Diagrammatic aspect of the Nielsen identity

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Nielsen Identity

- Unphysical parameters: gauge dependent.
- Physical observables: gauge independent. (Guaranteed by the Nielsen identity)

Nielsen Identity

- $\delta\Gamma[\phi] = i\int d^4x \int d^4y \frac{\delta\Gamma}{\delta\phi_i}\langle \delta_g \phi_i(x) c(x) \bar{c}(y) \delta' F(y)\rangle_{1\mathrm{PI}}$

- At stationary point $\frac{\delta\Gamma}{\delta\phi_i} = 0$ (Solution to the equation of motion), then $\delta\Gamma[\phi] = 0$ when $\langle \delta_g \phi_i(x) c(x) \bar{c}(y) \delta' F(y)\rangle_{1\mathrm{PI}}$ is finite (Unlikely to be infinite)

Nielsen Identity

- R_ξ (or $\overline{R}_\xi$)Gauge:
- $\xi \frac{\partial \Gamma}{\partial \xi} = -\int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x),$
- $C_R(x) = -\frac{i}{2}\int d^4y \langle \varphi(x) c(x) \overline{c}(y)(\partial_\mu A^\mu + g\xi(v+\phi)\varphi)\rangle_{1\mathrm{PI}}.$
- The above comes from non-perturbative path-integral analysis.

Nielsen Identity

- Perturbative analysis?
- Proof of the Nielsen identity perturbatively?
- Benefit: a preview of the structures of the perturbative terms before calculate straightforwardly.

Coleman-Weinberg Potential

Order unbalance???

$$\xi \frac{\partial \Gamma}{\partial \xi} = - \int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$

One-loop

Tree-level!!!

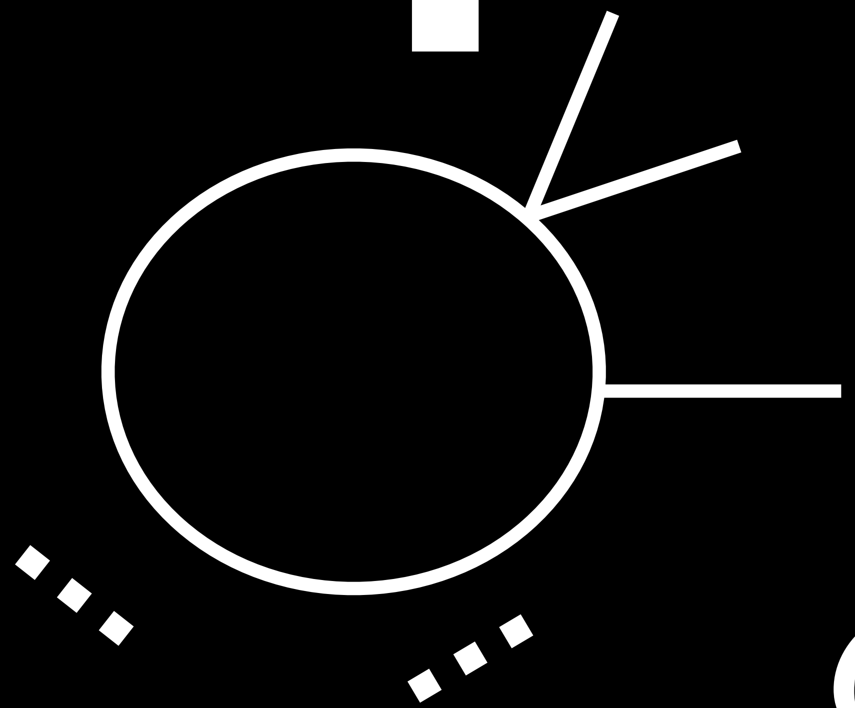
One-loop

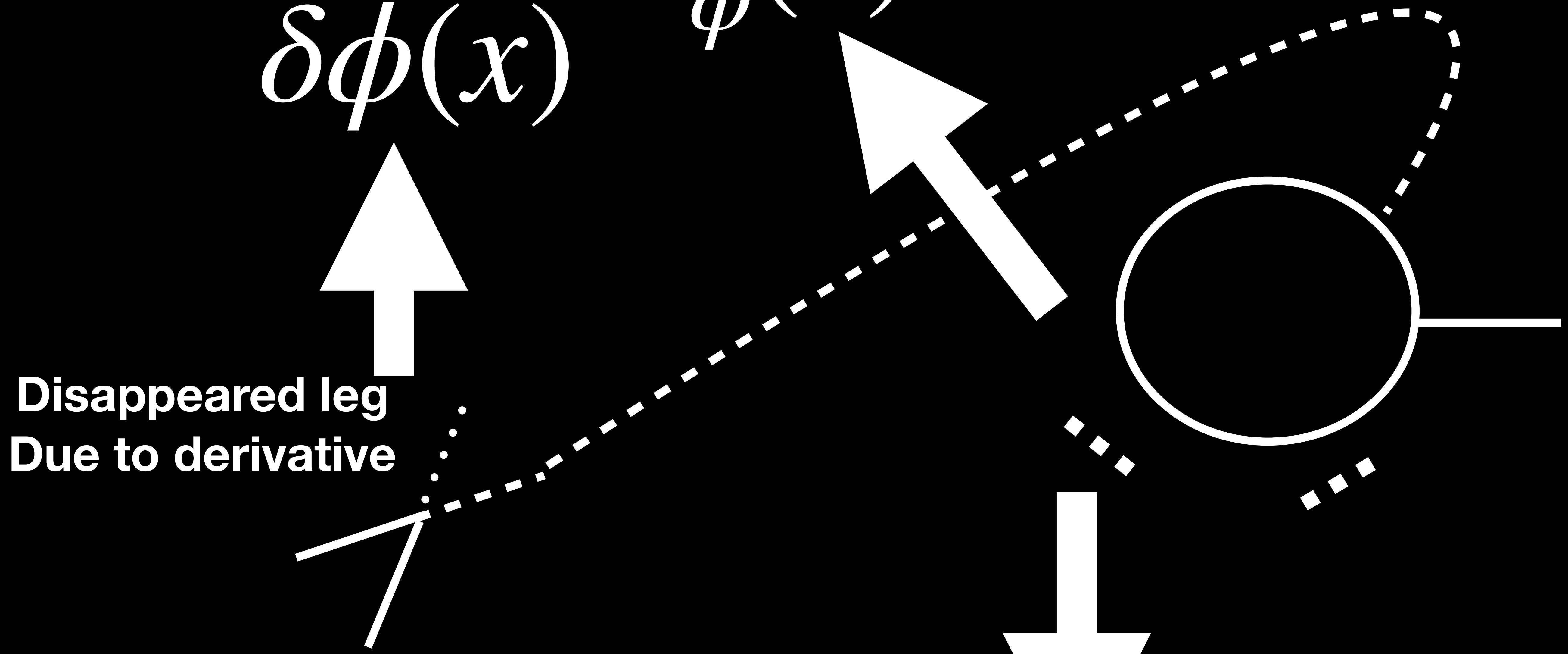
$$C_R(x) = -\frac{i}{2} \int d^4y \langle \varphi(x) c(x) \overline{c}(y) (\partial_\mu A^\mu + g\xi(v+\phi)\varphi) \rangle_{1PI}$$

Coleman-Weinberg Potential

More intuitive.

$$\xi \frac{\partial \Gamma}{\partial \xi} = -\int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$



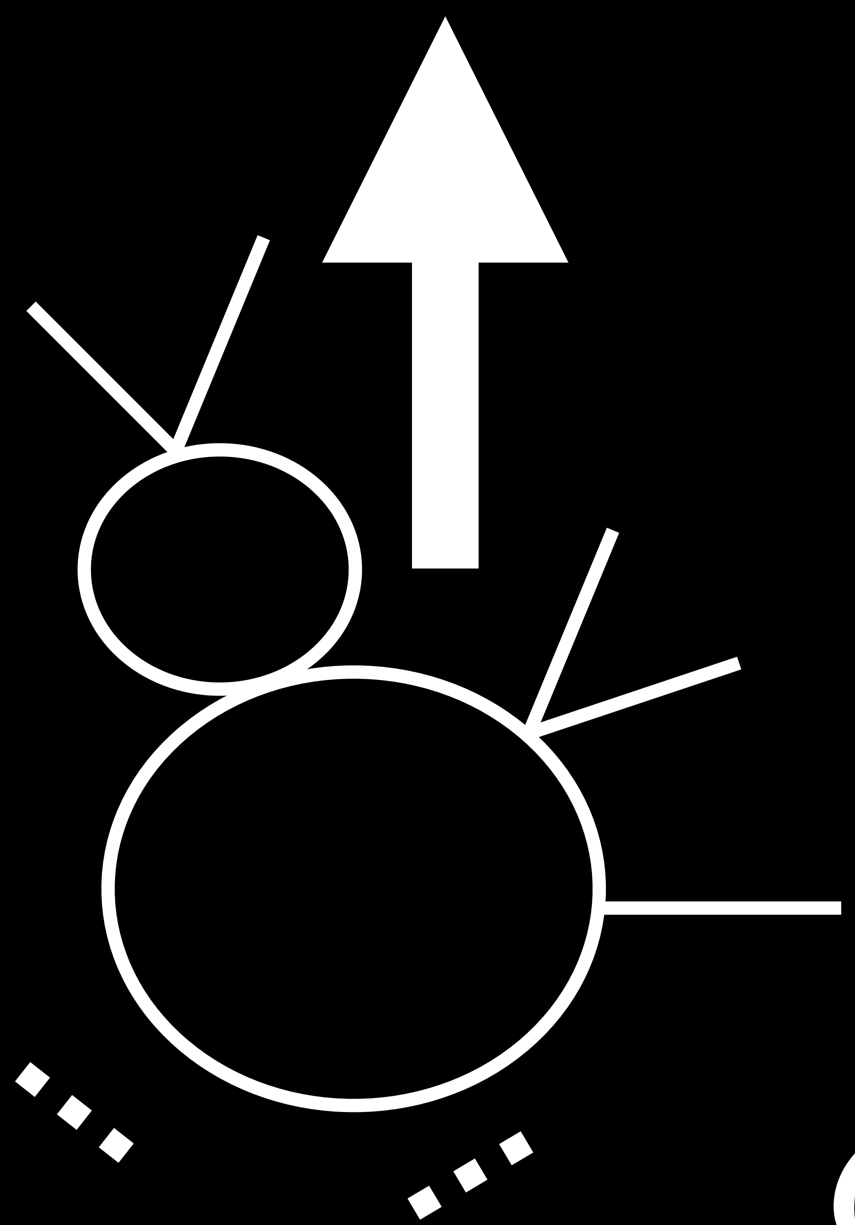


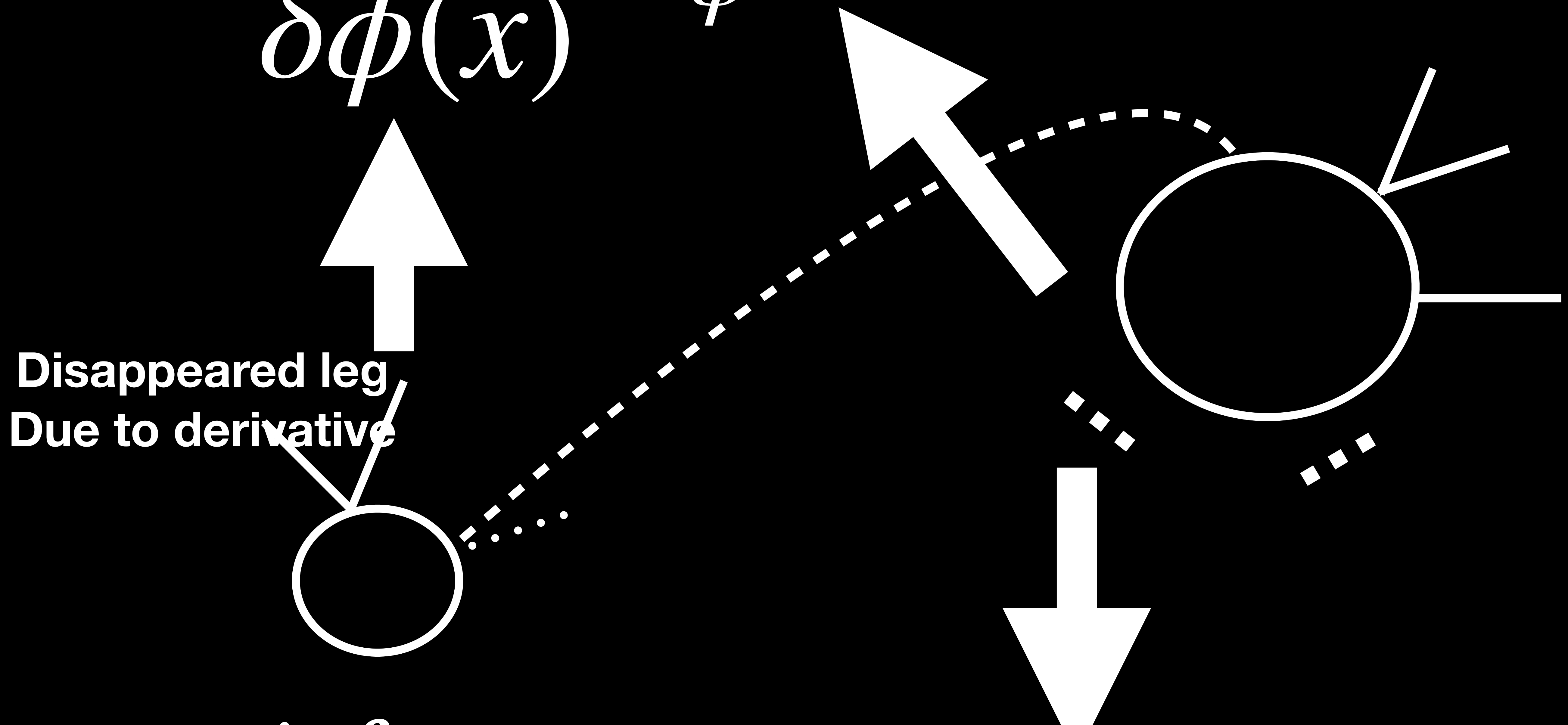
$$C_R(x) = -\frac{i}{2}\int d^4y \langle \varphi(x) c(x) \overline{c}(y) (\partial_\mu A^\mu + g\xi(v+\phi)\varphi)\rangle_{\text{1PI}}$$

More general loop results

More intuitive.

$$\xi \frac{\partial \Gamma}{\partial \xi} = - \int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$





$$C_R(x) = -\frac{i}{2} \int d^4y \langle \varphi(x) c(x) \overline{c}(y) (\partial_\mu A^\mu + g\xi(v + \phi)\varphi) \rangle_{1PI}$$

More general loop results

$$\xi \frac{\partial \Gamma}{\partial \xi} = - \int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$

Loop unbalance!

At least one loop

less then $\xi \frac{\partial \Gamma}{\partial \xi}$

At least one loop!

$$C_R(x) = -\frac{i}{2} \int d^4y \langle \varphi(x) c(x) \overline{c}(y) (\partial_\mu A^\mu + g\xi(v+\phi)\varphi) \rangle_{\text{1PI}}$$

$\hbar$ Expansion

$$\xi \frac{\partial \Gamma}{\partial \xi} = - \int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$

$$C = \hbar C_1 + \hbar^2 C_2 + \ldots$$

$$\Gamma = \Gamma_0 + \hbar \Gamma_1 + \hbar^2 \Gamma_2 + \ldots$$

At least one $\hbar$

less then $\xi \frac{\partial \Gamma}{\partial \xi}$

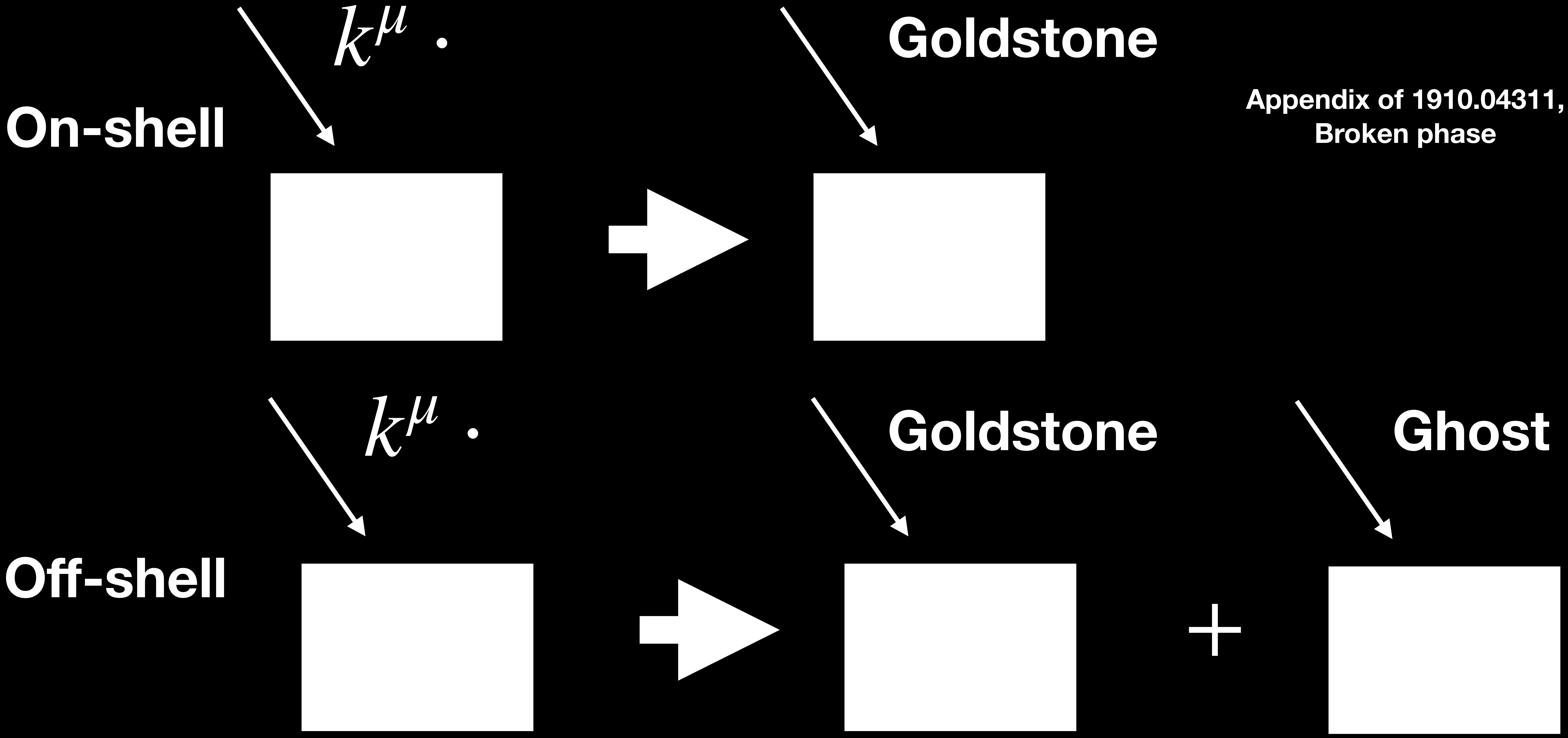
Target: $\hbar$ balance

- I want to acquire a method to calculate the effective potential satisfying the Nielsen identity up to all $\hbar$ orders. No $\hbar$ order unbalance.
- Not accomplished yet, however I studied the structure of the Nielsen identity.

Diagrammatic proof of the Nielsen identity

- Aim: Proof of this identity diagrammatically. (arXiv:2203.06876)
- Complete proof: too technical, so I make a sketchy description here.
- Previous discussions of the Nielsen identity were mainly based upon the path integral methods.
- Perturbative discussions: Integrate up all the loop momentums before verifying the identity.
- Inspired by the diagrammatic proof of the W-T identity on Peskin's book (unbroken phase), I study the diagrammatic structure of the Nielsen identity **before** the loop integrating processes.

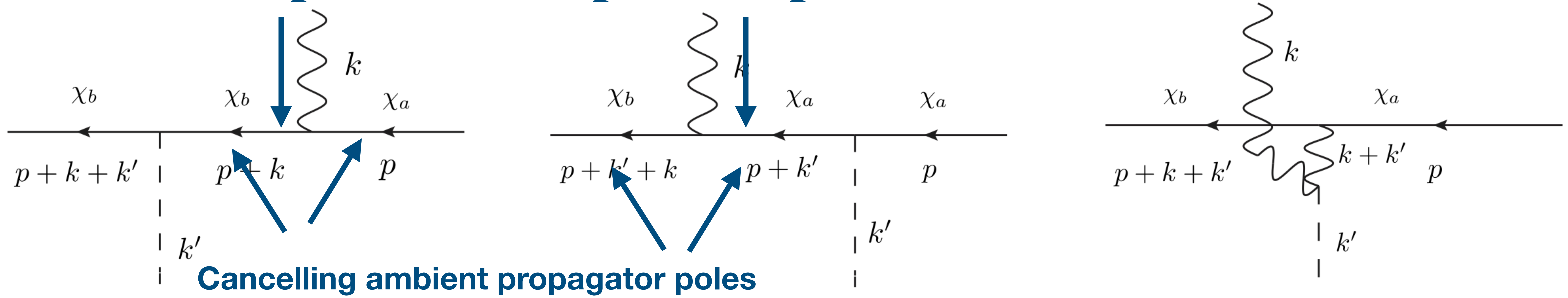
Diagrammatic aspect of the W-T identity



Diagrammatic aspect of the W-T identity

An example of basic trick:

$$k^{\mu} \cdot = (p+k)^{\mu} - p^{\mu} = (p+k+k')^{\mu} - (k+k')^{\mu}$$

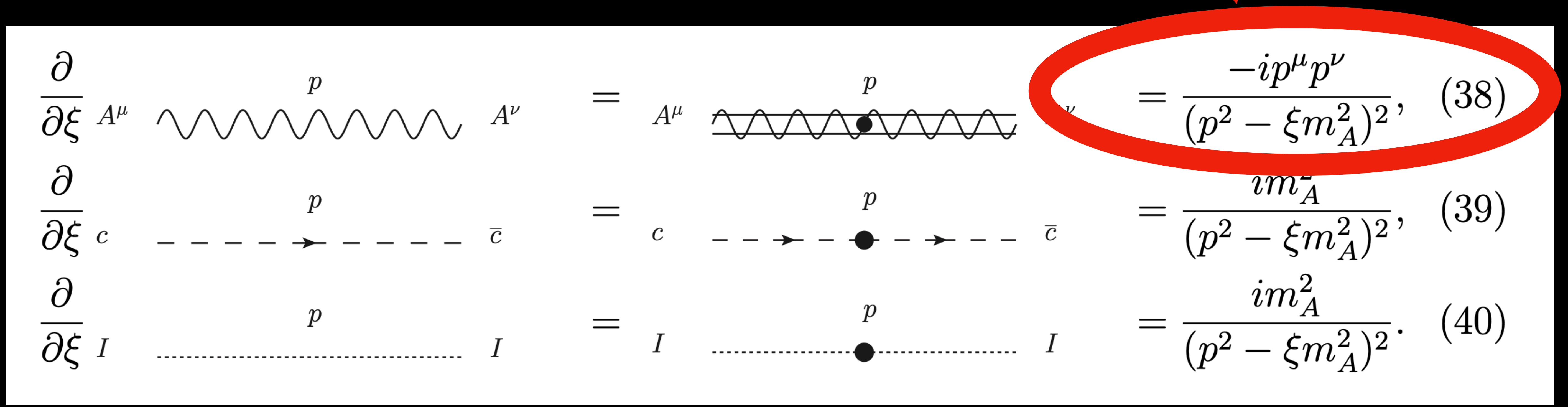


Cancelling among different terms, as the vector propagator floats among different insersions.

Casting the trick to the Nielsen Identity

$$\xi \frac{\partial \Gamma}{\partial \xi} = - \int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$

$p^\mu \cdot$ **Generated**

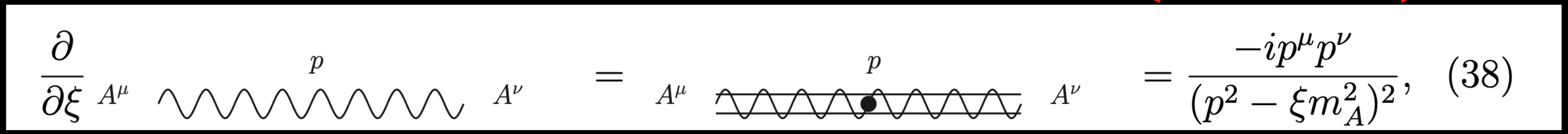


Differentiating the vector-/Goldstone-/ghost-propagators and ξ-dependent coupling constants.

Casting the trick to the Nielsen Identity

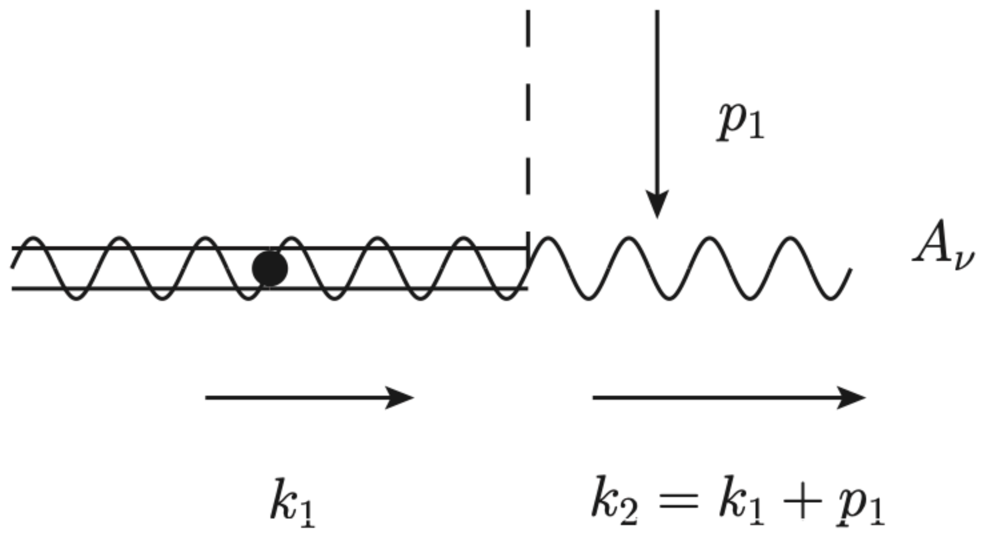
$p^\mu \cdot$ **becomes Goldstone/Ghost**

Spreading this processes successively



Prolonging!!!

Casting the trick to the Nielsen Identity

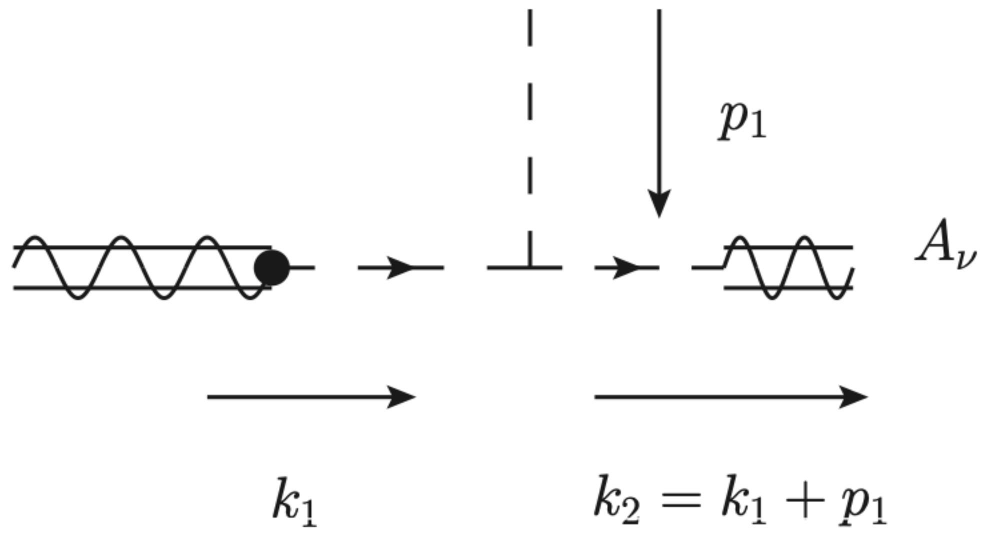


$$= 2ig^2 v k_1^\mu \frac{-i}{k_2^2 - m_A^2}\left[g_{\mu\nu} - \frac{k_{2\mu}k_{2\nu}}{k_2^2 - \xi m_A^2}(1-\xi)\right].$$

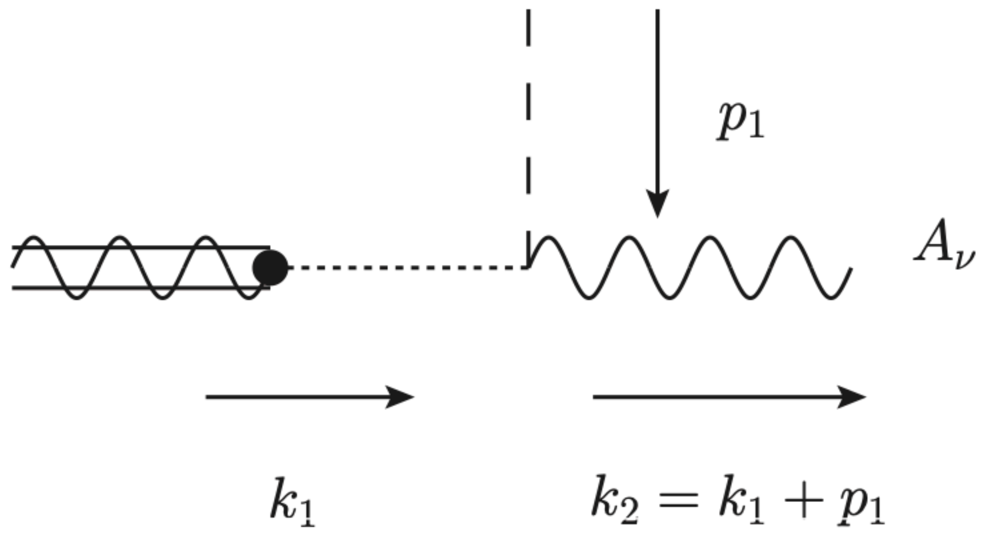
$$2ig^2 v(k_2^\mu - p_1^\mu)\frac{-i}{k_2^2 - m_A^2}\left[g_{\mu\nu} - \frac{k_{2\mu}k_{2\nu}}{k_2^2 - \xi m_A^2}(1-\xi)\right]$$

$$= (-2ig^2 v\xi)\frac{i}{k_2^2 - \xi m_A^2}k_{2\nu} - im_A \cdot 2gp_1^\mu \frac{-i}{k_2^2 - m_A^2}\left[g_{\mu\nu} - \frac{k_{2\mu}k_{2\nu}}{k_2^2 - \xi m_A^2}(1-\xi)\right].$$

$=$



$- im_A$



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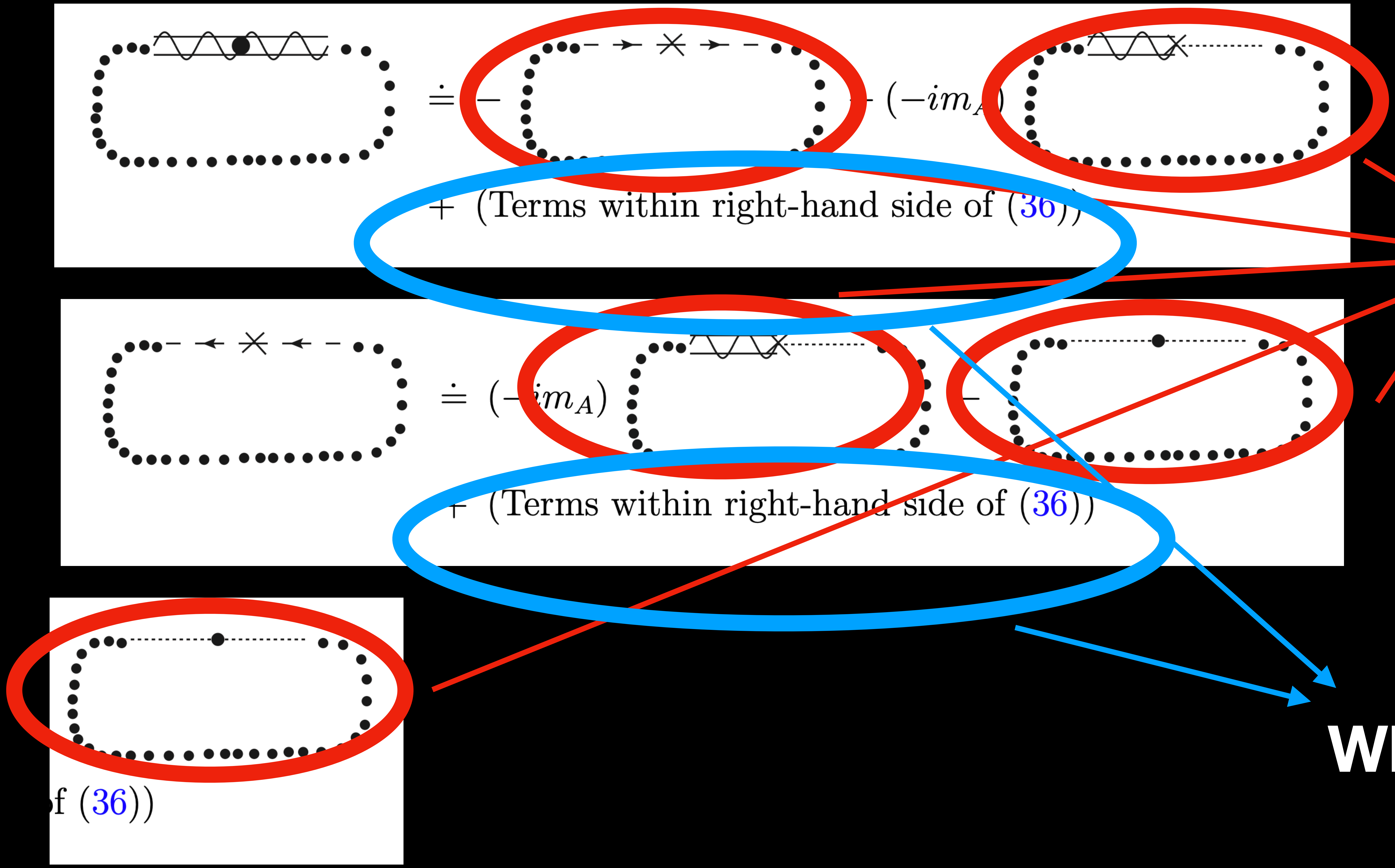
Casting the trick to the Nielsen Identity

- Enumerate all diagrammatical patterns that the vector-Goldstone chain can encounter, and finally,

- Vector-Goldstone chain can become a ghost chain + a chain started with a Goldstone boson.

- Similarly, ghost chain and a chain started with a Goldstone boson can be restored to a vector-Goldstone chain.

Proving the Nielsen identity



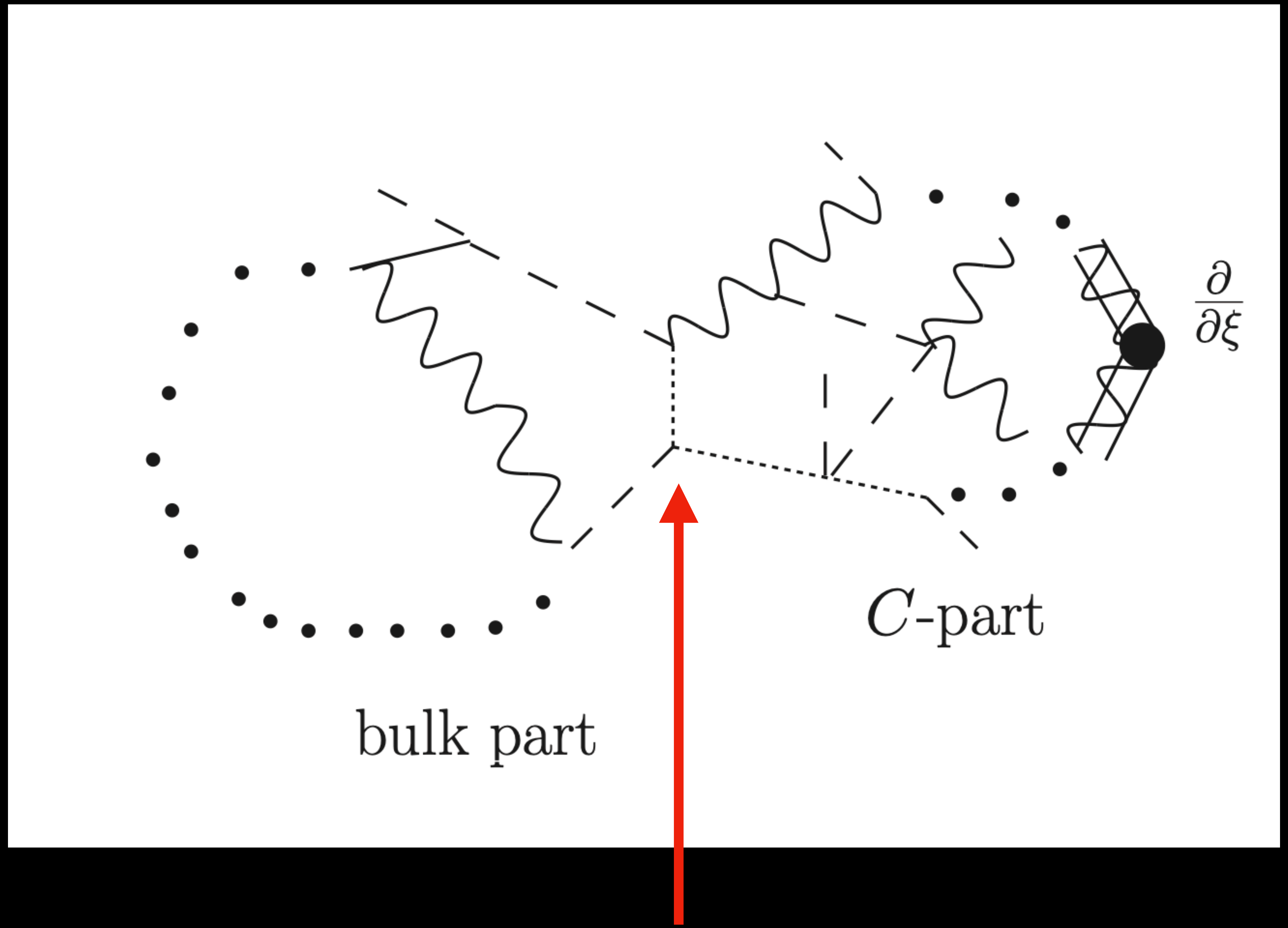
These terms finally cancel each other

What are they?

Proving the Nielsen identity

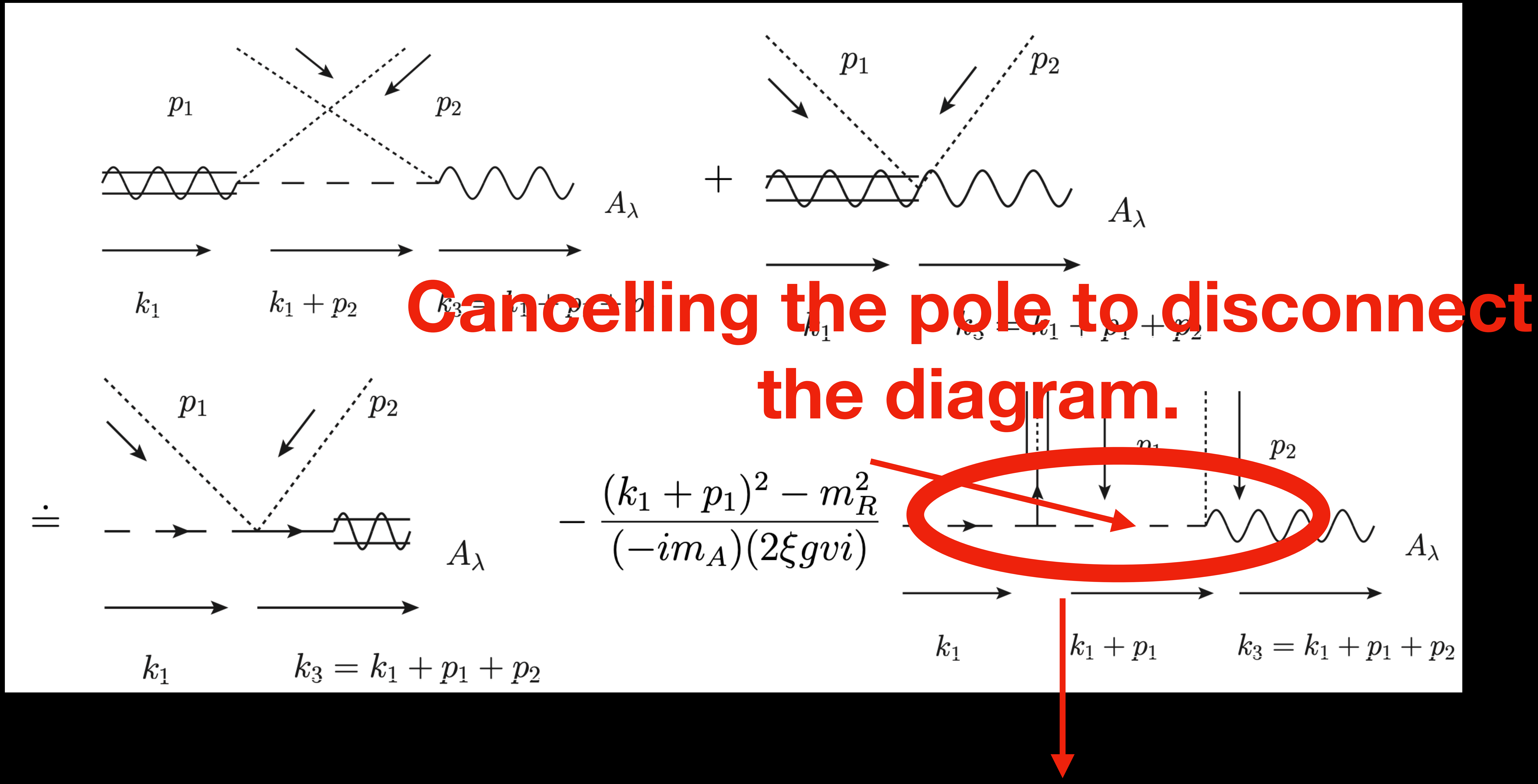
- The Ward-Takahashi identity is about the connected diagrams.
- The Nielsen identity works with the 1PI connected diagrams.
- Therefore, some diagrams which are expected to cancel the other terms in the proof of the WT identity does not appear as 1PI connected diagrams!

Gourd structure



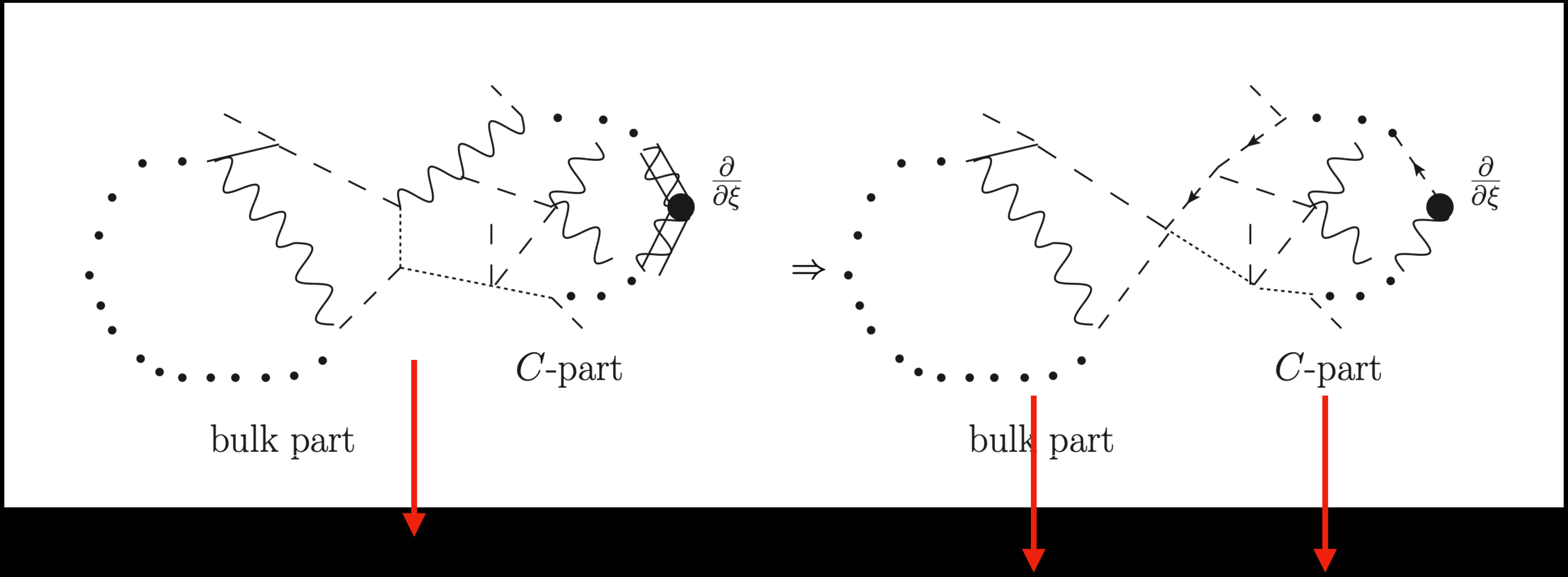
Only one common side or one common vertex to connect two different parts of one diagram!

Gourd structure



If this appears to be the waist of the gourd, no 1PI appears to cancel this term!

Gourd structure



$$\xi \frac{\partial \Gamma}{\partial \xi} = -\int d^4x \frac{\delta \Gamma}{\delta \phi(x)} C_\phi(x)$$

Summary and Future Prospect

- Nielsen identity is successfully “proved” up to all perturbative orders.

- Right-hand side contributions are naturally at least one loop less than left-hand side contributions.

- Future prospect: abandon some less important cancelling terms to accomplish effective potential up to all loop orders?

Thank you!