



$\Xi_c - \Xi'_c$ mixing from Lattice QCD

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Phys.Lett.B 841 (2023) 137941 and arXiv:2309.05432

In collaboration with Wei Wang, Qi-An Zhang, etc

CLHCP2023

OUTLINE

- Motivation
- $\Xi_c - \Xi'_c$ mixing from two-point correlation functions
- An improved method to extract $\Xi_c - \Xi'_c$ mixing
- Summary and outlook

Motivation

► Experimental studies

$$B_{\text{Belle}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%, \quad \text{Phys. Rev. Lett. } \mathbf{127} \text{ no. 12, (2021)}$$

$$B_{\text{ALICE}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.48 \pm 0.25 \pm 0.40 \pm 0.72)\%. \quad \text{Phys. Rev. Lett. } \mathbf{127} \text{ no. 27, (2021)}$$

2022 Review of Particle Physics (using only the Belle measurement as input)

$$B_{\text{PDG}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.04 \pm 0.24)\%. \quad \text{PTEP } \mathbf{2022} \text{ (2022) 083C01.}$$

► Theoretical studies

$$B_{\text{Lattice}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.38 \pm 0.30 \pm 0.32)\% \quad \text{Zhang et al., (2022)}$$

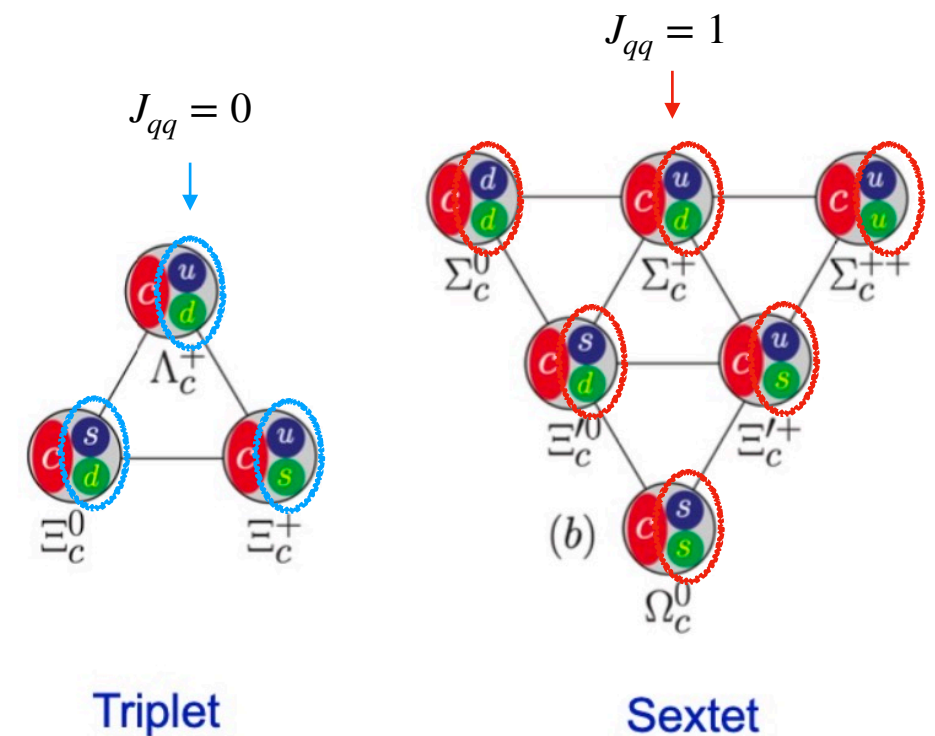
Method	$B(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)$	
light-front quark model	$(3.49 \pm 0.95)\%$	Geng, Liu, and Tsai, 2021
QCD sum rules	$(3.4 \pm 1.7)\%$	Zhao, 2021
rel. quark model	$2.38\%*$	Faustov and Galkin, 2019
SU(3)	$(3.0 \pm 0.3)\%*$	Geng et al., 2019
light-front quark model	$1.35\%*$	Zhao, 2018
SU(3)	$(4.87 \pm 1.74)\%*$	Geng et al., 2018
SU(3)	$(11.9 \pm 1.6)\%*$	Geng et al., 2017
light-cone QCD sum rules	$(7.26 \pm 2.54)\%*$	Azizi, Sarac, and Sundu, 2012
QCD sum rules	$2.4\%*$	Liu and Huang, 2010

Big discrepancy!

A competitive explanation is through the $\Xi_c - \Xi'_c$ mixing

$\Xi_c - \Xi'_c$ mixing

- ◆ In **heavy quark limit**, heavy baryon with one **charm quark** can be classified according to angular momentum J_{qq} of light quark system.



- ◆ Mixing effect can be described by a 2×2 matrix

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi'_c\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\Xi_c^{\bar{3}}\rangle \\ |\Xi_c^6\rangle \end{pmatrix}$$

Energy eigenstates
Flavor eigenstates

$$T_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{+'}}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^{0'}}{\sqrt{2}} \\ \frac{\Xi_c^{+'}}{\sqrt{2}} & \frac{\Xi_c^{0'}}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}$$

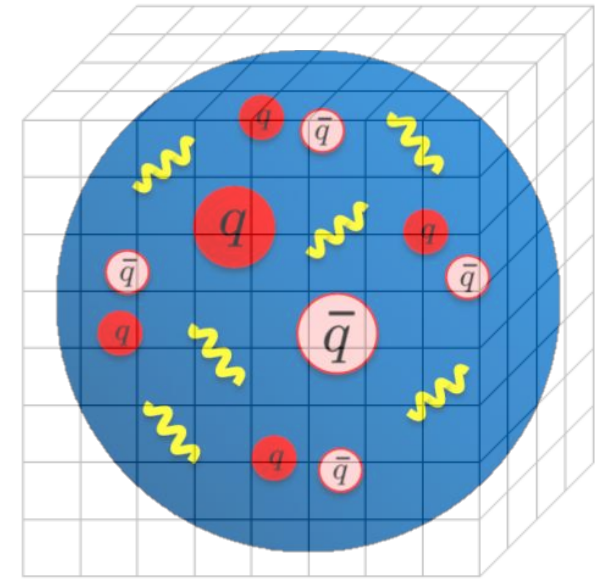
$\Xi_c - \Xi'_c$ mixing

◆ Some results from different methods about the mixing angle

Sum rule	$5.5^\circ \pm 1.8^\circ$	<i>Phys. Rev. D 83, 016008 (2011);</i>
HQET	$8.12^\circ \pm 0.80^\circ$	<i>Nucl. Phys. A 1008, 122139, (2021);</i>
Quark model	$16.27^\circ \pm 2.30^\circ$	<i>Phys. Rev. D 105, 096011 (2022);</i>
	$24.66^\circ \pm 0.90^\circ$	<i>Phys. Lett. B 838, 137736, (2023);</i> <i>Phys. Lett. B 839, 137831, (2023);</i>
Lattice QCD	Negligibly small	<i>Phys. Rev. D 90, 094507, (2014).</i>

Lattice simulation

$$S_E^{latt} = \underbrace{\sum \frac{6}{g^2} \text{ReTr}(1 - U_P)}_{\text{wilson gauge action}} + \sum_q \underbrace{\bar{q}(D_\mu^{lat} \gamma_\mu + am_q)q}_{\text{lattice fermion action}}$$



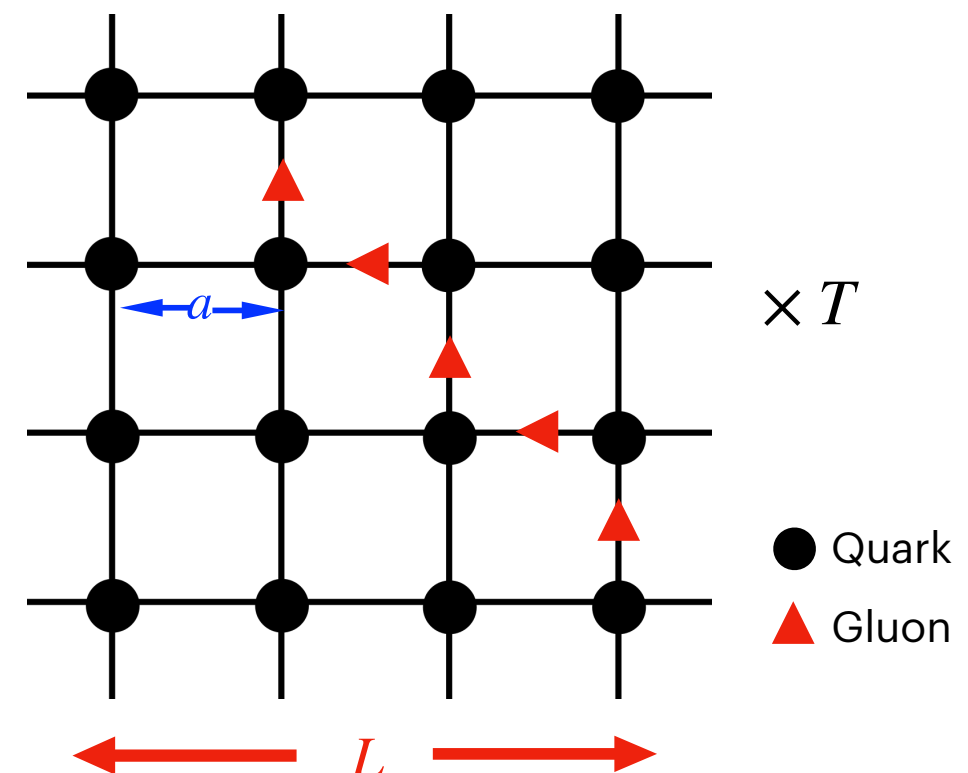
- ◆ Charm quark on discrete lattice:
consider both **IR** and **UV** effects:

$$m_\pi L \gtrsim 4, \quad \text{and} \quad a^{-1} \gg \text{mass scale}$$

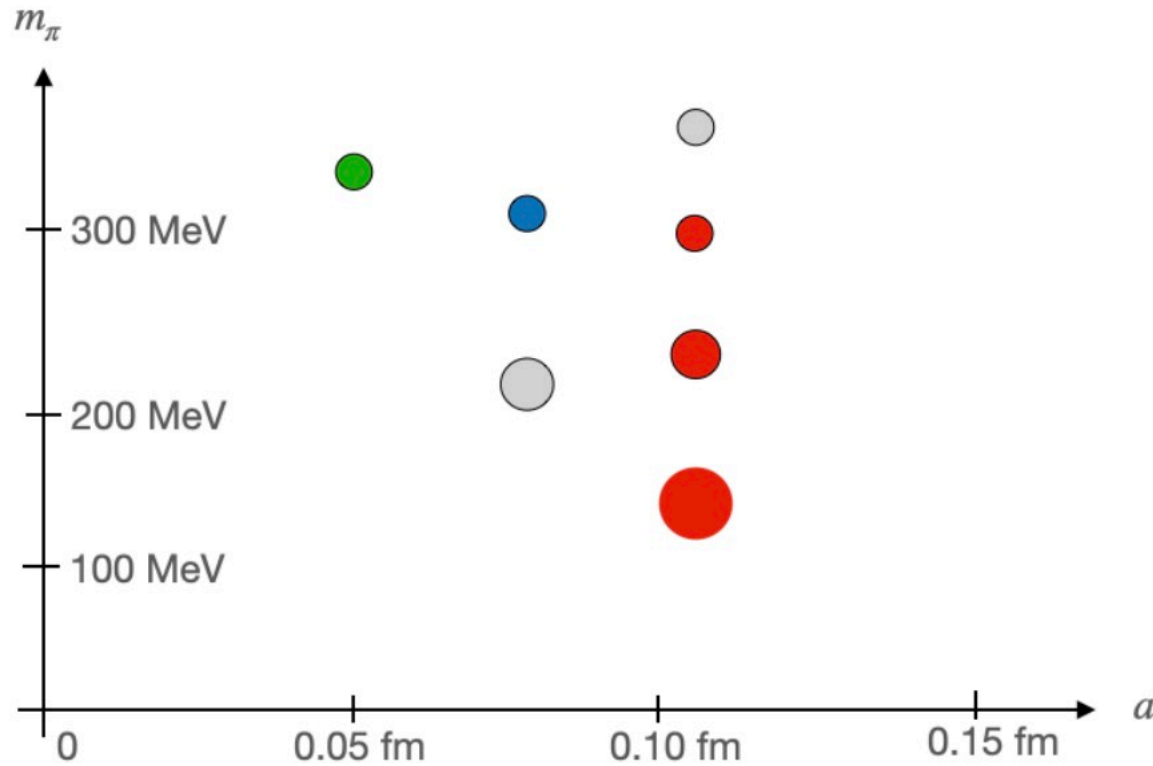
for $m_\pi = m_\pi^{\text{phy}} \sim 140\text{MeV}$, and $m_c \simeq 1.3\text{GeV}$,

$$L \gtrsim 5.6\text{fm}$$

$$a^{-1} \gg 1.3\text{GeV} \simeq (0.15\text{fm})^{-1}$$



CLQCD ensembles



$\Xi_c - \Xi'_c$ mixing from Lattice QCD

H.Liu et al., Phys.Lett.B 841 (2023) 137941

An improved method to determine the $\Xi_c - \Xi'_c$ mixing

H. Liu et al., arXiv:hep-ph/2309.05432

Ensemble	β	$L^3 \times T$	a (fm)	m_l^b	m_s^b	m_c^b	m_π	N_{meas}
C11P14L		$48^3 \times 96$		-0.2825	-0.2310	0.4800	135	203×48
C11P22M	6.20	$32^3 \times 64$	0.108	-0.2790	-0.2310	0.4800	222	451×20
C11P29S		$24^3 \times 72$		-0.2770	-0.2315	0.4780	284	432×26
C08P30S	6.41	$32^3 \times 96$	0.080	-0.2295	-0.2010	0.2326	297	653×26
C06P30S	6.72	$48^3 \times 144$	0.055	-0.1850	-0.1687	0.0770	312	136×80

$\Xi_c - \Xi'_c$ mixing from LQCD

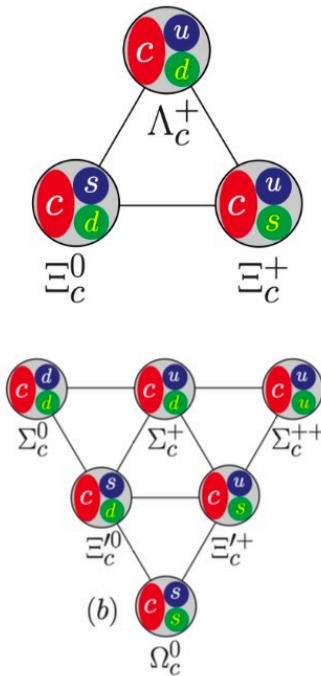
◆ Baryonic operators of $SU(3)_F$ eigenstates

$$O_{SU(3)}^{\bar{3}} = \epsilon^{abc} (q^{Ta} C \gamma_5 s^b) P_+ c^c,$$

$J = 0$

$$O_{SU(3)}^6 = \epsilon^{abc} (q^{Ta} C \vec{\gamma} s^b) \cdot \vec{\gamma} \gamma_5 P_+ c^c$$

$J = 1$



◆ Build the 2x2 correlation function matrix in the lattice calculation

$$\mathcal{C}(t, t_0) = \sum_{\vec{x}} \begin{pmatrix} \left\langle O_p^{\bar{3}}(\vec{x}, t) \bar{O}_w^{\bar{3}}(\vec{0}, t_0) \right\rangle & \left\langle O_p^{\bar{3}}(\vec{x}, t) \bar{O}_w^6(\vec{0}, t_0) \right\rangle \\ \left\langle O_p^6(\vec{x}, t) \bar{O}_w^{\bar{3}}(\vec{0}, t_0) \right\rangle & \left\langle O_p^6(\vec{x}, t) \bar{O}_w^6(\vec{0}, t_0) \right\rangle \end{pmatrix}$$

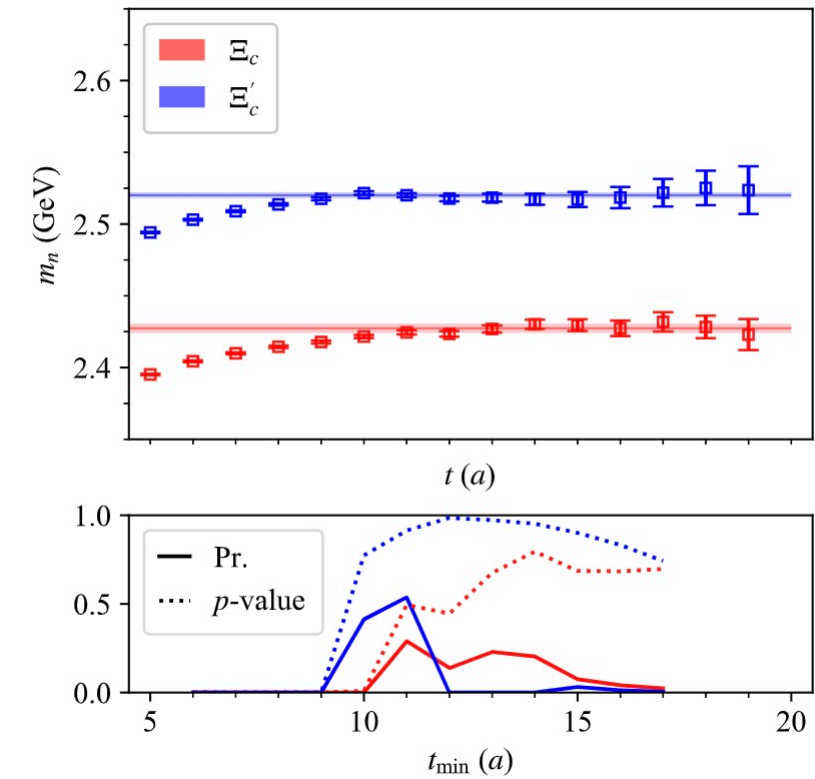
Mixing angle from generalized eigenvalue problem(GEVP)

◆ Solving the generalized eigenvalue problem:

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$

◆ Mass from eigenvalues

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)} \left(1 + c_1 e^{-\Delta E(t-t_r)} \right)$$

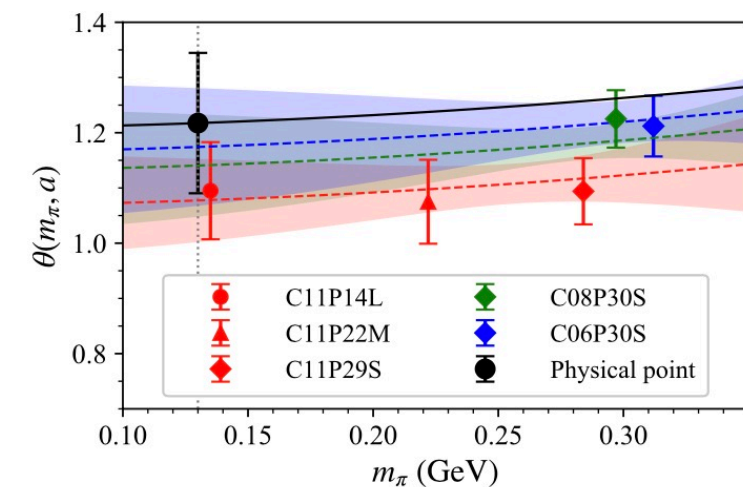


Model average method

◆ Mixing angle from eigenvectors

$$v_1 = \sqrt{\frac{B_p^2}{A_p^2 \cot^2 \theta + B_p^2}} \begin{pmatrix} \frac{A_p}{B_p} \cot \theta \\ 1 \end{pmatrix},$$

$$v_2 = \sqrt{\frac{B_p^2}{A_p^2 \tan^2 \theta + B_p^2}} \begin{pmatrix} -\frac{A_p}{B_p} \tan \theta \\ 1 \end{pmatrix},$$



Mixing angle from generalized eigenvalue problem(GEVP)

◆ Solving the generalized eigenvalue problem:

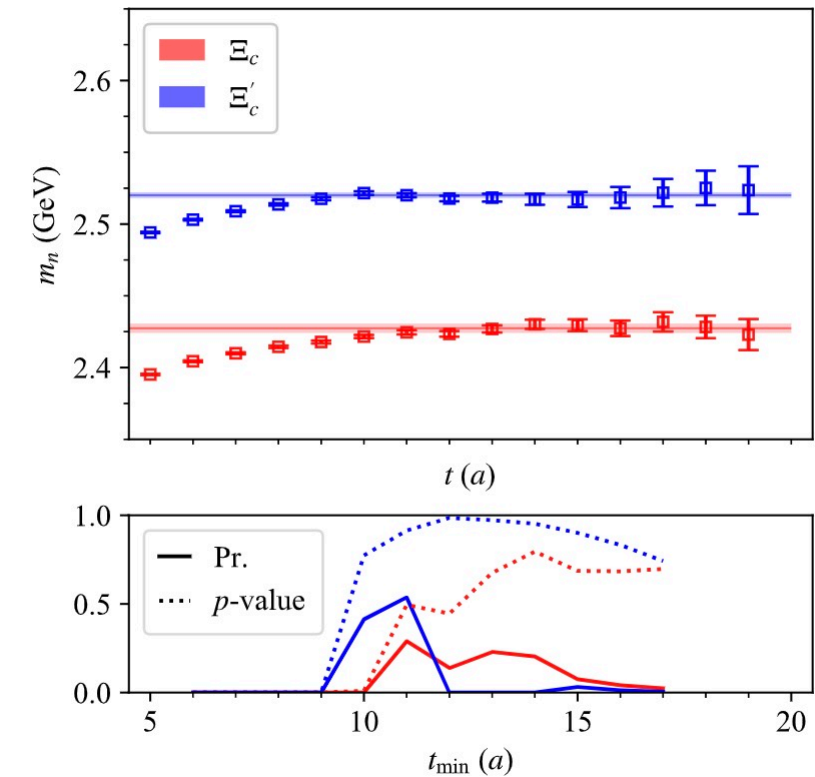
$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$



◆ Mass from eigenvalues

$$\mathcal{C}(t_r)^{-1}\mathcal{C}(t)v_n(t) = \lambda_n(t)v_n(t)$$

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)} \left(1 + c_1 e^{-\Delta E(t-t_r)} \right)$$

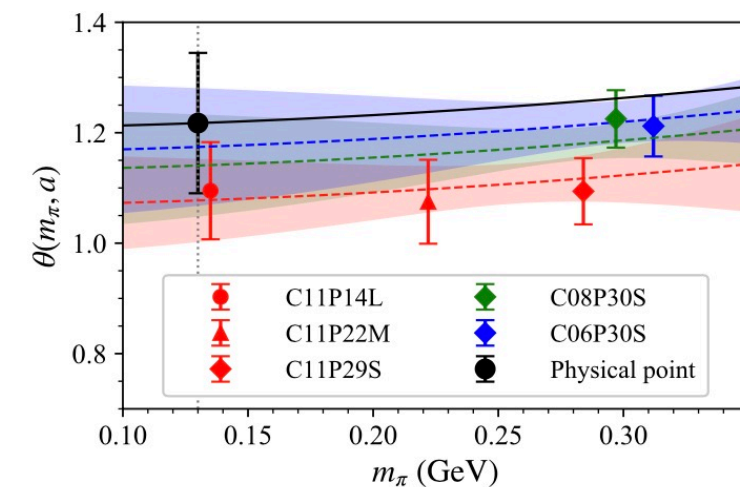


Model average method

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Mixing angle from generalized eigenvalue problem(GEVP)

◆ Solving the generalized eigenvalue problem:

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◆ Mass from eigenvalues

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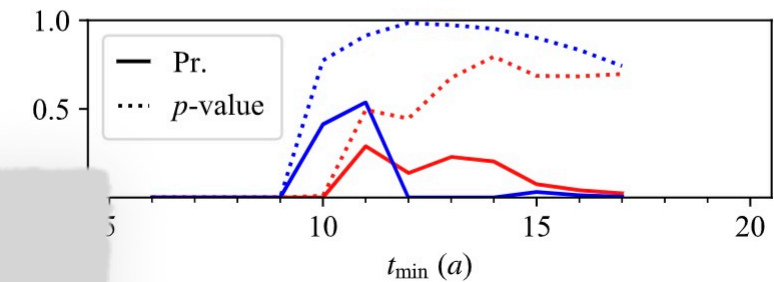
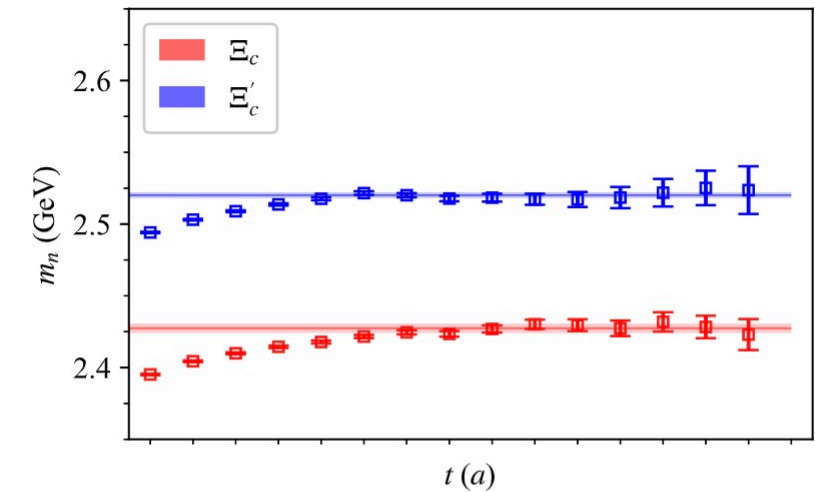
$$\langle a \rangle = \sum_M \langle a \rangle_M \text{Pr}(M|D),$$

$$\text{Pr}(M|D) \approx \exp \left[-\frac{1}{2} (\chi_{\text{aug}}^2(\mathbf{a}^*) + 2k + 2N_{\text{cut}}) \right]$$

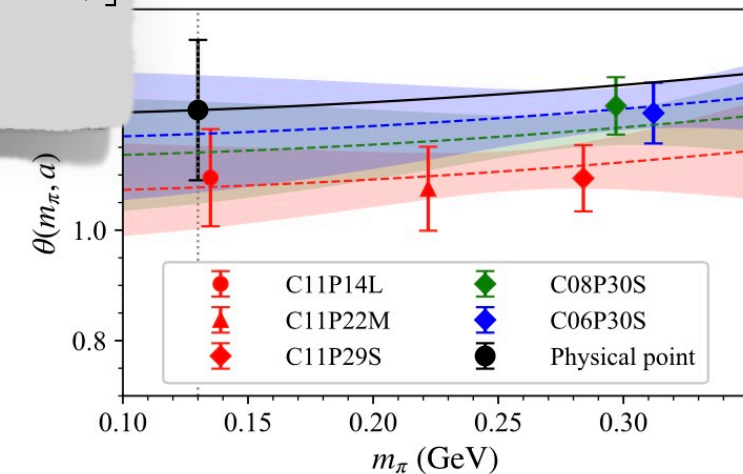
◆ Mixing angle from e

$$v_1 = \sqrt{\frac{B_p^2}{A_p^2 \cot^2 \theta + B_p^2}} \begin{pmatrix} \frac{A_p}{B_p} \cot \theta \\ 1 \end{pmatrix},$$

$$v_2 = \sqrt{\frac{B_p^2}{A_p^2 \tan^2 \theta + B_p^2}} \begin{pmatrix} -\frac{A_p}{B_p} \tan \theta \\ 1 \end{pmatrix},$$



Model average method



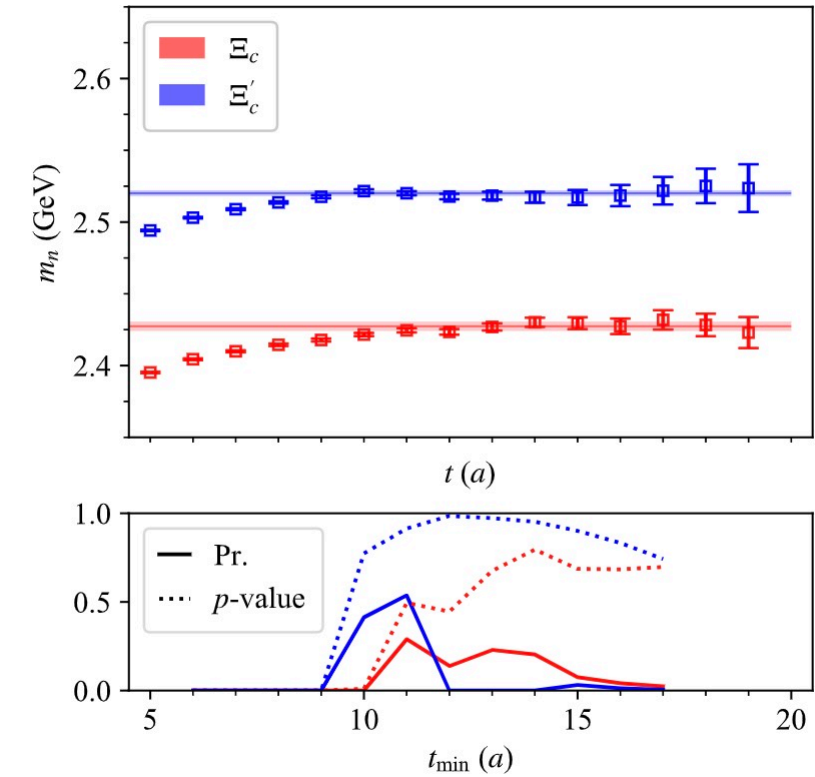
Mixing angle from generalized eigenvalue problem(GEVP)

◆ Solving the generalized eigenvalue problem:

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t)$$

◆ Mass from eigenvalues

$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)} \left(1 + c_1 e^{-\Delta E(t-t_r)} \right)$$

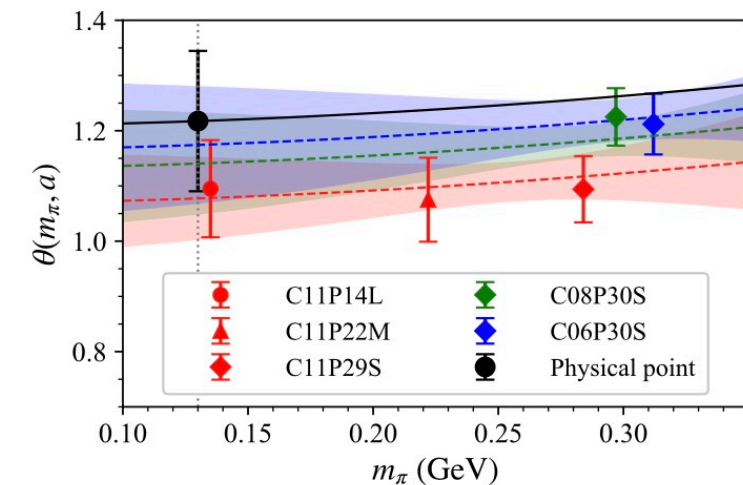


Model average method

◆ Mixing angle from eigenvectors

$$v_1 = \sqrt{\frac{B_p^2}{A_p^2 \cot^2 \theta + B_p^2}} \begin{pmatrix} \frac{A_p}{B_p} \cot \theta \\ 1 \end{pmatrix},$$

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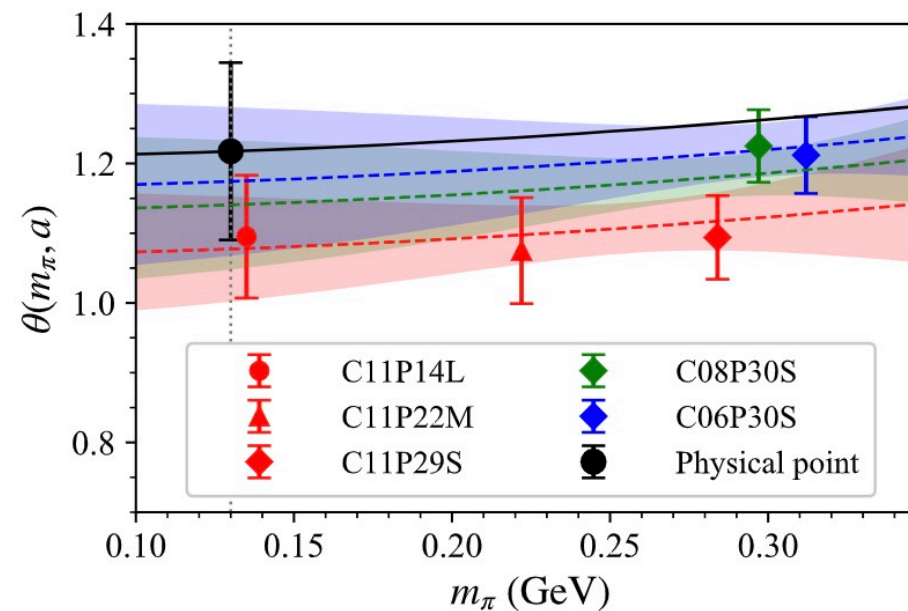
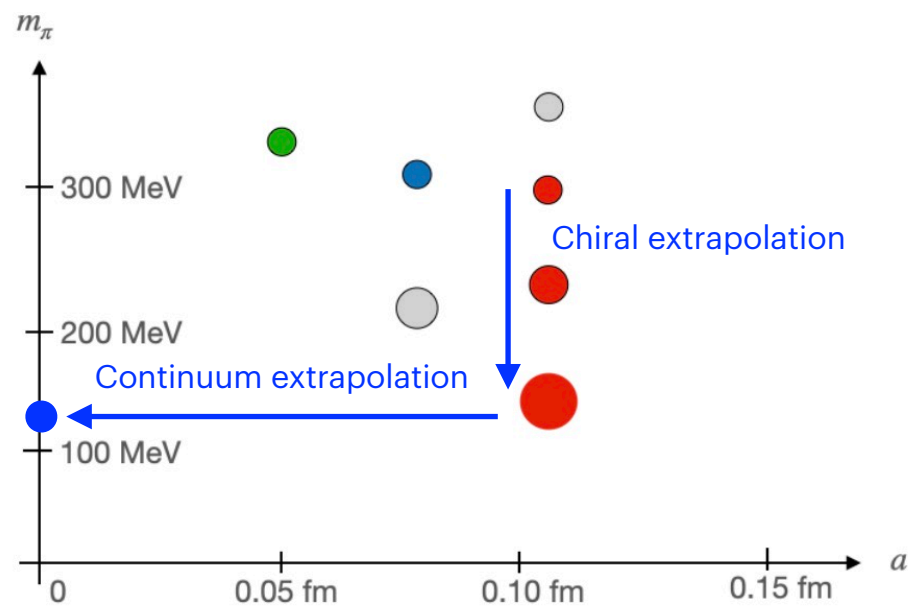


Chiral and continuum extrapolation

◆ Extrapolation formula:

$$\theta(m_\pi, a) = \theta_{\text{phy}} + c_1 (m_\pi^2 - m_{\pi, \text{phy}}^2) + c_2 a^2,$$

$$m_n(m_\pi, a) = m_{n, \text{phy}} + c_1 (m_\pi^2 - m_{\pi, \text{phy}}^2) + c_2 a^2$$

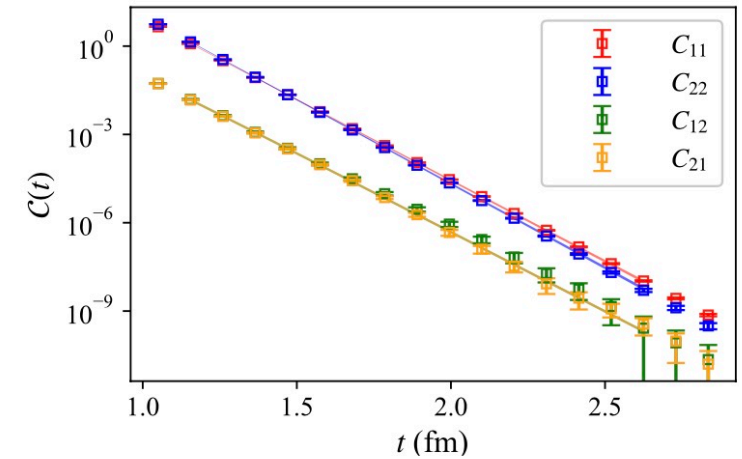


$$\theta = (1.22 \pm 0.13 \pm 0.01)^\circ$$

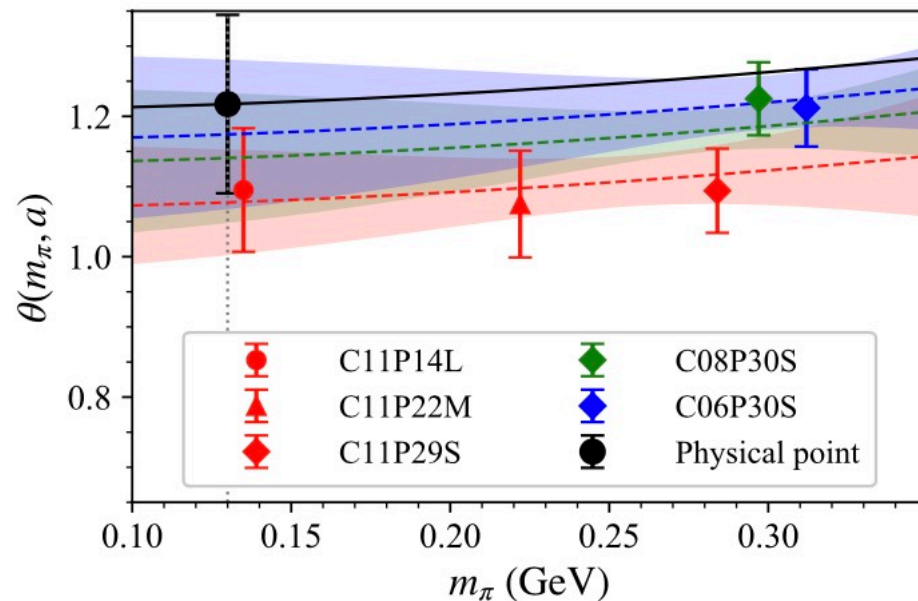
Mixing angle from correlated joint fit

- ◆ Insert the mass eigenstates and consider the excited state contributions are greatly suppressed, parametrization form of correlation function matrix elements:

$$\begin{aligned}
 C_{11}(t, t_0) &= A_p A_w^\dagger \left[\frac{\cos^2 \theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} + \frac{\sin^2 \theta}{2m_{\Xi'_c}} e^{-m_{\Xi'_c}(t-t_0)} \right] \\
 C_{12}(t, t_0) &= A_p B_w^\dagger \left[\frac{\cos \theta \sin \theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos \theta \sin \theta}{2m_{\Xi'_c}} e^{-m_{\Xi'_c}(t-t_0)} \right], \\
 C_{21}(t, t_0) &= B_p A_w^\dagger \left[\frac{\cos \theta \sin \theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos \theta \sin \theta}{2m_{\Xi'_c}} e^{-m_{\Xi'_c}(t-t_0)} \right], \\
 C_{22}(t, t_0) &= B_p B_w^\dagger \left[\frac{\sin^2 \theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} + \frac{\cos^2 \theta}{2m_{\Xi'_c}} e^{-m_{\Xi'_c}(t-t_0)} \right]
 \end{aligned}$$



- ◆ One can extract the mixing parameters from a joint analysis of both diagonal and non-diagonal terms.



	m_{Ξ_c} (GeV)	$m_{\Xi'_c}$ (GeV)	θ (°)	$\chi^2/\text{d.o.f}$	fit range (fm)
C11P14L	2.4256(19)	2.5196(22)	1.083(30)	0.96	1.19 – 2.81
C11P22M	2.4380(27)	2.5351(30)	0.988(49)	1.0	1.19 – 2.92
C11P29S	2.4587(27)	2.5536(29)	1.002(50)	1.1	1.19 – 3.24
C08P30S	2.4753(21)	2.5809(26)	1.080(42)	0.95	1.20 – 2.40
C06P30S	2.4695(37)	2.5815(48)	1.021(67)	1.2	1.32 – 2.40
Extrapolated	$2.4380(68)_{\text{stat}}(403)_{\text{syst}}$	$2.5562(74)_{\text{stat}}(422)_{\text{syst}}$	$1.20(9)_{\text{stat}}(2)_{\text{syst}}$		
Exp. data [19]	$2.46794^{+0.00017}_{-0.00020}$	2.5784 ± 0.0005	—		

m_c dependence

★ In HQET, the mixing would vanish in the heavy-quark limit.

★ Valence quark mass is tunable in lattice QCD.

◆ Solve the baryon masses and mixing angle from **different charm quark masses**;

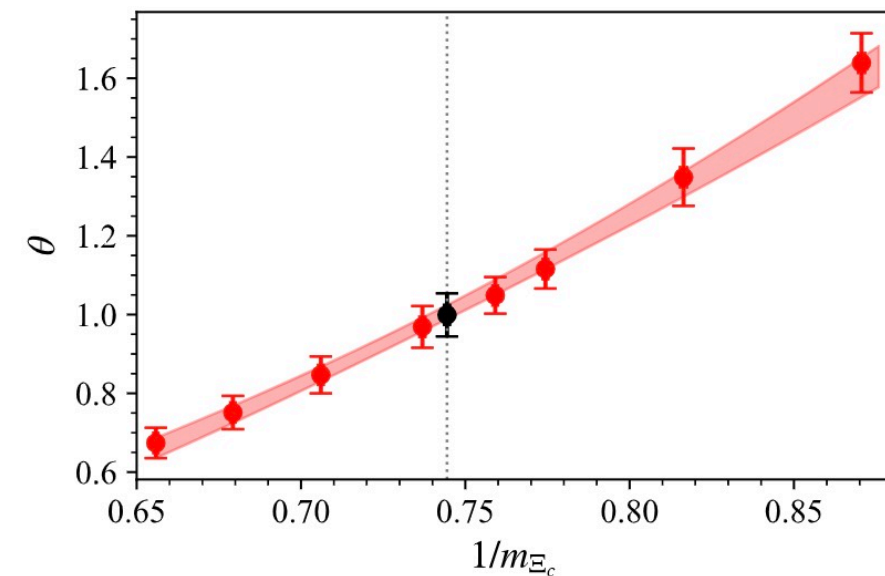
◆ Then fit the m_c dependence of θ :

$$\theta = \frac{B_1}{m_{\Xi_c}} + \frac{B_2}{m_{\Xi_c}^2}$$

$$B_1 = -2.78(52)\text{GeV}$$

$$B_2 = 12.9(1.3)\text{GeV}^2$$

$$\chi^2/\text{d.o.f} = 0.11.$$



m_c^b	0.2	0.3	0.4	0.44	0.478	0.5	0.6	0.7	0.8
m_{Ξ_c} (GeV)	2.0987(25)	2.2380(28)	2.3594(26)	2.4069(26)	2.4587(27)	2.4793(29)	2.5878(30)	2.6898(30)	2.7859(31)
$m_{\Xi_c'}$ (GeV)	2.1834(24)	2.3249(29)	2.4514(24)	2.4999(24)	2.5536(29)	2.5718(29)	2.6823(29)	2.7859(30)	2.8835(30)
θ (°)	1.639(75)	1.349(73)	1.116(49)	1.049(46)	1.002(50)	0.969(53)	0.847(47)	0.751(42)	0.674(39)

An improved method of $\Xi_c - \Xi'_c$ mixing

◆ The QCD Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - M)\psi$$

with

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$\Delta\mathcal{L} = -\bar{s}(m_s - m_u)s.$$

◆ Therefore, the Hamiltonian is correspondingly derived as

$$H = \int d^3\vec{x} \left[\frac{\partial\mathcal{L}}{\partial\dot{\psi}(\vec{x})} \dot{\psi}(\vec{x}) + \frac{\partial\mathcal{L}}{\partial\dot{\bar{\psi}}(\vec{x})} \dot{\bar{\psi}}(\vec{x}) - \mathcal{L} \right] \\ \equiv H_0 + \Delta H,$$

with

$$\Delta H = (m_s - m_u) \int d^3\vec{x} \bar{s}s(\vec{x}).$$

SU(3) flavor
symmetry breaking

Energy(mass) eigenstates and Flavor eigenstates

$$M_E(\vec{p}) \equiv \int \frac{d^3\vec{p}'}{(2\pi)^3} \times \begin{pmatrix} \langle \Xi_c(\vec{p}) | H | \Xi_c(\vec{p}') \rangle & \langle \Xi_c(\vec{p}) | H | \Xi'_c(\vec{p}') \rangle \\ \langle \Xi'_c(\vec{p}) | H | \Xi_c(\vec{p}') \rangle & \langle \Xi'_c(\vec{p}) | H | \Xi'_c(\vec{p}') \rangle \end{pmatrix}$$



$$M_E(\vec{p}=0) \equiv \begin{pmatrix} 2m_{\Xi_c}^2 & 0 \\ 0 & 2m_{\Xi'_c}^2 \end{pmatrix}.$$

Rotation

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi'_c\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\Xi_c^{\bar{3}}\rangle \\ |\Xi_c^6\rangle \end{pmatrix}$$

$$\begin{aligned} M_{F,11} &= 2\cos^2\theta m_{\Xi_c}^2 + 2\sin^2\theta m_{\Xi'_c}^2, \\ M_{F,12} &= 2\cos\theta\sin\theta(m_{\Xi_c}^2 - m_{\Xi'_c}^2), \\ M_{F,21} &= 2\cos\theta\sin\theta(m_{\Xi_c}^2 - m_{\Xi'_c}^2), \\ M_{F,22} &= 2\sin^2\theta m_{\Xi_c}^2 + 2\cos^2\theta m_{\Xi'_c}^2, \end{aligned}$$

$$\begin{aligned} M_F(\vec{p}) &\equiv \int \frac{d^3\vec{p}'}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p}) | H | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^{\bar{3}}(\vec{p}) | H | \Xi_c^6(\vec{p}') \rangle \\ \langle \Xi_c^6(\vec{p}) | H | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^6(\vec{p}) | H | \Xi_c^6(\vec{p}') \rangle \end{pmatrix} \\ &= \int \frac{d^3\vec{p}'}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p}) | (H_0 + \Delta H) | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^{\bar{3}}(\vec{p}) | \Delta H | \Xi_c^6(\vec{p}') \rangle \\ \langle \Xi_c^6(\vec{p}) | \Delta H | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^6(\vec{p}) | (H_0 + \Delta H) | \Xi_c^6(\vec{p}') \rangle \end{pmatrix}. \end{aligned}$$

$|\Xi_c^{\bar{3}/6}\rangle$ are eigenstate of H_0
under the $SU(3)_F$ symmetry



$$\begin{aligned} M_F(\vec{p}=0) &= \begin{pmatrix} 2m_{\Xi_c^{\bar{3}}}^2 & 0 \\ 0 & 2m_{\Xi_c^6}^2 \end{pmatrix} + (m_s - m_u) \\ &\times \begin{pmatrix} \langle \Xi_c^{\bar{3}} | \bar{s}s(\vec{x}=0) | \Xi_c^{\bar{3}} \rangle & \langle \Xi_c^{\bar{3}} | \bar{s}s(\vec{x}=0) | \Xi_c^6 \rangle \\ \langle \Xi_c^6 | \bar{s}s(\vec{x}=0) | \Xi_c^{\bar{3}} \rangle & \langle \Xi_c^6 | \bar{s}s(\vec{x}=0) | \Xi_c^6 \rangle \end{pmatrix} \end{aligned}$$

$$M_{F,11} = 2m_{\Xi_c^{\bar{3}}}^2 + (m_s - m_u)M_{\bar{s}s}^{\bar{3}-\bar{3}}$$

$$M_{F,22} = 2m_{\Xi_c^6}^2 + (m_s - m_u)M_{\bar{s}s}^{6-6}$$

$$M_{F,12} = (m_s - m_u)M_{\bar{s}s}^{\bar{3}-6}$$

$$M_{F,21} = (m_s - m_u)M_{\bar{s}s}^{6-\bar{3}}$$

Energy(mass) eigenstates and Flavor eigenstates

$$M_E(\vec{p}) \equiv \int \frac{d^3\vec{p}'}{(2\pi)^3} \times \begin{pmatrix} \langle \Xi_c(\vec{p}) | H | \Xi_c(\vec{p}') \rangle & \langle \Xi_c(\vec{p}) | H | \Xi'_c(\vec{p}') \rangle \\ \langle \Xi'_c(\vec{p}) | H | \Xi_c(\vec{p}') \rangle & \langle \Xi'_c(\vec{p}) | H | \Xi'_c(\vec{p}') \rangle \end{pmatrix}$$



$$M_E(\vec{p}=0) \equiv \begin{pmatrix} 2m_{\Xi_c}^2 & 0 \\ 0 & 2m_{\Xi'_c}^2 \end{pmatrix}.$$

Rotation

$$\begin{pmatrix} |\Xi_c\rangle \\ |\Xi'_c\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\Xi_c^{\bar{3}}\rangle \\ |\Xi_c^6\rangle \end{pmatrix}$$

$$\begin{aligned} M_{F,11} &= 2\cos^2\theta m_{\Xi_c}^2 + 2\sin^2\theta m_{\Xi'_c}^2, \\ M_{F,12} &= 2\cos\theta\sin\theta(m_{\Xi_c}^2 - m_{\Xi'_c}^2), \\ M_{F,21} &= 2\cos\theta\sin\theta(m_{\Xi_c}^2 - m_{\Xi'_c}^2), \\ M_{F,22} &= 2\sin^2\theta m_{\Xi_c}^2 + 2\cos^2\theta m_{\Xi'_c}^2, \end{aligned}$$

$$\sin 2\theta = \pm \frac{(m_s - m_u)M_{\bar{s}s}^{6-\bar{3}}}{m_{\Xi'_c}^2 - m_{\Xi_c}^2},$$

$$M_F(\vec{p}) \equiv \int \frac{d^3\vec{p}'}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p}) | H | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^{\bar{3}}(\vec{p}) | H | \Xi_c^6(\vec{p}') \rangle \\ \langle \Xi_c^6(\vec{p}) | H | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^6(\vec{p}) | H | \Xi_c^6(\vec{p}') \rangle \end{pmatrix}$$

$$= \int \frac{d^3\vec{p}'}{(2\pi)^3} \begin{pmatrix} \langle \Xi_c^{\bar{3}}(\vec{p}) | (H_0 + \Delta H) | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^{\bar{3}}(\vec{p}) | \Delta H | \Xi_c^6(\vec{p}') \rangle \\ \langle \Xi_c^6(\vec{p}) | \Delta H | \Xi_c^{\bar{3}}(\vec{p}') \rangle & \langle \Xi_c^6(\vec{p}) | (H_0 + \Delta H) | \Xi_c^6(\vec{p}') \rangle \end{pmatrix}.$$

$|\Xi_c^{\bar{3}/6}\rangle$ are eigenstate of H_0
under the $SU(3)_F$ symmetry



$$M_F(\vec{p}=0) = \begin{pmatrix} 2m_{\Xi_c^{\bar{3}}}^2 & 0 \\ 0 & 2m_{\Xi_c^6}^2 \end{pmatrix} + (m_s - m_u)$$

$$\times \begin{pmatrix} \langle \Xi_c^{\bar{3}} | \bar{s}s(\vec{x}=0) | \Xi_c^{\bar{3}} \rangle & \langle \Xi_c^{\bar{3}} | \bar{s}s(\vec{x}=0) | \Xi_c^6 \rangle \\ \langle \Xi_c^6 | \bar{s}s(\vec{x}=0) | \Xi_c^{\bar{3}} \rangle & \langle \Xi_c^6 | \bar{s}s(\vec{x}=0) | \Xi_c^6 \rangle \end{pmatrix}$$

$$M_{F,11} = 2m_{\Xi_c^{\bar{3}}}^2 + (m_s - m_u)M_{\bar{s}s}^{\bar{3}-\bar{3}}$$

$$M_{F,22} = 2m_{\Xi_c^6}^2 + (m_s - m_u)M_{\bar{s}s}^{6-6}$$

$$M_{F,12} = (m_s - m_u)M_{\bar{s}s}^{\bar{3}-6}$$

$$M_{F,21} = (m_s - m_u)M_{\bar{s}s}^{6-\bar{3}}$$

Extraction of the matrix elements

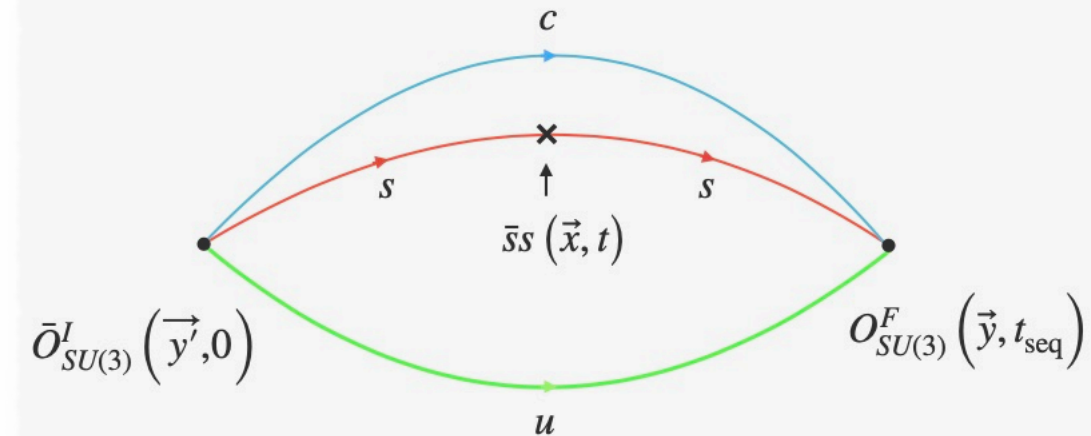
◆ We want

$$M_{\bar{s}s}^{F-I} \equiv \langle \Xi_c^F(\vec{p}=0) | \bar{s}s(x=0) | \Xi_c^I(\vec{p}'=0) \rangle,$$

◆ We construct and simulate

$$\begin{aligned} O_{SU(3)}^{\bar{3}} &= \epsilon^{abc} (q^{Ta} C \gamma_5 s^b) P_+ c^c, \\ O_{SU(3)}^6 &= \epsilon^{abc} (q^{Ta} C \vec{\gamma} s^b) \cdot \vec{\gamma} \gamma_5 P_+ c^c. \end{aligned}$$

Operators of SU(3)
flavor eigenstates $\bar{3}$ and 6



$$\left\{ \begin{aligned} C_3^{F-I}(t_{\text{seq}}, t) &= \int \frac{d^3 \vec{q}}{(2\pi)^3} \int d^3 \vec{y} d^3 \vec{y}' d^3 \vec{x} e^{i\vec{q} \cdot \vec{x}} T_{\gamma' \gamma} \langle O_{\gamma, SU(3)}^F(\vec{y}, t_{\text{seq}}) \bar{s}s(\vec{x}, t) \bar{O}_{\gamma', SU(3)}^I(\vec{y}', 0) \rangle, & \text{Three-point function} \\ C_2^{\bar{3}/6}(t) &= \int d^3 \vec{y} T_{\gamma' \gamma} \langle O_{\gamma, SU(3)}^{\bar{3}/6}(\vec{y}, t) \bar{O}_{\gamma', SU(3)}^{\bar{3}/6}(\vec{0}, 0) \rangle & \text{Two-point function} \end{aligned} \right.$$



Inserting the hadronic states and keeping the lowest two

$$\left\{ \begin{aligned} C_3^{F-I}(t_{\text{seq}}, t) &= \frac{M_{\bar{s}s}^{F-I}}{\sqrt{4m_{\Xi_c^I} m_{\Xi_c^F}}} f_{\Xi_c^I} f_{\Xi_c^F} m_{\Xi_c^I}^2 m_{\Xi_c^F}^2 e^{-(m_{\Xi_c^I} - m_{\Xi_c^F})t} e^{-m_{\Xi_c^F} t_{\text{seq}}} \left(1 + c_1 e^{-\Delta m_{\Xi_c^I} t}\right) \left(1 + c_2 e^{-\Delta m_{\Xi_c^F} (t_{\text{seq}} - t)}\right), \\ C_2^{\bar{3}/6}(t) &= f_{\Xi_c^{\bar{3}/6}}^2 m_{\Xi_c^{\bar{3}/6}}^4 e^{-m_{\Xi_c^{\bar{3}/6}} t} (1 + d_i e^{-\Delta m_{\Xi_c^{\bar{3}/6}} t}) \end{aligned} \right.$$

Extraction of the matrix elements

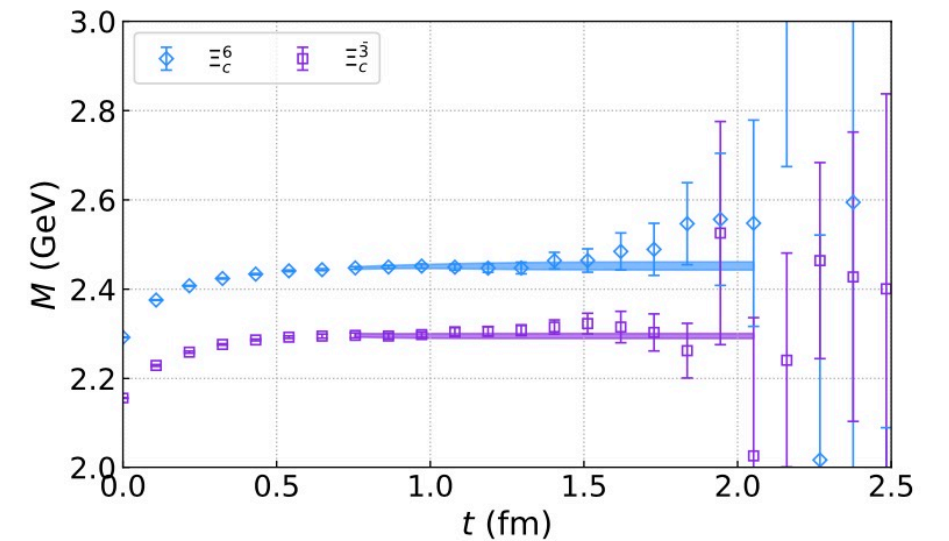
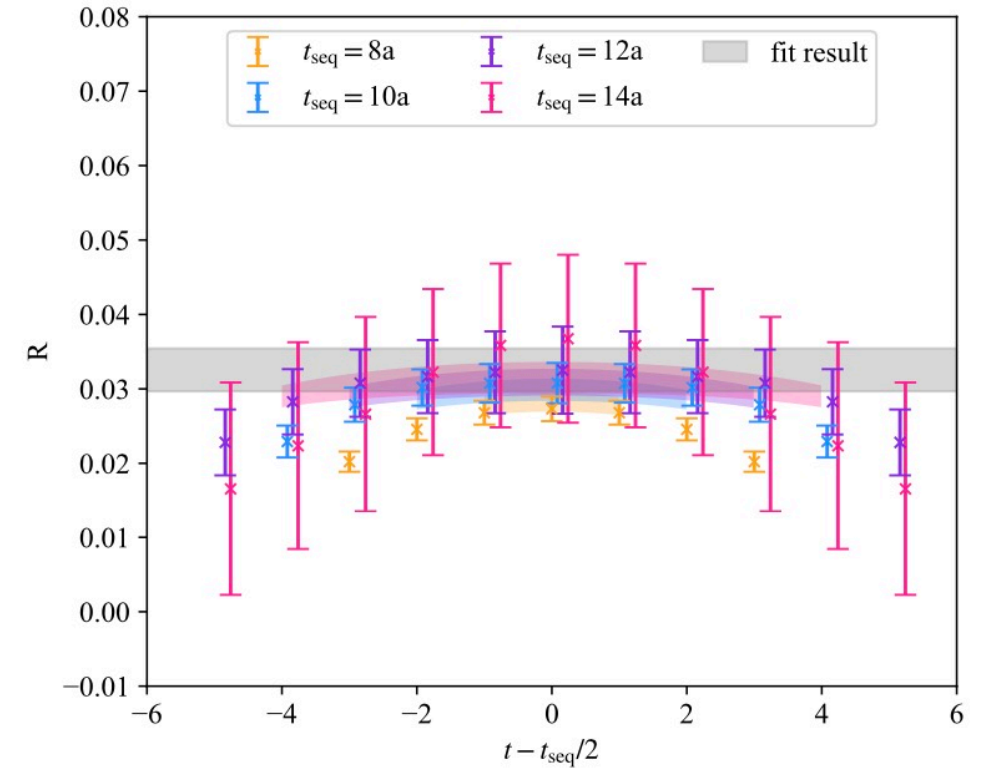
- ◆ Combining the 3pt and 2pt, one can remove the dependence on the decay constants

$$R = \sqrt{\frac{C_3^{FI}(t_{\text{seq}}, t) C_3^{FI}(t_{\text{seq}}, t_{\text{seq}} - t)}{C_2^I(t_{\text{seq}}) C_2^F(t_{\text{seq}})}}$$

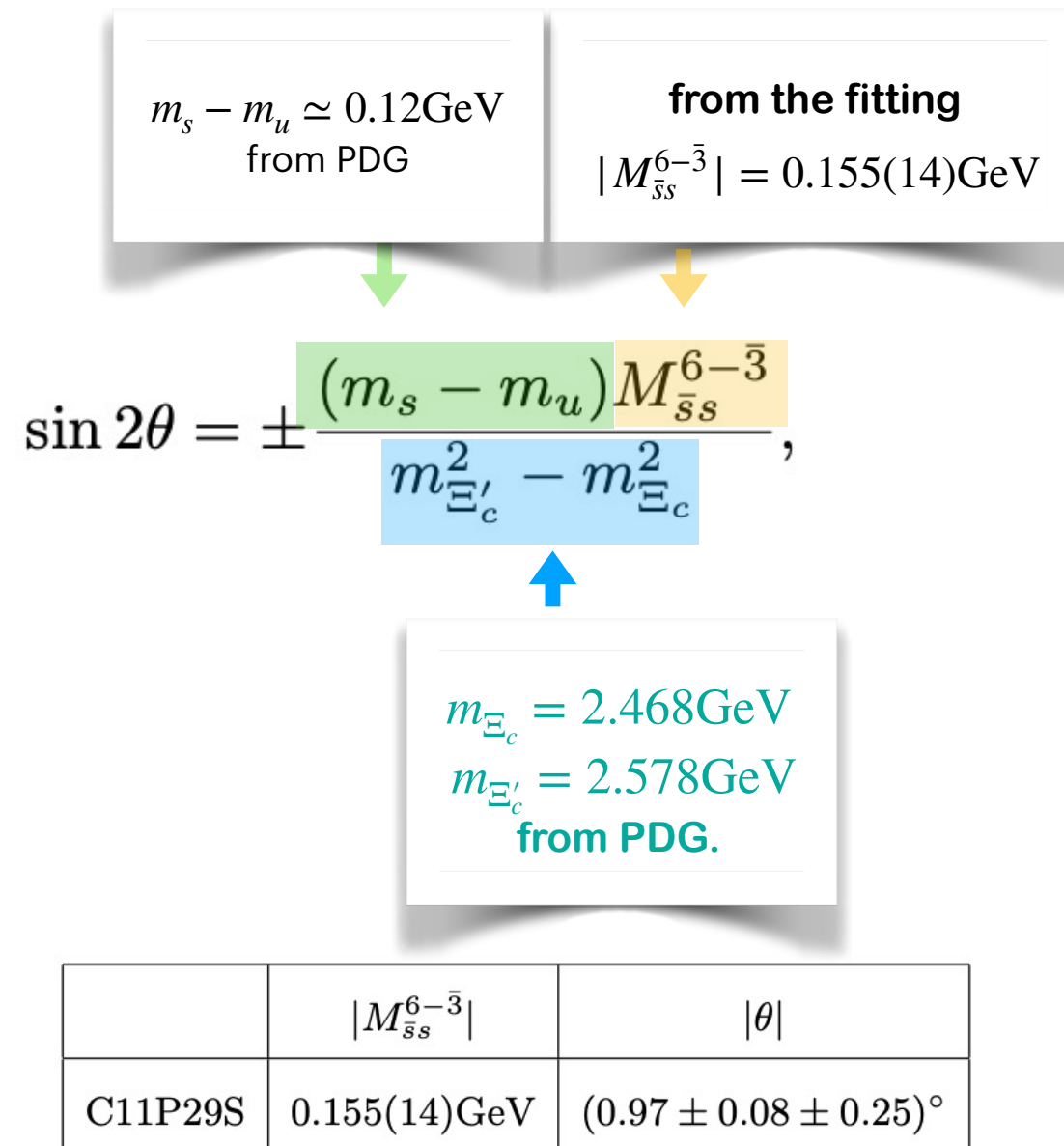
R can be parameterized as

$$R = \frac{|M_{ss}^{F-I}|}{2\sqrt{m_{\Xi_c^I} m_{\Xi_c^F}}} \left(\frac{(1 + c_1 e^{-\Delta m_{\Xi_c^I} t})(1 + c_1 e^{-\Delta m_{\Xi_c^I}(t_{\text{seq}} - t)})(1 + c_2 e^{-\Delta m_{\Xi_c^F} t})(1 + c_2 e^{-\Delta m_{\Xi_c^F}(t_{\text{seq}} - t)})}{(1 + d_1 e^{-\Delta m_{\Xi_c^F} t_{\text{seq}}})(1 + d_2 e^{-\Delta m_{\Xi_c^I} t_{\text{seq}}})} \right)^{1/2}$$

$$\simeq \frac{|M_{ss}^{F-I}|}{2\sqrt{m_{\Xi_c^I} m_{\Xi_c^F}}} \left(\frac{(1 + c_1 e^{-\Delta m_{\Xi_c^I} t} + c_2 e^{-\Delta m_{\Xi_c^F}(t_{\text{seq}} - t)})(1 + c_1 e^{-\Delta m_{\Xi_c^I}(t_{\text{seq}} - t)} + c_2 e^{-\Delta m_{\Xi_c^F} t})}{(1 + d_1 e^{-\Delta m_{\Xi_c^I} t_{\text{seq}}})(1 + d_2 e^{-\Delta m_{\Xi_c^F} t_{\text{seq}}})} \right)^{1/2},$$



Numerical results of the mixing angle



The mixing angle is about 1° and consistent with the previous lattice investigation.

$\theta = 0.04^\circ$ from QED correction (Z.F. Deng, Y.J. Shi, W. Wang, J. Zeng, arxiv: 2309.16386)

SUMMARY

We have explored the $\Xi_c - \Xi'_c$ mixing by two different methods

- ★ Calculate the two-point correlation matrix and adopt two independent methods to determine the mixing angle;
- ★ Develop an improved method to explore the mixing which arises from the SU(3) flavor symmetry breaking effects.

Our numerical results are consistent and are not able to explain the large SU(3) symmetry breaking in semileptonic charmed baryon decays.

OUTLOOK

- ☆ The two methods can be used to other interesting examples such as the $K_1(1270)$ and $K_1(1400)$ mixing which exhibit effects on multiple decay channels...

Thank you!

