Sphaleron in the Higgs Triplet Model

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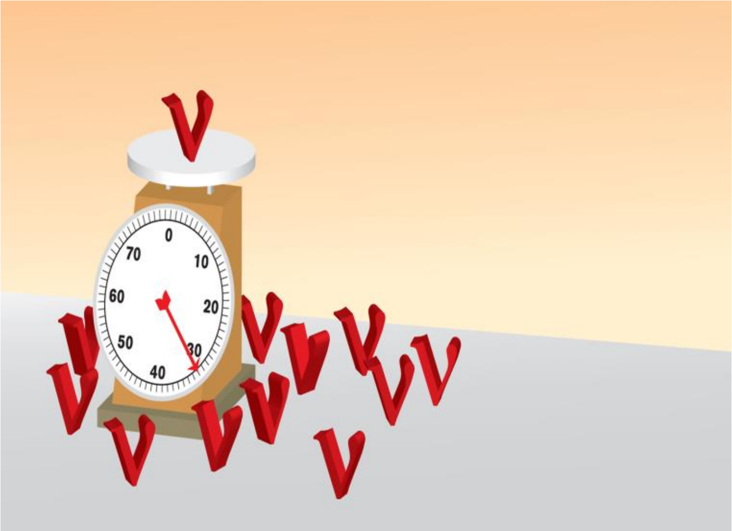
Outline

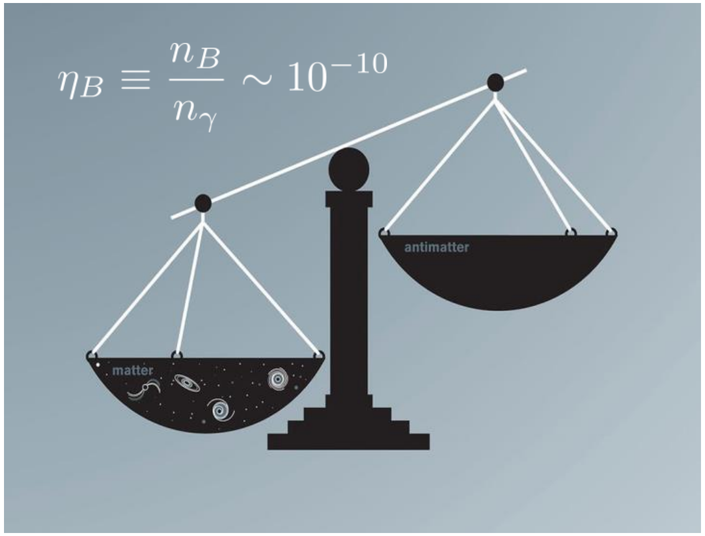
- Motivation
- Theoretical Setup and Sphaleron Ansatz
- Sphaleron Solution
- Summary and Outlook

Motivation

Two important **unsolved** problems in the Standard Model



origin of neutrino masses



matter-antimatter asymmetry

Can we accommodate **both** in a unified framework?

A successful attempt: **thermal leptogenesis**, SM + 3 singlet fermions

(naturally, new physics @ GUT scale) Fukugita, Yanagida, 1986

Motivation

Higgs triplet model (HTM)

Alternatives: extend the SM by one complex triplet scalar $\Delta \sim (1, 3, -1)$

$$\mathcal{L} \supset -\frac{1}{2}\overline{\ell_L} Y_\Delta \Delta i\sigma^2 \ell_L^c + \text{h.c.} \quad \Rightarrow \quad M_\nu = Y_\Delta v_\Delta$$

type-II seesaw mechanism

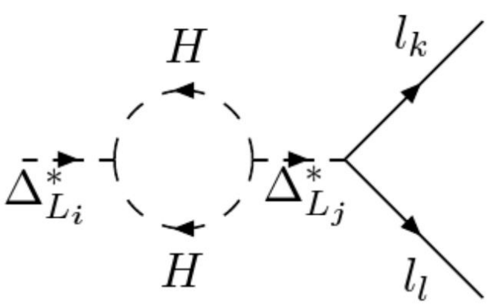
(naturally, new physics @ TeV scale)

Konetschny *et al.*, 1977
Magg *et al.*, 1980
Schechter *et al.*, 1980
Cheng *et al.*, 1980
Lazarides *et al.*, 1981
Mohapatra et al., 1981

Can the HTM explain the baryon asymmetry of the Universe?

triplet leptogenesis

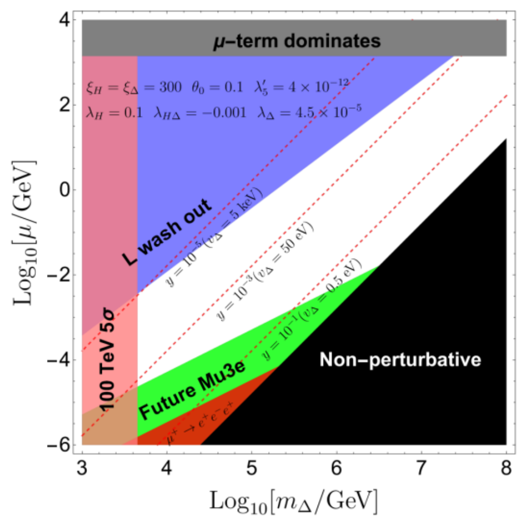
Ma *et al.*, 1998
Hambye *et al.*, 2003



At least two triplet scalars are needed!

Affleck-Dine mechanism

Barrie, Han, Murayama, 2021



electroweak baryogenesis

- The pattern of EW phase transition is modified by the triplet
- New sources of CP violation come from the leptonic sector

Can a successful EW baryogenesis be fulfilled in the HTM?

Prerequisite: working out the **sphaleron configuration** in the presence of the triplet scalar

Sphaleron in the Standard Model

- Vacuum structure of non-Abelian gauge theories is characterized by **integer numbers** (Chern-Simons numbers $N_{\rm CS}$) G. 't Hooft, 1976, Callan *et al.*, 1976 Jackiw *et al.*, 1976

$$N_{\rm CS} = -\frac{g^2}{16\pi^2}\int d^3x\ 2\epsilon^{ijk}\ {\rm Tr}\left[(\partial_i W_j)W_k - \frac{2}{3}igW_iW_jW_k\right]$$

- Chiral anomaly: $\Delta B = \Delta L = N_f \times \Delta N_{\rm CS}$

- Instanton rate: $\exp\left(-16\pi^2/g^2\right) \sim 10^{-160}$

- Sphaleron configuration: **saddle-point** solution of the energy functional

- Sphaleron rate (B-violating rate):

$$\Gamma_{\rm sph} \sim \exp\left(-E_{\rm sph}/T\right)\ , \quad T < T_{\rm EW}$$
$$\Gamma_{\rm sph} \sim \alpha_{\rm W}^5 T^4\ , \quad T > T_{\rm EW}$$

$$\alpha_{\rm W} \equiv g^2/(4\pi) \qquad E_{\rm sph} \sim 4\pi v/g \sim 5\ {\rm TeV}$$

Manton, 1983; Klinkhammer and Manton, 1984

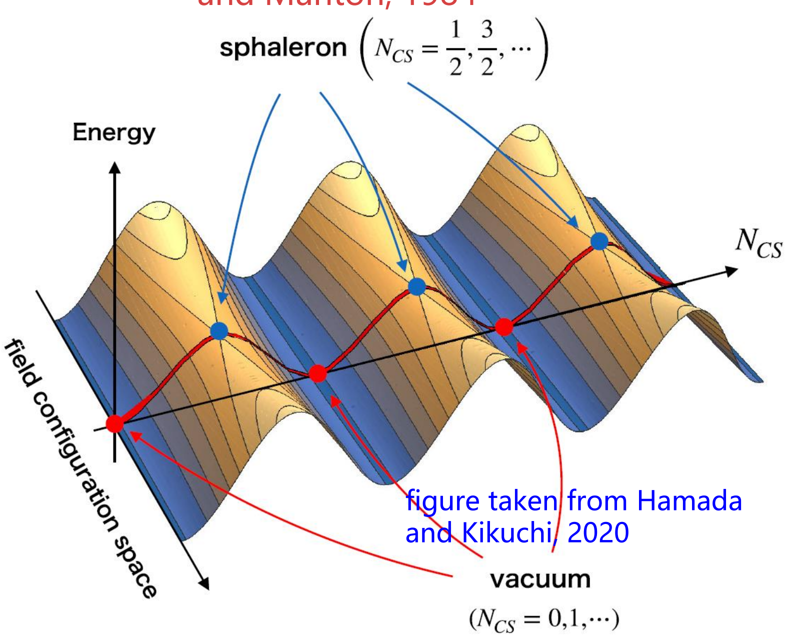


figure taken from Hamada and Kikuchi, 2020

Sphaleron in the Higgs Triplet Model

Assumptions:

- Neglect the contributions from fermion fields
- Neglect the effects from the finite Weinberg angle

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} \Delta^- & -\sqrt{2}\Delta^0 \\ \sqrt{2}\Delta^{--} & -\Delta^- \end{pmatrix}$$

Lagrangian in the HTM:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) + \frac{1}{2}\mathrm{Tr}\left[\left(D^\mu\Delta\right)^\dagger\left(D_\mu\Delta\right)\right] - V\left(\phi,\Delta\right)$$

$$\langle\phi\rangle = v_\phi/\sqrt{2}\,, \qquad \langle\Delta\rangle = v_\Delta\,, \qquad \sqrt{v_\phi^2 + 2v_\Delta^2} = v \approx 246\ \mathrm{GeV}$$

Energy functional:

$$\mathcal{H}\left[W_\mu,\phi,\Delta\right] = \frac{1}{2}g^{ik}g^{jl}\mathrm{Tr}\left(F_{ij}F_{kl}\right) + g^{ij}\left(D_i\phi\right)^\dagger\left(D_j\phi\right) + \frac{1}{2}g^{ij}\left[\left(D_i\Delta\right)^\dagger\left(D_j\Delta\right)\right] + V\left(\phi,\Delta\right)$$

$$E\left[W_\mu,\phi,\Delta\right] = \int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\theta\sin\theta\int_0^{\infty}\mathrm{d}r\,r^2\,\mathcal{H}\left[W_\mu,\phi,\Delta\right]$$

Sphaleron in the Higgs Triplet Model

$\mu \in [0, \pi]$

Non-contractible loops : $U(\mu, \theta, \varphi) = \begin{pmatrix} e^{\mathrm{i}\mu}(\cos\mu - \mathrm{i}\sin\mu\cos\theta) & e^{\mathrm{i}\varphi}\sin\mu\sin\theta \\ -e^{-\mathrm{i}\varphi}\sin\mu\sin\theta & e^{-\mathrm{i}\mu}(\cos\mu + \mathrm{i}\sin\mu\cos\theta) \end{pmatrix}$

$$U(\mu, \theta = 0, \varphi) = U(\mu = 0, \theta, \varphi) = U(\mu = \pi, \theta, \varphi) = \mathbf{1}$$

$U(\mu, \theta, \varphi)$ defines a map: $S^3 \to S^3 \qquad \pi_3(S^3) = \mathbb{Z}$

Minmax procedure: $\mu = 0$ or $\mu = \pi$: vacuum configuration (minimal energy)
$\mu = \pi/2$: sphaleron configuration (maximal energy)

Sphaleron ansatz:

$$W_j(\mu, r, \theta, \varphi) = -\frac{\mathrm{i}}{g} f(r) \underline{\partial_j U(\mu, \theta, \varphi) U^{-1}(\mu, \theta, \varphi)} \ , \quad j = \theta, \varphi \ ,$$

pure gauge

$$\phi(\mu, r, \theta, \varphi) = \frac{v_\phi}{\sqrt{2}} h(r) \underline{U(\mu, \theta, \varphi) \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \ ,$$

fundamental rep.

$$\Delta(\mu, r, \theta, \varphi) = v_\Delta h_\Delta(r) \underline{U(\mu, \theta, \varphi) \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} U^{-1}(\mu, \theta, \varphi)} \ ,$$

adjoint rep.

boundary conditions:

$$f(0) = h(0) = h_\Delta(0) = 0$$
$$f(\infty) = h(\infty) = h_\Delta(\infty) = 1$$

Sphaleron in the Higgs Triplet Model

Kinematic terms:

$$\frac{1}{2}g^{jk}g^{jl}\mathrm{Tr}\left(F_{ij}F_{kl}\right) = \frac{4}{g^2r^4}\sin^2\mu\left[2f^2\left(1-f\right)^2\sin^2\mu+r^2f'^2\right]$$

spherically symmetric

$$g^{ij}\left(D_i\phi\right)^\dagger\left(D_j\phi\right) = \frac{v_\phi^2}{2r^2}\left[2\left(1-f\right)^2h^2\sin^2\mu+r^2h'^2\right]$$

spherically symmetric

$$\frac{1}{2}g^{ij}\left[\left(D_i\Delta\right)^\dagger\left(D_j\Delta\right)\right] = \frac{v_\Delta^2}{2r^2}\left[\left(5-\cos 2\theta\right)\left(1-f\right)^2h_\Delta^2\sin^2\mu+r^2h_\Delta'^2\right]$$

non-spherically symmetric

Scalar potential:

$$V\left(\phi,\Delta\right) =\lambda\left(\phi^\dagger\phi\right)^2-\kappa^2\phi^\dagger\phi+\frac{1}{2}M_\Delta^2\mathrm{Tr}\left(\Delta^\dagger\Delta\right)-\left(\lambda_\Delta M_\Delta\phi^{\mathrm{T}}\epsilon\Delta\phi+\mathrm{h.c.}\right)$$
$$+\frac{\lambda_1}{4}\left[\mathrm{Tr}\left(\Delta^\dagger\Delta\right)\right]^2+\frac{\lambda_2}{4}\mathrm{Tr}\left[\left(\Delta^\dagger\Delta\right)^2\right]+\lambda_3\left(\phi^\dagger\phi\right)\mathrm{Tr}\left(\Delta^\dagger\Delta\right)+\lambda_4\phi^\dagger\Delta\Delta^\dagger\phi$$

8 real parameters

but not all of them are relevant to the sphaleron configuration

VEVs determined from:

$$\frac{\partial}{\partial v_\phi}V\left(v_\phi,v_\Delta\right) = \left(-\kappa^2+\lambda v_\phi^2-2\lambda_\Delta M_\Delta v_\Delta+\lambda_3 v_\Delta^2\right)v_\phi = 0$$

$$\frac{\partial}{\partial v_\Delta}V\left(v_\phi,v_\Delta\right) = -\lambda_\Delta M_\Delta v_\phi^2+M_\Delta^2 v_\Delta+(\lambda_1+\lambda_2)v_\Delta^3+\lambda_3 v_\phi^2 v_\Delta = 0$$

Sphaleron Equations of Motion

Sphaleron energy: $\xi \equiv gvr \approx 8.1 \times \left(\frac{r}{10^{-15}\ \mathrm{cm}}\right)$ $\beta \equiv v^2/v_\phi^2 = 1 + 2\varrho_3$

$$
\begin{aligned}
E_{\mathrm{sph}} = \frac{4\pi v}{g}\int_0^\infty \mathrm{d}\xi \Bigg\{ & 4f'^2 + \frac{8}{\xi^2} f^2 (1-f)^2 + \frac{1}{\beta}(1-f)^2 h^2 + \frac{1}{2\beta}\xi^2 h'^2 \\
& + \frac{\xi^2}{4\beta^2}\left[(\varrho_1 - \varrho_2)\left(1-h^2\right)^2 + \varrho_2\left(h^2 - h_\Delta\right)^2\right] + \frac{\varrho_3}{6\beta}\left[3\xi^2 h_\Delta'^2 + 16 h_\Delta^2 (1-f)^2\right] \\
& + \frac{\xi^2}{4\beta^2}\left[2\left(\varrho_4 - \varrho_1 + \varrho_2\right)\left(1-h^2\right) - \left(2\varrho_3\varrho_5 - \varrho_2\right)\left(1-h_\Delta^2\right)\right] \\
& + \frac{\xi^2}{2\beta^2}\left(\sqrt{2\varrho_2\varrho_3\varrho_5} - \varrho_2\right)\left(1 - h^2 h_\Delta\right) + \frac{\xi^2}{2\beta^2}\left(\varrho_1 - \varrho_4 - \sqrt{2\varrho_2\varrho_3\varrho_5}\right)\left(1 - h^2 h_\Delta^2\right) \\
& - \frac{\xi^2}{4\beta^2}\left(\varrho_1 - \varrho_4 - \varrho_3\varrho_5 - \sqrt{\varrho_2\varrho_3\varrho_5/2}\right)\left(1 - h_\Delta^4\right)\Bigg\}\,.
\end{aligned}
$$

$$
\begin{aligned}
\varrho_1 &\equiv \lambda/g^2 \\
\varrho_2 &\equiv 2\lambda_\Delta^2/g^2 \\
\varrho_3 &\equiv v_\Delta^2/v_\phi^2 \\
\varrho_4 &\equiv \kappa^2/\left(g^2 v_\phi^2\right) \\
\varrho_5 &\equiv M_\Delta^2/\left(g^2 v_\phi^2\right)
\end{aligned}
$$

sphaleron configuration in the HTM is determined by 5 independent parameters

Equations of motion:

$$
\begin{aligned}
\xi^2 f'' &= 2f(1-f)(1-2f) - \frac{\xi^2}{4\beta}(1-f)h^2 - \frac{2\varrho_3}{3\beta}\xi^2(1-f)h_\Delta^2 \\
\left(\xi^2 h'\right)' &= 2(1-f)^2 h - \frac{\xi^2}{\beta}\Big[(\varrho_1 - \varrho_2)h\left(1-h^2\right) - \varrho_2 h\left(h^2 - h_\Delta\right) \\
& \quad + \left(\varrho_4 - \varrho_1 + \varrho_2\right)h + \left(\sqrt{2\varrho_2\varrho_3\varrho_5} - \varrho_2\right)h h_\Delta + \left(\varrho_1 - \varrho_4 - \sqrt{2\varrho_2\varrho_3\varrho_5}\right)h h_\Delta^2\Big] \\
\varrho_3\left(\xi^2 h_\Delta'\right)' &= \frac{16}{3}\varrho_3(1-f)^2 h_\Delta - \frac{\varrho_2\xi^2}{2\beta}\left(h^2 - h_\Delta\right) + \frac{\xi^2}{2\beta}\Big[\left(2\varrho_3\varrho_5 - \varrho_2\right)h_\Delta - \left(\sqrt{2\varrho_2\varrho_3\varrho_5} - \varrho_2\right)h^2 \\
& \quad - 2\left(\varrho_1 - \varrho_4 - \sqrt{2\varrho_2\varrho_3\varrho_5}\right)h^2 h_\Delta + 2\left(\varrho_1 - \varrho_4 - \varrho_3\varrho_5 - \sqrt{\varrho_2\varrho_3\varrho_5/2}\right)h_\Delta^3\Big]
\end{aligned}
$$

boundary conditions:

$f(0) = h(0) = h_\Delta(0) = 0$

$f(\infty) = h(\infty) = h_\Delta(\infty) = 1$

Constraints on Parameters

$$\varrho_1 \equiv \lambda/g^2$$
$$\varrho_2 \equiv 2\lambda_\Delta^2/g^2$$
$$\varrho_3 \equiv v_\Delta^2/v_\phi^2$$
$$\varrho_4 \equiv \kappa^2/\left(g^2 v_\phi^2\right)$$
$$\varrho_5 \equiv M_\Delta^2/\left(g^2 v_\phi^2\right)$$

- **EW precision measurements on triplet VEV:**

$$\varrho_3 = v_\Delta^2/v_\phi^2 \lesssim 10^{-3} \qquad \varrho_4 \approx \varrho_1 - \frac{1}{2}\varrho_3\varrho_5\left(2 + \sqrt{\frac{2\varrho_2}{\varrho_3\varrho_5}}\right)$$

- **Bounded-from-below conditions and unitarity**

$$\frac{\lambda_3}{g^2} \approx \sqrt{\frac{\varrho_2\varrho_5}{2\varrho_3}} - \varrho_5$$

$$0 < \varrho_1 \leqslant \frac{4\pi}{g^2}\,, \quad -\sqrt{\frac{4\pi}{g^2}\varrho_1} < \sqrt{\frac{\varrho_2\varrho_5}{2\varrho_3}} - \varrho_5 \leqslant \frac{4\pi}{g^2}\,, \quad \varrho_1 - \varrho_3\varrho_5 - \sqrt{\varrho_2\varrho_3\varrho_5/2} > 0$$

$$-\sqrt{\frac{4\pi}{g^2}\varrho_1} < \frac{\lambda_3+\lambda_4}{g^2} \leqslant \frac{4\pi}{g^2}\,, \quad |2\lambda_3 + 3\lambda_4| \leqslant 8\pi\,, \quad |2\lambda_3 - \lambda_4| \leqslant 8\pi$$

- **Higgs mass** $m_h \approx 125 \text{ GeV}$

$$\frac{\lambda_4}{g^2} \approx \varrho_5 \pm \frac{1}{(2\varrho_3)^{3/4}}\sqrt{\left(\varrho_1 - \frac{m_h^2}{2g^2v_\phi^2}\right)\left(\sqrt{\varrho_2\varrho_5} - \frac{\sqrt{2\varrho_3}m_h^2}{g^2v_\phi^2}\right)}$$

- **Collider constraints** $m_{H^{\pm\pm}} \gtrsim 350 \text{ GeV}$

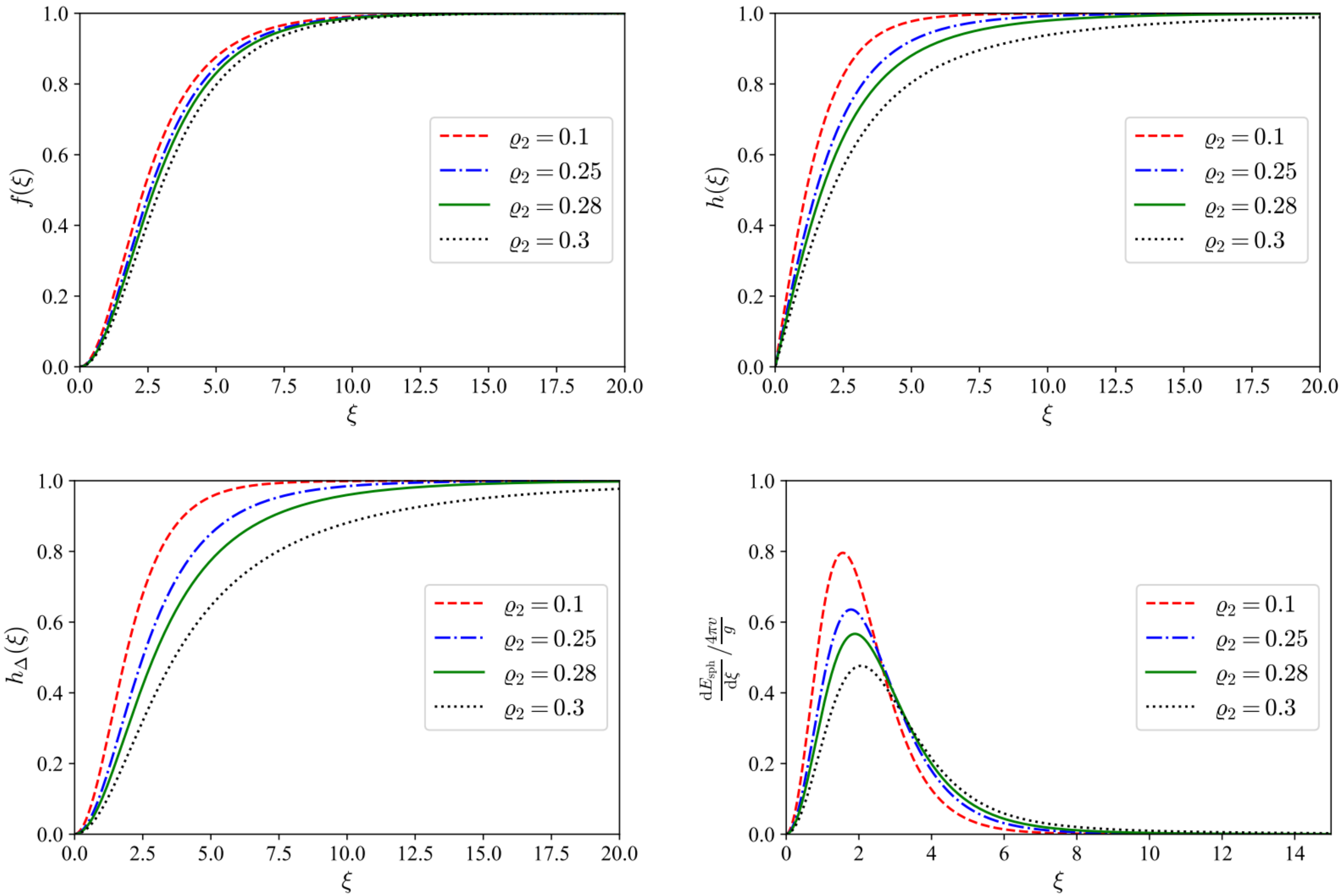
$$m_{H^{\pm\pm}}^2 \approx g^2 v_\phi^2\left(\sqrt{\frac{\varrho_2\varrho_5}{2\varrho_3}} - \frac{\lambda_4}{g^2}\right) \quad \Rightarrow \quad \sqrt{\frac{\varrho_2\varrho_5}{2\varrho_3}} - \frac{\lambda_4}{g^2} \gtrsim 4.8$$

- **Charged lepton flavor violation**

$$M_\Delta v_\Delta \gtrsim 10^2 \text{ GeV}\cdot\text{eV} \quad \Rightarrow \quad \varrho_3\varrho_5 \gtrsim 10^{-24}$$

Sphaleron Field Configuration

$$V(\phi,\Delta)=\lambda\left(\phi^{\dagger}\phi\right)^{2}-\kappa^{2}\phi^{\dagger}\phi+\frac{1}{2}M_{\Delta}^{2}\mathrm{Tr}\left(\Delta^{\dagger}\Delta\right)-\left(\lambda_{\Delta}M_{\Delta}\phi^{\mathrm{T}}\epsilon\Delta\phi+\mathrm{h.c.}\right)$$



sphaleron configuration scale:

restricted within a narrow region

$$\xi\lesssim 10 \iff r\lesssim 10^{-15}\ \mathrm{cm}$$

$$\varrho_2\equiv 2\lambda_{\Delta}^{2}/g^{2}$$

larger trilinear coupling:

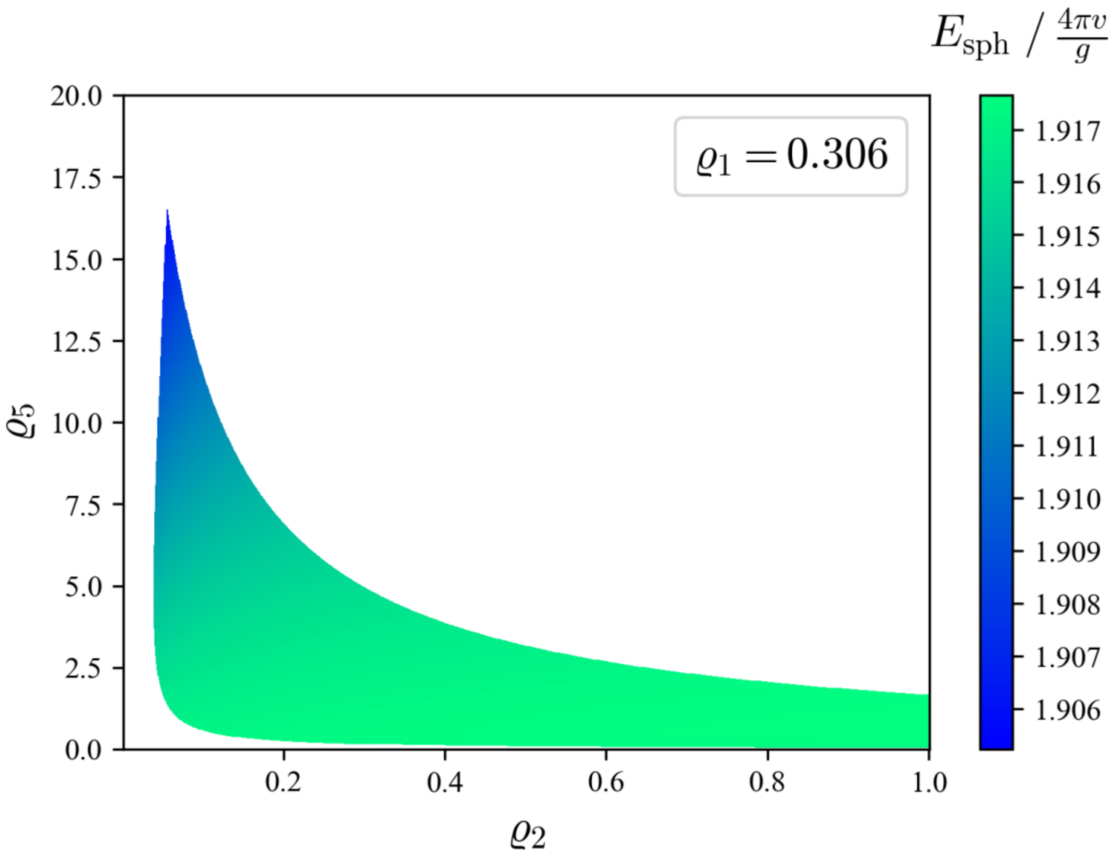
- tend to vacuum configuration more slowly
- diffuse the distribution of the sphaleron energy density, decrease the sphaleron energy

Sphaleron Energy

$\rho_3 = 10^{-3}$

$$\varrho_4 \approx \varrho_1 - \frac{1}{2}\varrho_3\varrho_5\left(2 + \sqrt{\frac{2\varrho_2}{\varrho_3\varrho_5}}\right)$$

allowed parameter space in the HTM:



In the SM:

$$\varrho_1^{\mathrm{SM}} = m_h^2/\left(2g^2v_\phi^2\right) \approx 0.306$$

$$E_{\mathrm{sph}}^{\mathrm{SM}} \approx 1.92 \times \frac{4\pi v}{g} \approx 9\ \mathrm{TeV}$$

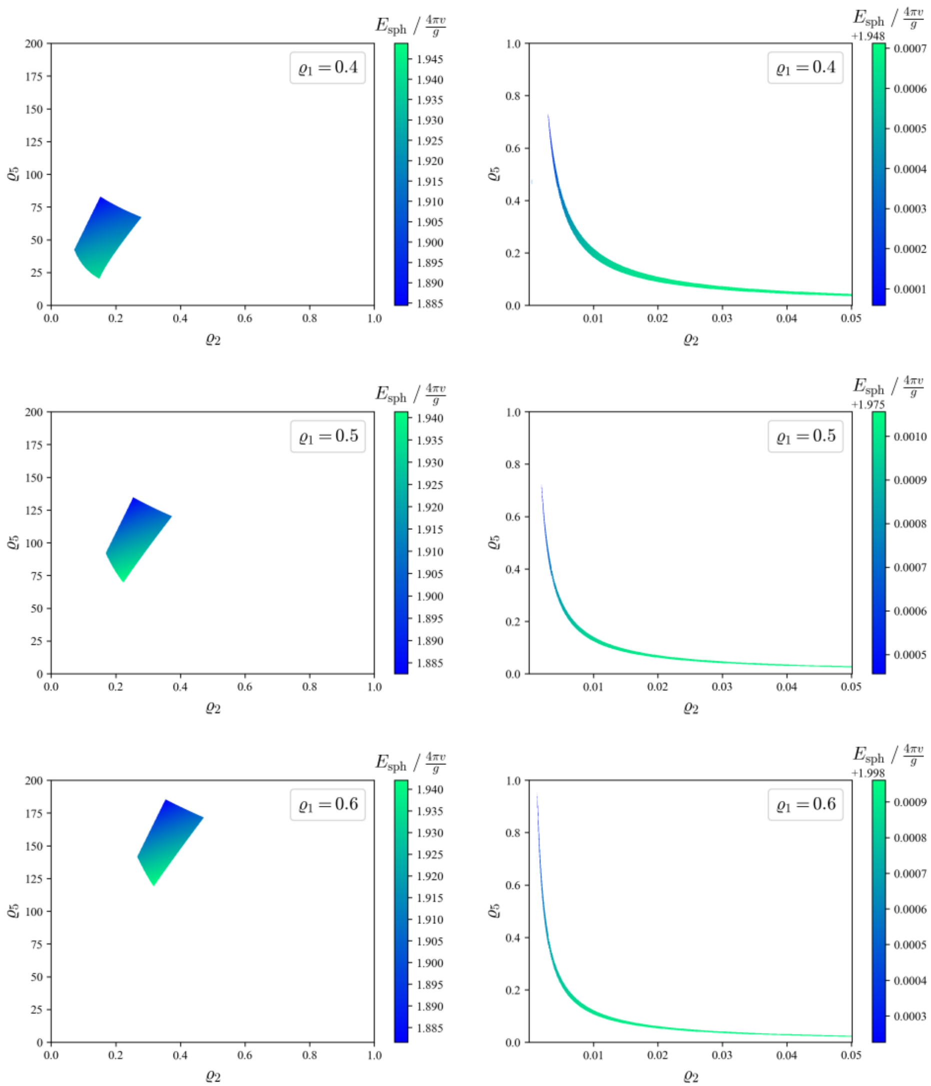
In the HTM:

larger trilinear coupling or heavier triplet scalar $\Longleftrightarrow$ smaller sphaleron energy

Sphaleron Energy

$$\varrho_1^{\rm SM} = m_h^2 / \left(2g^2 v_\phi^2\right) \approx 0.306$$

$$E_{\rm sph}^{\rm SM} \approx 1.92 \times 4\pi v / g$$



Region A **Region B**

parameter space starts to split into two parts as $\varrho_1 > 0.306$

Region A: large trilinear coupling and heavy triplet scalar $M_\Delta \gtrsim 1\ \text{TeV}$

Region B: small trilinear coupling and light triplet scalar $M_\Delta \lesssim 1\ \text{TeV}$ (narrow band, testable by future collider searches)

sphaleron energy in **Region A**:

$$1.88 \times 4\pi v/g \lesssim E_{\rm sph} \lesssim 1.97 \times 4\pi v/g$$

sphaleron energy in **Region B**:

$$1.92 \times 4\pi v/g \lesssim E_{\rm sph} \lesssim 2.48 \times 4\pi v/g$$

enhanced up to 30% compared with the SM case!

Summary

$$\begin{aligned} \varrho_1 &\equiv \lambda/g^2 \\ \varrho_2 &\equiv 2\lambda_\Delta^2/g^2 \\ \varrho_3 &\equiv v_\Delta^2/v_\phi^2 \\ \varrho_4 &\equiv \kappa^2/\left(g^2 v_\phi^2\right) \\ \varrho_5 &\equiv M_\Delta^2/\left(g^2 v_\phi^2\right) \end{aligned}$$

- The HTM provides a unified framework to explain both the neutrino masses and the baryon asymmetry of the Universe.
- To carry out a consistent EW baryogenesis in the HTM, we calculate the sphaleron configuration in the presence of a triplet.
- The sphaleron configuration in the HTM depends on 5 parameters: the quartic coupling parameter ϱ_1 would increase the sphaleron energy; the trilinear coupling parameter ϱ_2 , the VEV ratio parameter ϱ_3 , and the triplet mass parameter ϱ_5 would decrease the sphaleron energy.
- Basically, the difference of the sphaleron energy between the SM and the HTM is suppressed by the small triplet VEV. Interestingly, there still exists some narrow parameter space where the sphaleron energy could be enhanced by 30% compared with the SM case. Such narrow space can be tested by future collider searches.

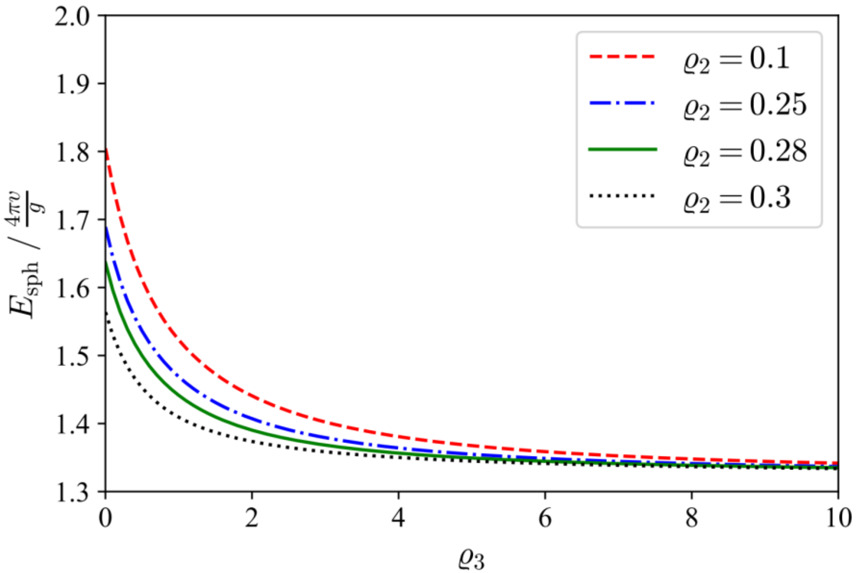
Future Extensions

◻ Finite temperature effects:

$$E_{\text{sph}}(T) = E_{\text{sph}} \frac{v(T)}{v}$$

$$v(T) = \sqrt{v_\phi^2(T) + 2v_\Delta^2(T)}$$

$\varrho_3 = v_\Delta^2 / v_\phi^2$ is not suppressed by experiments at finite temperature



◻ Effects of leptonic CP violation on EW baryogenesis:

The triplet coupling violates the lepton number and brings about new sources of CP violation.

This may have sizable effects on EW baryogenesis (collision between the triplet and the bubble wall will propagate new CP asymmetries) .

Thank you!

Q&A