

Double-mixing CP Violation

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Based on arXiv: 2301.05848

Outline

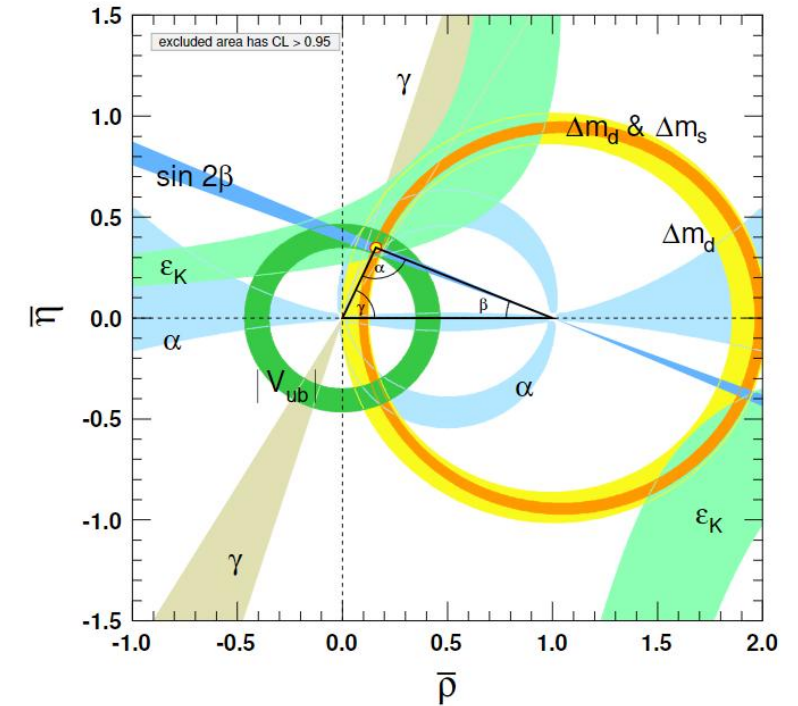
- Background and motivation
- Theoretical framework
- Numerical results
- CKM phase extraction
- Summary

CP violation

- SM precision test

Unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



[PDG, 2023]

- A necessary condition for baryogenesis, the process of dynamically generating the **matter-antimatter asymmetry** of the universe.

Search for new CPV

How?

- Baryon number violation
- **C and CP violation**
- Thermodynamic non equilibrium

Sakharov conditions

[A.D.Sakharov et al, Fiz.5,32-35 (1967)]

CP violation observables

- Common CPV observables

- ✓ Direct CP violation

$$\left| \frac{\bar{A}_{\bar{f}}}{A_f} \right| \neq 1 \iff \Gamma(M \rightarrow f) \neq \Gamma(\bar{M} \rightarrow \bar{f})$$

- ✓ CPV in mixing (indirect CPV)

$$\left| \frac{q}{p} \right| \neq 1 \iff \Gamma(M^0 \rightarrow \bar{M}^0) \neq \Gamma(\bar{M}^0 \rightarrow M^0)$$

- ✓ CPV in interference between a decay with and without **initial** mixing

$$(M^0 \rightarrow f) + (M^0 \rightarrow \bar{M}^0 \rightarrow f)$$

- CPV in interference between a decay with and without **final** mixing

$$(P \rightarrow M^0) + (P \rightarrow \bar{M}^0 \rightarrow M^0)$$

[Wang, Yu, Li, *PRL* 119(2017)181802]

New type of CPV

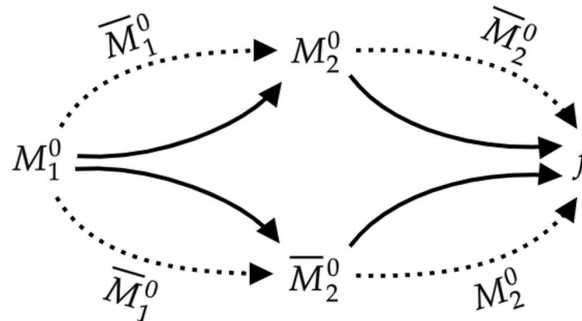
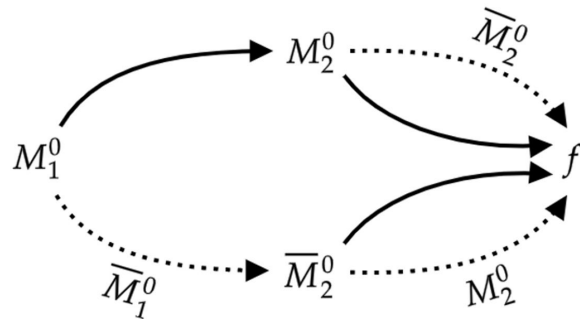
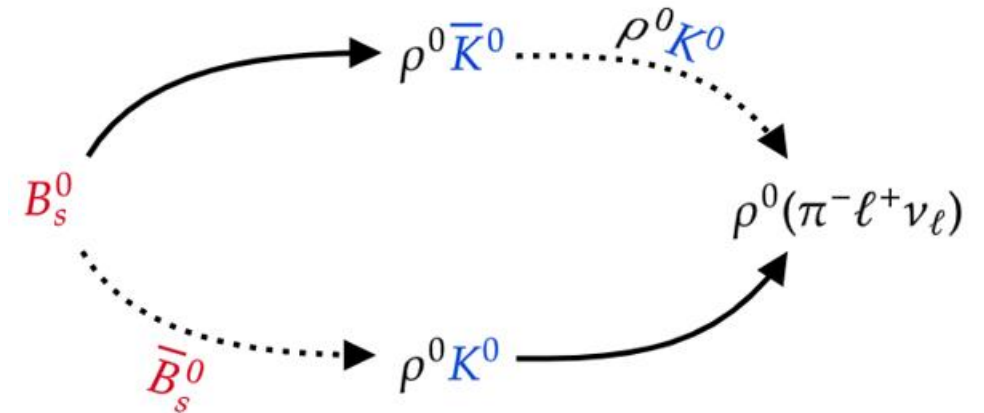
Upper path $B_S^0 \rightarrow \rho^0 \bar{K}^0 \rightarrow \rho^0 K^0 \rightarrow \rho^0(\pi^- \ell^+ \nu_\ell)$

Lower path $B_S^0 \rightarrow \bar{B}_S^0 \rightarrow \rho^0 K^0 \rightarrow \rho^0(\pi^- \ell^+ \nu_\ell)$

- Induced by interferences between **two mixing processes** in one cascade decay.

- More complicated cases

Consider $B_S^0 \rightarrow \rho^0 K \rightarrow \rho^0(\pi^- \ell^+ \nu_\ell)$



[Shen, **WJS**, Qin, 2301.05848]

Double mixing CPV

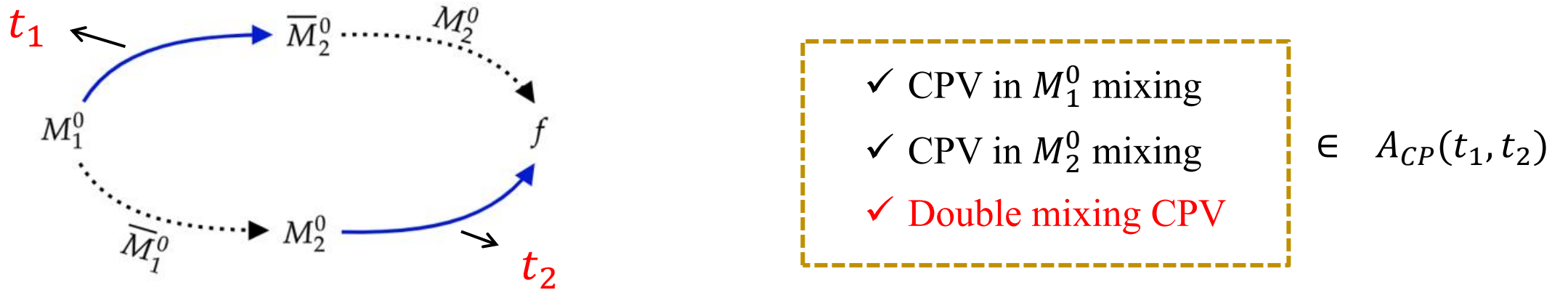
Adv:

- It does not require nonzero strong phases, providing opportunities to directly extract weak phases **without strong pollution**.
- Strong phases can be **extracted from experimental data** without theoretical input.
- The two-dimensional time-dependent CP asymmetry can be analyzed.  $A_{CP}(t_1, t_2)$
- The **double mixing CPV** can be very **significant** and practically measurable by experiments.

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CP asymmetry



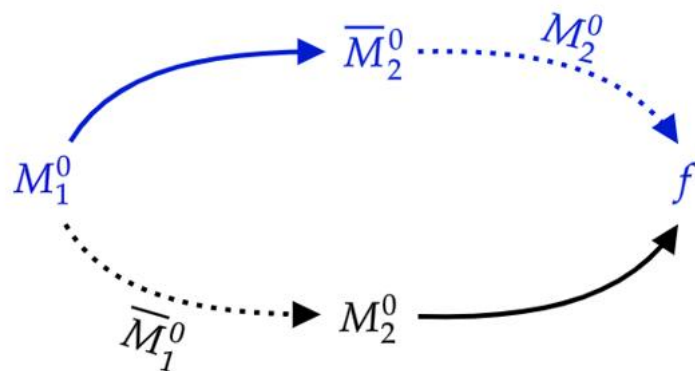
- The two-dimensional time-dependent CP asymmetry is defined as

$$A_{CP}(t_1, t_2) \equiv \frac{\Gamma_f(t_1, t_2) - \Gamma_{\bar{f}}(t_1, t_2)}{\Gamma_f(t_1, t_2) + \Gamma_{\bar{f}}(t_1, t_2)} = \frac{N(t_1, t_2)}{D(t_1, t_2)}$$

Theoretical framework

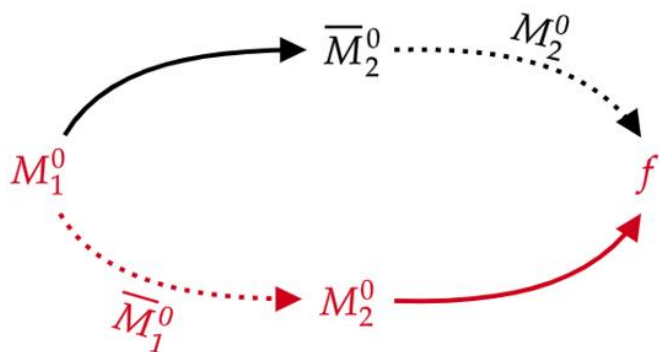
✓ Take $B_s^0 \rightarrow \rho^0 \bar{K}^0 \rightarrow \rho^0 (\pi^- \ell^+ \nu_\ell)$ as an example (penguin ≈ 0)

$$\begin{aligned} |M^0(t)\rangle &= g_+(t)|M^0\rangle - \frac{q}{p}g_-(t)|\bar{M}^0\rangle \\ |\bar{M}^0(t)\rangle &= g_+(t)|\bar{M}^0\rangle - \frac{p}{q}g_-(t)|M^0\rangle \end{aligned}$$



$$\propto |g_{1,+}(t_1)|^2 |g_{2,-}(t_2)|^2 \left(\frac{|p_2|^2}{|q_2|^2} - \frac{|q_2|^2}{|p_2|^2} \right)$$

CP violation in M_2^0 mixing

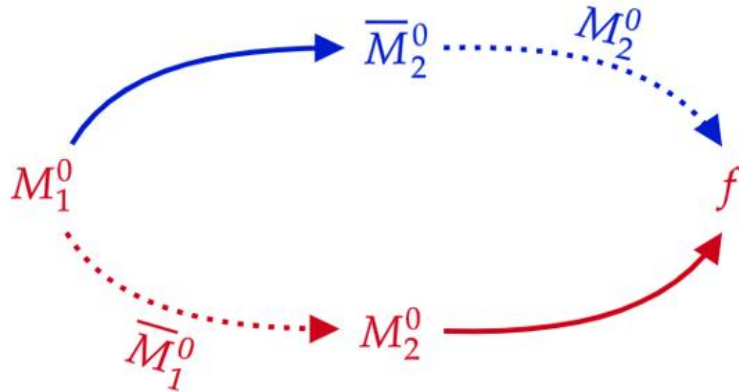


$$\propto |g_{1,-}(t_1)|^2 |g_{2,+}(t_2)|^2 \left(\frac{|q_1|^2}{|p_1|^2} - \frac{|p_1|^2}{|q_1|^2} \right)$$

CP violation in M_1^0 mixing

Double mixing CPV

- ✓ Take $B_s^0 \rightarrow \rho^0 \bar{K}^0 \rightarrow \rho^0 (\pi^- \ell^+ \nu_\ell)$ as an example (penguin ≈ 0)



K_S, K_L interference

$$A_h(t_1, t_2) \propto -e^{-\Gamma_1 t_1} \sinh \frac{\Delta \Gamma_1 t_1}{2} [2S_{n2}(t_2) \sin(\phi_1 + \phi_2 + 2\delta)]$$

$$A_n(t_1, t_2) \propto e^{-\Gamma_1 t_1} \sin \Delta m_1 t_1 [2S_{h2}(t_2) \sin(\phi_1 + \phi_2 + 2\delta)]$$

$K_S + K_L$

$$q_1/p_1 = |q_1/p_1| e^{-i\phi_1}$$

$$q_2/p_2 = |q_2/p_2| e^{-i\phi_2}$$

$$\langle \rho^0 \bar{K}^0 | B_s^0 \rangle = \langle \rho^0 K^0 | \bar{B}_s^0 \rangle e^{2i\delta}$$

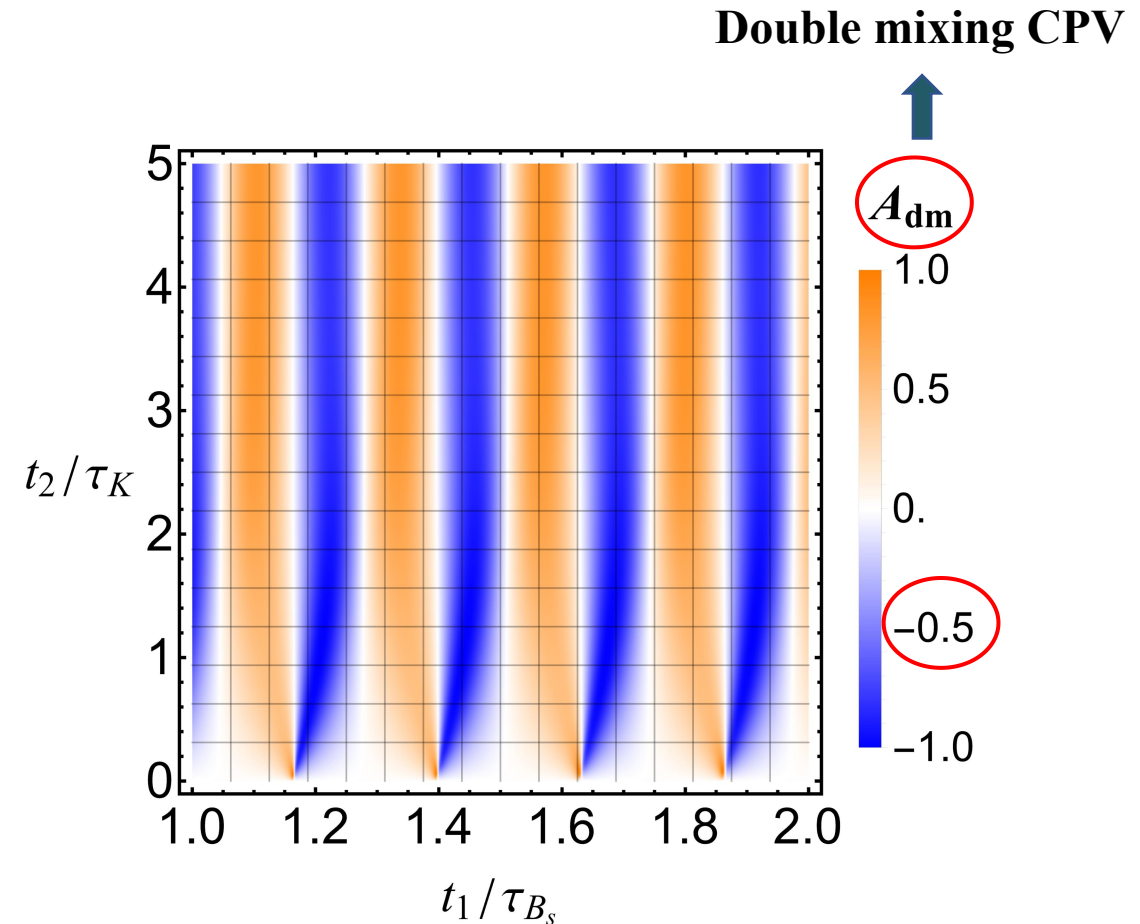
- With ϕ_1, ϕ_2, δ the relevant **weak phases**.

Outline

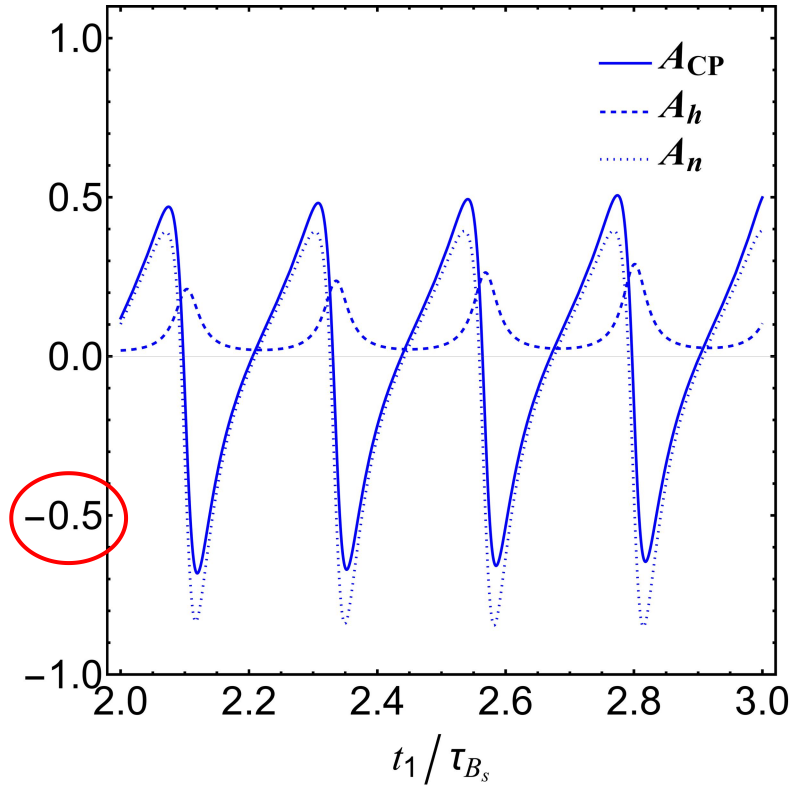
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t_1, t_2 Dependence

- $B_S^0 \rightarrow \rho^0 \bar{K}^0 \rightarrow \rho^0 (\pi^- \ell^+ \nu_\ell)$
- The $A_{CP}(t_1, t_2)$ dependence on t_1 and t_2 .
- The magnitude of the peak values can be larger than **50%**.

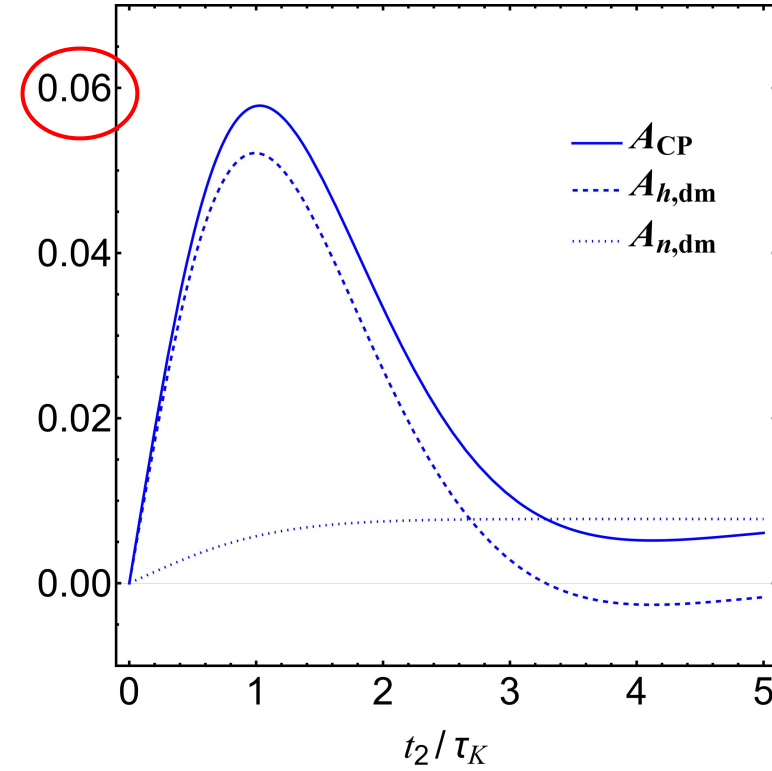


t_1 Dependence



- Integrate out the t_2 from 0 to τ_K

t_2 Dependence



- Integrate out the t_1 from τ_{B_s} to $5\tau_{B_s}$

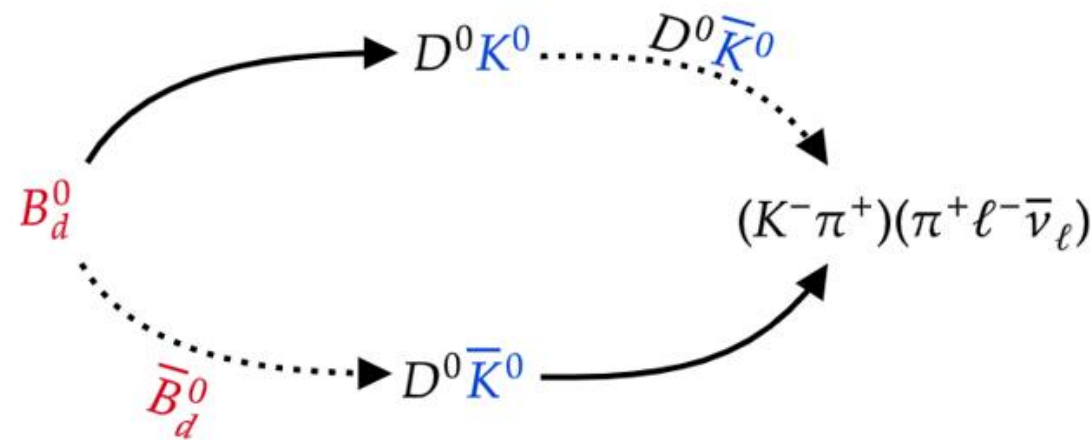
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CKM phase extraction

✓ Take $B_d^0 \rightarrow D^0 K^0 \rightarrow (K^- \pi^+)(\pi^+ \ell^- \bar{\nu}_\ell)$ as an example (penguin = 0)

$$\begin{aligned} \langle D^0 \bar{K}^0 | \bar{B}_d^0 \rangle &= \langle \bar{D}^0 K^0 | B_d^0 \rangle e^{i\theta_1} \\ \langle D^0 K^0 | B_d^0 \rangle &= \langle \bar{D}^0 K^0 | B_d^0 \rangle r_B e^{i(\delta + \theta_2)} \end{aligned}$$



- $A_{CP}(t_1, t_2) \propto \sin \omega'$
- **Parameters:** r_B , δ , $\underbrace{\phi_1 - \phi_2 + \theta_2 - \theta_1}_{\omega'} \approx 2\beta + \gamma$
- **Related phases:** $\theta_2 \rightarrow \gamma$, $\phi_1 \rightarrow 2\beta$

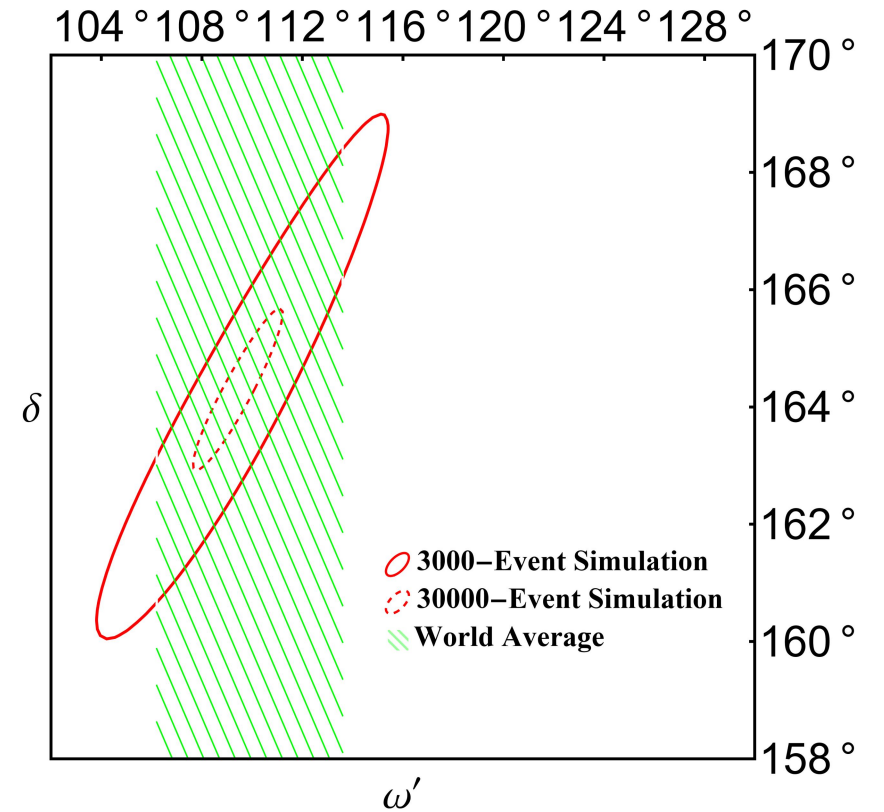
$$\begin{aligned} q_B/p_B &= |q_B/p_B| e^{-i\phi_1} \\ q_K/p_K &= |q_K/p_K| e^{-i\phi_2} \end{aligned}$$

CKM phase extraction

✓ Take $B_d^0 \rightarrow D^0 K^0 \rightarrow (K^- \pi^+)(\pi^+ \ell^- \bar{\nu}_\ell)$ as an example (penguin = 0)

Number of Events	Parameter	Fitted Value	Input Value
3000	r_B	0.370 ± 0.015	0.366
	ω'	$(109.1 \pm 5.7)^\circ$	$(110 \pm 4)^\circ$
	δ	$(163.9 \pm 4.4)^\circ$	164°
30000	r_B	0.365 ± 0.004	0.366
	ω'	$(109.4 \pm 1.8)^\circ$	$(110 \pm 4)^\circ$
	δ	$(164.2 \pm 1.4)^\circ$	164°

$$\omega' = \phi_1 - \phi_2 + \theta_2 - \theta_1 \quad 109.9^\circ \pm 3.7^\circ \text{ (Experiment)}$$



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Summary

- A novel type of CP violation effect, the **double-mixing** CP asymmetry, is proposed.
- It does not require nonzero strong phases.
- The double-mixing CP asymmetry can be very **significant** in some decay modes.
$$B_s^0 \rightarrow \rho^0 \bar{K}^0 \rightarrow \rho^0 (\pi^- \ell^+ \nu_\ell)$$
- Strong phases can be **extracted from experimental data** without theoretical input.
$$B_d^0 \rightarrow D^0 K^0 \rightarrow (K^- \pi^+) (\pi^+ \ell^- \bar{\nu}_\ell)$$

Thanks !

Appendix

Case 1 : $B_s^0 \rightarrow \rho^0 \bar{K}^0 \rightarrow \rho^0 (\pi^- \ell^+ \nu_\ell)$

$$\begin{aligned} S_h(t_2) &= \frac{e^{-\Gamma_2 t_2}}{2} [-2 \sin(\Delta m_2 t_2) \sin(\phi_1 + \phi_2 + 2\delta) \\ &\quad + \sinh \frac{\Delta \Gamma_2 t_2}{2} \left(\left| \frac{q_1}{p_1} \right| \left| \frac{p_2}{q_2} \right| - \left| \frac{p_1}{q_1} \right| \left| \frac{q_2}{p_2} \right| \right) \\ &\quad \times \cos(\phi_1 + \phi_2 + 2\delta)] , \\ S_n(t_2) &= \frac{e^{-\Gamma_2 t_2}}{2} [2 \sinh \frac{\Delta \Gamma_2 t_2}{2} \sin(\phi_1 + \phi_2 + 2\delta) \\ &\quad + \sin(\Delta m_2 t_2) \left(\left| \frac{q_1}{p_1} \right| \left| \frac{p_2}{q_2} \right| - \left| \frac{p_1}{q_1} \right| \left| \frac{q_2}{p_2} \right| \right) \\ &\quad \times \cos(\phi_1 + \phi_2 + 2\delta)] , \end{aligned}$$

$$C'_+(t_2) = 2|g_{2,-}(t_2)|^2 ,$$

$$C'_-(t_2) = 2|g_{2,+}(t_2)|^2 ,$$

$$S'_h(t_2) = e^{-\Gamma_2 t_2} \sinh \frac{\Delta \Gamma_2 t_2}{2} \cos(\phi_1 + \phi_2 + 2\delta) ,$$

$$S'_n(t_2) = e^{-\Gamma_2 t_2} \sin(\Delta m_2 t_2) \cos(\phi_1 + \phi_2 + 2\delta) ,$$

Appendix

$$e^{2i\delta} = -\frac{V_{ub}^* V_{ud}}{V_{ub} V_{ud}^*} \quad CP|\rho^0 K^0\rangle = -|\rho^0 \bar{K}^0\rangle$$

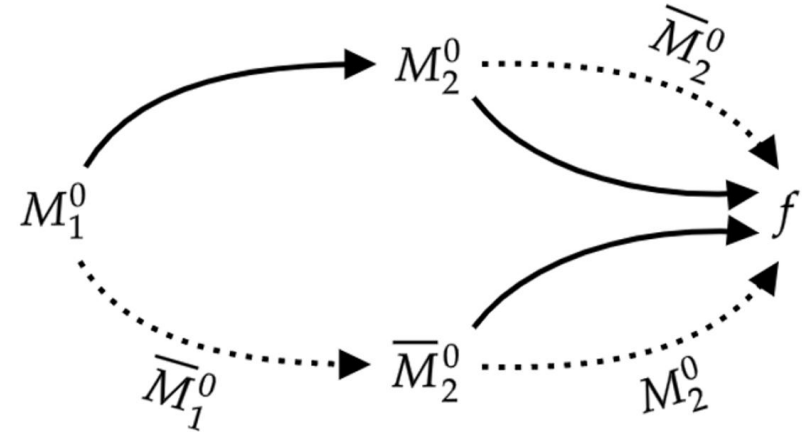
$$e^{-i\phi_1} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}, \quad e^{-i\phi_2} = \frac{V_{cd}^* V_{cs}}{V_{cb} V_{cs}^*}.$$

TABLE I: The input parameters and their values.

Parameter	Value
$ q_1/p_1 $	1.0003 ± 0.0014 [21]
ϕ_1	$(-2.106 \pm 0.135)^\circ$ [22]
$x_1 = 2\Delta m_1/(\Gamma_{B_s,L} + \Gamma_{B_s,S})$	27.01 ± 0.10 [22]
$y_1 = \Delta\Gamma_1/(\Gamma_{B_s,L} + \Gamma_{B_s,S})$	-0.064 ± 0.003 [22]
$ q_2/p_2 $	0.996774 ± 0.000019 [22]
ϕ_2	$(0.176 \pm 0.001)^\circ$ [22]
$x_2 = 2\Delta m_2/(\Gamma_{K,L} + \Gamma_{K,S})$	0.946 ± 0.002 [22]
$y_2 = \Delta\Gamma_2/(\Gamma_{K,L} + \Gamma_{K,S})$	-0.996506 ± 0.000016 [22]
2δ	$(-48.907 \pm 3.094)^\circ$ [22]

$$|M_{H,L}\rangle = p|M^0\rangle \mp q|\bar{M}^0\rangle$$

$$g_{\pm}(t) = \frac{1}{2} \left[e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t} \right]$$



Appendix

Case 2 : $B_d^0 \rightarrow D^0 K^0 \rightarrow (K^- \pi^+)(\pi^+ \ell^- \bar{\nu}_\ell)$

$$A_{\text{CP}}(t_1, t_2) = \frac{e^{-\Gamma_1 t_1} \sinh \frac{\Delta\Gamma_1 t_1}{2} S_h(t_2) + e^{-\Gamma_1 t_1} \sin(\Delta m_1 t_1) S_n(t_2)}{|g_{1,+}(t_1)|^2 r^2 C'_+(t_2) + |g_{1,-}(t_1)|^2 C'_-(t_2) + e^{-\Gamma_1 t_1} \sinh \frac{\Delta\Gamma_1 t_1}{2} S'_h(t_2) + e^{-\Gamma_1 t_1} \sin(\Delta m_1 t_1) S'_n(t_2)} ,$$

$$S_h(t_2) = -e^{-\Gamma_2 t_2} r \sin \omega' [\sin \delta \sinh \frac{\Delta\Gamma_K}{2} t_2 + \cos \delta \sin \Delta m_K t_2] ,$$

$$S_n(t_2) = e^{-\Gamma_2 t_2} r \sin \omega' [\cos \delta \sinh \frac{\Delta\Gamma_K}{2} t_2 - \sin \delta \sin \Delta m_K t_2] ,$$

$$S'_h(t_2) = e^{-\Gamma_2 t_2} r \cos \omega' [\cos \delta \sinh \frac{\Delta\Gamma_K}{2} t_2 - \sin \delta \sin \Delta m_K t_2] ,$$

$$S'_n(t_2) = e^{-\Gamma_2 t_2} r \cos \omega' [\sin \delta \sinh \frac{\Delta\Gamma_K}{2} t_2 + \cos \delta \sin \Delta m_K t_2] .$$