Analysis of Matter Growth Perturbations and Cosmographic Parameters in Specific Modified Gravity

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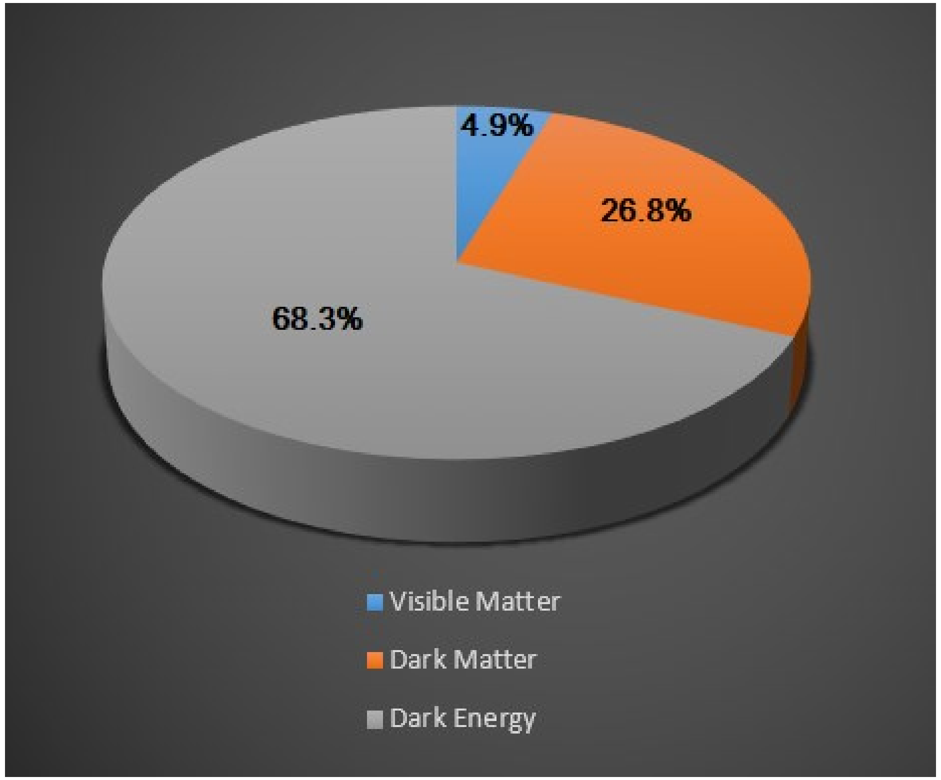
Dark Sector of the Universe (Components)

Dark Matter (DM)

Baryonic DM	Non-baryonic DM
It is visible, as it can emit or absorb light	It is invisible, as it can not emit or absorb light
Can be seen with eyes, telescope, microscope or any other instrument.	It can be feel only due to its gravitational properties.



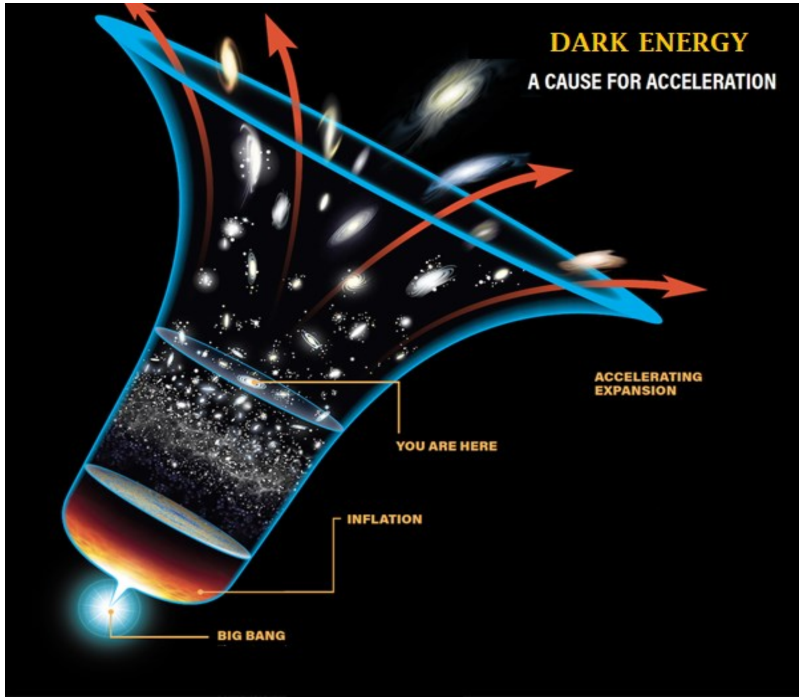
Planck 2018

[Riess, A. G. et al. (1998). Astron. J. 116, 1009; Perlmutter, S. et al. (1999). Astrophys. J. 517, 565]

Dark Sector of the Universe (Components)

Dark Energy (DE)

- Its hypothetical form of energy
- having enough negative pressure
- Responsible for the accelerated expansion



(Photo Credit: astronomy.com)

Observational Evidence

- Observation of galaxies flying away from us in all directions.
- Remnant of the Big Bang known as CMBR.

[Riess, A. G. et al. (1998). Astron. J. 116, 1009; Perlmutter, S. et al. (1999). Astrophys. J. 517, 565]

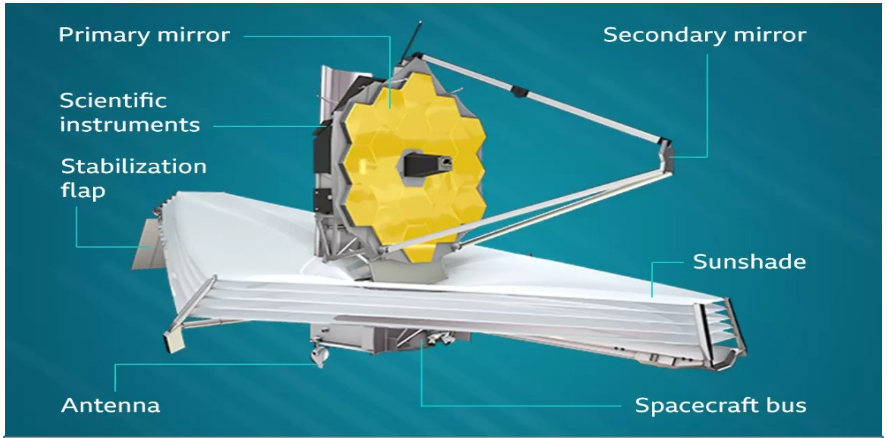
Recent Development

The James Webb Space Telescope (JWST)

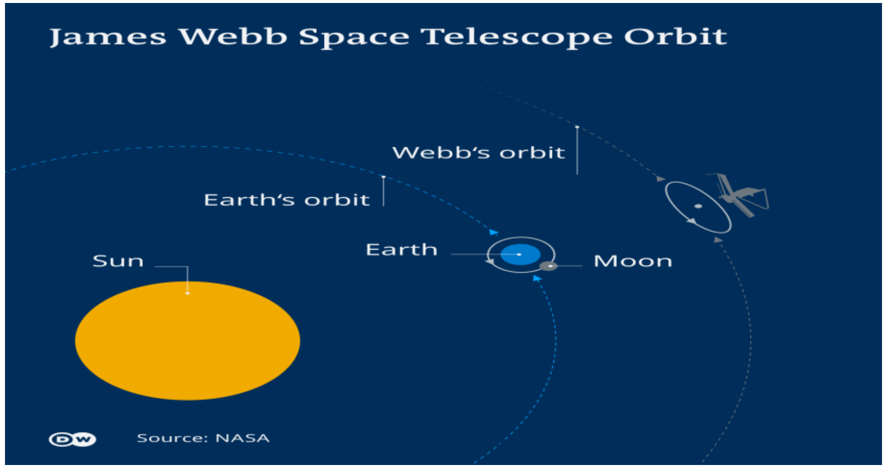
- The JWST is made of 18 hexagonal-shaped gold plated mirrors each having a diameter of 1.32 m.
- The JWST was launched on 25 December 2021 by a joint venture of NASA, ESA and CSA.
- It arrived at its destination the Sun–Earth L2 Lagrange point in January 2022.
- The first image was released to the public via a press conference on 11 July 2022.

Expected Goals of JWST

- To unfold the mysteries of ancient universe specially the formation of first star.
- The current status of expanding universe.



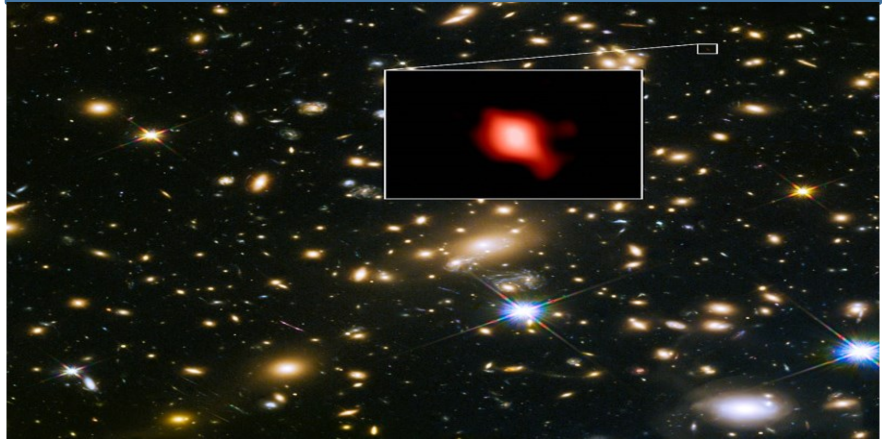
(Photo Credit: https://bbc.com



(Photo Credit: https://www.dw.com/

Some Important Discoveries by JWST

Smallest galaxy named JD-1



Clear image of the galaxy M74 (32 MLY).

Thousands young stars in a region Tarantula Nebula

Stephan Quintet, a group of five galaxies.

Photos credit: NASA

Growth Matter Perturbation

Two important aspects depend on DM in the evolution of the Universe.

These aspects are:

- DM is responsible to produce appropriate gravity that rotate the galaxies cluster.
- DM has an important part in matter growth perturbations which is the source for structure formation in the early phase of the Universe.

Modified Theories of Gravity (Motivation)

If GR is so successful, why we need modified theories of gravity?

Problems in GR

- No quantum description
- Dark matter and dark energy
- Exotic matter in wormholes
- Existence of singularities

→ Need for modified theories of gravity!

How can we modify the theory of GR?

- Modification in Curvature
- Modification in Torsion
- Others Modifications

Curvature and Torsion based Modifications

Curvature based modification

$$S = \int d^r x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \implies S = \int d^r x \sqrt{-g} \left(\frac{f(R)}{2\kappa} + \mathcal{L}_m \right), \quad (4)$$

[Buchdahl, H. A. (1970). Mon. Not. Roy. Astro. Soc. 150, 1]

where $f(R)$ is a general function depending upon Ricci curvature.

Torsion based modification

$$S = \int d^r x \sqrt{-g} \left(\frac{T}{2\kappa} + \mathcal{L}_m \right) \implies S = \int d^r x \sqrt{-g} \left(\frac{f(T)}{2\kappa} + \mathcal{L}_m \right), \quad (5)$$

[Zheng, R., Huang, Q. G. (2011). JCAP 03, 002]

where $f(T)$ is a general function depending upon torsion scalar.

Scale Factor and Geometry of the Universe

FRW metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right). \qquad (6)$$

$a(t) > 0$ scale factor of the universe used to measure the scale of expansion of the universe.

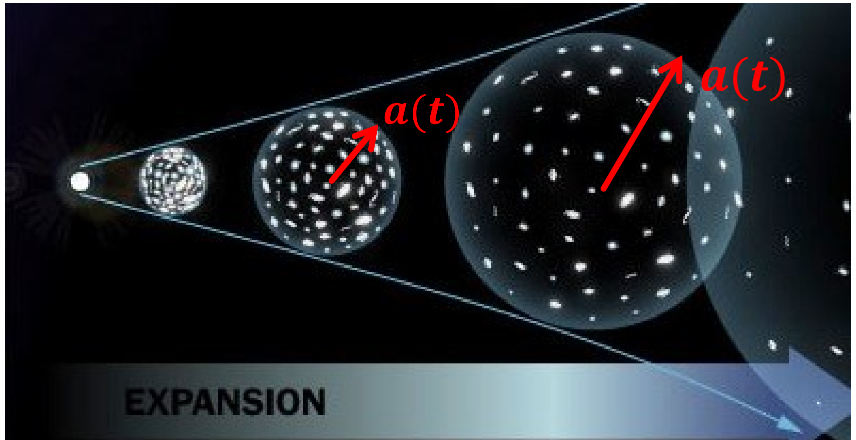
k is the spatial curvature used to define the geometry of the universe.

- $k = -1$, defines open universe (hyperbolic geometry).
- $k = 0$, gives flat universe (Euclidean geometry).
- $k = 1$, means closed universe (spherical geometry).

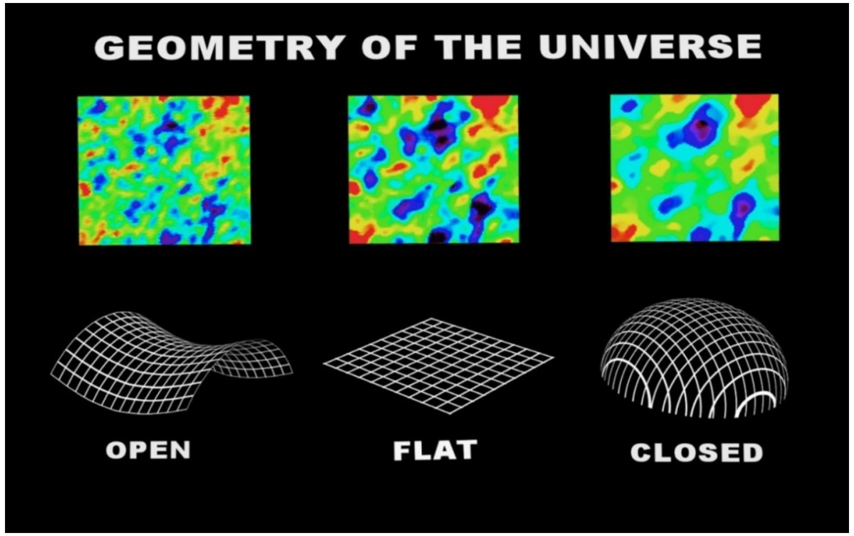
Friedmann equations:

$$\left.\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right). \end{aligned}\right\} \qquad (7)$$

where $H = \frac{\dot{a}}{a}$ is called Hubble parameter.



Scale factor in an expanding universe.



Research Work

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Regular Article - Theoretical Physics

Matter growth perturbations and cosmography in modified torsion cosmology

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Basic Field Equations

The field equations for FRW spacetime with perfect fluid matter distribution in the framework of non-zero torsion gravity are given by *[D. Kranas, C.G. Tsagas, J.D. Barrow, D. Iosifidis, Eur. Phys. J. C **79**, 341 (2019)]*

$$3H^2 = (\rho_m + \rho_D) - 12\phi(\phi + H),$$

$$\dot{H} + H^2 = -\frac{1}{6}\{\rho_m + \rho_D + 3(P_m + P_D)\} - 2(\dot{\phi} + \phi H),$$

where dot represents the time derivative, ϕ is the scalar field, ρ_m, ρ_D, P_m and P_D are the energy densities and pressures of matter and DE. We consider the expression for ϕ as

$$\phi = \lambda H,$$

where λ defines the strength of torsional effects and restricted as *[M. Cruz, F. Izaurita, S. Lepe, Eur. Phys. J. C **80**, 559 (2019)]*

$$\lambda \in [-0.005813, 0.019370].$$

Objective

In the following, we will consider two dark matter models and discuss the

(i) density contrast parameter,

(ii) growth function,

(iii) cosmographic parameters.

Density contrast parameter:

$$\delta_m = \frac{\rho_m^c}{\rho_m} - 1 = \frac{\delta\rho_m}{\rho_m}.$$

where ρ_m^c is the energy density of spherically perturbed cloud and ρ_m represents the background density.

Growth function:

$$f(a) = \frac{d\ln\delta_m}{dx}$$

where $x = \ln a$.

Cosmographic parameters:

Matter density: $\Omega_m(z) = \frac{\Omega_{m,0}}{E^2(z)}(1+z)^{3+2\lambda}.$

Energy density: $\Omega_D(z) = \frac{\Omega_{D,0}}{E^2(z)}(1+z)^{-4\lambda}$

Deceleration parameter: $q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{(1+z)}{H(z)}\frac{dH}{dz}$

Jerk parameter: $j = \frac{1}{aH^3}\frac{d^3a}{dt^3} = q(2q+1) + (1+z)\frac{dq}{dz}$

Model 1

Model 1: Pressureless matter with vacuum DE

$$P_m = 0 \quad \text{and} \quad \omega_D = -1$$

Energy conservation equations and solutions

$$\dot{\rho}_m + (3+2\lambda)H\rho_m = 0,$$
$$\dot{\rho}_D - 4\lambda H\rho_D = 0,$$

$$\rho_m = \rho_{m,0}a^{-(3+2\lambda)},$$
$$\rho_D = \rho_{D,0}a^{4\lambda},$$

Field equations take the form

$$3H^2 = \rho_m + \rho_D - 12\phi(\phi + H),$$
$$\dot{H} + H^2 = -\frac{1}{6}(\rho_m + \rho_D + 3P_D) - 2(\dot{\phi} + \phi H),$$

First field equation along with scalar field expression gives

$$3H^2 = \frac{\rho_m + \rho_D}{(2\lambda+1)^2}.$$

In terms of redshift parameter, we obtain the Hubble parameter and its normalized form for Model 1 as follow

$$H^2(z) = \frac{\rho_{m,0}(1+z)^{3+2\lambda} + \rho_{D,0}(1+z)^{-4\lambda}}{3(2\lambda+1)^2}$$

$$E(z) = \frac{H}{H_0} = \sqrt{\Omega_m (1+z)^{3+2\lambda} + \Omega_D (1+z)^{-4\lambda}}.$$

Model 1: Density Contrast

For the analysis of the growth of perturbation, we use spherical collapse (SC) approach by considering a spherically symmetric perturbed region with radius a_p and homogenous density $\rho_m^c(t)$ for which $\rho_m^c(t) - \rho_m(t) = \delta\rho_m$. For matter dominant era of the Universe, the region which is denser expanded slowly as compared to whole Universe. The conservation equation in the case of spherically perturbed region is given by *[A.H. Ziaie, H. Moradpour, H. Shahbani, Eur. Phys. J. Plus **135**, 916 (2020)]*

$$\dot{\rho}_m^c + (3+2\lambda)h\rho_m^c = 0,$$

where $h = \frac{\dot{a}_p}{a_p}$ = local rate of expansion of perturbed spherical region of radius a_p. Using first field equation, linear regime for $\delta\rho_m$, early phase of Universe in which $\frac{\rho_D}{\rho_m} < 1$ and after some simplifications, we get

$$\delta_m'' + \left(\frac{3-2\lambda}{2a} + \frac{3(1+2\lambda)}{2a}\frac{\rho_D}{\rho_m}\right)\delta_m' + \frac{(3+2\lambda)(1+2\lambda)}{2a^2}\left(\frac{\rho_D}{\rho_m} - 1\right)\delta_m = 0.$$

Here prime denotes derivative with respect to a. For $\lambda = 0$ and in the absence of $\rho_D = 0$ in the early Universe, the above equation reduces to standard cosmology equation *[L.R. Abramo, R.C. Batista, L. Liberto, R. Rosenfeld, JCAP **1**, 012 (2007)]*.

Model 1: Density Contrast

In terms of redshift function, by applying chain rules, we obtain

$$\delta_m' = -(1+z)^2 \frac{d\delta_m}{dz}, \qquad \delta_m'' = (1+z)^4 \frac{d^2\delta_m}{dz^2} + 2(1+z)^3 \frac{d\delta_m}{dz}.$$

Taking into account these equations in previous differential equation, we obtain the following initial value problem

$$\frac{d^2\delta_m}{dz^2} + \frac{1+2\lambda}{2(1+z)}\left(1 - 3\frac{\Omega_{D,0}}{\Omega_{m,0}}(1+z)^{-3(1+2\lambda)}\right)\frac{d\delta_m}{dz} + \frac{(3+2\lambda)(1+2\lambda)}{2(1+z)^2}\left(\frac{\Omega_{D,0}}{\Omega_{m,0}}(1+z)^{-3(1+2\lambda)} - 1\right)\delta_m = 0,$$

with initial conditions *[B. Farsi, A. Sheykhi, Phys. Rev. D **106**, 024053 (2022)]*

$$\delta_m|_{(z=z_i)} = 0.0001, \quad \text{and} \quad \left.\frac{d\delta_m}{dz}\right|_{(z=z_i)} = -\frac{\delta_m|_{(z=z_i)}}{1+z_i},$$

here $\delta_m(z_i)$ is the initial value of density contrast.

Model 1: Density Contrast

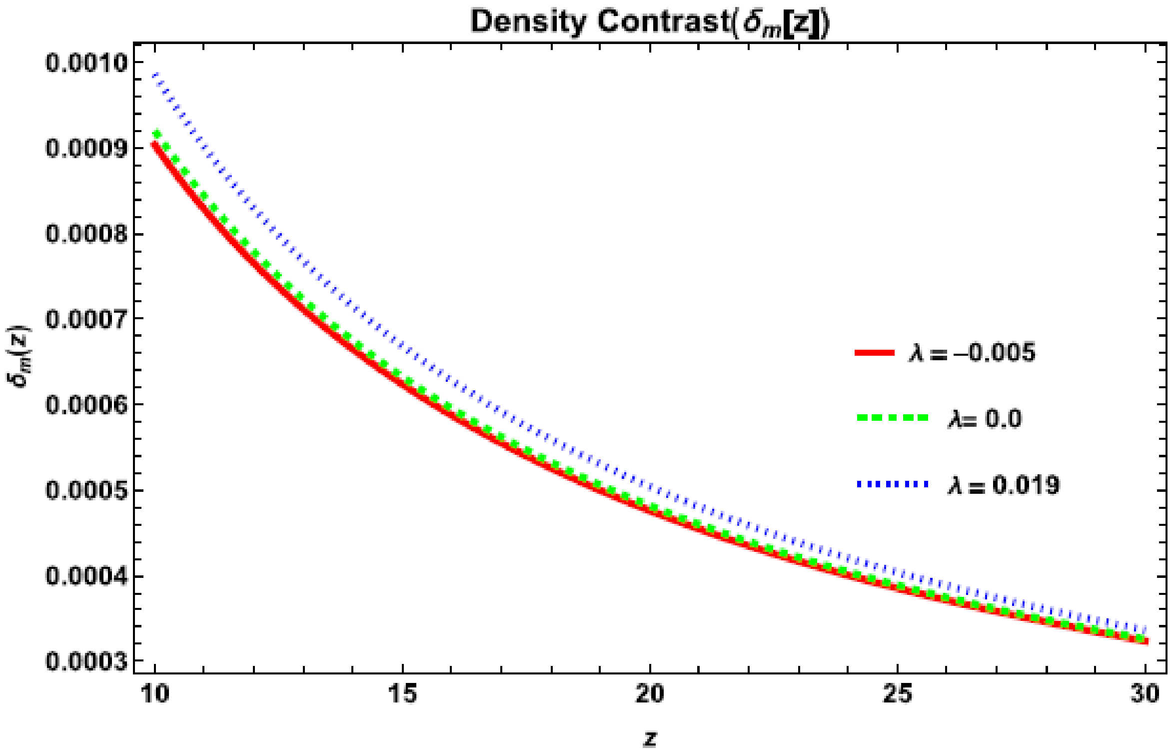

Figure 1: Plot between density contrast and redshift parameter for Model 1.

$\Omega_{m,0} = 0.7$
$\Omega_{D,0} = 0.3$

Density contrast increases from high to low redshifts.

For positive choice of λ, the growth of density contrast is greater and for its negative choice, the growth of density contrast is lower than the standard ΛCDM model $(\lambda = 0)$.

Our observations are, with the increase of parameter λ the matter growth perturbation increases that physically implies the effect of non-zero torsion field on matter growth perturbation and here the density contrast is positive and $\delta_m < 1$. Therefore, it yields that region under consideration for matter growth perturbation will expand with the expansion of Universe in lieu of gravitational collapse.

Model 1: Growth Function

To discuss the growth of matter perturbations, we evaluate the growth function $f(a) = \frac{d \ln \delta_m}{dx}$,

by applying chain rules, we obtain $\delta'_m = \frac{\delta_m f}{a}, \quad \delta''_m = \frac{\delta_m}{a^2}\frac{df}{dx} + \frac{f^2 \delta_m}{a^2} - \frac{\delta_m f}{a^2}.$

In this case, we obtain the following differential equation

$$f^2 - (1+z)\frac{df}{dz} + \left(-1 + \frac{3-2\lambda}{2} + \frac{3(1+2\lambda)}{2}\frac{\Omega_{D,0}}{\Omega_{m,0}}(1+z)^{-3(1+2\lambda)} \right) f$$
$$= \frac{(3+2\lambda)(1+2\lambda)}{2}\left(1 - \frac{\Omega_{D,0}}{\Omega_{m,0}}(1+z)^{-3(1+2\lambda)}\right).$$

We solve the above equation numerically for growth function for three different values of λ.

Model 1: Growth Function

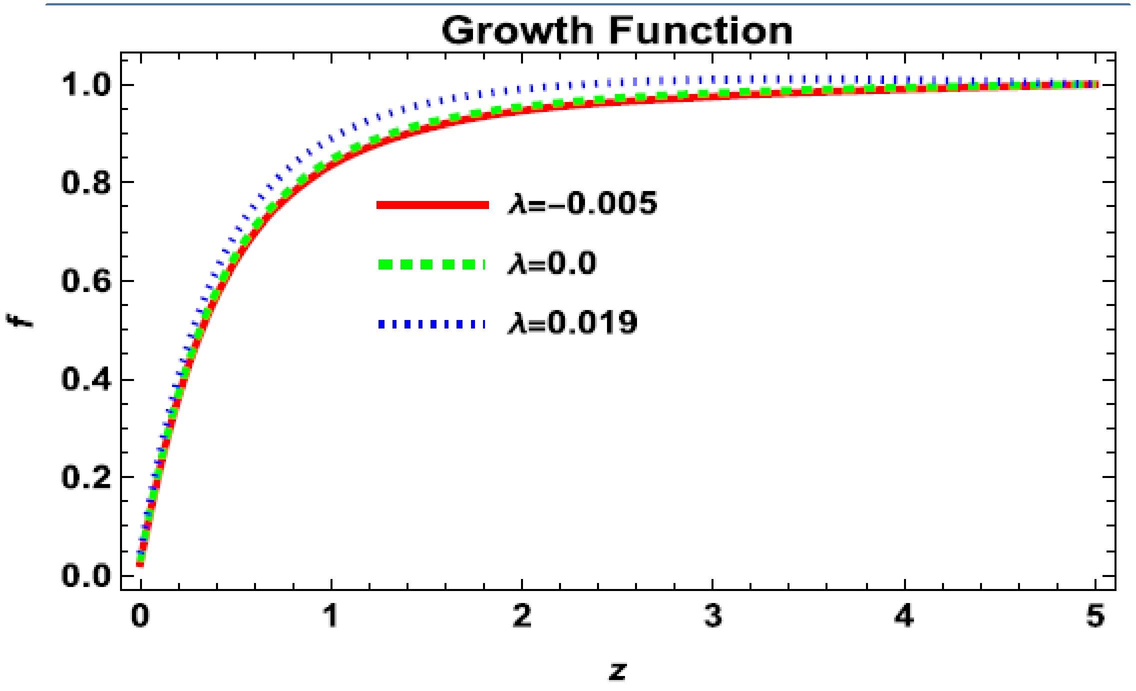

Figure 2: Plot between growth function and redshift parameter for Model 1.

$\Omega_{m,0} = 0.7$
$\Omega_{D,0} = 0.3$

This figure indicates that at high redshifts, the matter growth function approaches unity.

The trajectory for $\lambda = 0.019$ approaches unity with high growth perturbations as compared to $\lambda = 0$ (ΛCDM for GR).

Similarly, the matter growth perturbations for $\lambda = 0$ is little higher than $\lambda = -0.005$.

Hence, the role of non-zero torsion field affects the growth function in matter growth perturbations.

Model 1: Cosmographic Parameters

Matter density: $$\Omega_m(z) = \frac{\Omega_{m,0}}{E^2(z)}(1+z)^{3+2\lambda}.$$

Energy density: $$\Omega_D(z) = \frac{\Omega_{D,0}}{E^2(z)}(1+z)^{-4\lambda}.$$

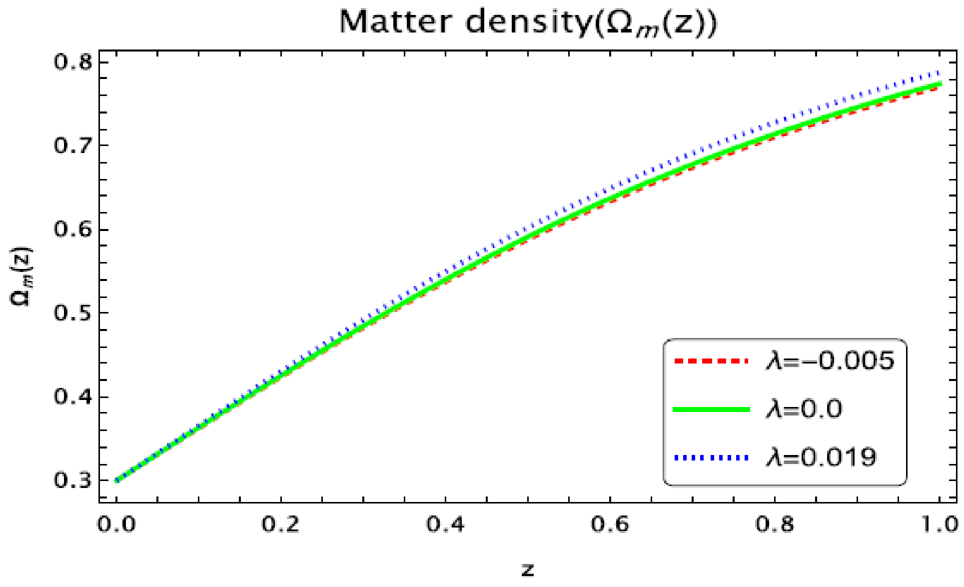

Figure-3: With the increase of parameter λ, the density abundance of matter increases. All three trajectories of $\Omega_m(z)$ at present $z = 0$ show same behavior.

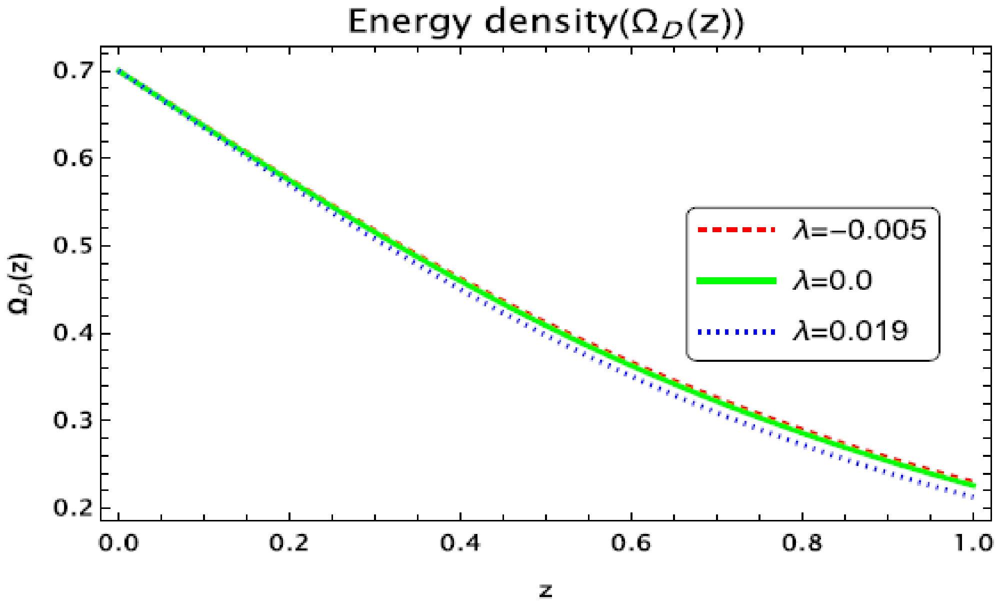

Figure-4: With smaller values of parameter λ, the plot of density abundance of DE reaches faster than the higher choice of λ to present-day value of $\Omega_{D,0}$ from high redshift.

Model 1: Cosmographic Parameters

Deceleration Parameter:

$$q=-1+\frac{(3+2\lambda)(1+z)^{3+2\lambda}\Omega_{m,0}-4\lambda(1+z)^{-4\lambda}\Omega_{D,0}}{2(\Omega_{m,0}(1+z)^{3+2\lambda}+\Omega_{D,0}(1+z)^{-4\lambda})}.$$

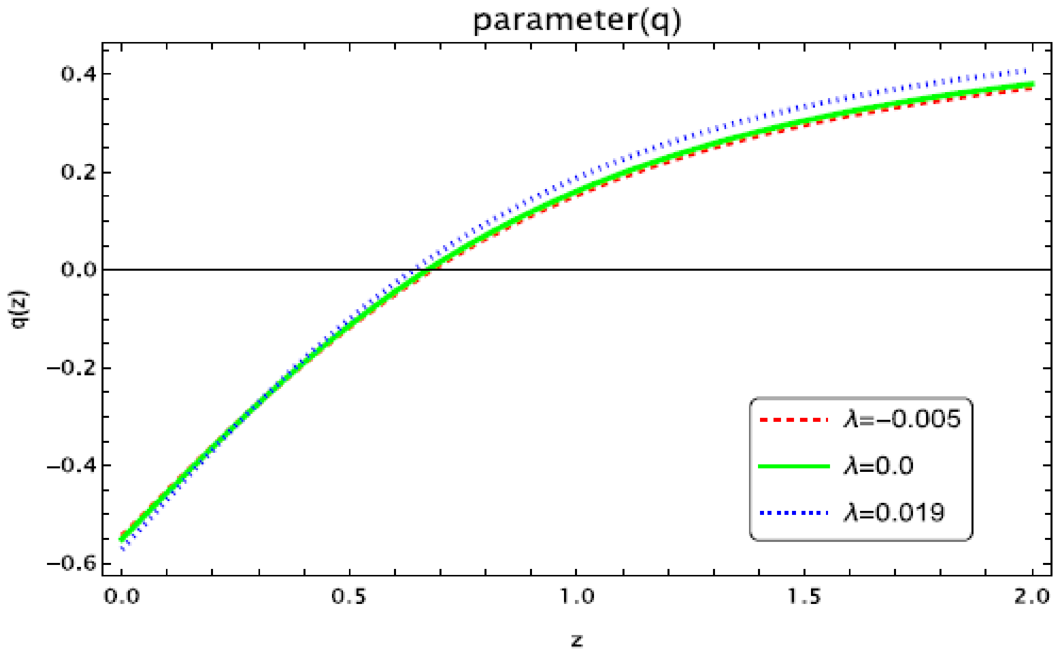

Figure 5: Plot between deceleration and redshift parameters.

We observe that the Universe faces a transition from deceleration phase $(q > 0)$ for $z > z_t$ to acceleration phase $(q < 0)$ for $z < z_t$, here z_t is the transition point on the redshift. Figure-5 shows that with different values of λ, the transition from decelerating phase to accelerating phase appears at low redshift signifies around $z = z_t \approx 0.6$. At the present stage, the deceleration parameter approaches to $q \approx -0.6$. This behavior is compatible with the recent observational data *[S. Capozziello, A. Ruchika, A. Sen, MNRAS* ***484****, 484 (2019)].*

Model 1: Cosmographic Parameters

Jerk Parameter:

$$
\begin{aligned}
j &= \left(-1+\frac{((3+2\lambda)(1+z)^{3+2\lambda}\Omega_{m,0}-4\lambda(1+z)^{-4\lambda}\Omega_{D,0})}{2(\Omega_{m,0}(1+z)^{3+2\lambda}+\Omega_{D,0}(1+z)^{-4\lambda})}\right)\\
&\times\left(-1+\frac{((3+2\lambda)(1+z)^{3+2\lambda}\Omega_{m,0}-4\lambda(1+z)^{-4\lambda}\Omega_{D,0})}{(\Omega_{m,0}(1+z)^{3+2\lambda}+\Omega_{D,0}(1+z)^{-4\lambda})}\right)\\
&+\frac{1}{2N^2}\Big(((3+2\lambda)^2(1+z)^{3+2\lambda}\Omega_{m,0}+16\lambda^2(1+z)^{-4\lambda}\Omega_{D,0})\\
&\times N-M(((3+2\lambda)(1+z)^{3+2\lambda}\Omega_{m,0}-4\lambda(1+z)^{-4\lambda}\Omega_{D,0}))\Big),
\end{aligned}
$$

where $M=(3+2\lambda)(1+z)^{3+2\lambda}\Omega_{m,0}-4\lambda(1+z)^{-4\lambda}\Omega_{D,0}$ and $N=(1+z)^{3+2\lambda}\Omega_{m,0}+(1+z)^{-4\lambda}\Omega_{D,0}$.

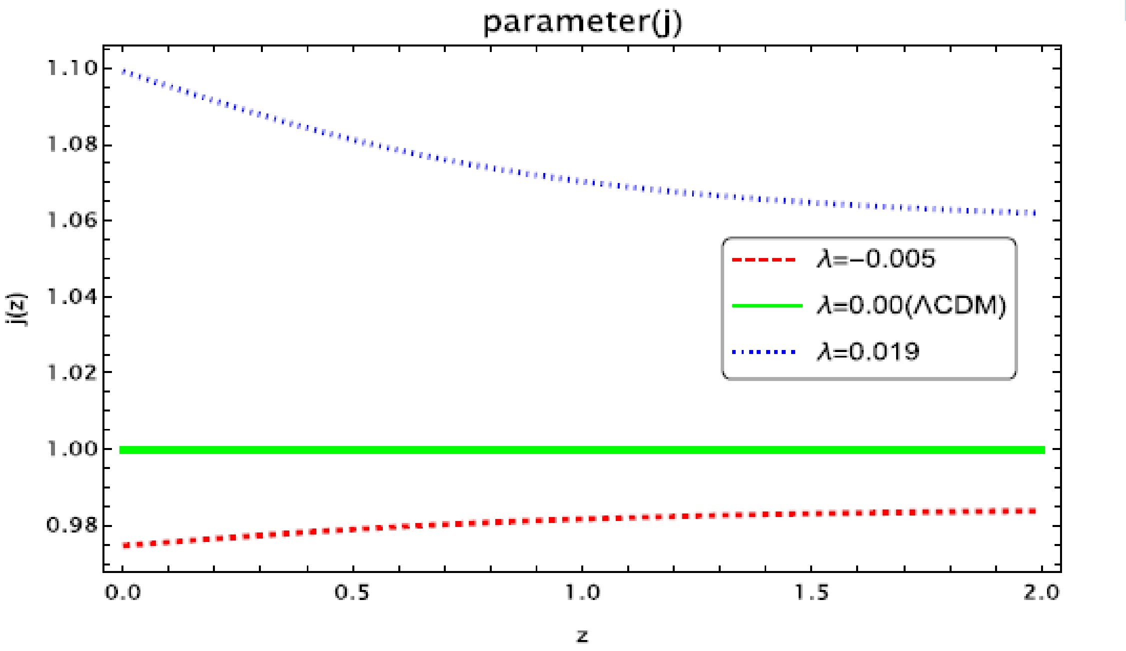

Figure-6 indicates that the deviation of behavior of jerk parameter for $\lambda = -0.005$ and $\lambda = 0.019$ from ΛCDM $(j = 1)$ is due to presence of non-zero torsion. However, our results are consistent with observational constraints [A. A. Mamon, K. Bamba Eur. Phys. J. C (2018) 78:862].

Model 2

Model 2: Non-zero pressure matter with vacuum DE

$$P_m = \alpha\,\rho_m \quad \text{and} \quad \omega_D = -1$$

Energy conservation equations and solutions

$$\dot{\rho}_m + (3+2\lambda)H\rho_m + 3\alpha H(1+2\lambda)\rho_m = 0,$$
$$\dot{\rho}_D - 4\lambda H\rho_D = 0,$$

$$\rho_m = \rho_{m,0}(1+z)^{((3+2\lambda)+3\alpha(1+2\lambda))},$$
$$\rho_D = \rho_{D,0}(1+z)^{-4\lambda}.$$

Field equations take the form

$$3H^2 = \frac{\rho_m + \rho_D}{(1+2\lambda)^2},$$
$$\frac{\ddot{a}}{a} = -\frac{(1+3\alpha)\rho_m}{6(1+2\lambda)} + \frac{2\rho_D}{6(1+2\lambda)}.$$

In terms of redshift parameter, we obtain the Hubble parameter for Model 2 as follows

$$H(z) = H_0(z)((\Omega_{m,0}(1+z)^{((3+2\lambda)+3\alpha(1+2\lambda))} + \Omega_{D,0}(1+z)^{-4\lambda})^{\frac{1}{2}}.$$

Model 2: Density Contrast

Density Contrast:

$$\frac{d^2\delta_m}{dz^2} + \frac{1}{(1+z)}\left[\frac{(1+3\alpha)(1+2\lambda)}{2} - \frac{3(1+2\lambda)(1+\alpha)}{2}\frac{\Omega_{D,0}}{\Omega_{m,0}}P\right]\frac{d\delta_m}{dz}$$
$$-\frac{((3+2\lambda)+3\alpha(1+2\lambda))(1+3\alpha)(1+2\lambda)}{2(1+z)^2}\left[1-\frac{\Omega_{D,0}}{\Omega_{m,0}}P\right]\delta_m = 0.$$

where $P = (1+z)^{-3(1+2\lambda)(1+\alpha)}$.

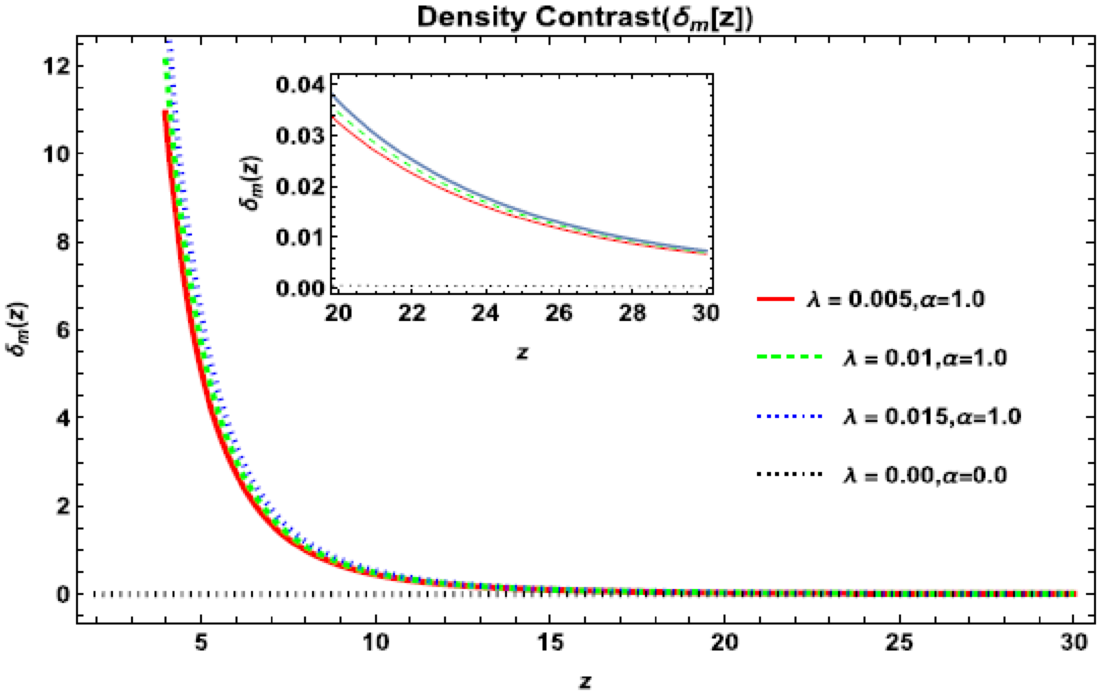

Figure 7: Plot between density contrast and redshift parameter for Model 2.

Figure-7 shows that density contrast grows from higher redshift to low redshift, particularly in the range $5 < z < 8$, density contrast grows very fast for non-zero pressure and non-zero torsion as compared to zero torsion (i.e $\lambda = 0$) with pressureless matter. Physically, it implies that matter growth of perturbations are very high due to non-zero pressure of DM as well as non-zero torsion. As a result, the perturbed region under consideration is going to expand more faster than the overall Universe's expansion force. Eventually this region will collapse under its own gravitational force and form new large scale matter or galaxy.

Model 2: Growth Function

Growth Function:

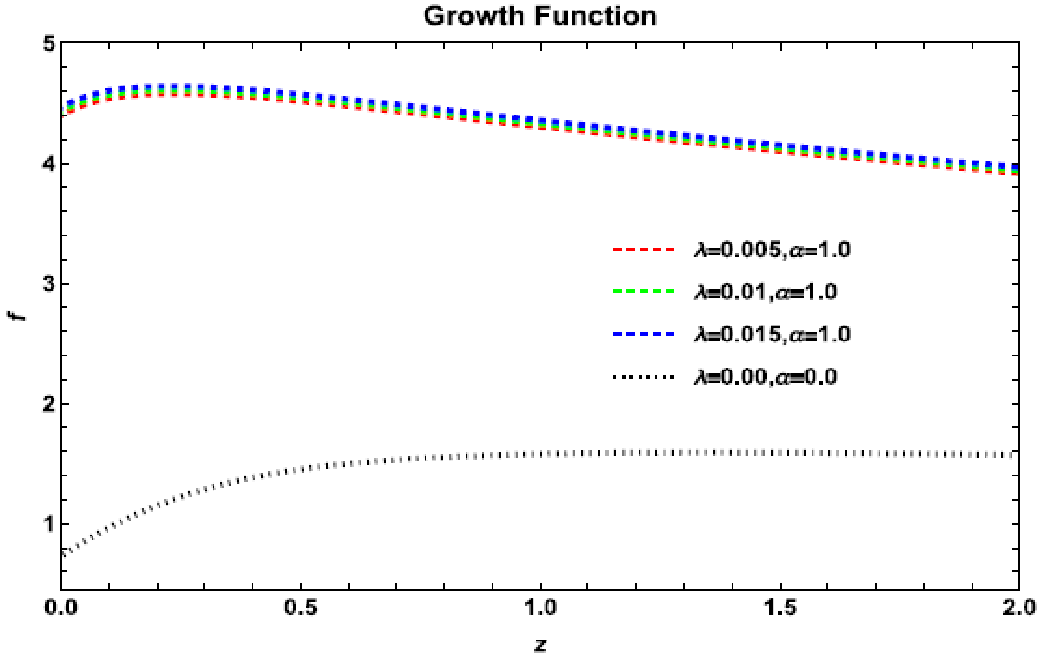

Figure 8: Plot between growth function and redshift parameter for Model 2.

$$f^2-(1+z)\frac{df}{dz}+\left(-1-\frac{(4-(1+3\alpha)(1+2\lambda))}{2}+\frac{(3+3\alpha)(1+2\lambda)}{2}\frac{\Omega_{D,0}}{\Omega_{m,0}}A\right)f=\left(\frac{((3+2\lambda)+3\alpha(1+2\lambda))(1+3\alpha)(1+2\lambda)}{2}\right)\left(1-\frac{\Omega_{D,0}}{\Omega_{m,0}}A\right),$$

where $A=(1+z)^{(-4\lambda-((3+2\lambda)+3\alpha(1+2\lambda)))}$.

It is clear from Figure-8 that the behavior of the growth function for three different non-zero choices of λ with DM having non-zero pressure deviates from $\lambda=0$ with pressureless matter. The deviation in the profiles of the growth function is due to matter growth perturbations.

Concluding Remarks

- It is found that results about density contrasts and growth functions in modified torsion gravity are feasible and consistent with the literature for both models.

- Cosmographic parameters are also consistent with the recent observational constraints.


Thank you