# Relativistic Bondi Accretion and the BEC Dark Matter Spike


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# Outline

- Motivation
- Relativistic Bondi Accretion
- Bose-Einstein Condensate (BEC) Dark Matter
- BEC Dark Matter Spikes Around Supermassive Black Holes
- Summary & Conclusion

# Motivation

- Supermassive black holes
- Accretion processes
- Dark matter

General Relativity (Gravity)

Black hole Accretion

Bondi Accretion (Astrophysics)

Hydrodynamic Limit

Dark Matter Properties (Particle Physics)

# Relativistic Bondi Accretion

## The Bondi Problem

Armitage, Astron. Geophys. **61**, 2.40-2.42 (2020)

It is amusing that

the study of accretion was initiated by speculation about

a system where accretion is not taking place – the Sun

- Spherically symmetric accretion:

$\dot{M} = 4\pi\lambda_s \frac{M^2}{a_\infty^3}\rho_\infty$, where $\lambda_s$ depends on the adiabatic index of gas

- Usually $\lambda_s \sim \mathcal{O}(1)$ in Newtonian gravity

- $\rho_\infty$ (density), $a_\infty$ (sound speed) are evaluated at $r_\infty$ far away from Bondi radius $r_{\rm B} = 2M/a_\infty^2$ (radius of influence)

Bondi and Hoyle, Mon. Not. Roy. Astron. Soc. **104** (1944) 273
Bondi, Mon. Not. Roy. Astron. Soc. **112** (1952) 195

# Relativistic Bondi Accretion

**Relativistic Bernoulli equation** → Why relativistic?

1. Accretion rate changes significantly when matter becomes radiation-like

2. Matter distribution near the BH horizon

$$\begin{cases} \dot{M} \equiv 4\pi r^2 \rho_0 u = \text{constant}, \rightarrow \text{ continuity} \\ \text{where } \rho_0 = mn \text{ the rest mass density and } u \text{ the radial velocity of the inflow} \\ \left(\frac{p+\rho}{\rho_0}\right)^2 \left(1 + u^2 - \frac{2M}{r}\right) = \left(\frac{p_\infty + \rho_\infty}{\rho_{0,\infty}}\right)^2 \rightarrow \text{ energy conservation} \end{cases}$$

with the boundary condition: $u \rightarrow 0$ at $r \rightarrow r_\infty \gg r_B$.

At sonic horizon, $u_s^2 = \dfrac{a_s^2}{1+3a_s^2} = \dfrac{M}{2r_s}$.

Teukolsky and Shapiro (1983)

# Bose-Einstein Condensate (BEC) Dark Matter

## Self-interacting Dark Matter

- DM with self-interaction? Spergel & Steinhardt, PRL, **84**, 3760 (2000)
  Tulin & Yu, Phys. Rept. **730**, 1-57 (2018)
- Core-cusp, diversity,... Kamada, Kaplinghat, Pace & Yu, PRL, **119**, 111102 (2017)
  Ren, Kwa, Kaplinghat & Yu, PRX, **9**, 031020 (2019)
  Kaplinghat, Ren & Yu, JCAP, **06**, 027 (2020)
- The origin of supermassive black holes at high redshifts Balberg & Shapiro, PRL, **88**, 101301 (2002);
  Balberg, Shapiro & Inagaki, ApJ, **568**, 475 (2002)
  Feng, Yu & Zhong, Astrophys. J. Lett. **914**, L26 (2021);
  JCAP **05** (2022) 036
- Observational constraints on self-interaction strength from dwarf galaxies
  $\sigma/m \sim \mathcal{O}(1)\ \mathrm{cm}^2/\mathrm{g}$ Kaplinghat, Tulin & Yu, PRL, **116**, 041302 (2016)
- The simplest model is a canonical scalar field with $\lambda\phi^4$ and $\lambda > 0$ (repulsive)
  $$\rightarrow \sigma = \frac{\lambda^2}{64\pi m^2}$$

# Bose-Einstein Condensate (BEC) Dark Matter

## The Equation of State (EoS)

$$\mathscr{L} = -\frac{1}{2}g^{\mu\nu}\phi^*_{;\mu}\phi_{;\nu} - V(|\phi|)$$

$$V(|\phi|) = \frac{1}{2}m^2|\phi|^2 + \frac{\lambda}{4!}|\phi|^4$$

$$\lambda > 0 \text{ (repulsive)}$$

- Dark matter halos as big boson stars

Colpi et al., Phys. Rev. Lett. **57** (1986) 2485-2488
Ferreira, Astron. Astrophys. Rev. **29**, no.1, 7 (2021)

- Constraints from cosmology

Li et al., Phys. Rev. D **89** (2014) 8, 083536

- In fast oscillation limit, the EoS is simple

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

$$\frac{p}{\rho_B} = \frac{4}{9}\left[\left(1 + \frac{3\rho}{4\rho_B}\right)^{1/2} - 1\right]^2$$

$$\begin{cases} \rho \simeq 3p \gg \rho_B \text{ (radiation fluid)} \\ p \propto \rho^2 \text{ (condensate fluid) if } p \ll \rho \ll \rho_B \end{cases}$$

with $\underline{\rho_B = \frac{3m^4}{2\lambda} = \frac{3.48}{\lambda}\left(\frac{m}{\text{GeV}}\right)^4 \times 10^{20} \text{ kg m}^{-3}}$ $\Rightarrow$ two parameters $(m, \lambda)$

# BEC Dark Matter Spikes Around Supermassive Black Holes

## Dark Matter Properties

- Dark matter develops a **density spike** around a central black hole due to **accretion.**
- Black hole shadow might be blurred due to dark matter **annihilations.**

Lacroix et al., Phys. Rev. D **96**, 063008 (2017)

- The central spike follows **power law** profiles depending on the dark matter properties:

**Cold dark matter** $\propto r^{-7/3}$; **Coulomb-like self-interacting dark matter** $\propto r^{-7/4}$.

Shapiro and Paschalidis, Phys. Rev. D **89**, 023506 (2014)

What about the BEC dark matter?

# BEC Dark Matter Spikes Around Supermassive Black Holes

## Relativistic Bondi Accretion

Sound speed: $a^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_{\text{ad}} = \frac{p}{\rho + p}\gamma = \frac{1}{3}\left(1 - \frac{1}{\sqrt{1 + 3\rho/4\rho_B}}\right) \rightarrow \gamma$ (adiabatic index)

First law of thermodynamics: $\left(\frac{\partial \rho}{\partial \rho_0}\right)_{\text{ad}} = \frac{\rho + p}{\rho_0} \rightarrow \rho_0 = mn$

Relativistic Bernoulli equation: $\begin{cases} \dot{M} = 4\pi r^2 \frac{8a^2}{1-3a^2}\sqrt{\frac{1-a^2}{1-3a^2}}\rho_B u = \text{constant} & \text{[continuity]} \\ \left(\frac{1-a^2}{1-3a^2}\right)^2\left(1 + u^2 - \frac{2M}{r}\right) = \left(\frac{1-a_\infty^2}{1-3a_\infty^2}\right)^2 & \text{[energy conservation]} \end{cases}$

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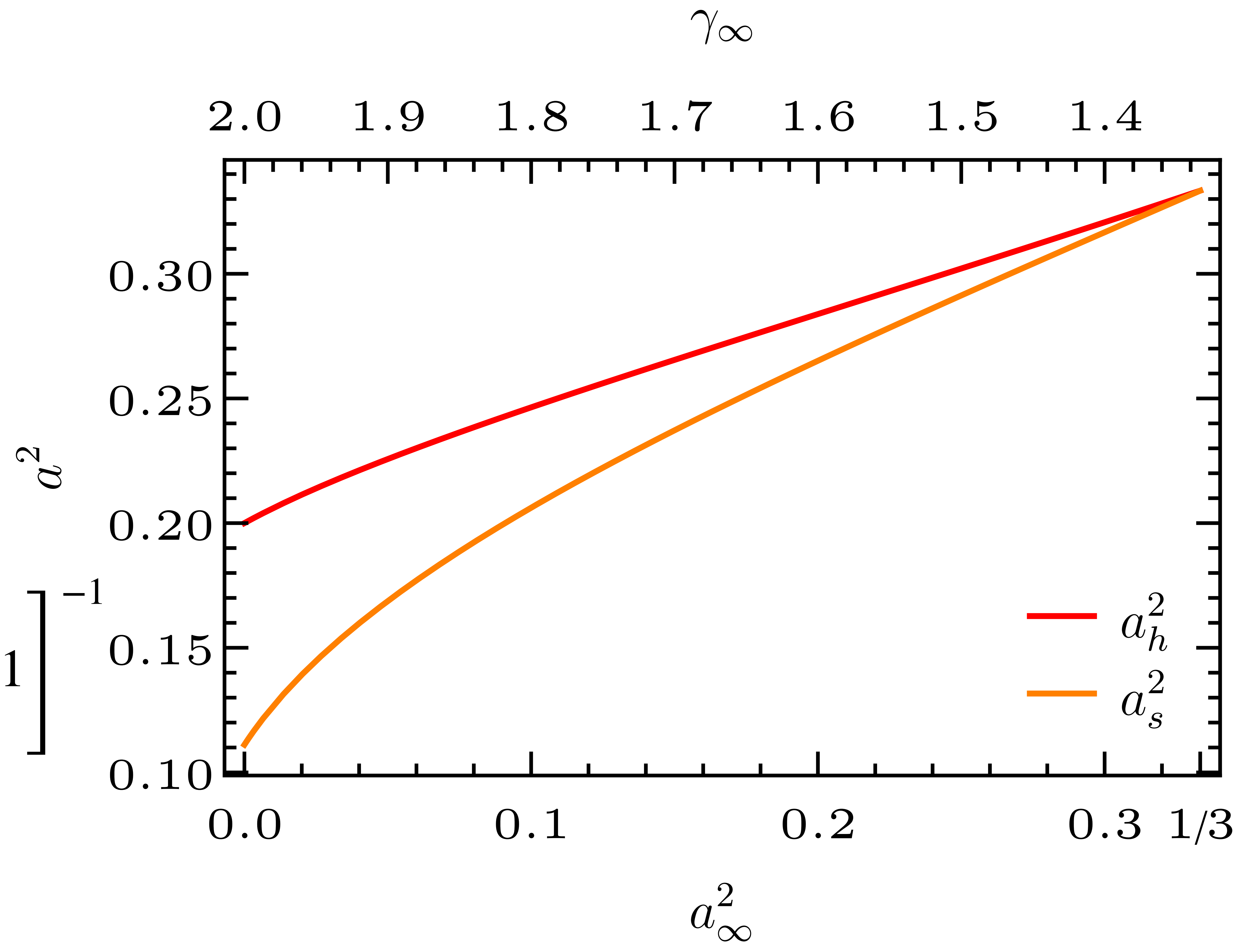
# BEC Dark Matter Spikes Around Supermassive Black Holes

## Sound Speeds

- At sonic horizon: $a_s$
- At event horizon: $a_h$

$$\begin{cases} a_s^2 = \dfrac{1 - 3a_\infty^2 + \sqrt{1 + 66a_\infty^2 - 63a_\infty^4}}{18(1 - a_\infty^2)} \\ a_h^2 = \dfrac{1}{3} - \dfrac{1}{3}\left[\dfrac{3}{16}\left(\dfrac{1+3a_s^2}{1-3a_s^2}\right)^{3/2}\sqrt{\dfrac{1-a_s^2}{a_s^2}}\sqrt{\dfrac{1-3a_\infty^2}{1-a_\infty^2}}+1\right]^{-1} \end{cases}$$

$$\Rightarrow \frac{1}{5} \le a_h^2 < \frac{1}{3}, \quad \text{and} \quad \frac{1}{9} \le a_s^2 < \frac{1}{3}$$

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# BEC Dark Matter Spikes Around Supermassive Black Holes

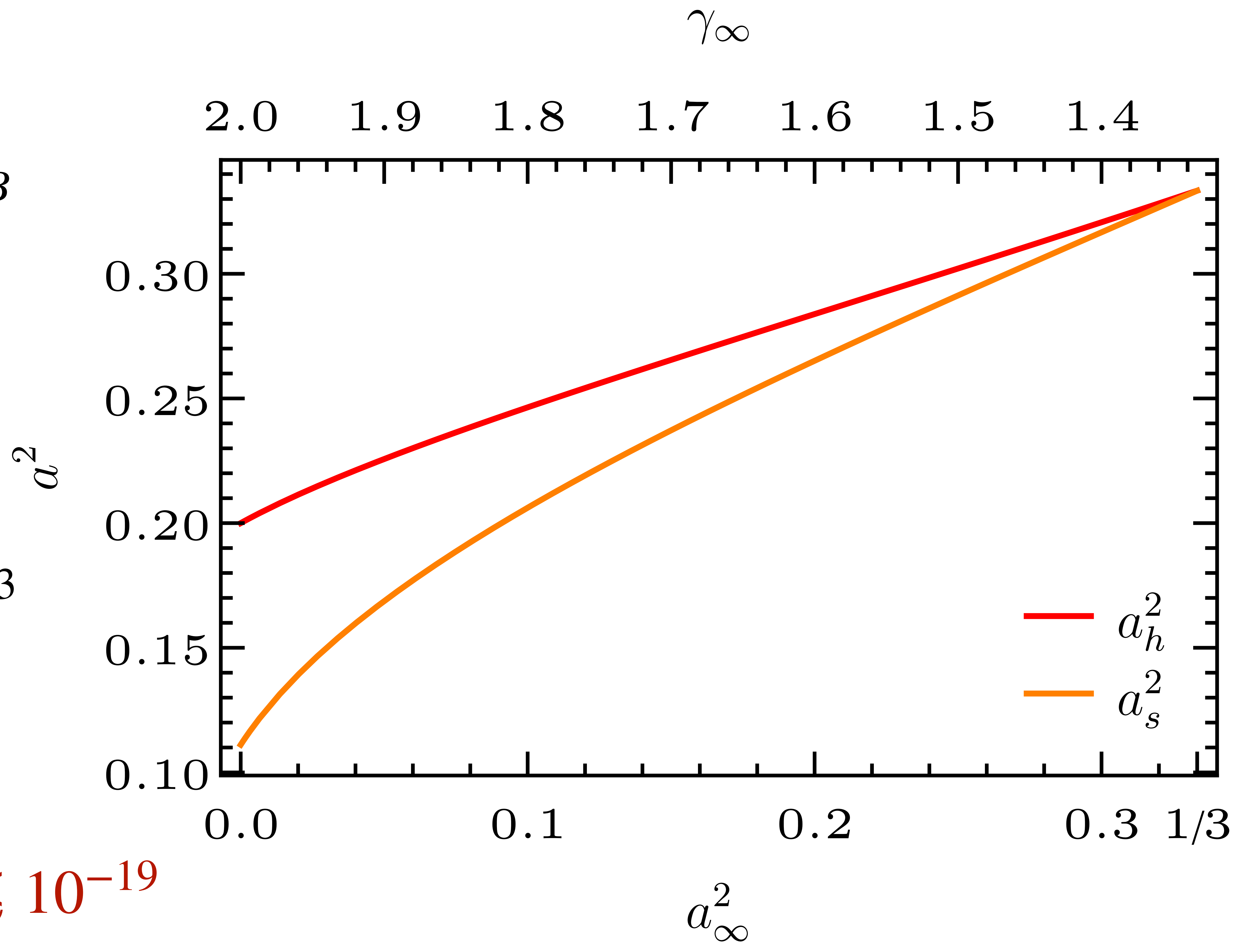
## Fixing the Model Parameters

- Velocity dispersion, $v_{\text{dis},\infty}^2 \sim a_\infty^2$ if $\rho_\infty \ll \rho_B$

$$\Rightarrow \rho_B \simeq \frac{3}{16}\frac{\rho_\infty}{v_{\text{dis},\infty}^2} \text{ if } v_{\text{dis},\infty}^2 \ll 1.$$

- On galactic scale, $\rho_\infty \sim 10^{-2}\ \mathrm{M_\odot\ pc^{-3}}$ and $v_{\text{dis},\infty} \sim 10^2\ \mathrm{km\ s^{-1}} \Rightarrow \rho_B \sim 10^4\ \mathrm{M_\odot\ pc^{-3}}$
- Combined with the constraint $\sigma/m \sim \lambda^2/m^3 \sim \mathcal{O}(1)\ \mathrm{cm^2/g}$

$$\Rightarrow 10^{-7}\ \mathrm{eV} \lesssim m \lesssim 10^{-5}\ \mathrm{eV},\ 10^{-22} \lesssim \lambda \lesssim 10^{-19}$$

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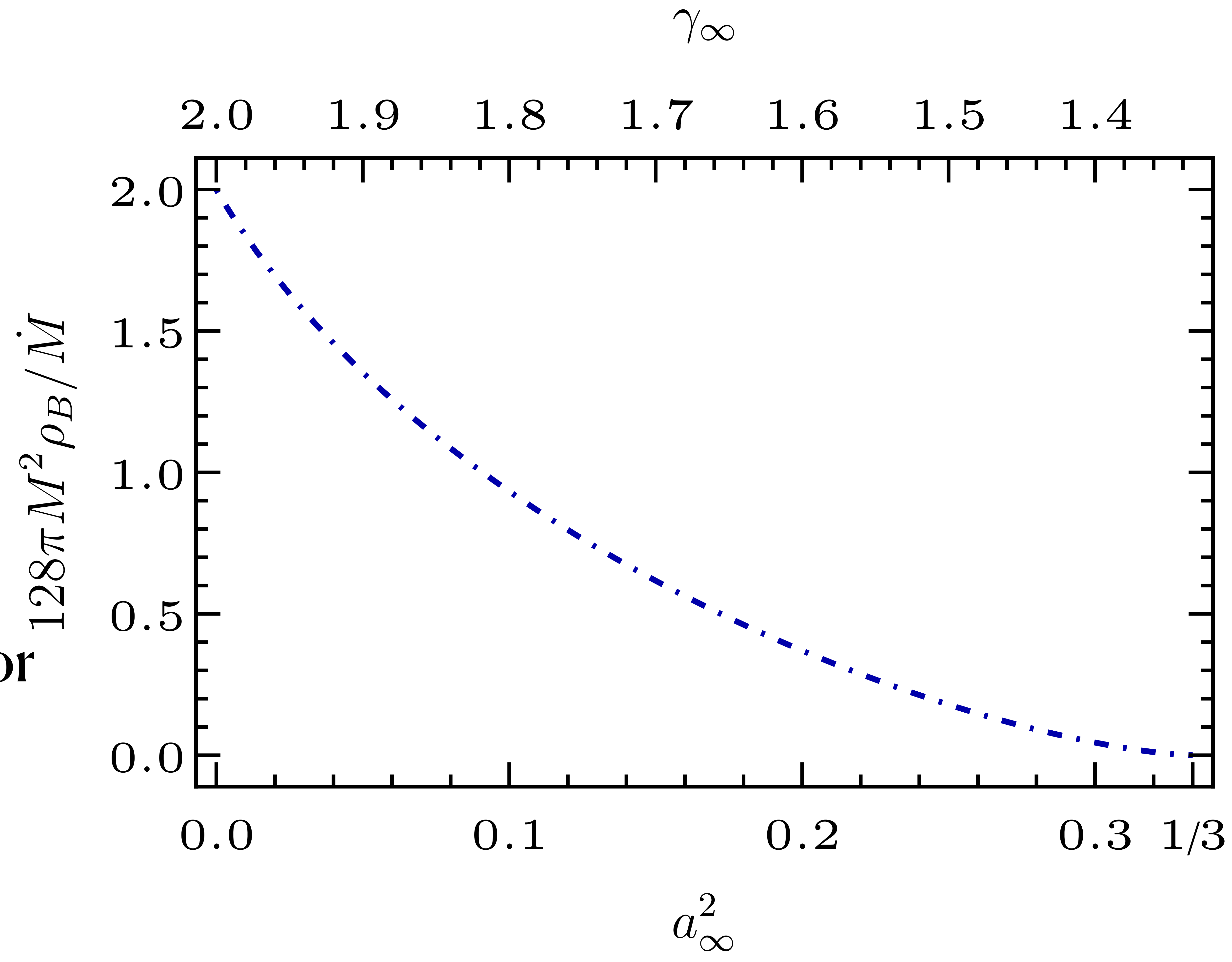
# BEC Dark Matter Spikes Around Supermassive Black Holes

## Accretion Rate

$$\dot{M} = 8\pi \left( \frac{1+3a_s^2}{1-3a_s^2} \right)^{3/2} \sqrt{\frac{1-a_s^2}{a_s^2}} M^2 \rho_B$$

- $64\pi M^2 \rho_B \leq \dot{M} < \infty$ and $\dot{M} \to \infty$ as $a_s^2 \to 1/3$

$\dot{M}_{\rm min} = 64\pi M^2 \rho_B \simeq 1.41 \times 10^{-9}\ \mathrm{M_\odot\ yr^{-1}}$ for $M = 10^6\ \mathrm{M_\odot}$ black holes at galactic centers, which is **negligible** compared to the standard Eddington accretion of baryons.

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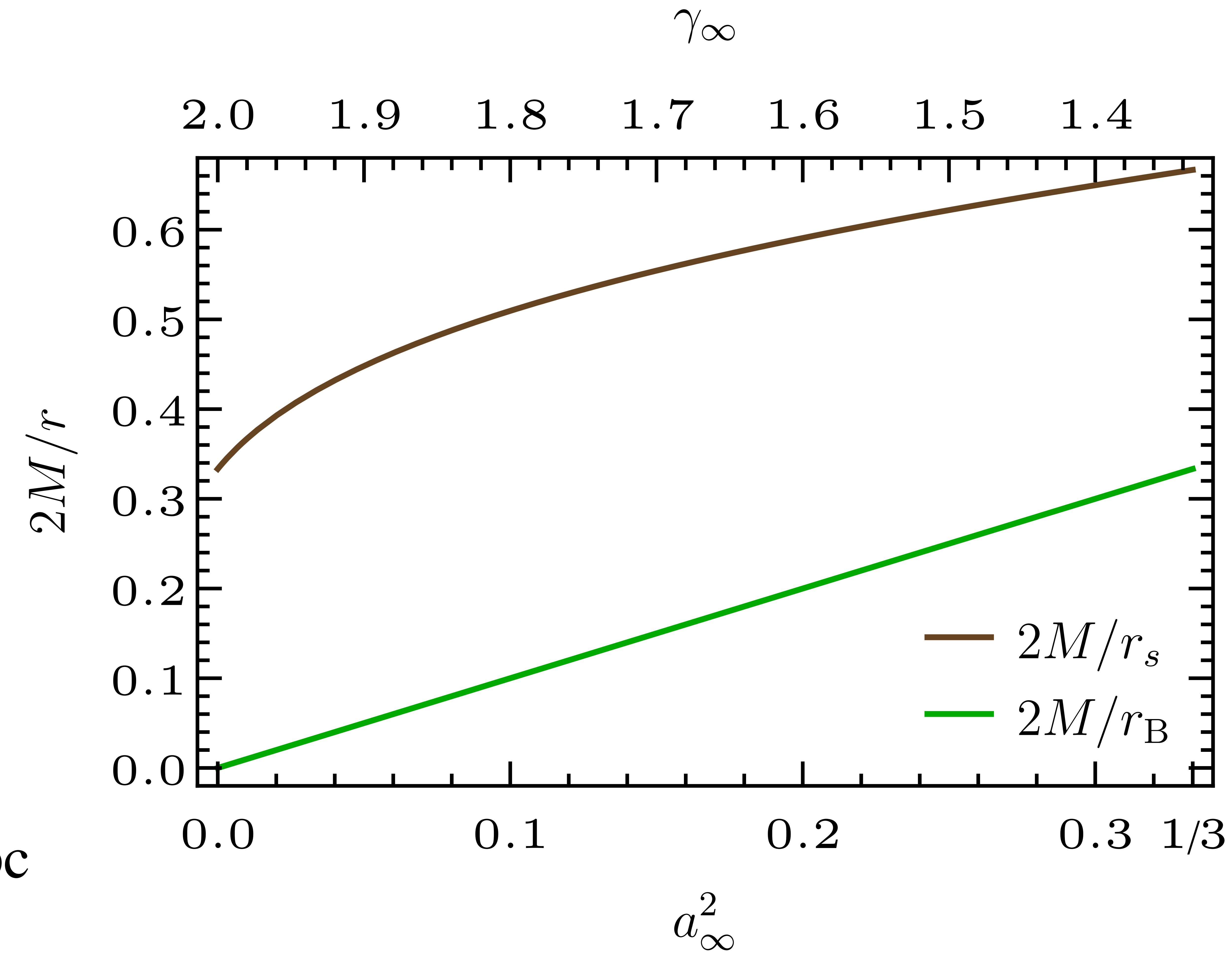
# BEC Dark Matter Spikes Around Supermassive Black Holes

## Characteristic Distances

- Sonic horizon: $r_s$
- Bondi radius: $r_{\rm B}$
- $3M \leq r_s \leq 6M \leq r_{\rm B} < \infty$
- $3M \rightarrow$ Photon sphere
- $6M \rightarrow$ Inner most stable circular orbit

On galactic scales, $r_{\rm B} = 2M/a_\infty^2 \simeq \left(\dfrac{M}{10^6\ {\rm M}_\odot}\right)$ pc

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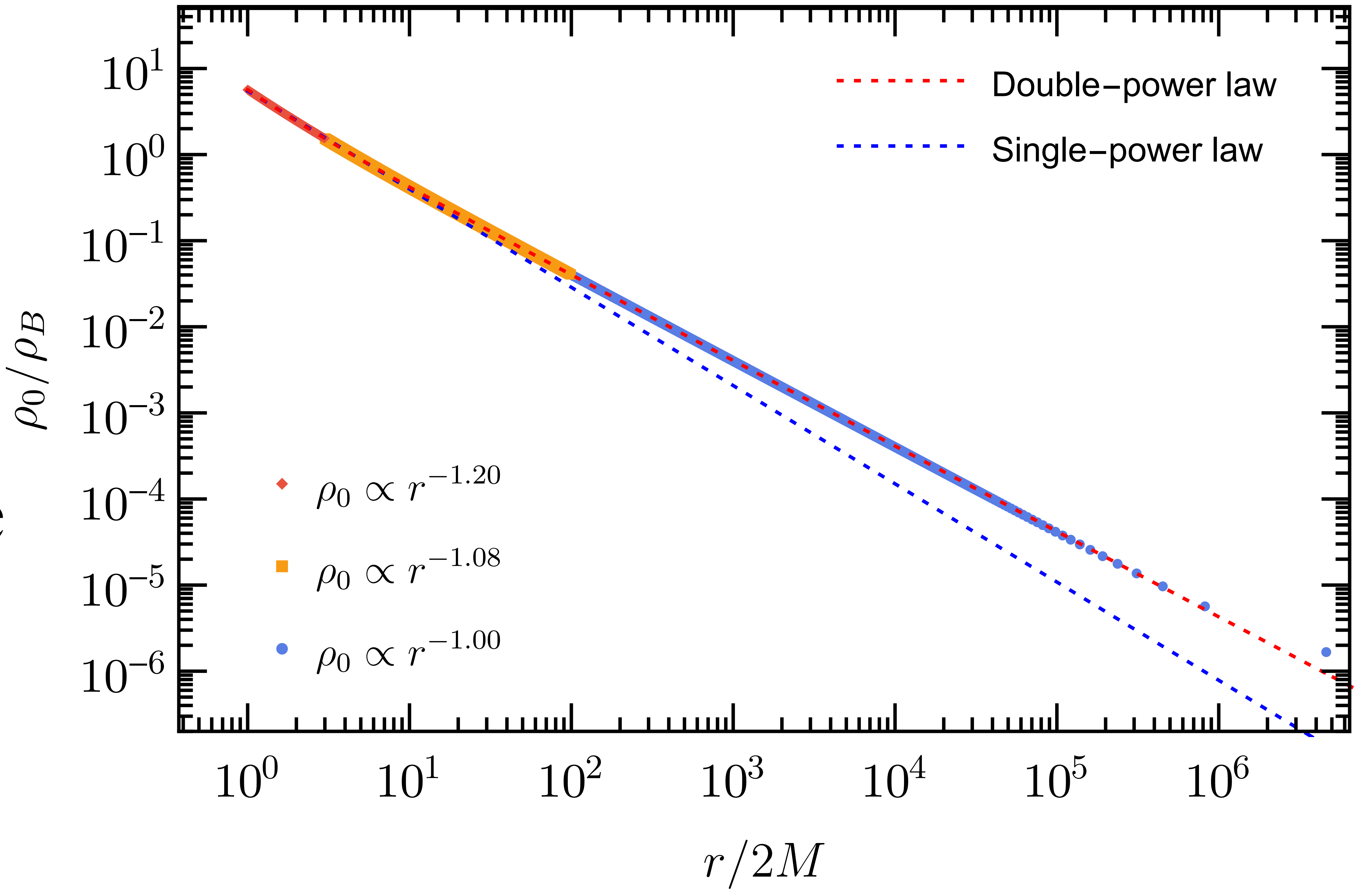
# BEC Dark Matter Spikes Around Supermassive Black Holes

## Density Spike Profile

- The profile **cannot** be fitted by single-power law within Bondi radius, while double-power law gives a **better** fit.

- The **repulsive** self-interactions of **scalar dark matter (BEC)** reshape the spikes to be $\propto r^{-1.2}$ near the event horizon; while it approaches $\propto r^{-1}$ *asymptotically*, which matches the **cusp** in **NFW profile**.

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# Summary & Conclusion

- We solve the **relativistic Bondi accretion** for BEC dark matter around a black hole.
- The **repulsive** self-interactions of **BEC dark matter** reshape the spikes to be $\propto r^{-1.2}$, which can be verified observationally.

General Relativity (Gravity)

Black hole Accretion

Bondi Accretion (Astrophysics)

Hydrodynamic Limit

Canonical Scalar with $\lambda\phi^4$ (Particle Physics)

Thanks for your attention!!