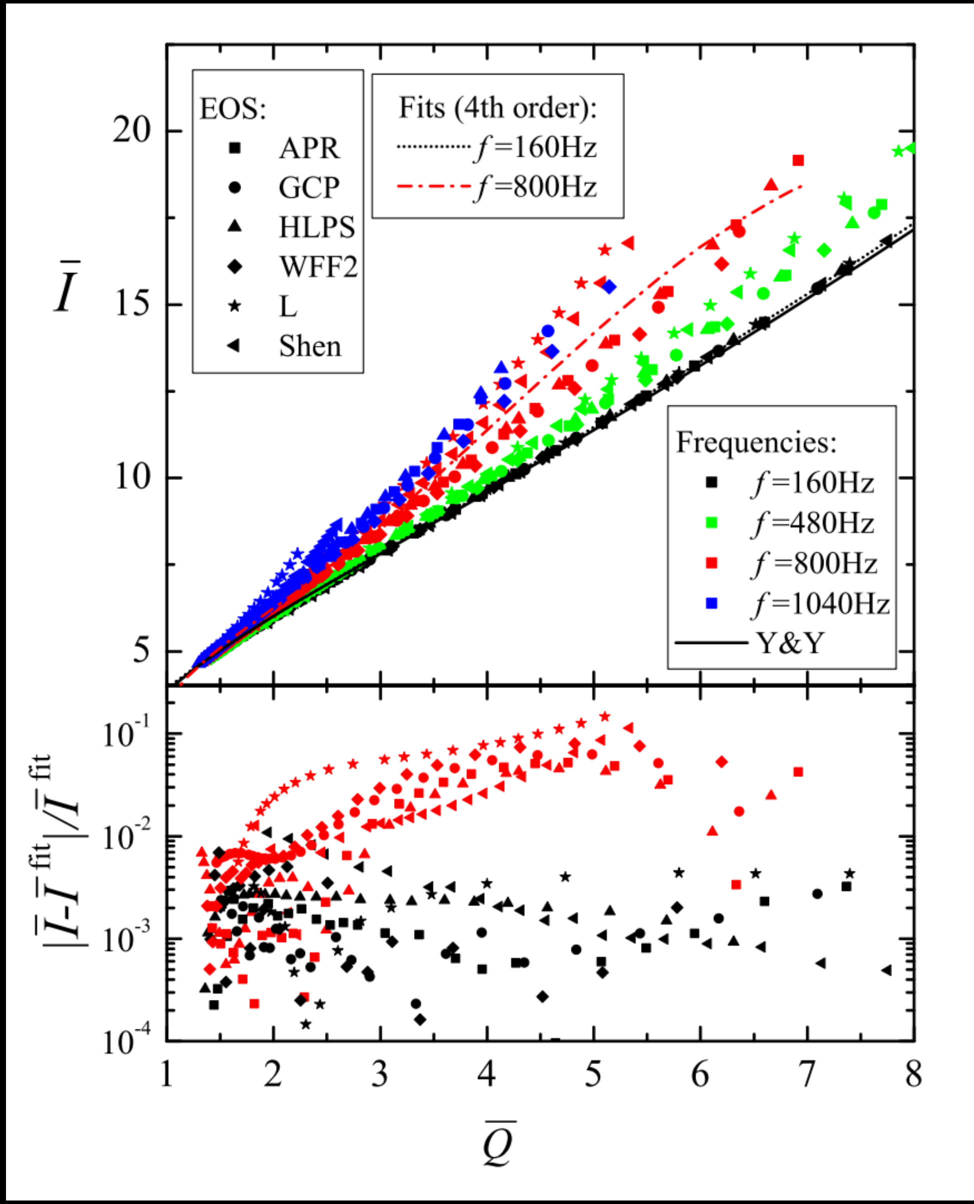
QSs: $\frac{e_s^*}{\hat{R}} = -0.0434 + 1.211 \ln(\bar{I} - 0.471)$

Gao, Shao, Steinhoff 2023

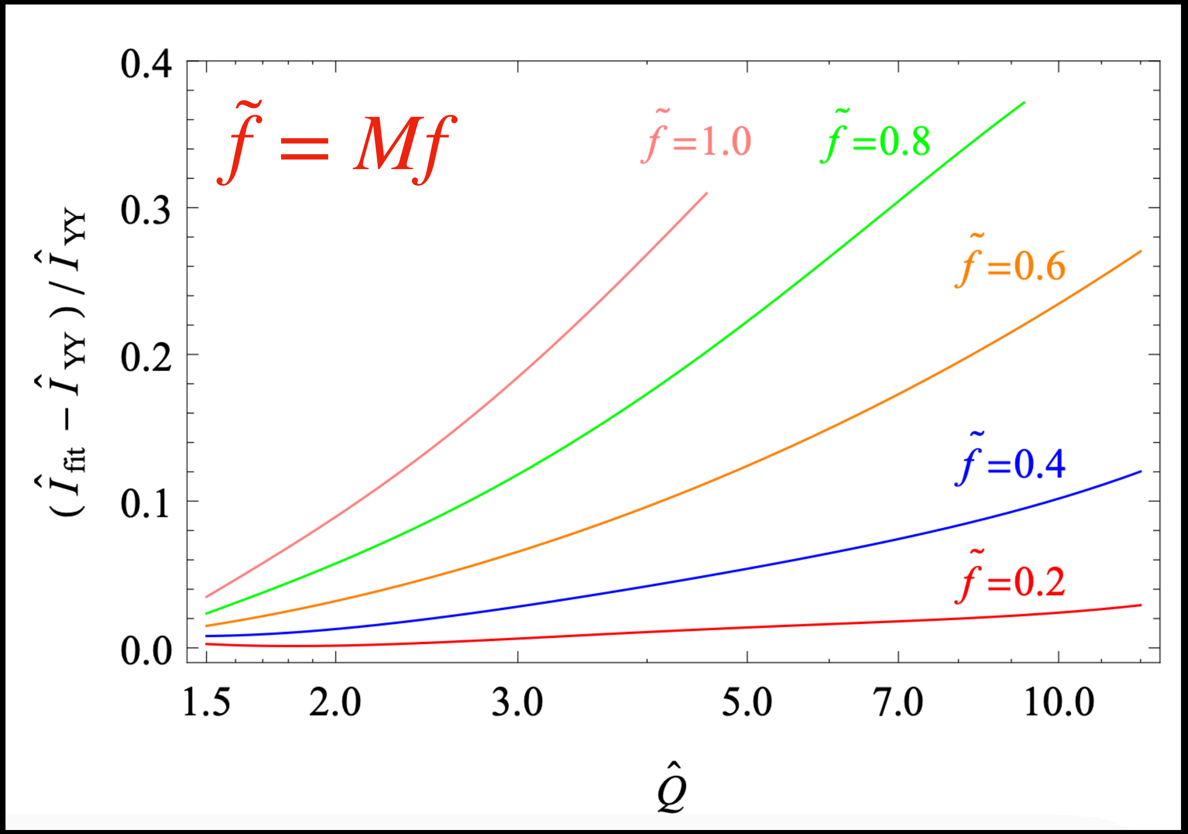
Eccentricity-MoI universal relation

Universal relation for fast rotation?

Learn from I-Q relation!

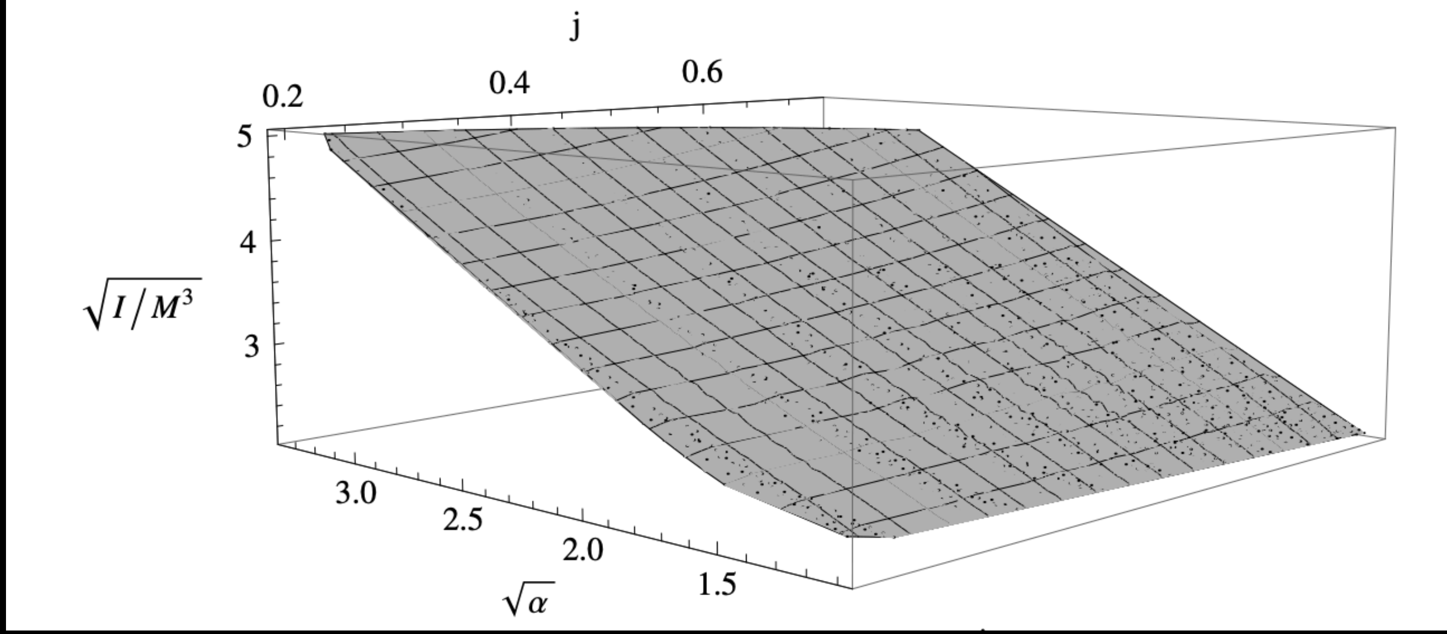


Doneva et al. 2014



Chakrabarti et al. 2014

Papas & Apostolatos 2014



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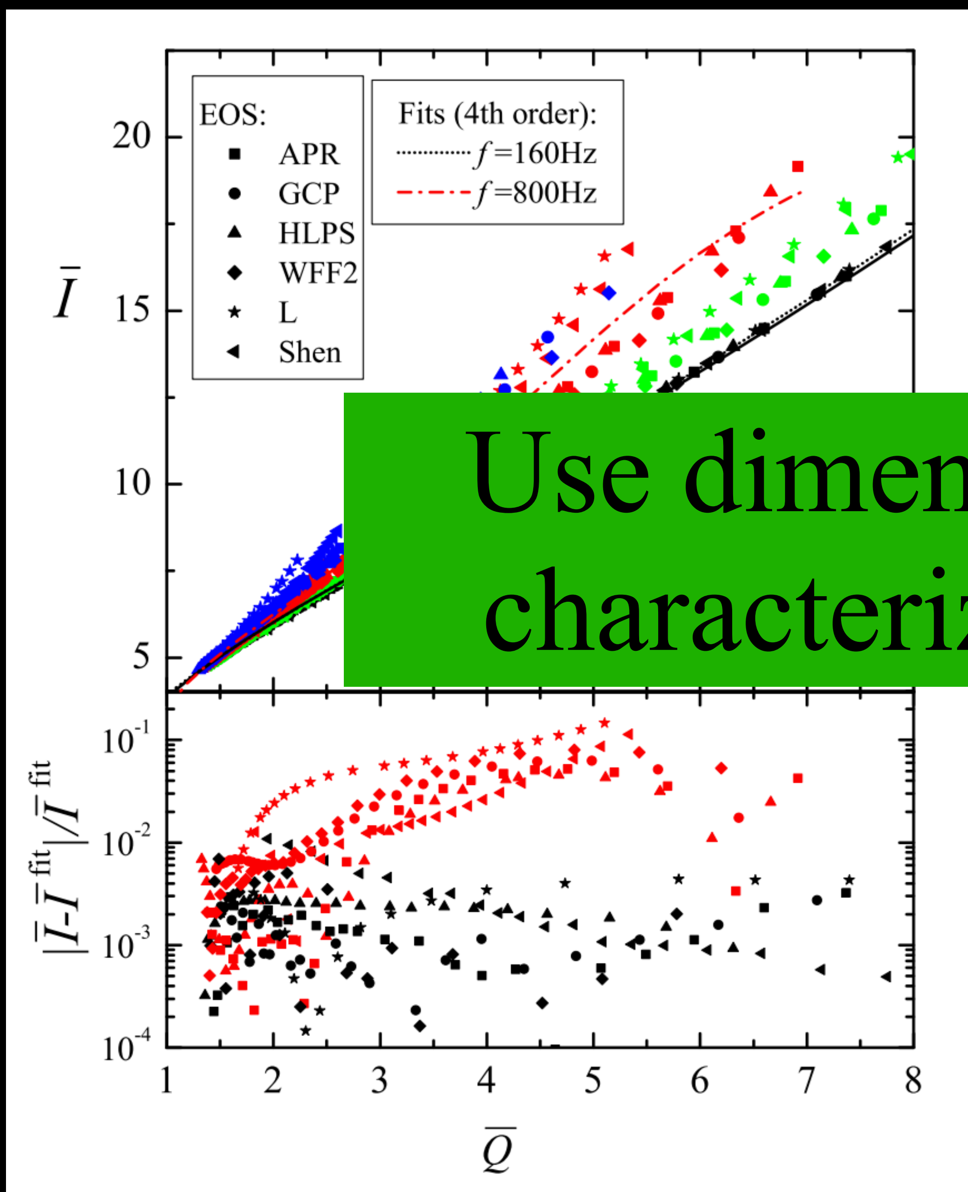
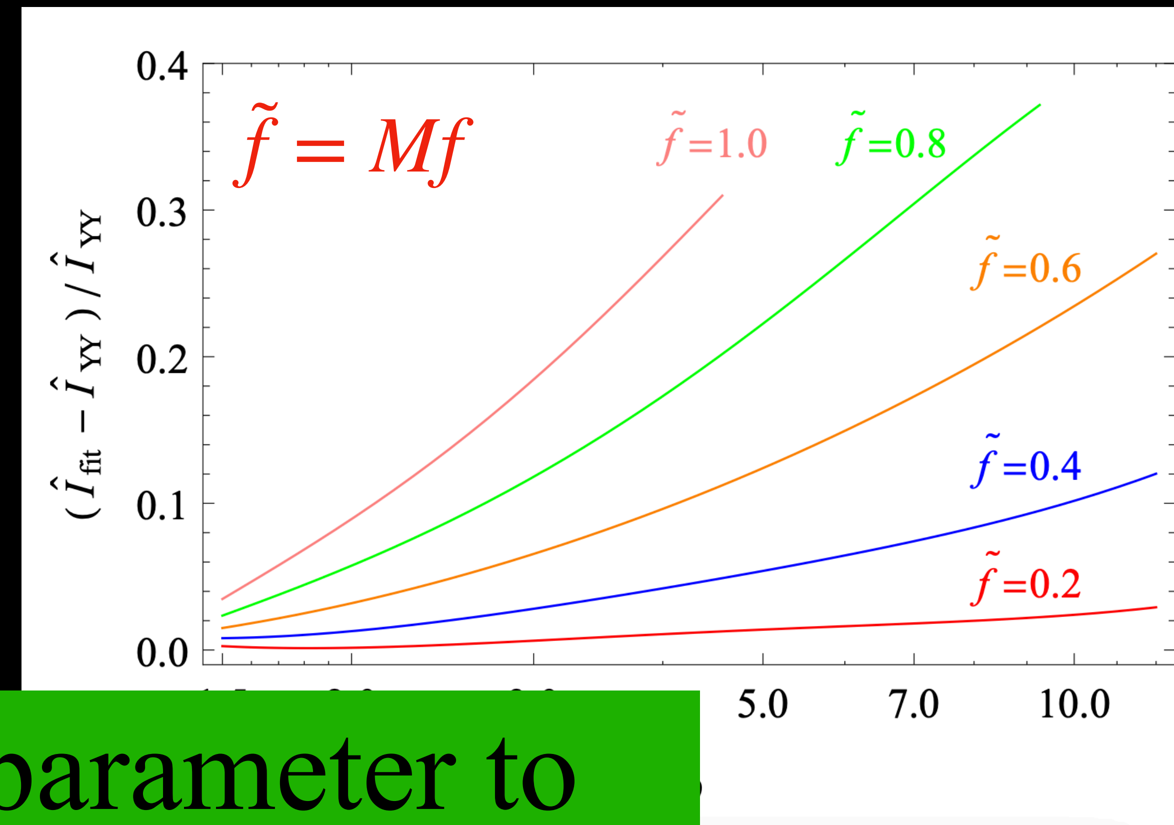


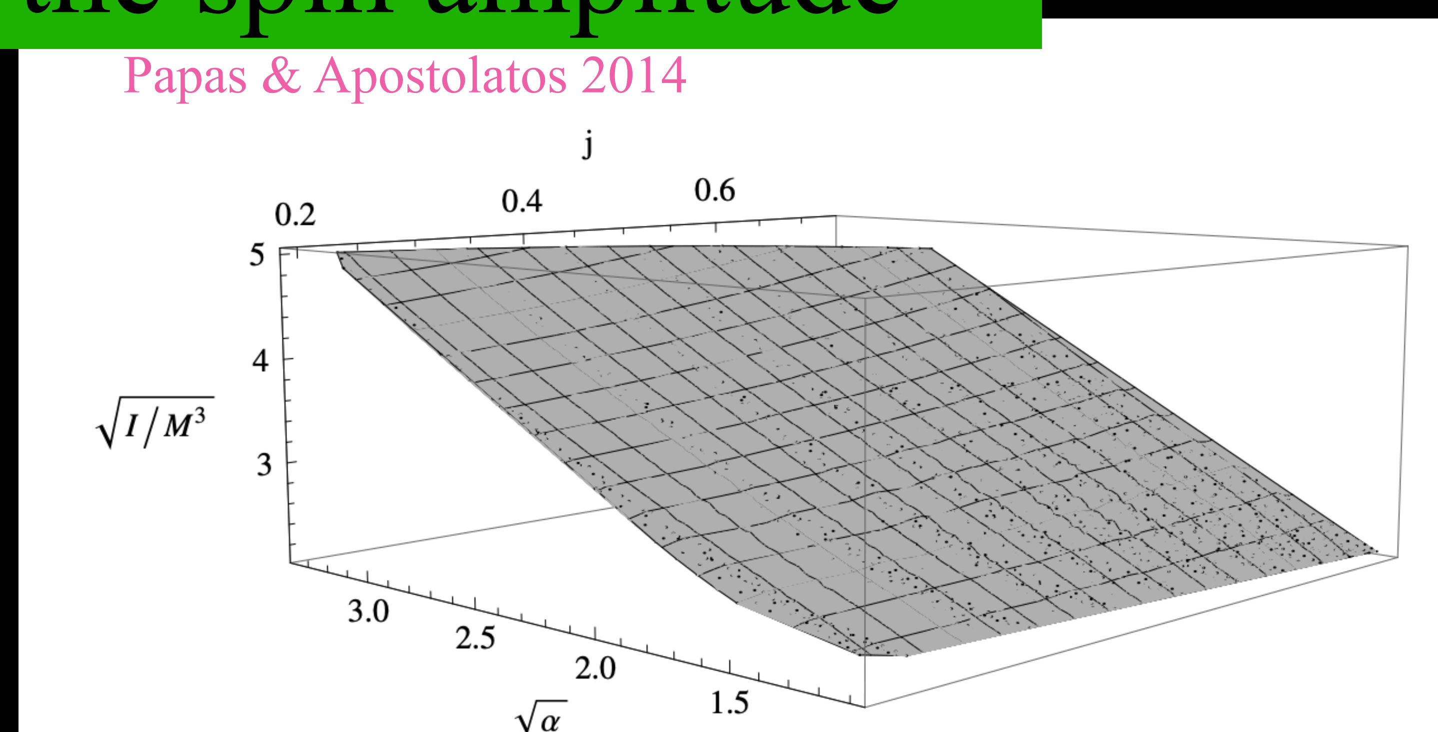
Doneva et al. 2014



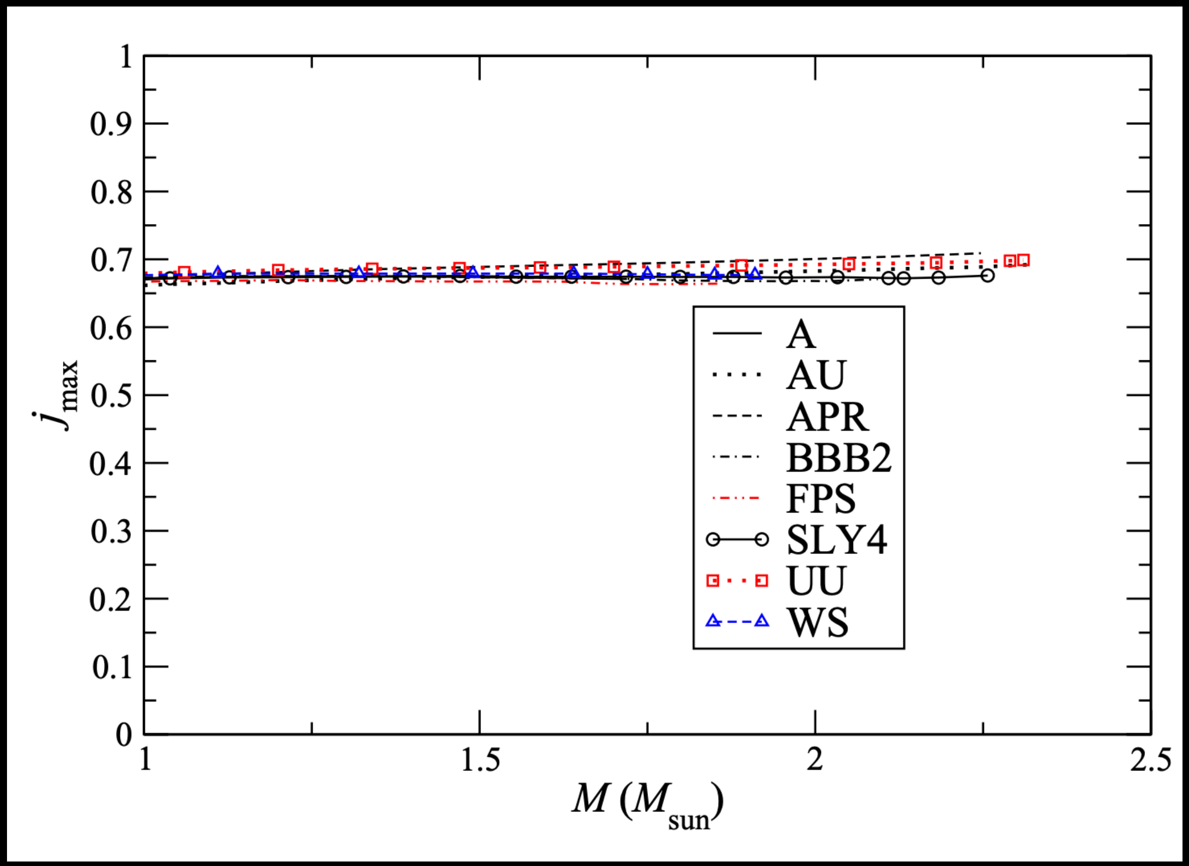
Use dimensionless parameter to characterize the spin amplitude

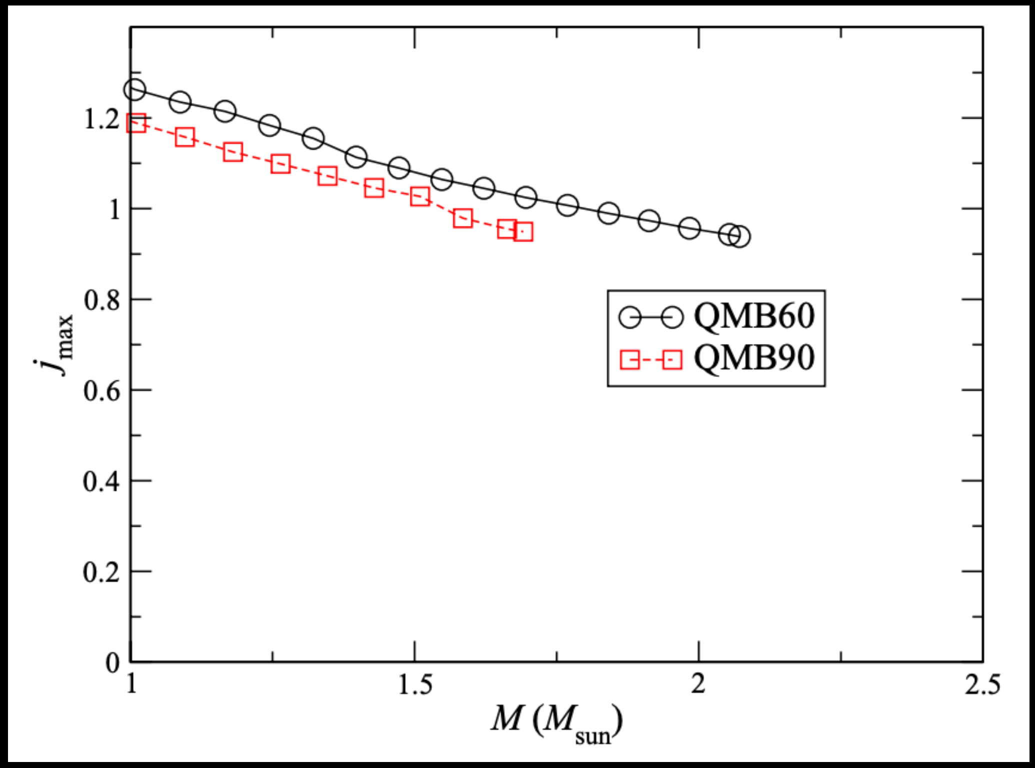
krabarti et al. 2014

Papas & Apostolatos 2014

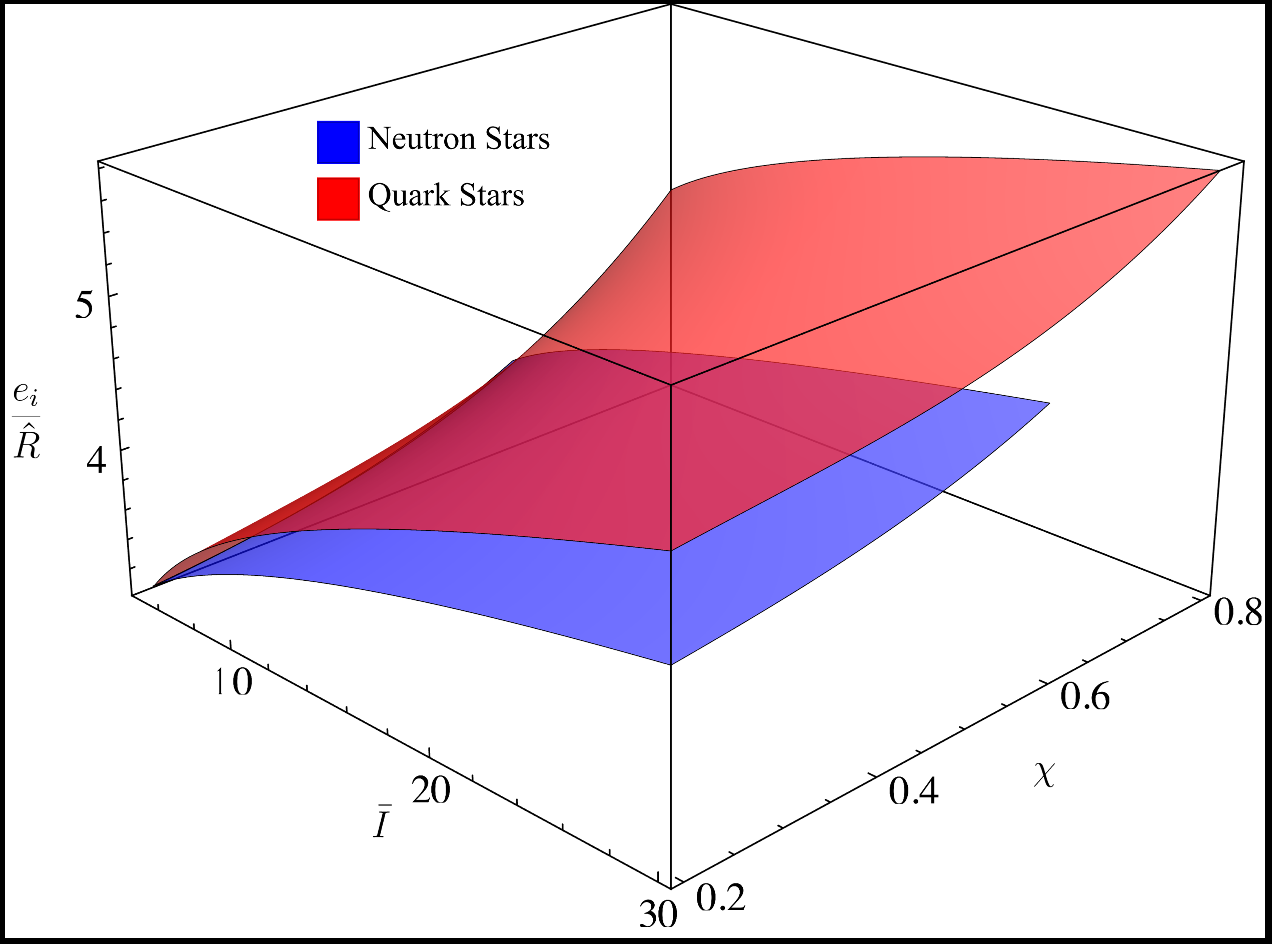


Universal relation for the eccentricity





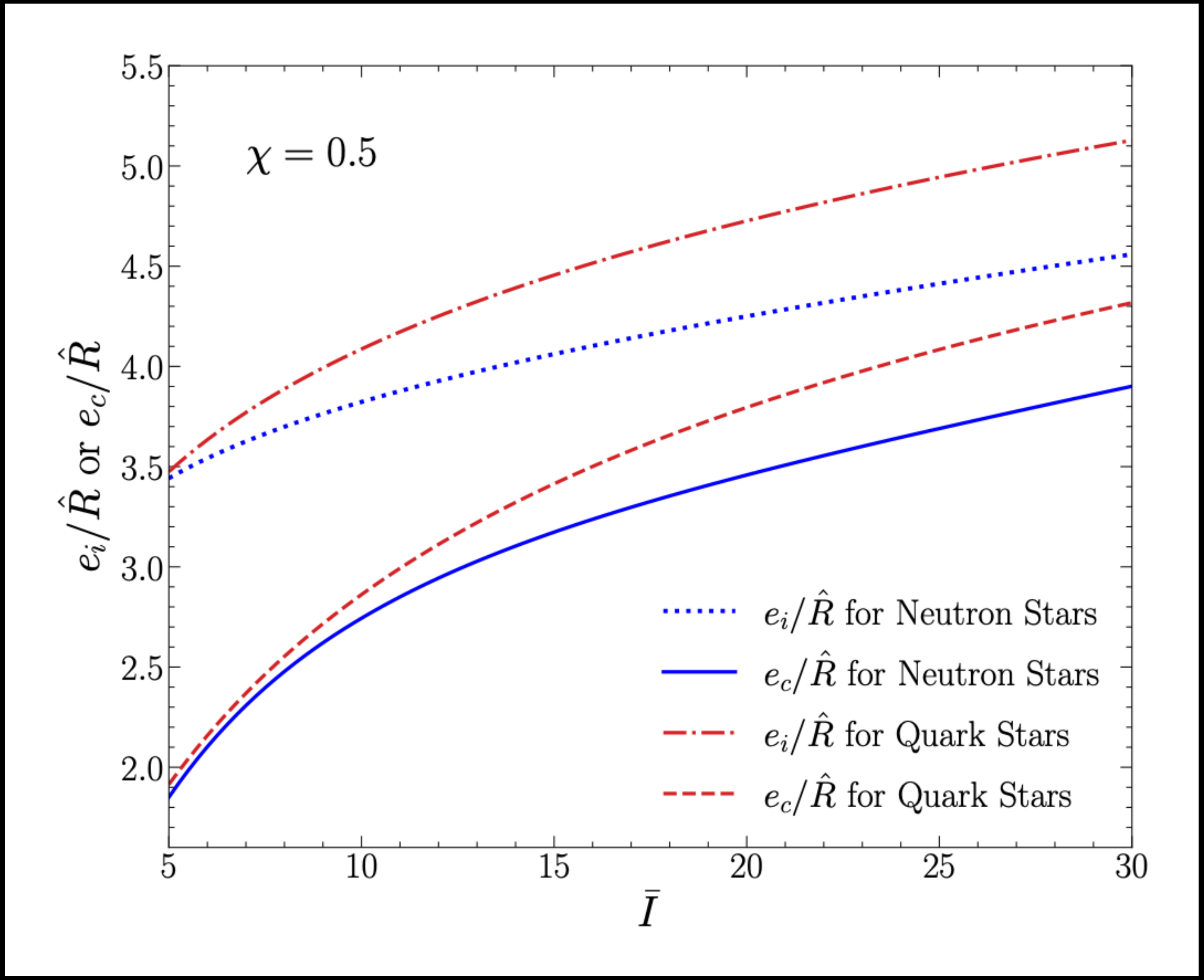
Fast rotation from numerical relativity



Gao, Shao, Steinhoff 2023

Universal relation for the eccentricity

$$\mathrm{d}s^2 = -e^{2\nu}\mathrm{d}t^2 + B^2 r^2 \sin^2\theta e^{-2\nu}(\mathrm{d}\phi - \omega \mathrm{d}t)^2 + e^{2(\xi-\nu)}\left(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2\right)$$



$$\bar{r} = e^{-\nu(r,\theta)}B(r,\theta)r\sin\theta = r_c(r,\theta)\sin\theta$$

$$R^c_{\mathrm{eq}} = r_c\left(R^i_{\mathrm{eq}}, \theta = \frac{\pi}{2}\right), \quad R^c_{\mathrm{p}} = r_c\left(R^i_{\mathrm{p}}, \theta = 0\right)$$

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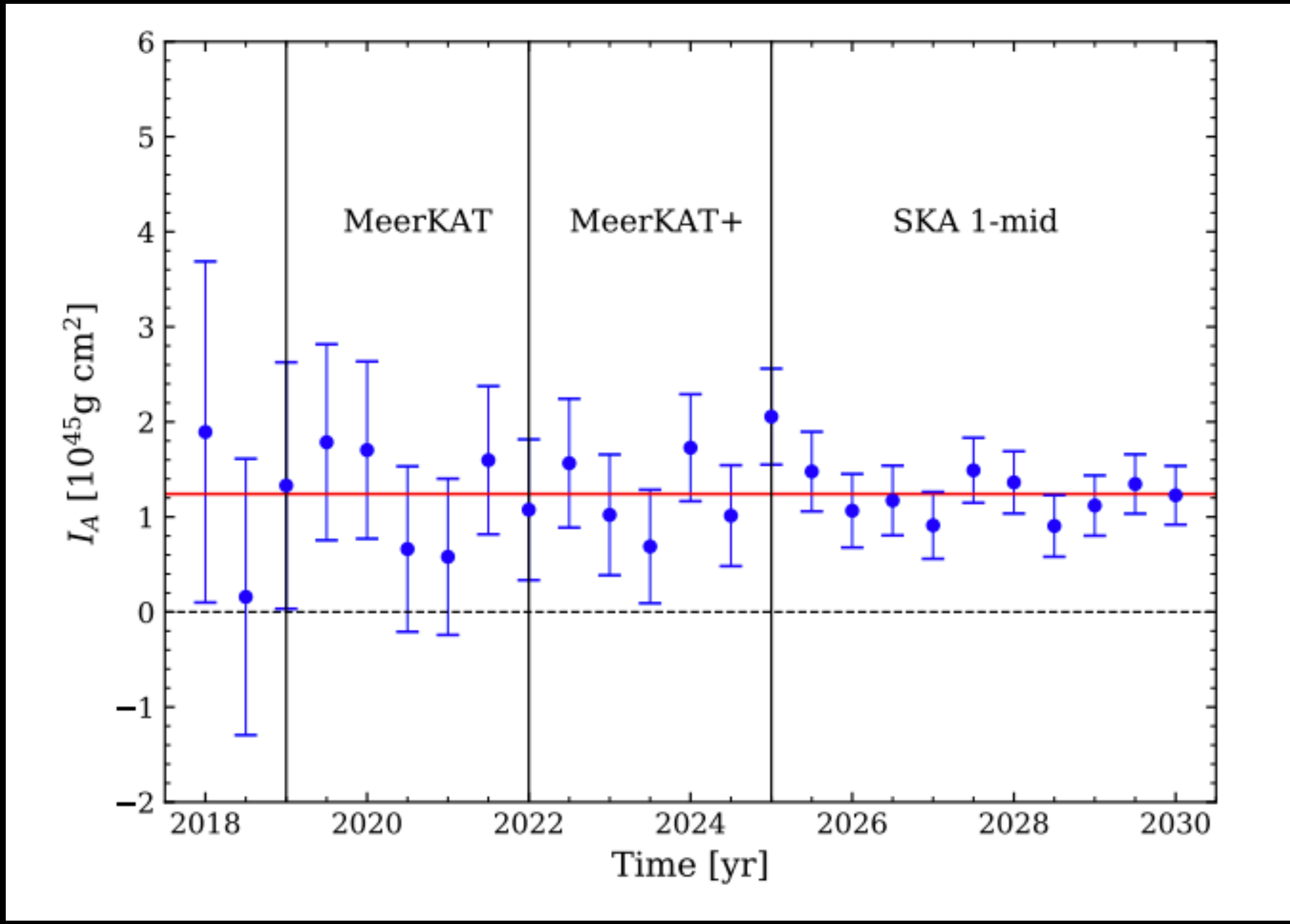
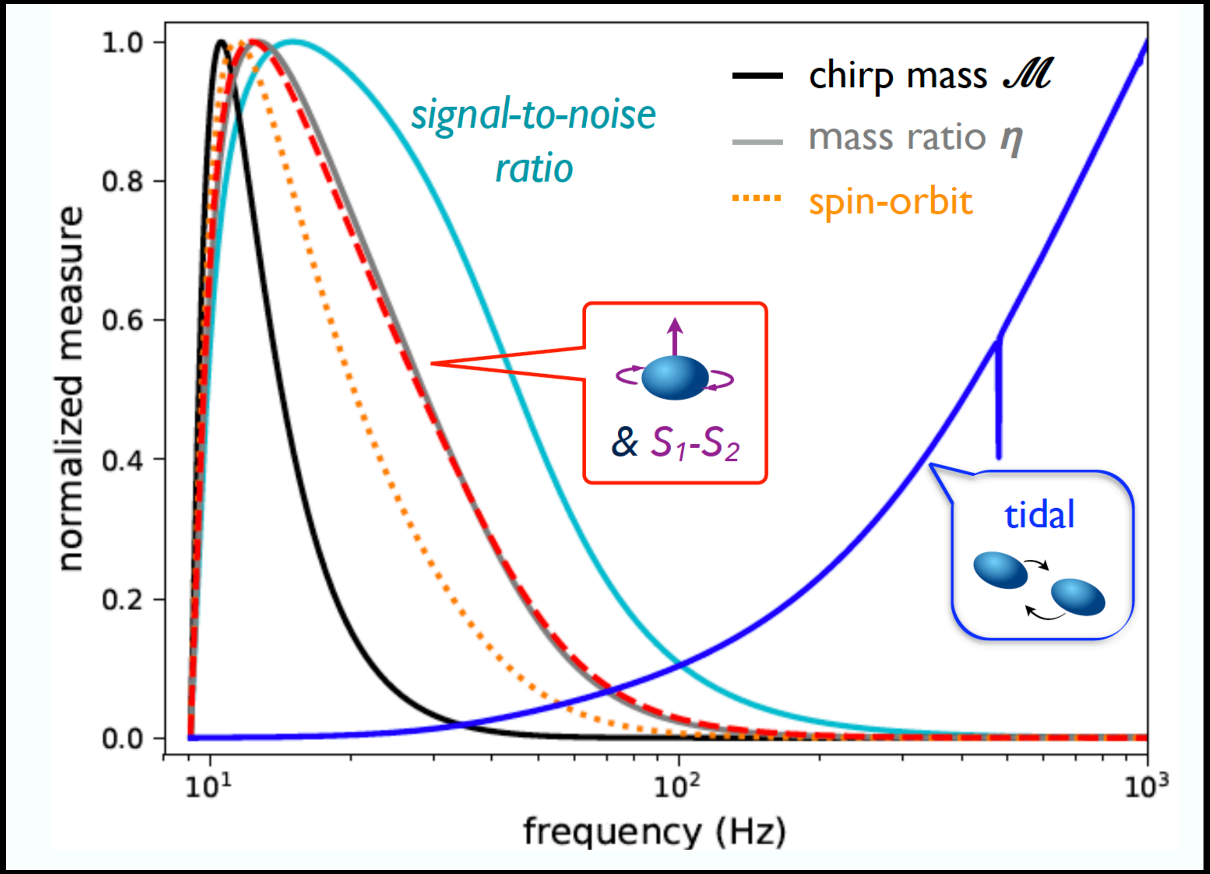
Summary and outlook

- We studied the structures of static, rotating, and tidally deformed NSs

Hu et al. 2022



Harry & Hinderer 2018

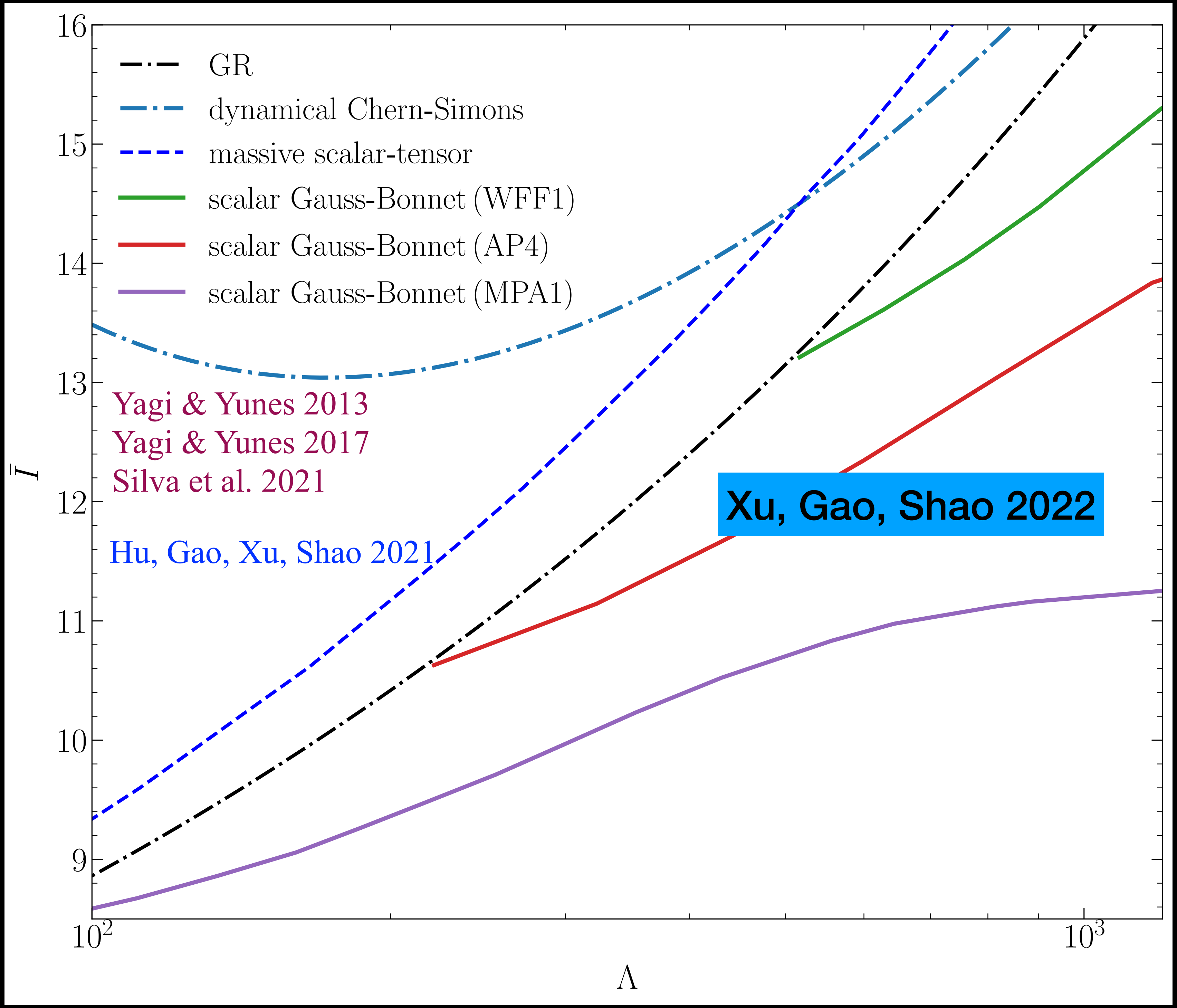


- More accurate measurements of moment of inertia at $\mathcal{O}(10\%)$ and tidal deformability at $\mathcal{O}(1\%)$, and tight constraints on the mass quadrupole

Liu & Shao 2022

Summary and outlook

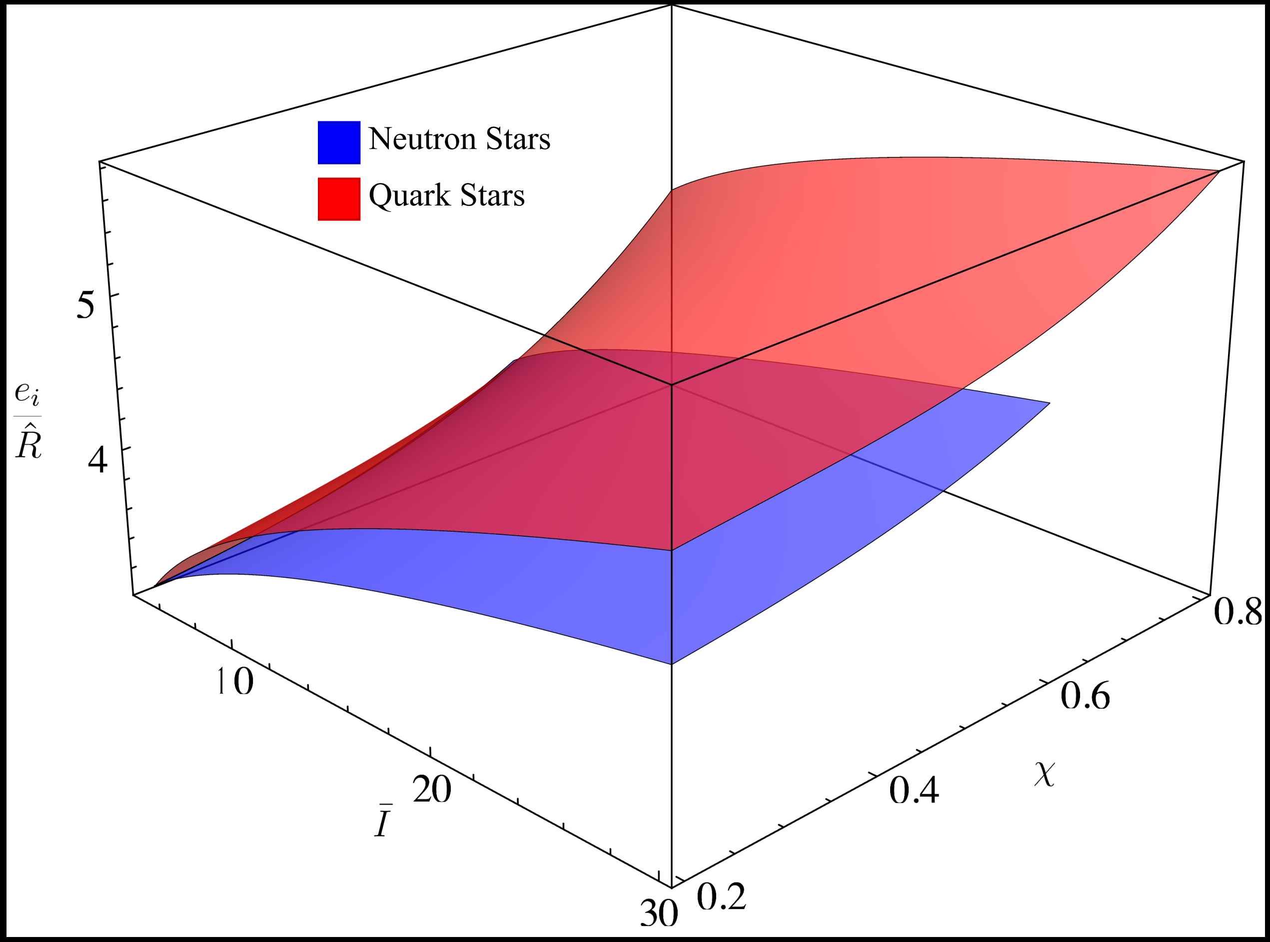
- We studied the I-Love relation in massive ST gravity for the first time

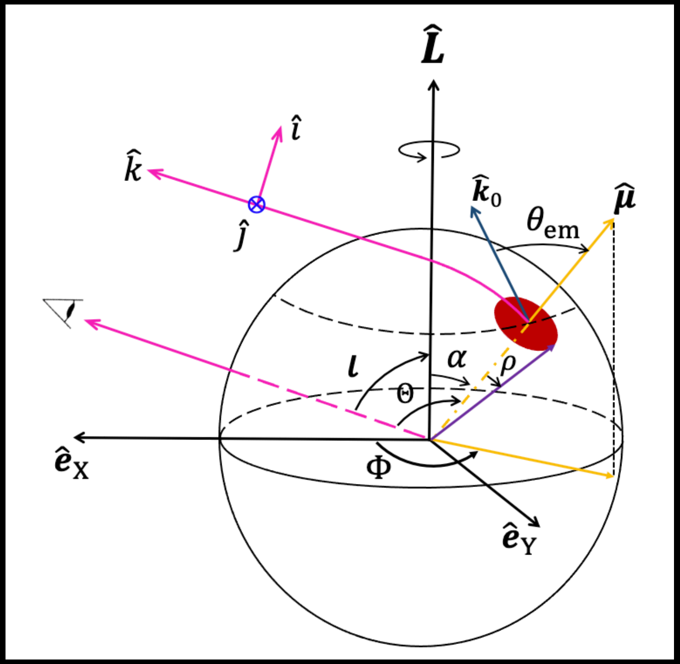


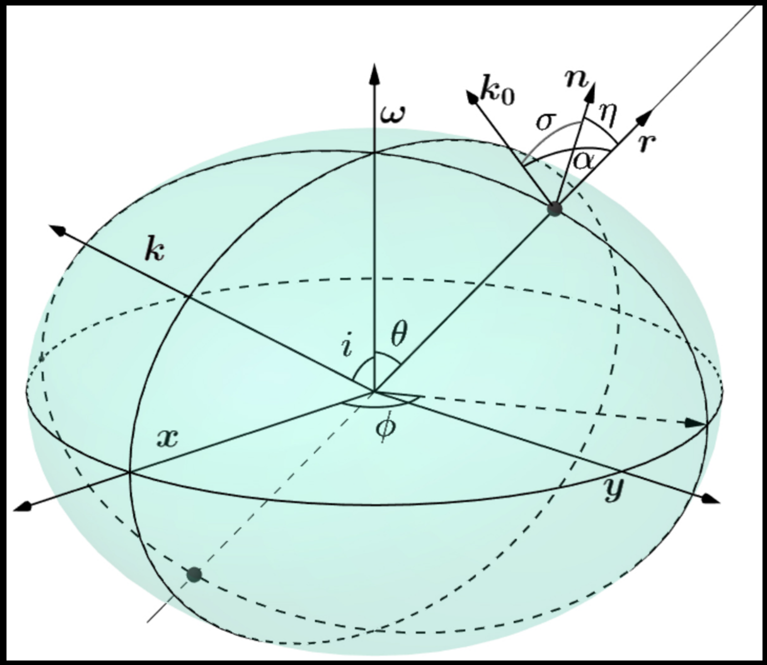
Summary and outlook

- We discovered a new universal relation between the eccentricity and MoI

Oblate-Schwarzschild approximation







Extract eccentricity from X-ray timing

Thank you for listening!