Comparing Characteristic Gravitational Lensing Features in Black Hole Spacetimes of the Plebanski-Demianski Class

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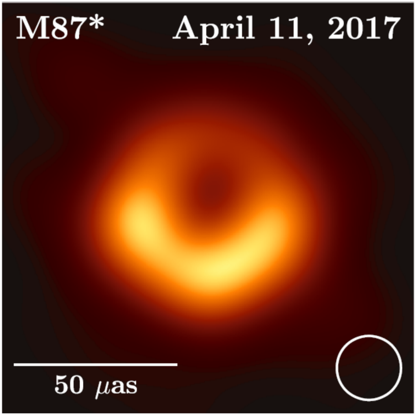
Outline of the Talk

Event Horizon Telescope Images

M87



Milky Way

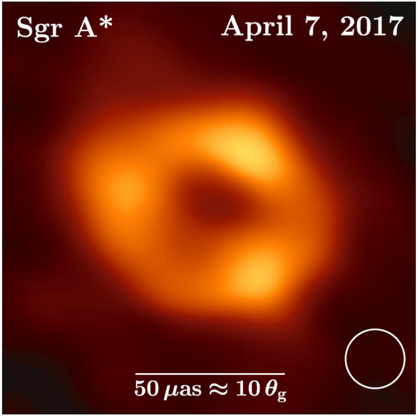


Figure: Event Horizon Telescope images of the shadows of the supermassive black holes in the centres of the galaxy M87 (left) and the Milky Way (right). Taken from Fig. 3 in EHT *et al.* (2019) and Fig. 3 in EHT *et al.* (2022).

Why Using Analytical Methods?

- one large class of black hole spacetimes in general relativity: Plebanski-Demianski metric (Plebanski and Demianski, 1976)
    - exact solution to Einstein's electrovacuum field equation with cosmological constant
    - includes Schwarzschild and Kerr metrics
    - equations of motion for light rays are separable and exactly analytically solvable
- benefits of using analytical methods:
    - arbitrarily precise
    - can reduce calculation time
    - allow very high resolution calculations
- in this talk: application to calculation of lensing features due to
    - acceleration $\rightarrow$ C-metric
    - gravitomagnetic charge $\rightarrow$ NUT metric
    - spin $\rightarrow$ Kerr metric (preliminary results)

Equations of Motion

- the equations of motion of the Plebanski-Demianski metric read, see e.g., Grenzebach *et al.* (2015)

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = (r^2+a^2+n^2-2anC)\frac{(r^2+a^2+n^2-2anC)E-aL_z}{Q(r)} + \sigma(\vartheta)\frac{L_z-\sigma(\vartheta)E}{\sin^2\vartheta P(\vartheta)},$$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 = ((r^2+a^2+n^2-2anC)E-aL_z)^2 - Q(r)K,$$

$$\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}\lambda}\right)^2 = P(\vartheta)K - \frac{(\sigma(\vartheta)E-L_z)^2}{\sin^2\vartheta},$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\lambda} = a\frac{(r^2+a^2+n^2-2anC)E-aL_z}{Q(r)} + \frac{L_z-\sigma(\vartheta)E}{\sin^2\vartheta P(\vartheta)}.$$

- where the Mino parameter λ (Mino, 2003) is related to the affine parameter s by

$$\frac{\mathrm{d}\lambda}{\mathrm{d}s} = \frac{\Omega(r,\vartheta)^2}{\rho(r,\vartheta)}$$

- all results shown in the following were obtained using solutions in terms of elementary and Jacobi's elliptic functions and Legendre's elliptic integrals

Observer Geometry

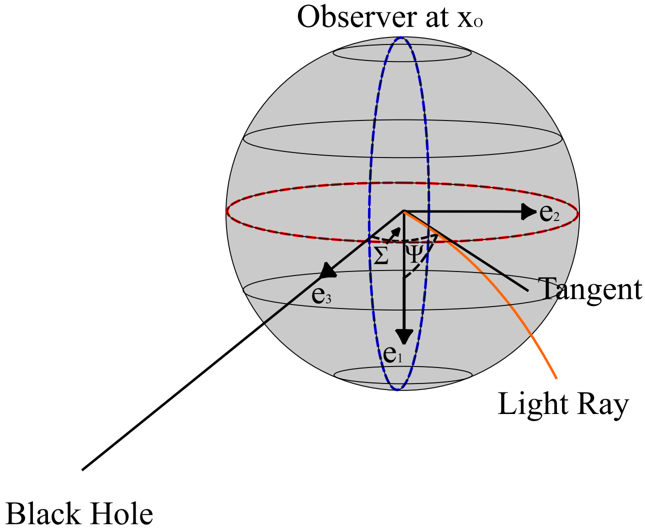


Figure: Illustration of the lens-observer geometry and the tetrad vectors e_1, e_2, and e_3.

Outline of Calculation Procedure

- in this talk we focus on:
  - lens equation: defines map from the celestial sphere of the observer at (x_O^μ) to a two sphere of light sources S_L^2 with radius coordinate r_L [see, e.g., Frittelli and Newman [1999] and Perlick (2004)]

$$(\Sigma, \Psi) \rightarrow (\vartheta_L(\Sigma, \Psi), \varphi_L(\Sigma, \Psi))$$

  - travel time $T(\Sigma, \Psi) = t_L(\Sigma, \Psi) - t_O$
  - redshift $z(\Sigma, \Psi)$
- calculation procedure using analytic solutions:
  - step 1: calculate Mino parameter $\lambda_L < \lambda_O = 0$ from fixed $r_O < r_L$
  - step 2: determine number of turning points of the ϑ motion
  - step 3: calculate $\vartheta_L(\Sigma, \Psi)$ and $\varphi_L(\Sigma, \Psi)$ (for $\varphi_O = 0$)
  - step 4: calculate travel time $T(\Sigma, \Psi)$ (for $t_O = 0$)
  - step 5: calculate redshift from r_O, ϑ_O, r_L, and $\vartheta_L(\Sigma, \Psi)$

Lens Map: Sphere of Light Sources

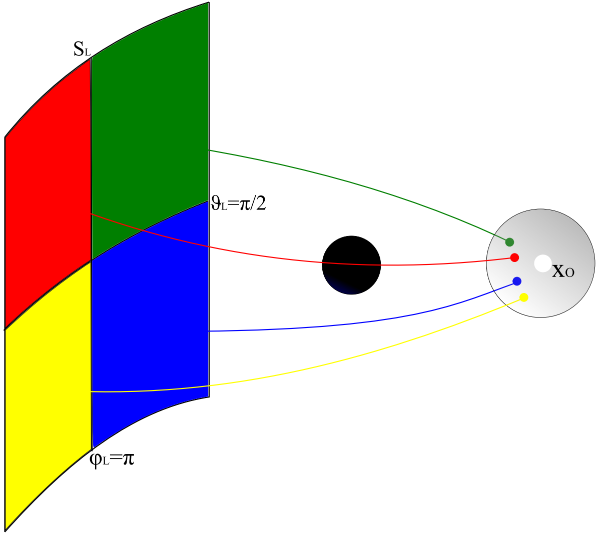


Figure: Illustration of the lens map following the colour convention in Bohn *et al.* (2015).

Lens Map: Accelerating Black Hole

Schwarzschild Metric | C-Metric $\alpha = 1/(30m)$

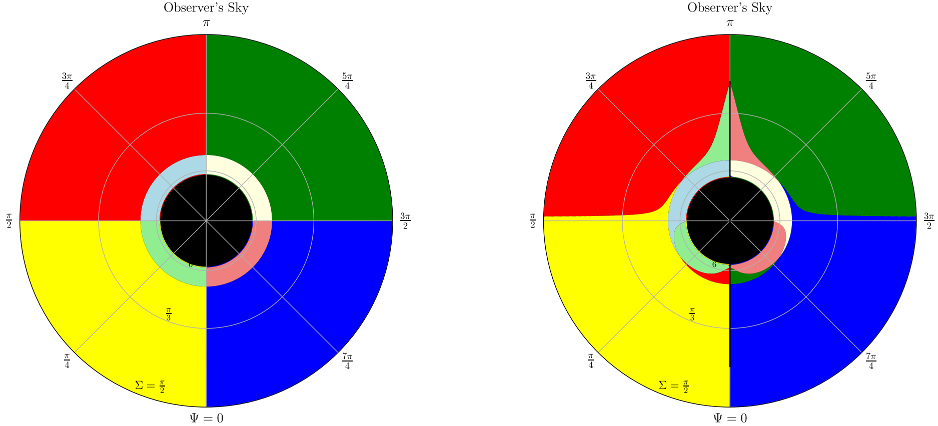
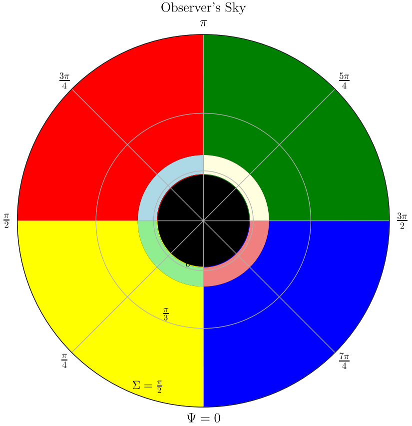


Figure: Lens maps for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Lens Map: Black Hole with Gravitomagnetic Charge

Schwarzschild Metric



NUT Metric $n = m/2$ and $C = 1$

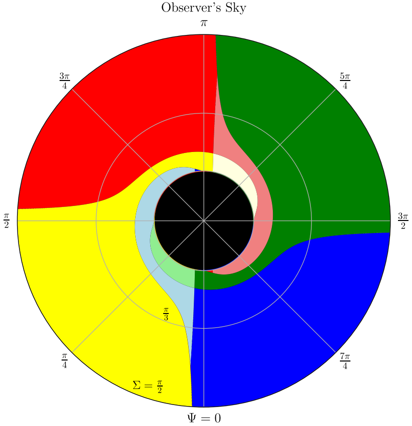
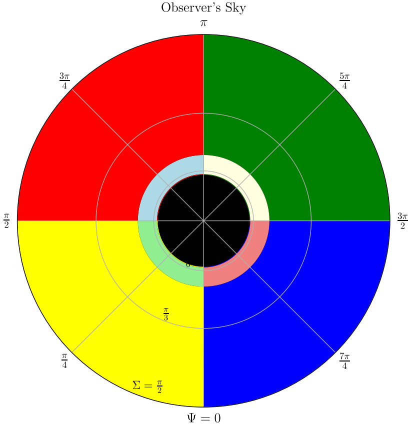


Figure: Lens maps for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Lens Map: Black Hole with Spin

Schwarzschild Metric



Kerr Metric $a = 95m/100$

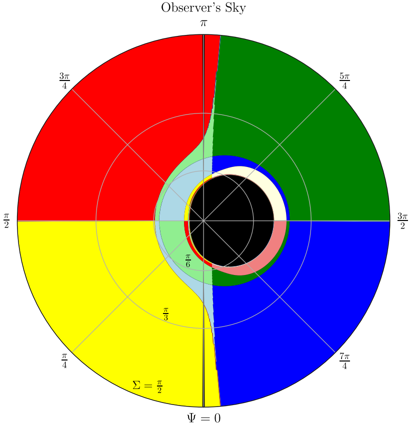


Figure: Lens maps for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Redshift: Accelerating Black Hole $\alpha = 1/(30m)$

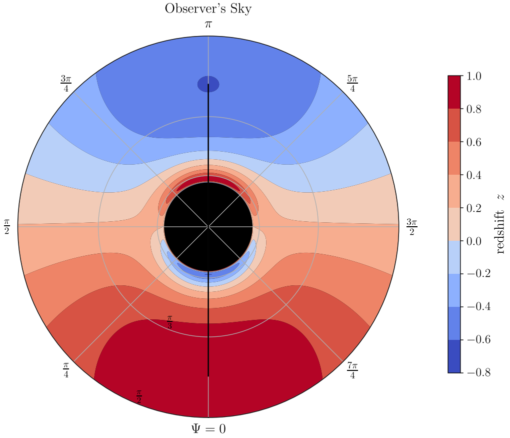


Figure: Redshift map for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

- Schwarzschild metric: $z = -0.057$, NUT metric: $z = -0.059$

Redshift: Black Hole with Spin $a = 95m/100$ - Light Sources on t Lines

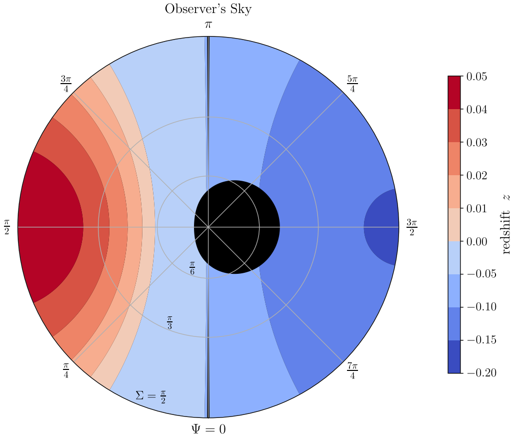


Figure: Redshift map for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Redshift: Black Hole with Spin $a = 95m/100$ - Dragged Light Sources

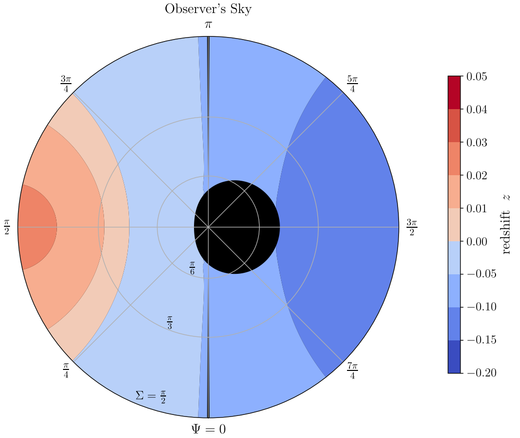
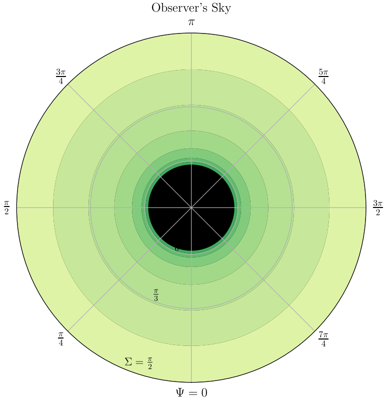


Figure: Redshift map for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Travel Time: Accelerating Black Hole

Schwarzschild Metric



C-Metric $\alpha = 1/(30m)$

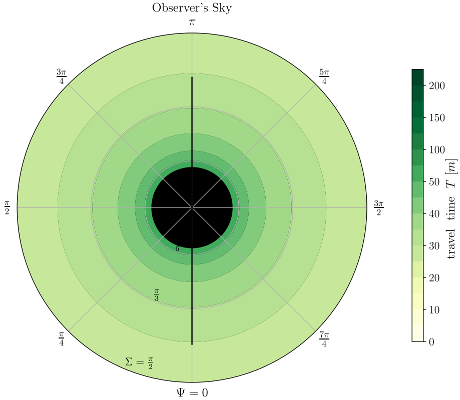


Figure: Travel time maps for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Travel Time: Black Hole with Gravitomagnetic Charge

Schwarzschild Metric | NUT Metric $n = m/2$ and $C = 1$

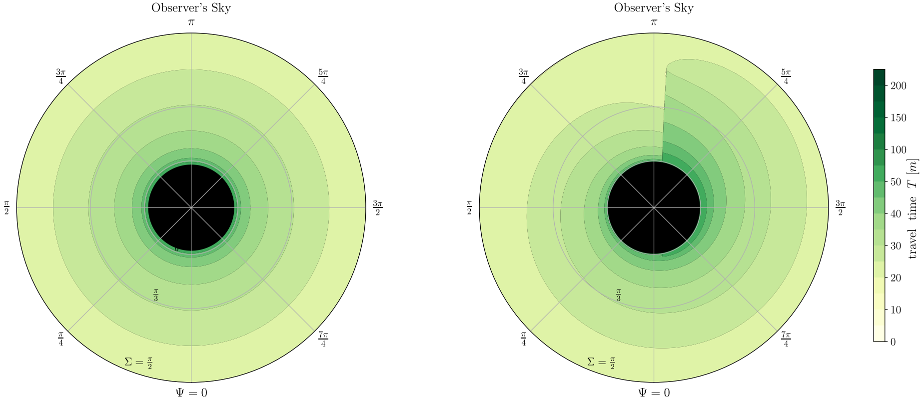


Figure: Travel time maps for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Travel Time: Black Hole with Spin

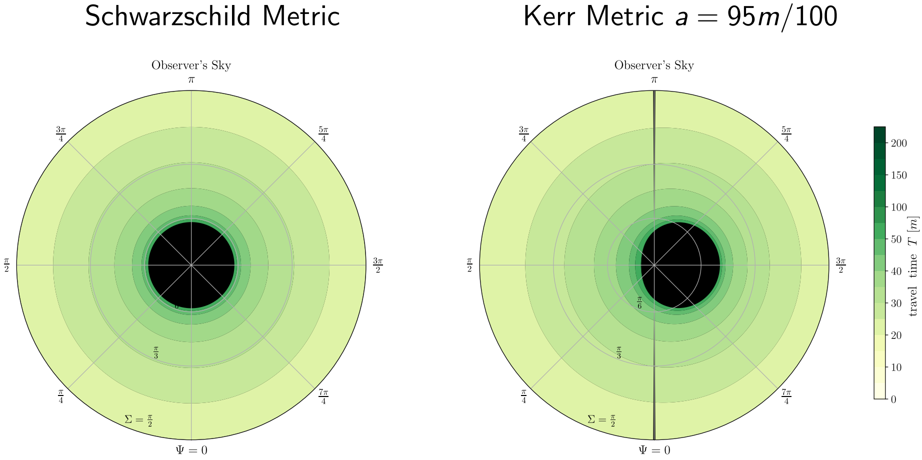


Figure: Travel time maps for $r_O = 10m$, $\vartheta_O = \pi/2$, and $r_L = 20m$.

Summary & Outlook on Applications

- acceleration parameter: circular shadow, lens map has asymmetry with respect to equatorial plane, redshift is function on observer's celestial sphere
- gravitomagnetic charge: circular shadow, twist in lens map, discontinuity and spiral-shaped pattern in travel time maps
- spin: noncircular shadow, images up to fourth order visible, redshift is function on celestial sphere, asymmetry in travel time maps
- potential applications in astrophysics:
  - tracing emission from accretion disks around black holes
  - tracing electromagnetic radiation emitted by pulsars orbiting supermassive black holes

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