Factorization of Non-Global LHC Observables and Resummation of Super-Leading Logarithms

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Based on PRL127(2021)212002 and arXiv:2307.06359

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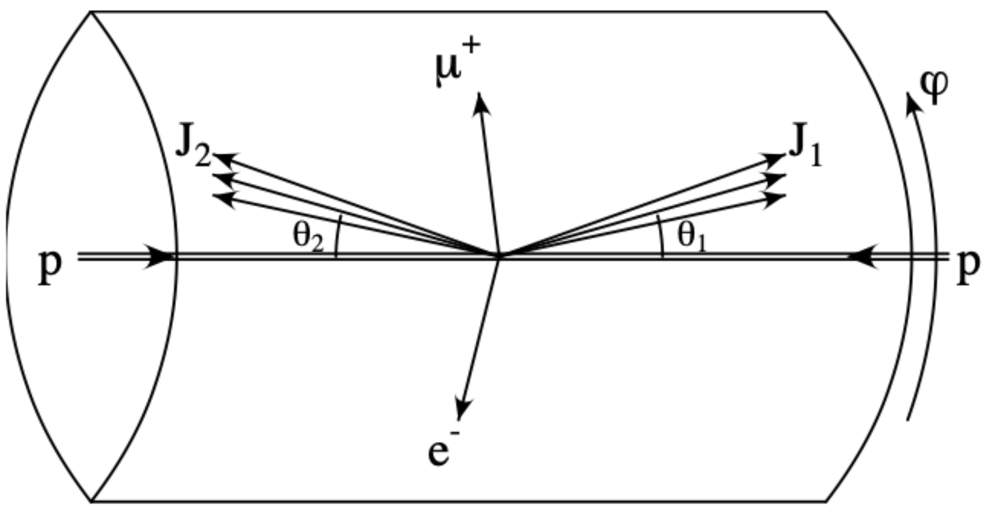
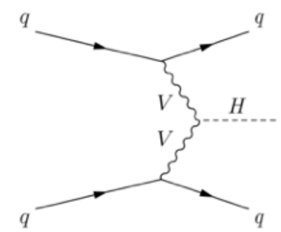
Central jet veto in Higgs production via VBF

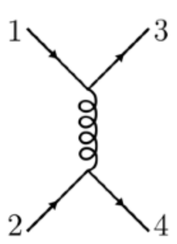


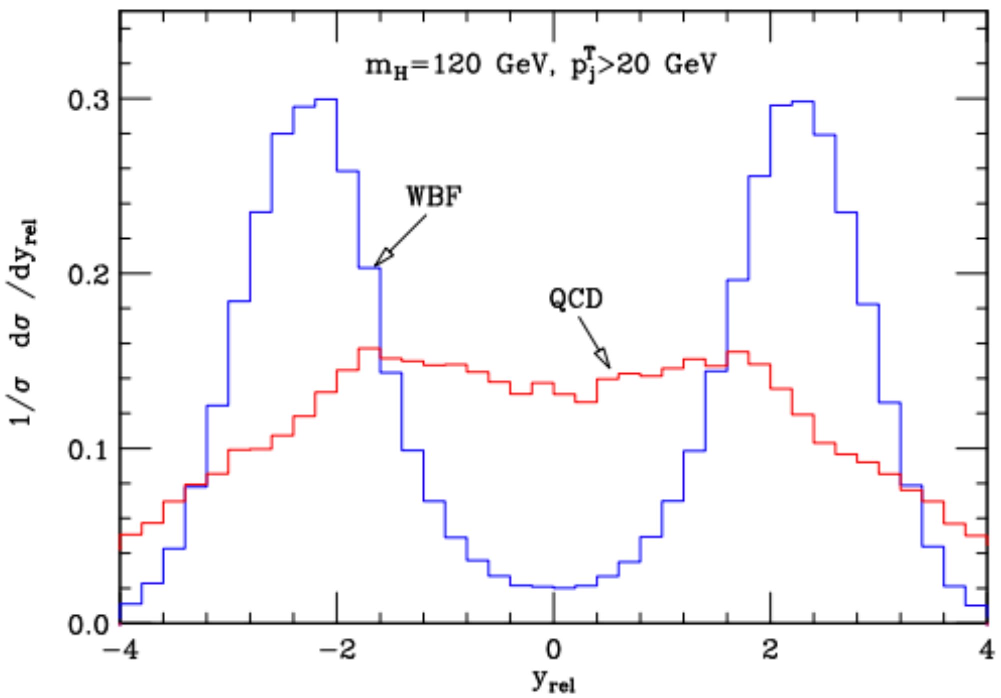
VBF signature:



- **Energetic jets in the forward and backward directions**
- **Large rapidity separation and large invariant mass of two tagged jets**
- **Little radiation in the central-rapidity region**

- **Major QCD backgrounds: t-channel color octet exchange**
- **Central jet veto can suppresses QCD background**
- **Central jet veto: no extra jets between tagging jets**





Del Duca, Frizzo, Maltoni '05

Jet veto & QCD resummation

- Due to existence of a small scale $p_T^{\rm veto}$, the fixed order calculations are unreliable
- QCD resummation is necessary, the large log should be resumed to all order
- Standard jet veto resummation for gg->H processes
  - Rapidity cut independent

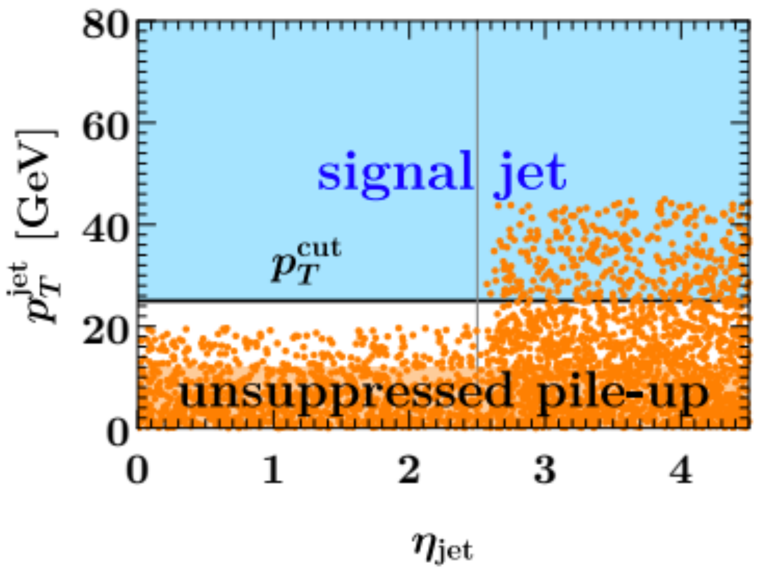
    Banfi, Monni, Salam, Zanderighi ’12;

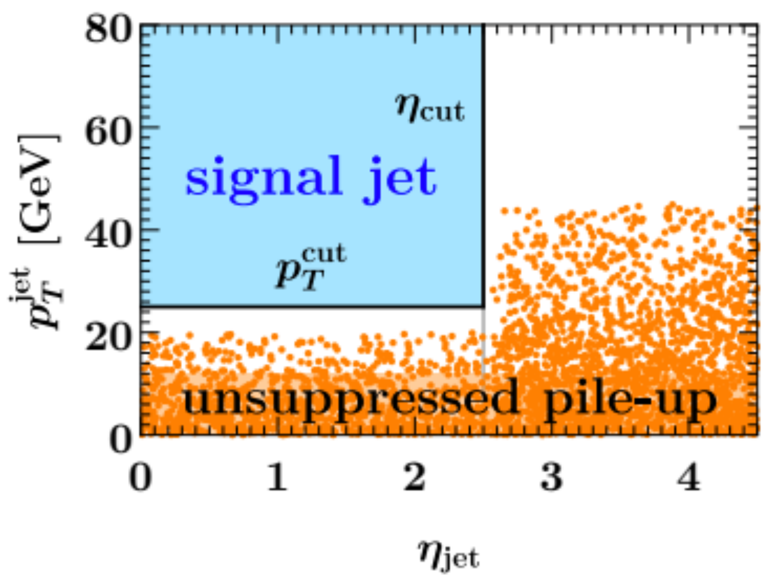
    Becher, Neubert, Rothen ’12, ’13;

    Stewart, Tackmann, Walsh, Zuberi ’12, ’13

    NNLO gluon Beam function (Bell, Brune, Das, DYS, Wald 2311.XXXXX)
  - Rapidity cut dependent

    Michel, Pietrulewicz, Tackmann ’18
  - Jet veto in VBF: Superleading Logs
    - **Four-loop** Forshaw, Kyrieleis, Seymour ’06
    - **Five-loop** Keates, Seymour ‘09
    - **All-order** Becher, Neubert, DYS ’21 + Stillger ‘23





Courtesy of Johannes Michel

Jet veto & QCD resummation

- **Due to existence of a small scale p_T^{veto}, the fixed order calculations are unreliable**
- **QCD resummation is necessary, the large log should be resumed to all order**
- **Standard jet veto resummation for gg->H processes**
  - **Rapidity cut independent**

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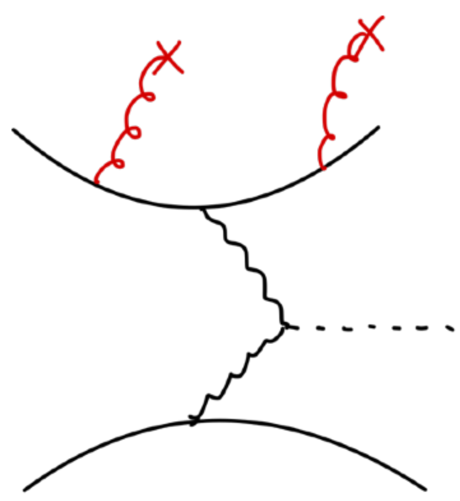
    NNLO gluon Beam function (Bell, Brune, Das, DYS, Wald 2311.XXXXX)



  - **Rapidity cut dependent**

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  - **Jet veto in VBF: Superleading Logs**
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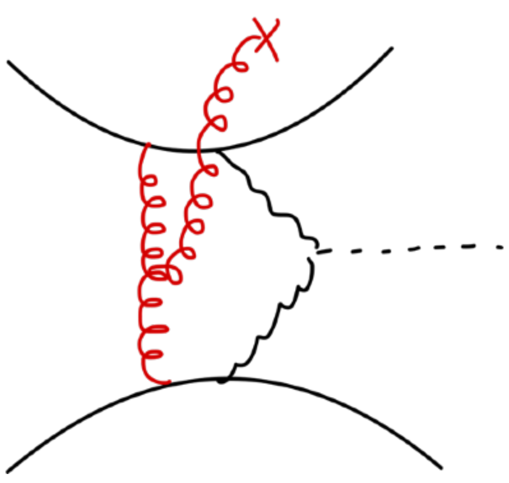
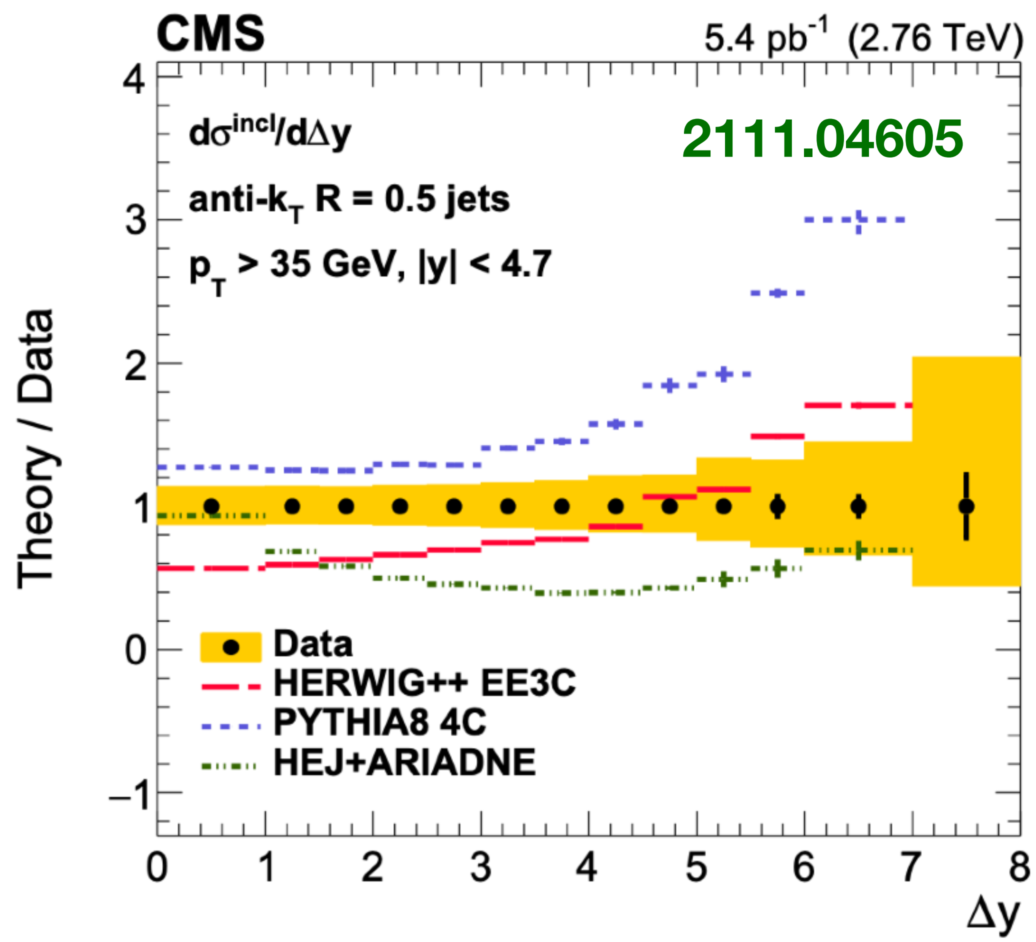
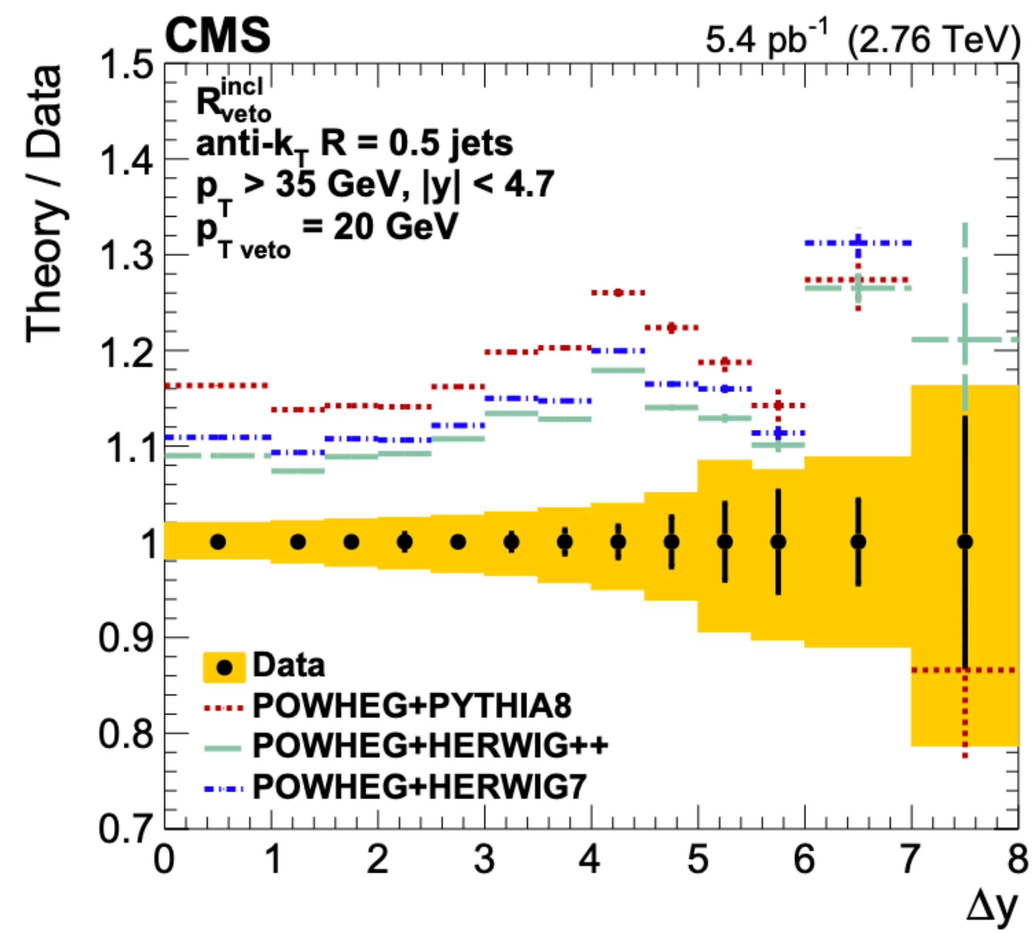
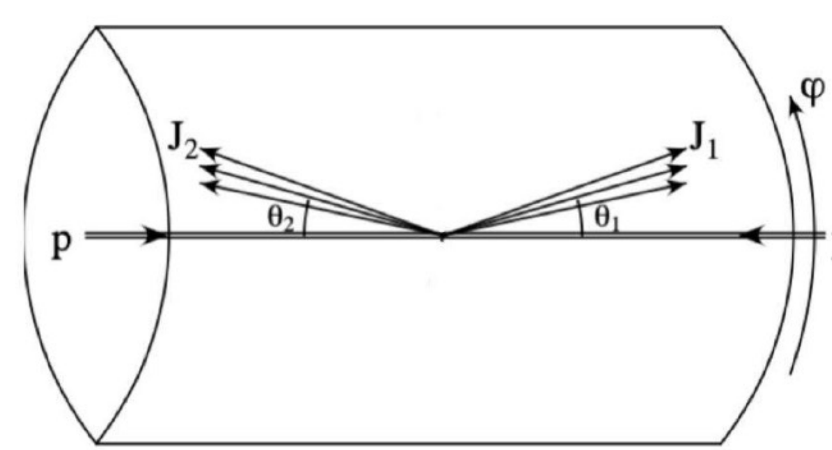


Dijet events with large rapidity gap at the LHC

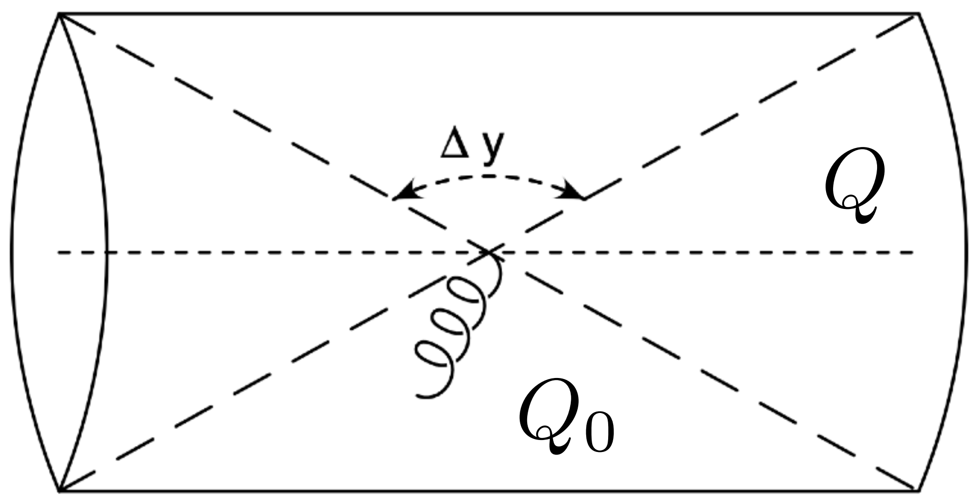




None of the DGLAP-based Monte Carlo generators using LO or NLO calculations can provide a complete description of all measured cross sections and their ratios



Central jet veto at the LHC

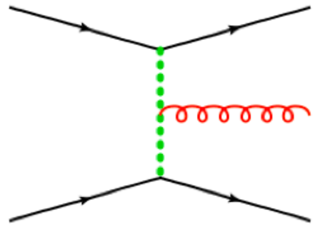


leading logs:

$$e^+e^-, ep: \quad \alpha_s^n \ln^n\left(\frac{Q}{Q_0}\right) \qquad \textbf{Soft Logs}$$

$$pp: \qquad \cdots \qquad + \alpha_s^3 (i\pi)^2 \ln^3\left(\frac{Q}{Q_0}\right) \times \alpha_s^n \ln^{2n}\left(\frac{Q}{Q_0}\right)$$

- **Such events was originally suggested on the basis of color flow considerations in QCD Bjorken '93**
- **Global Logs resummation is first done by Oderda & Sterman '98**
- **Forshaw, Kyrieleis, Seymour '06 have analyzed the effect of Glauber phases in non-global observables directly in QCD**
  - **Collinear logarithms starting at 4 loops: Super-leading logs**



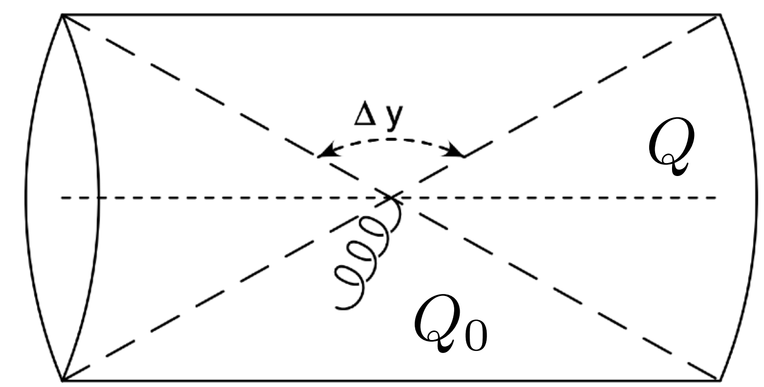
- **Leading order resummation is unknown, process dependence is unknown,**
  - **At forth order there are 1,746,272 diagrams !!!**
- **We apply renormalization-group approach and obtain the all-order results of leading SLLs Becher, Neubert, DYS '21 PRL + Stillger '23**

An effective field theory for jet processes (e^+e^-)

Becher, Neubert, Rothen, DYS '16 PRL

EFT contains two modes:

$$\begin{aligned} \text{hard:} \quad & p_h \sim \quad Q\,(1,1,1) \\ \text{soft:} \quad & p_s \sim \quad Q_0(1,1,1) \end{aligned}$$



1. no collinear singularity, only single logs

2. method of region to verify at two-loop level

Hard parton described by collinear field $\Phi_i \in \{\chi_i, \bar{\chi}_i, \mathcal{A}^{\mu}_{i\perp}\}$

gauge invariant: $\chi_i(0) = W_i^\dagger(\bar{n}_i)\,\frac{\not{n}_i\bar{\not{n}}_i}{4}\,\psi_i(0)$

Perform decoupling transformation: $\Phi_i = \boldsymbol{S}_i(n_i)\,\Phi_i^{(0)}$ $\quad \boldsymbol{S}_i(n_i) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n_i\cdot A_s^a(sn_i)\,\boldsymbol{T}_i^a\right)$

Evaluates the matrix element of the operator with one collinear particle

$$\langle 0|\,\chi_j^{(0)}(0)\,|p_i\rangle = \delta_{ij}\,u(p_i)$$

Factorization of jet cross sections at hadron colliders

(Balsiger, Becher, DYS '18, Becher, Neubert, DYS '21 PRL)

$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, x_1, x_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle .$$

Hard function

m hard partons along fixed directions $\{n_1, \ldots, n_m\}$

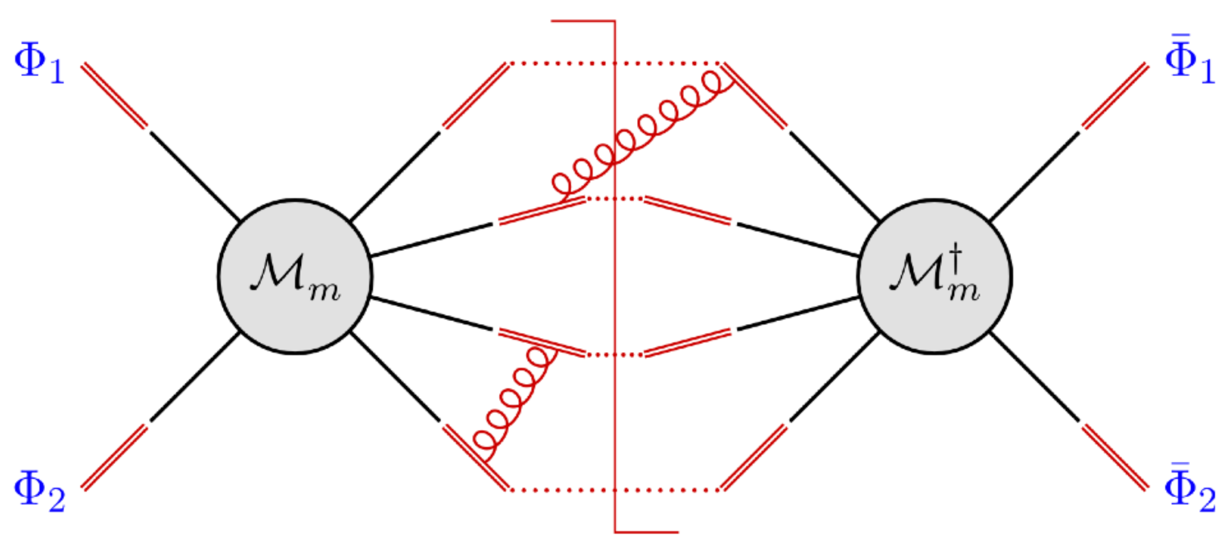
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

$$\boldsymbol{\mathcal{H}}_m = \frac{1}{2x_1 x_2 s} \prod_{i=3}^{m} \int \frac{dE_i\, E_i^{d-3}}{\tilde{c}^{\epsilon}\,(2\pi)^2} |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})| \\ \times (2\pi)^d\, 2\, \delta(\bar{n}_1 \cdot p_{\rm tot} - x_1\sqrt{s})\, \delta(\bar{n}_2 \cdot p_{\rm tot} - x_2\sqrt{s})\, \delta^{(d-2)}(p_{\rm tot}^{\perp})\, \Theta_{\rm hard}(\{\underline{n}\})$$

Collinear+Soft functions

squared amplitude with *m* Wilson lines

$$\boldsymbol{\mathcal{W}}_m = \sum_{X_s} \mathcal{P}^{(1)}_{\bar{\alpha}\alpha}\, \mathcal{P}^{(2)}_{\bar{\beta}\beta}\, \langle H_1(p_1) H_2(p_2)|\, \bar{\Phi}_1^{\bar{\alpha}}(t_1\bar{n}_1)\, \bar{\Phi}_2^{\bar{\beta}}(t_2\bar{n}_2)\, \boldsymbol{S}_1^{\dagger}(n_1) \ldots \boldsymbol{S}_m^{\dagger}(n_m)\, |X_s\rangle \\ \times \langle X_s|\, \boldsymbol{S}_1(n_1) \ldots \boldsymbol{S}_m(n_m)\, \Phi_1^{\alpha}(0)\, \Phi_2^{\beta}(0)\, |H_1(p_1) H_2(p_2)\rangle\, \theta(Q_0 - E_{\rm out}^{\perp}) ,$$

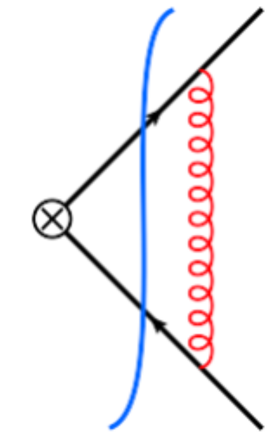


One-loop anomalous dimension:

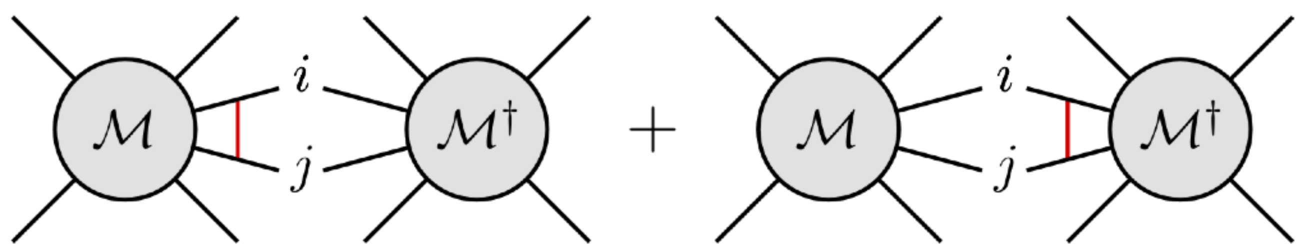
$$\Gamma^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\begin{aligned} \boldsymbol{V}_m = & -2 \sum_{(ij)} \int \frac{d\Omega(n_k)}{4\pi} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) W_{ij}^k \left[\Theta_{\text{in}}^{n\bar{n}}(k) + \Theta_{\text{out}}^{n\bar{n}}(k) \right] \\ & + 2\, i\pi \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \Pi_{ij} , \\ \boldsymbol{R}_m = & 4 \sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\, \Theta_{\text{in}}(n_{m+1}) \, . \end{aligned}$$

$\Pi_{ij} = 1$ **if both incoming or outgoing**



$$\mathcal{H}_m \, \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



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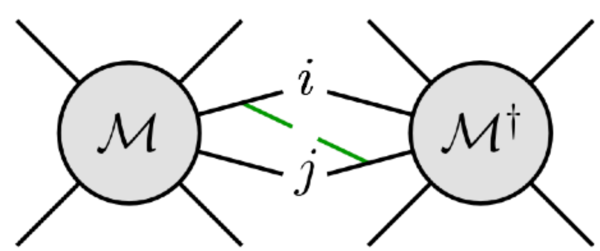

One-loop anomalous dimension:

$$\begin{aligned}\boldsymbol{V}_m =& -2\sum_{(ij)}\int\frac{d\Omega(n_k)}{4\pi}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}+\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)W_{ij}^{k}\left[\Theta_{\text{in}}^{n\bar{n}}(k)+\Theta_{\text{out}}^{n\bar{n}}(k)\right]\\ &+2\,i\pi\sum_{(ij)}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}-\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\Pi_{ij}\,,\\ \boldsymbol{R}_m =& 4\sum_{(ij)}\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\,W_{ij}^{m+1}\,\Theta_{\text{in}}(n_{m+1})\,.\end{aligned}$$

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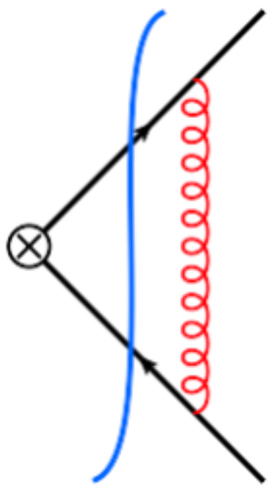


Imaginary part of the anomalous dimension:

For e+e-:

$$\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j=-\sum_i\mathbf{T}_i^2=-\sum_i C_i$$

For pp:

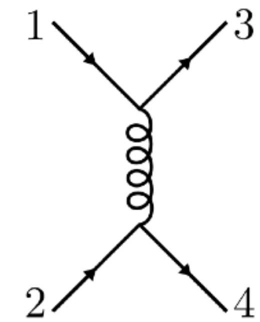
$$\begin{aligned}\sum_{(ij)}\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\Pi_{ij}&=2\,\boldsymbol{T}_1\cdot\boldsymbol{T}_2+\sum_{i=3}^{m}\boldsymbol{T}_i\cdot(-\boldsymbol{T}_1-\boldsymbol{T}_2-\boldsymbol{T}_i)\\ &=2\,\boldsymbol{T}_1\cdot\boldsymbol{T}_2+(\boldsymbol{T}_1+\boldsymbol{T}_2)\cdot(\boldsymbol{T}_1+\boldsymbol{T}_2)-\sum_{i=3}^{m}C_i^2\\ &=\boxed{4\,\boldsymbol{T}_1\cdot\boldsymbol{T}_2}+C_1^2+C_2^2-\sum_{i=3}^{m}C_i^2\end{aligned}$$

Matrix in color space

Super-leading logs from RG evolution

(Becher, Neubert, DYS, '21 PRL)

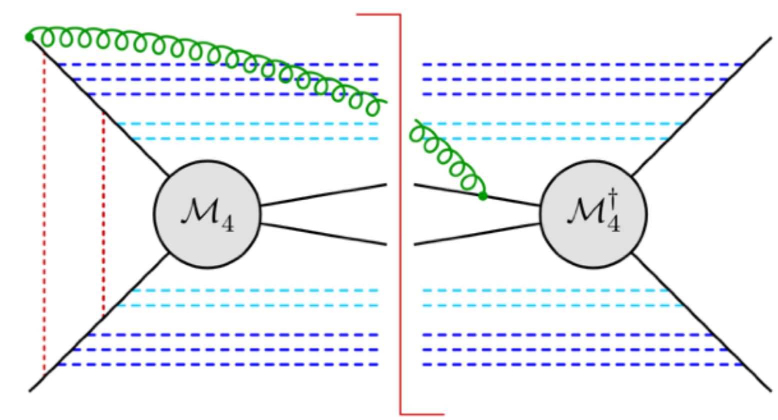
E.g. Consider the processes $q_1\, q_2' \to q_3\, q_4'$



LO hard function: $\mathcal{H}_4 = t^a_{\alpha_3\alpha_1} t^a_{\alpha_4\alpha_2} t^b_{\beta_1\beta_3} t^b_{\beta_2\beta_4} \sigma_0$

Expand the expansion kernel

$$\boldsymbol{\mathcal{H}}(\mu_h) \star \boldsymbol{U}(\mu_h, \mu_s) = \boldsymbol{\mathcal{H}}(\mu_h) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \boldsymbol{\mathcal{H}}(\mu_h) \star \boldsymbol{\Gamma}^H(\mu_1) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_1}^{\mu_h} \frac{d\mu_2}{\mu_2} \boldsymbol{\mathcal{H}}(\mu_h) \star \boldsymbol{\Gamma}^H(\mu_2) \star \boldsymbol{\Gamma}^H(\mu_1) + \dots$$



The first non-zero contribution of has factor arises at 3-loop order

$$S^{(3)} = \langle \boldsymbol{\mathcal{H}}_4 \boldsymbol{V}^I \boldsymbol{V}^I (\overline{\boldsymbol{V}}_4 + \overline{\boldsymbol{R}}_4) \rangle \left(\frac{\alpha_s}{4\pi}\right)^3 \frac{1}{3!} \ln^3\left(\frac{Q}{\mu}\right) = -\left(\frac{\alpha_s}{4\pi}\right)^3 \frac{16\, C_F}{3} \pi^2 L_Q^3\, J_1\, \sigma_0$$

Super-leading logs at 4-loop order:

$$J_1 = 2\Delta Y\, \text{sign}(\eta_J)$$

$$S_0^{(4)} = \left\langle \boldsymbol{\mathcal{H}}_4 \boldsymbol{V}^I \left[\sum_{i=1}^{4} \boldsymbol{V}_i^L \boldsymbol{V}^I (\overline{\boldsymbol{V}}_4 + \overline{\boldsymbol{R}}_4) + \sum_{i=1}^{4} \boldsymbol{R}_i^L \boldsymbol{V}^I (\overline{\boldsymbol{V}}_5 + \overline{\boldsymbol{R}}_5) \right] \right\rangle \left(\frac{\alpha_s}{4\pi}\right)^4 \frac{(-2)}{5!} \ln^5\left(\frac{Q}{\mu}\right)$$

All-order results of leading Super-Leading Logs

(Becher, Neubert, DYS ’21 PRL + Stillger ’23)

All-order structure: Kampe de Feriet function (a two-variable generalization of the generalized hypergeometric series)

$$\Sigma(v,w) = \sum_{m=0}^{\infty}\sum_{r=0}^{\infty} \frac{(1)_{m+r}\,(1)_m\,\left(\frac{1}{2}\right)_r}{(2)_{m+r}\left(\frac{5}{2}\right)_{m+r}}\,\frac{(-w)^m\,(-vw)^r}{m!\,r!}$$

$$w = \frac{N_c\alpha_s(\bar{\mu})}{\pi}\ln^2\left(\frac{\mu_h}{\mu_s}\right)$$

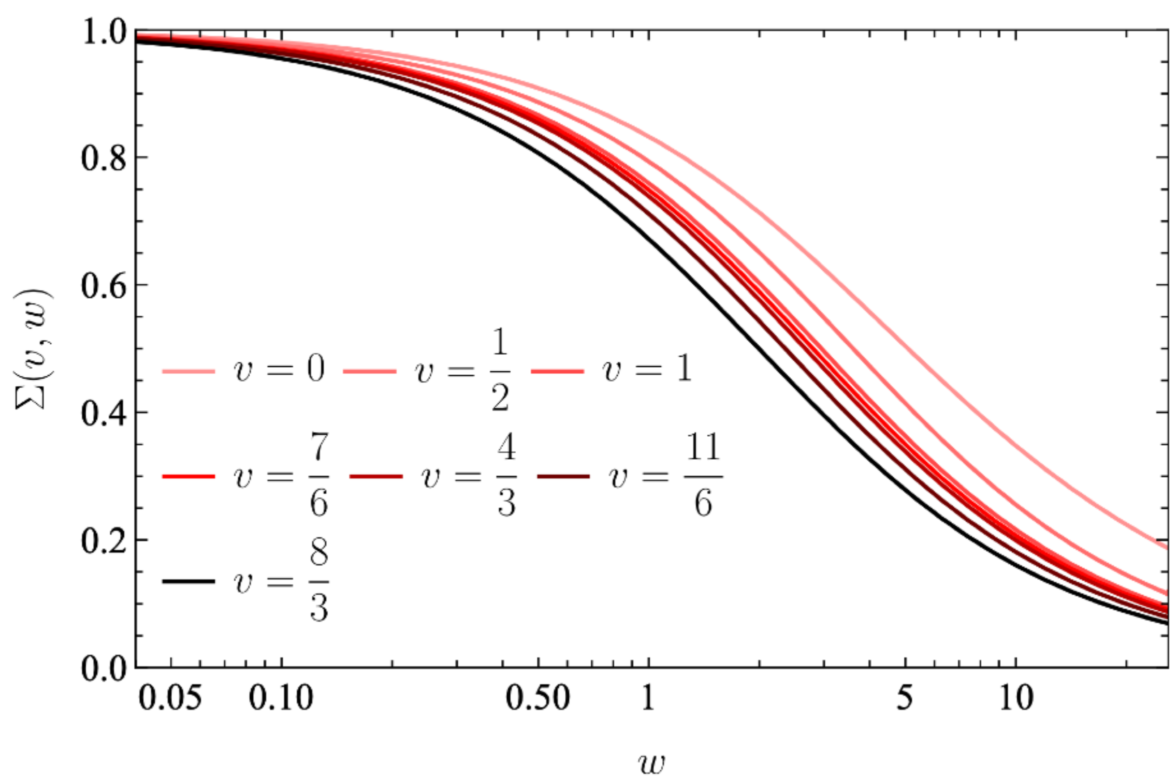
$$= {}^{1+1}F_{2+0}\left(\begin{matrix} 1\,:\,1,\,\frac{1}{2}\,; \\ 2,\,\frac{5}{2}\,:\quad ; \end{matrix}\,,-w,-vw\right)$$

$$v_1 = \frac{1}{2}\,,\qquad v_2 = 1\,,\qquad v_{3,4} = \frac{3N_c \pm 2}{2N_c}\,,\qquad v_{5,6} = \frac{2\,(N_c \pm 1)}{N_c}$$

Sudakov suppression of the superleading logarithms is weaker than the one present for global observables

Global logs $\longrightarrow$ $e^{-\omega}$

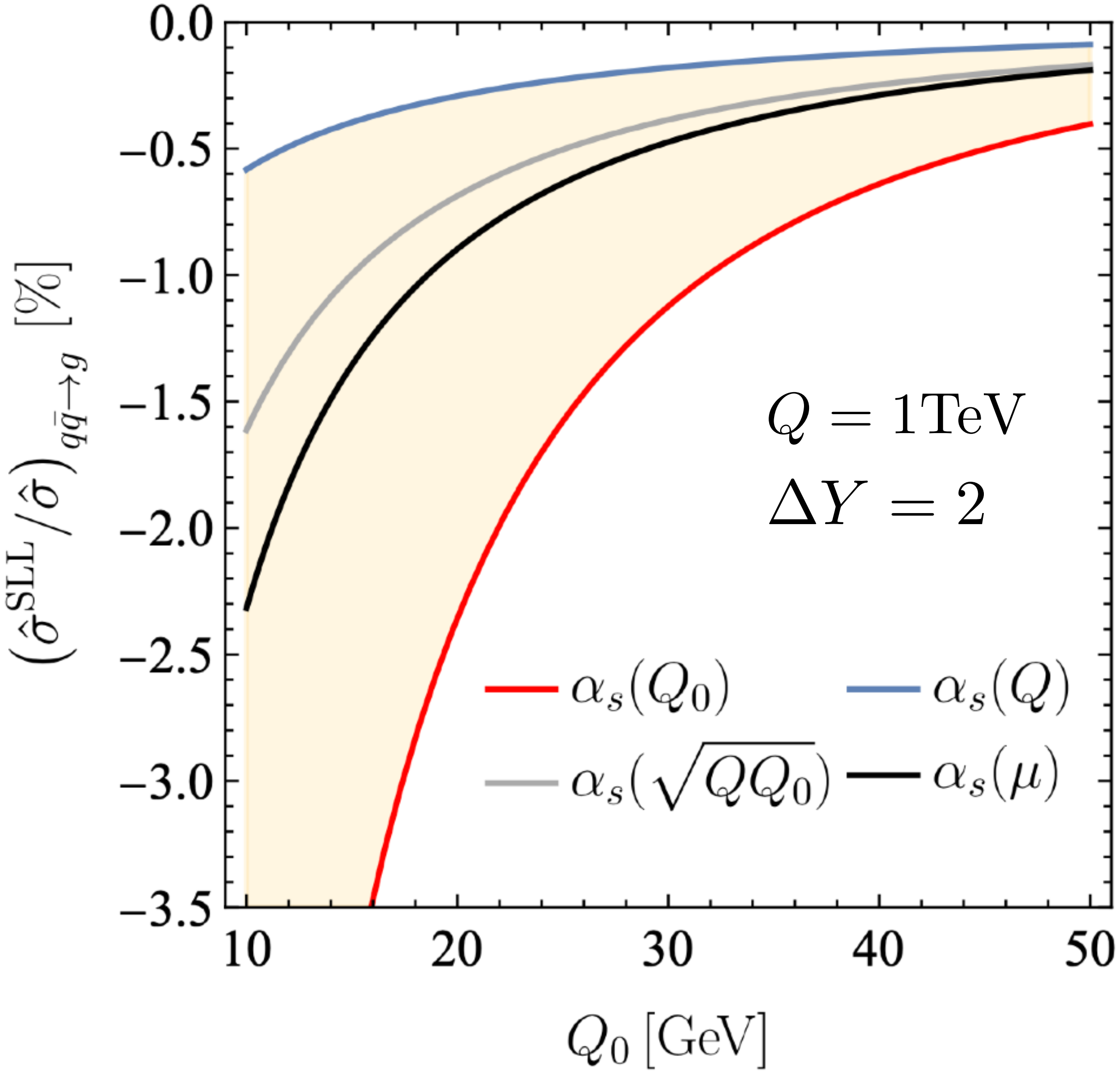
Superleading logs $\xrightarrow{\omega\to\infty}$ $\dfrac{1}{\omega}$



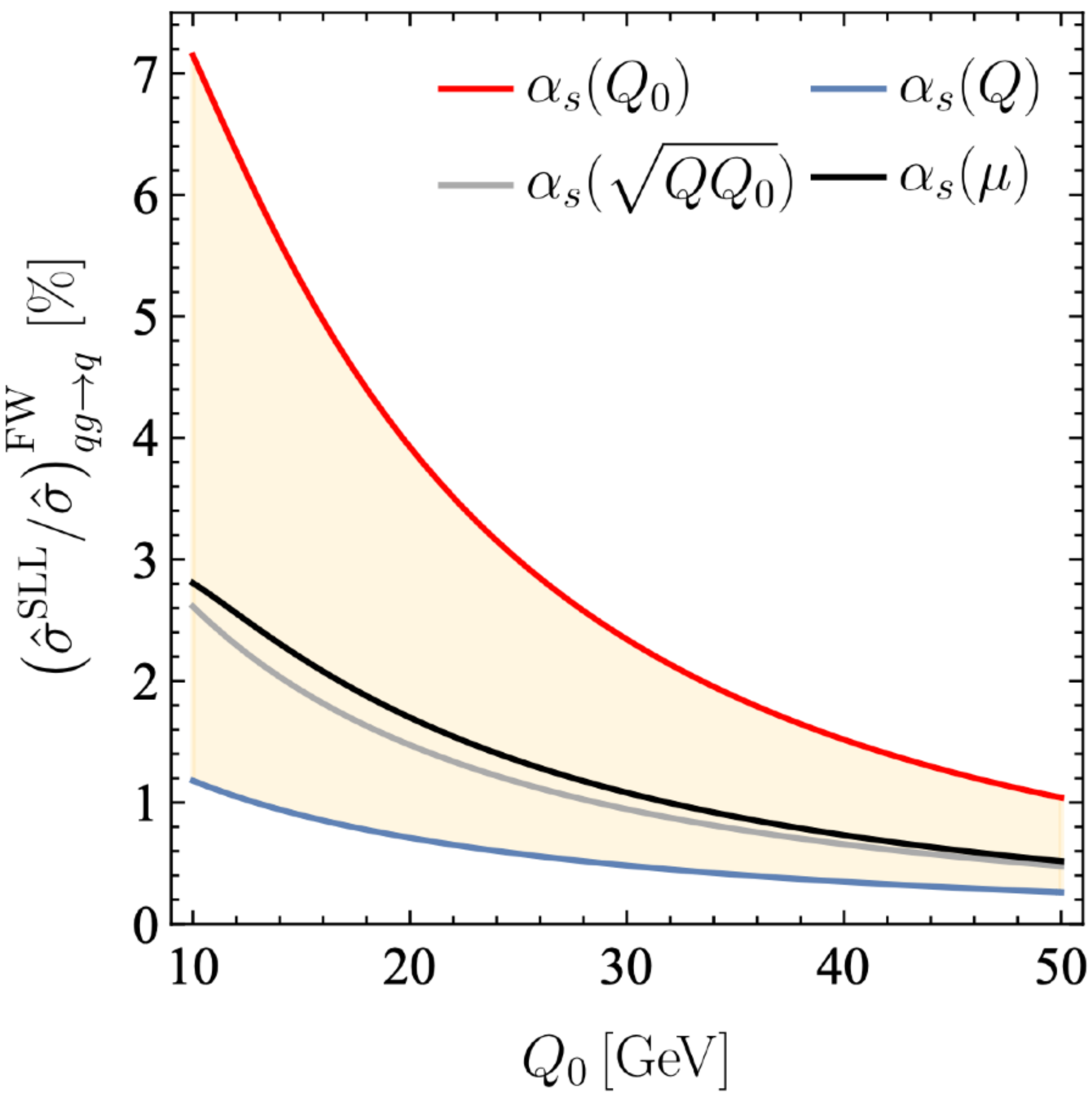
Numerical results

(Becher, Neubert, DYS, Stillger ’23)

$q\bar{q} \to gZ$



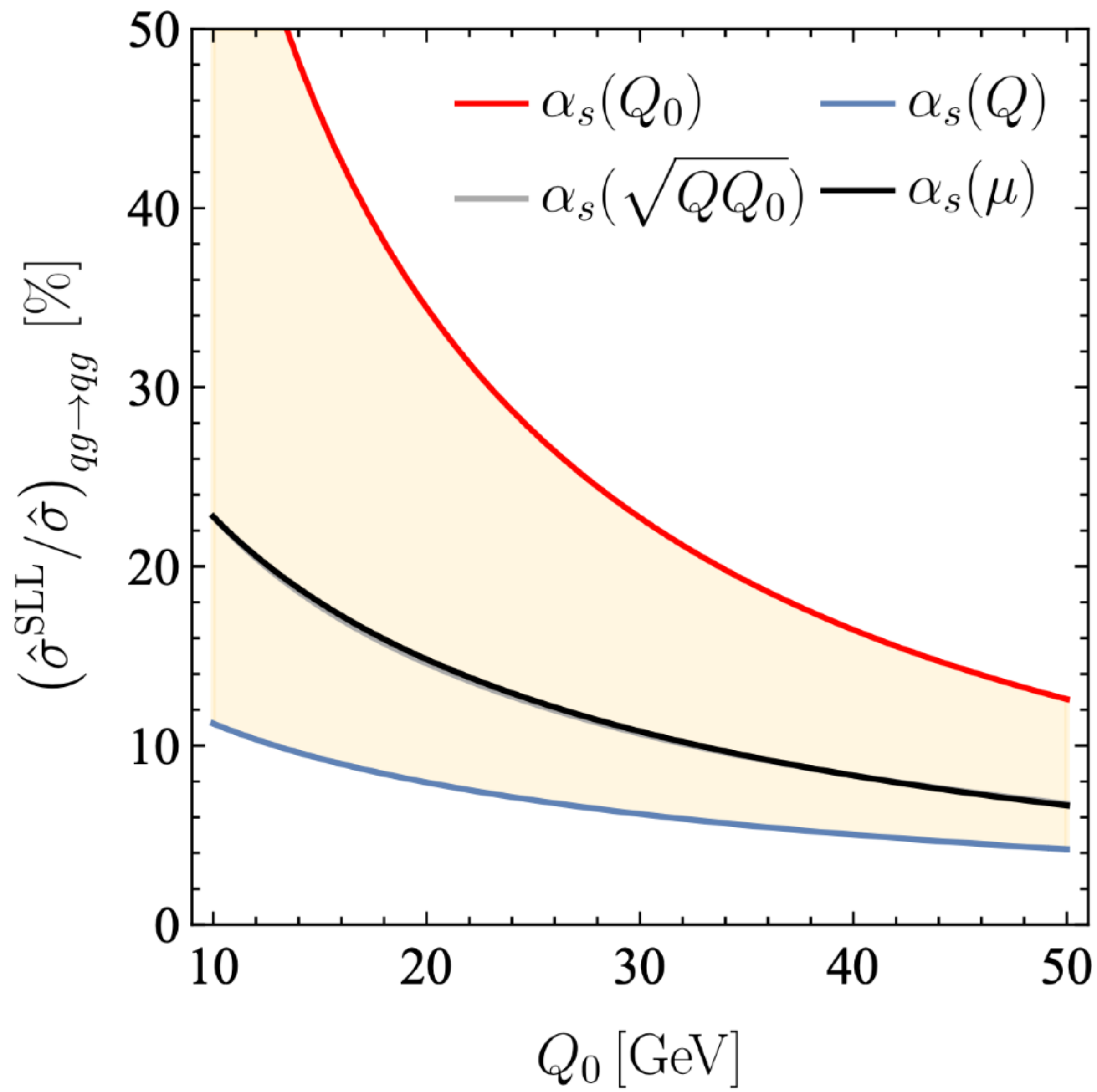
$qg \to qZ$



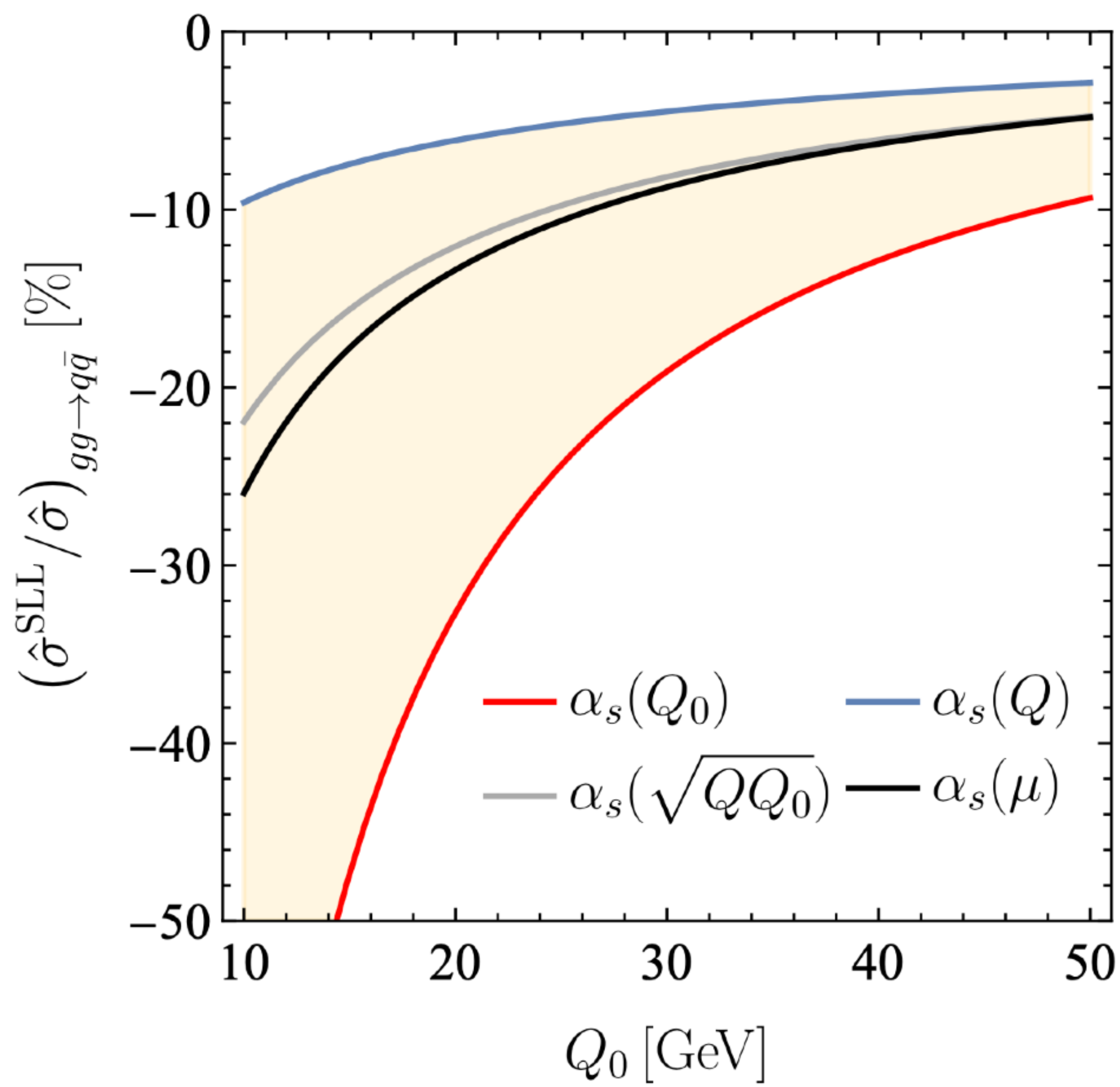
Numerical results

(Becher, Neubert, DYS, Stillger ’23)

$qg \to qg$



$gg \to q\bar{q}$



Summary and outlook

- **Factorization is at the heart of any quantitative prediction using pQCD**
- **We investigate naive factorization violation effects using the gap fraction of QCD jets at hadron colliders**

$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, x_1, x_2, s, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

- **The results are obtained by solving renormalization group equations order-by-order**

$$ {}^{1+1}F_{2+0}\left(\begin{matrix} 1 : 1, \frac{1}{2} ; \\ 2, \frac{5}{2} : \quad ; \end{matrix}, -w, -vw \right)$$

- **Subleading superleading logs? BFKL logs?**
- **Low energy theory from Glauber gluons ?**
- **Non-perturbative corrections?**

Thank you