Hybrid Renormalization for Quasi Distribution Amplitudes of A Light Baryon

Authors:Chao Han, Yushan Su, Wei Wang, Jia-lu Zhang

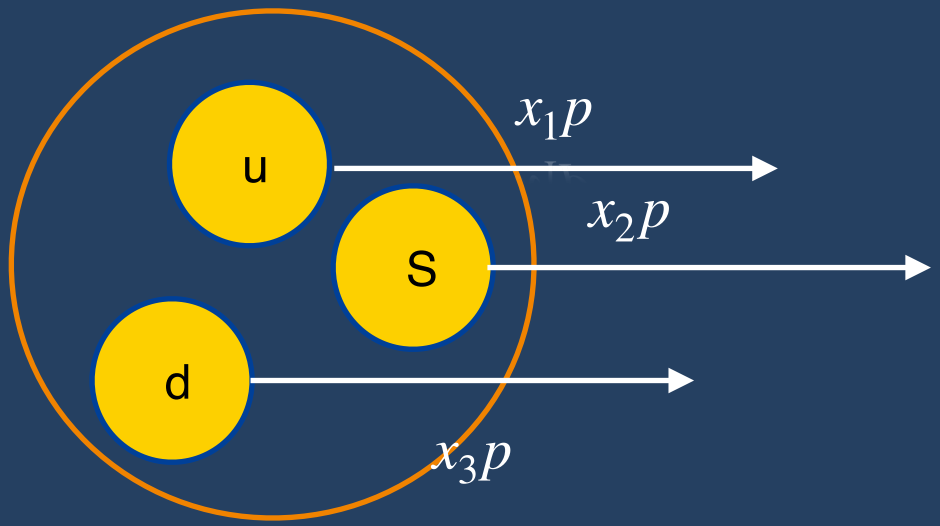
Speaker:Jia-Lu Zhang
Shanghai Jiaotong University

Contents

Introduction to LCDA\Quasi-DA

Physical Meaning of LCDA:

It describes the momentum fraction of each parton in the baryon.



$$x_1 + x_2 + x_3 = 1$$

The partonic interpretation has not been proven beyond tree level.

Introduction to LCDA\Quasi-DA

Operator definition of baryon LCDA in coordinate space:

$$M(z_1, z_2, z_3, P, \mu) = \left\langle 0 \left| \varepsilon^{ijk} u_\alpha^{i'}\left(\vec{z}_1\right) U_{i'i}\left(\vec{z}_1, \vec{z}_0\right) d_\beta^{j'}\left(\vec{z}_2\right) U_{j'j}\left(\vec{z}_2, \vec{z}_0\right) s_\gamma^{k'}\left(\vec{z}_3\right) U_{k'k}\left(\vec{z}_3, \vec{z}_0\right) \right| P(P, \lambda) \right\rangle$$

$$\left[a_1 n, a_0 n\right] = P\exp[ig\int_0^1 dt\left(x-y\right)_\mu A^\mu(tx+(1-x)y)]$$

$$f, g, h: quark\ \ flavour$$

$$\left| P(P, \lambda) \right\rangle : Baryon\ state$$

$$\vec{z}_1 = \frac{1}{\sqrt{2}}(z_1, 0, 0, z_1)$$

Introduction to LCDA\Quasi-DA

$$\left\langle 0 \left| u_\alpha(z_1)\, d_\beta(z_2)\, s_\gamma(z_3) \right| P(P,\lambda) \right\rangle$$

leading twist

$$= f_N\{(\not{P}C)_{\alpha\beta}(\gamma_5 u_\Lambda)_\gamma V(z_i P\cdot n) + (\not{P}\gamma_5 C)_{\alpha\beta}(u_\Lambda)_\gamma \underline{A(z_i P\cdot n)} + (i\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma_\mu\gamma_5 u_\Lambda)_\gamma T(z_i P\cdot n)$$

$$M(z_1, z_2, z_3, P^z, \mu) = \left\langle 0 \left| u^T(z_1)\, \Gamma d(z_2)\, s(z_3) \right| P(P,\lambda) \right\rangle_R \qquad \Gamma = C\gamma_5 \not{n}$$

Difficulty of LCDA: cannot be calculated perturbatively

Introduction to LCDA\Quasi-DA

A good way to calculate non-perturbative physics:

Lattice QCD

Flaw of Lattice QCD: only cannot solve light-like correlator

Matrix element which is calculable on Lattice: Quasi-DA

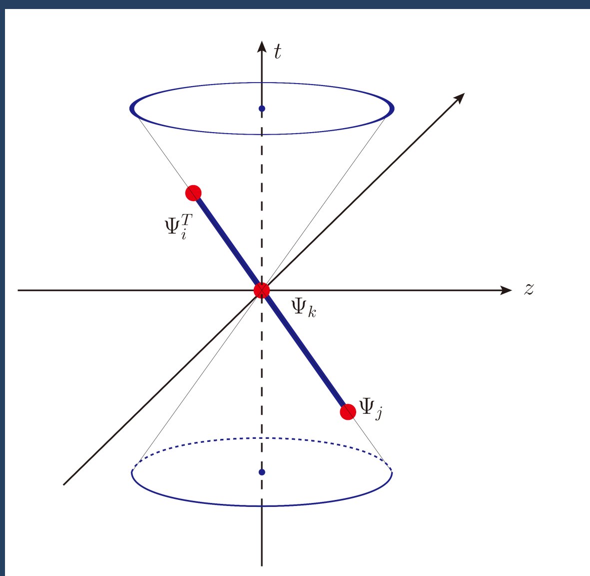
$$\tilde{M}(z_1, z_2, z_3, P^z, \mu) = \left\langle 0 \left| u^T(z_1)\, \tilde{\Gamma} d(z_2)\, s(z_3) \right| P(P, \lambda) \right\rangle_R \qquad \tilde{\Gamma} = C\gamma_5 \not{n}_z$$

$$\vec{z} = (0,0,0,z)$$

Large Momentum Effective theory

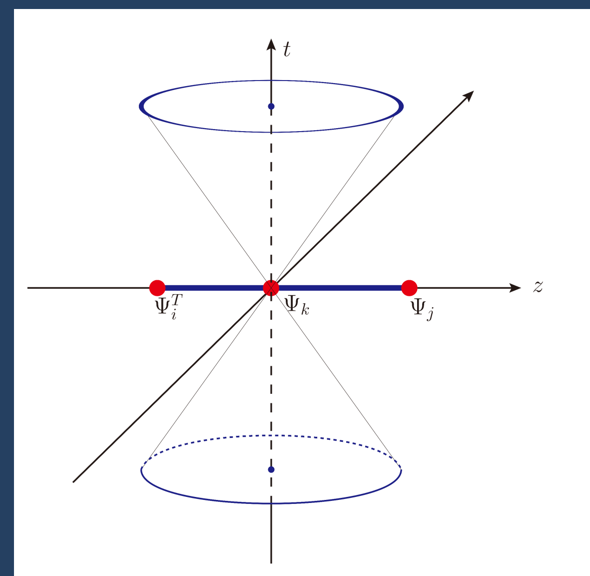
10.1103/PhysRevLett.110.262002

LCDA
Light-cone operator



relation ?

Quasi-DA
Equal-time operator



Large Momentum Effective theory

Baryon-DA

$$\tilde{\Phi}(x_1, x_2, P^z, \mu) = \int dy_1 dy_2 \mathcal{C}(x_1, x_2, y_1, y_2, P^z, \mu)\, \Phi(y_1, y_2, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{x_1 P^z}, \frac{\Lambda_{\mathrm{QCD}}}{x_2 P^z}, \frac{\Lambda_{\mathrm{QCD}}}{(1-x_1-x_2)\, P^z}\right)$$

The matching kernel C is perturbative calculable.

The matching kernel is valid for the same renormalization.

Large Momentum Effective theory

Procedure to calculate LCDA:

1. Using pQCD to calculate the matching kernel.
2. Calculate the quasi-DA using Lattice QCD.
3. Do the same renormalization on Quasi-DA
4. Convert the Quasi-DA calculated on lattice to LCDA.

Hybrid Renormalization

arXiv:2008.03886v2

Difficulty1: Regularization

Perturbative QCD.	v.s.	Lattice QCD
DR		Hard-cutoff

Difficulty2: Cannot introduce infrared effect

We need to extract perturbative information on Lattice.

Difficulty3: Perturbative calculation

Quasi-DA is divergent even after regular $\bar{M}S$ subtraction.

Hybrid Renormalization

arXiv:2008.03886v2

The previous method: RI/MOM

$$M^{RI/MOM}(x_1, x_2, \mu, P^2 = 0) = \frac{M^{\bar{M}S}(x_1, x_2, \mu, P^2 = 0)}{M^{\bar{M}S}(x_1, x_2, \mu, P^2 = -\rho)}$$

Linear divergence is not eliminated thoroughly.

It introduces extra infrared structure.

Hybrid Renormalization

1. Do ‘Self-renormalization’ to lattice matrix element

$$\tilde{M}^{\bar{M}S}(z_1, z_2, P^z) = \frac{\tilde{M}(z_1, z_2, a, P^z)}{Z_R(z_1, z_2, a)}$$

$$Z_R(z_1, z_2, a, \mu) = exp[(\frac{k}{a ln[a\Lambda_{QCD}]} - m_0)\tilde{z} + \frac{\gamma_0}{b_0} ln[\frac{ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{\bar{M}S}]}] + ln[1 + \frac{d}{ln(a\Lambda_{QCD})}] + f(z_1, z_2)a]$$

2. Do ‘Ratio-renormalization’ to both lattice matrix element and perturbative calculation.

$$\tilde{M}^{hybrid}(z_1, z_2, P^z) = \frac{\tilde{M}^{\bar{M}S}(z_1, z_2, P)}{\tilde{M}^{\bar{M}S}(z_1'(z_1, z_2), z_2'(z_1, z_2), P = 0)}$$

Hybrid Renormalization

arXiv:2008.03886v2

A scheme which deal with short and long length scale differently.

On lattice QCD side: Self-renormalization

A scheme convert factor should be included on Lattice QCD to convert to $\bar{M}S$ scheme

$$Z_R(z_1, z_2, a, \mu) = \exp\left[\left(\frac{k}{a\ln[a\Lambda_{\rm QCD}]} - m_0 \right) \tilde{z} + \frac{\gamma_0}{b_0} \ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\overline{\rm MS}}]} \right] + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\rm QCD})} \right] + f(z_1, z_2)a \right]$$

Linear divergence

Log divergence

Discrete effect

Hybrid Renormalization

arXiv:2008.03886v2

A scheme which deal with short and long length scale differently.

On lattice QCD side: Self-renormalization

A scheme convert factor should be included on Lattice QCD to convert to $\bar{M}S$ scheme

$$Z_R(z_1, z_2, a, \mu) = \exp\left[\left(\frac{k}{a\ln[a\Lambda_{\rm QCD}]} - m_0\right)\tilde{z} + \frac{\gamma_0}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\overline{\rm MS}}]}\right] + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\rm QCD})}\right] + f(z_1, z_2)a\right]$$

Linear divergence

Log divergence

Discrete effect

Claim: $$lim_{a\to 0}\frac{\tilde{M}(z_1, z_2, a, P^z)}{Z_R(z_1, z_2, a)} = lim_{\epsilon\to 0}\frac{\tilde{M}(z_1, z_2, \epsilon, P^z)}{Z_{\bar{M}S}(z_1, z_2, \epsilon)} = \tilde{M}^{\bar{M}S}(z_1, z_2, P^z)$$

Hybrid Renormalization

On pQCD side:
Quasi-DA is a plus function which contains divergence even after $\bar{M}S$

Real Diagram Virtual Diagram

$$M(x)_{\oplus}^{x_0} = M(x) - \delta(x - x_0)\int M(y)dy$$

$$lim_{x_{1/2}\to\infty}\tilde{M}(x_1, x_2, P, \epsilon) \simeq \frac{C}{x_{1/2}} \qquad \to \int dx_{1/2} M(x_{1/2})$$ is divergent at large x

We need to eliminate the $\frac{1}{x}$ behavior at large momentum.

Hybrid Renormalization

Where does $\frac{1}{x}$ come from ?

$$\tilde{M}(z_1, z_2, P^z, \epsilon) \supset log z_1^2, log z_2^2, log(z_1 - z_2)^2$$

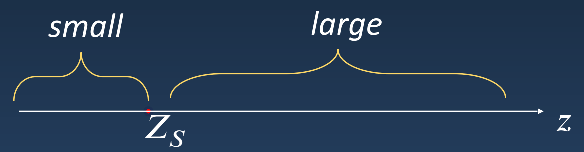
$$log z_1^2 \rightarrow \frac{1}{|x_1|}, log z_2^2 \rightarrow \frac{1}{|x_2|}, \log(z_1 - z_2)^2 \rightarrow \frac{1}{|x_1 - x_2|}$$

A simple solution: Ratio with 0 momentum matrix element when z is small

$$\tilde{M}^{ratio}(z_1, z_2, P, \epsilon) = \frac{\tilde{M}(z_1, z_2, P, \epsilon)}{\tilde{M}(z_1, z_2, 0, \epsilon)}$$

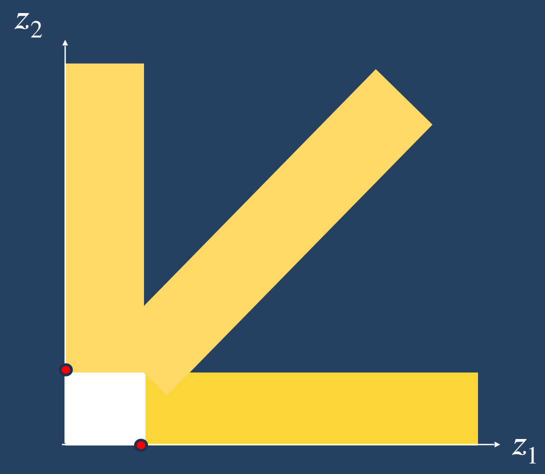
Hybrid Renormalization

In PDF case: only one length scale $\bar{\psi}(z)\Gamma\psi(0)$



1. Ratio with the zero momentum matrix element at the same point, when $z < z_s$.
2. Ratio with the zero momentum matrix element at point z_s when $z > z_s$.

In baryon-LCDA case: three length scale $\psi^T(z_1)\Gamma\psi(z_2)\psi(0) \rightarrow z_1 \;\; z_2 \;\; (z_1 - z_2)$



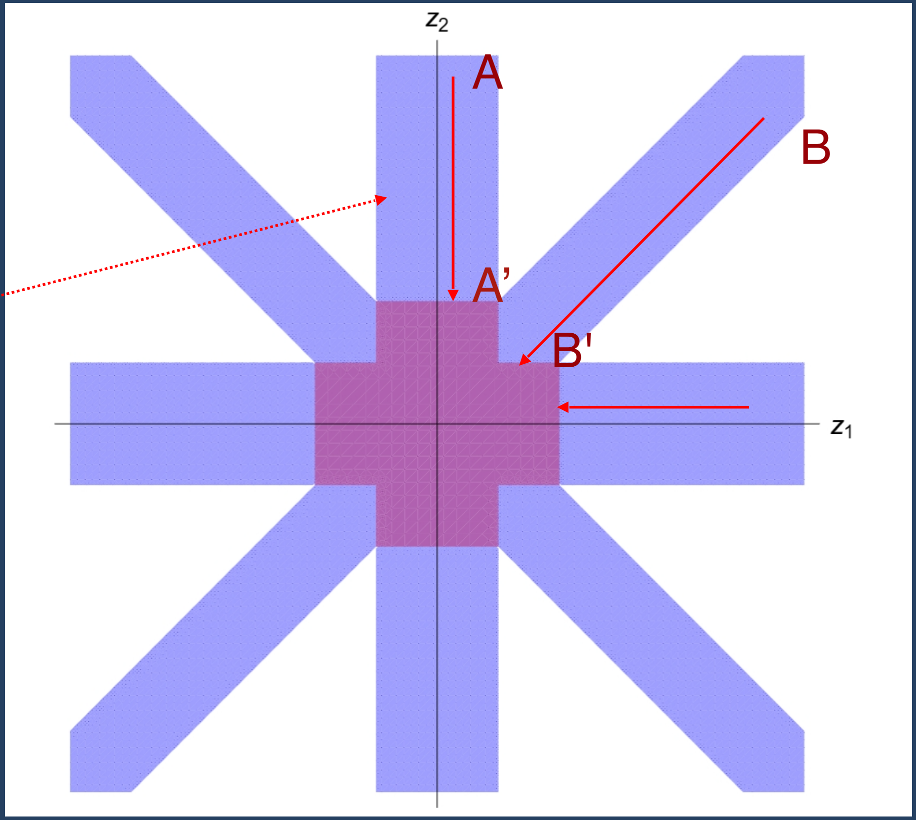
small and large scales are mixed in yellow area.
Matrix element should be renormalized separately.

Hybrid Renormalization

Up to one loop, the scale is separated: $A log z_1^2 + B log z_2^2 + C log(z_1 - z_2)^2$

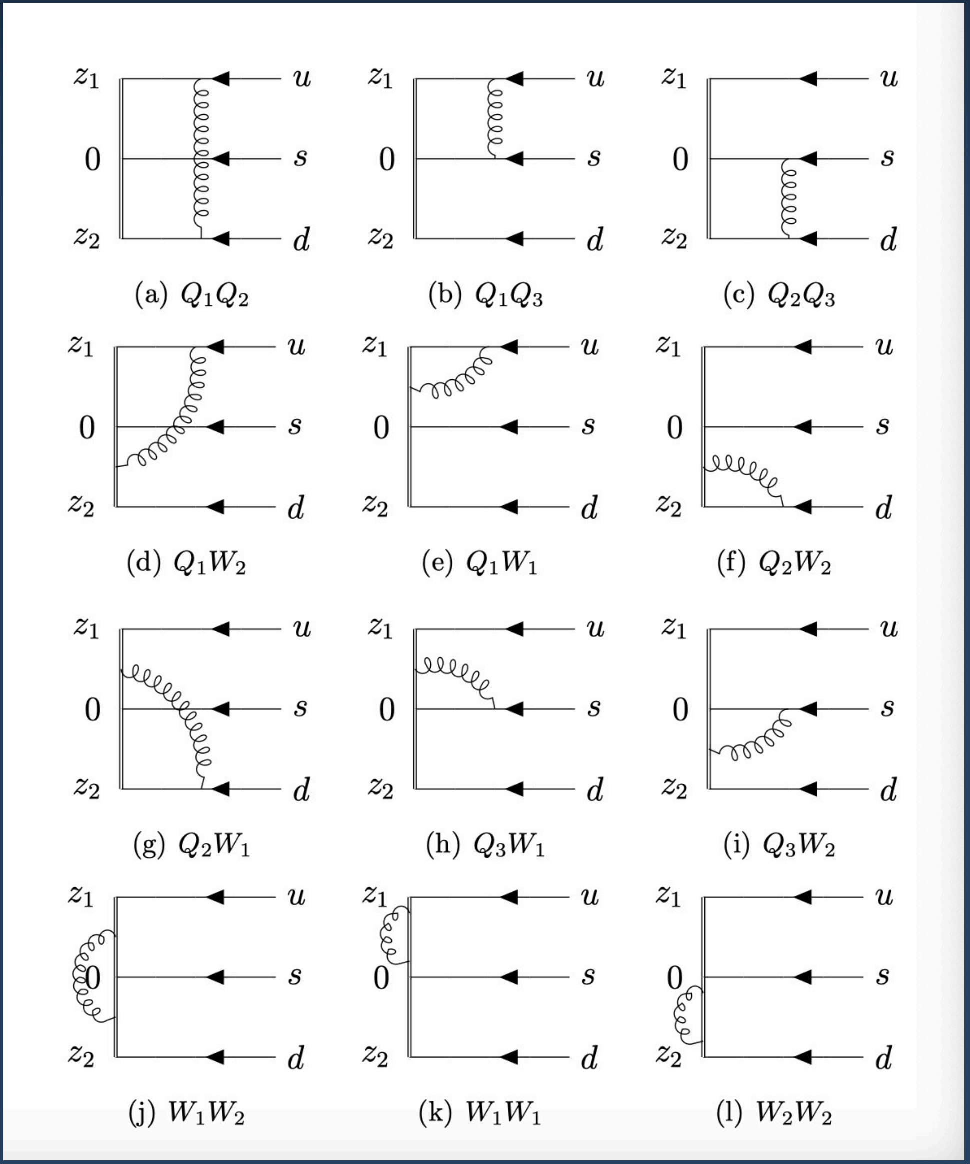
$$\tilde{q}^{hybrid}(z_1, z_2, P) = \frac{\tilde{q}^{\bar{M}S}(z_1, z_2, P)}{\tilde{q}^{\bar{M}S}(z_1'(z_1, z_2), z_2'(z_1, z_2), P = 0)}$$

$$\frac{\hat{M}_{\overline{\rm MS}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\rm MS}}(z_1, {\rm sign}(z_2) 2z_s, 0, 0, \mu)}\theta(z_s - |z_1|)\theta(|z_2| - 2z_s)$$



Result and Outlook

One-loop calculation for Quasi-DA under $\overline{MS}$ using pQCD



Result and Outlook

One-loop calculation:

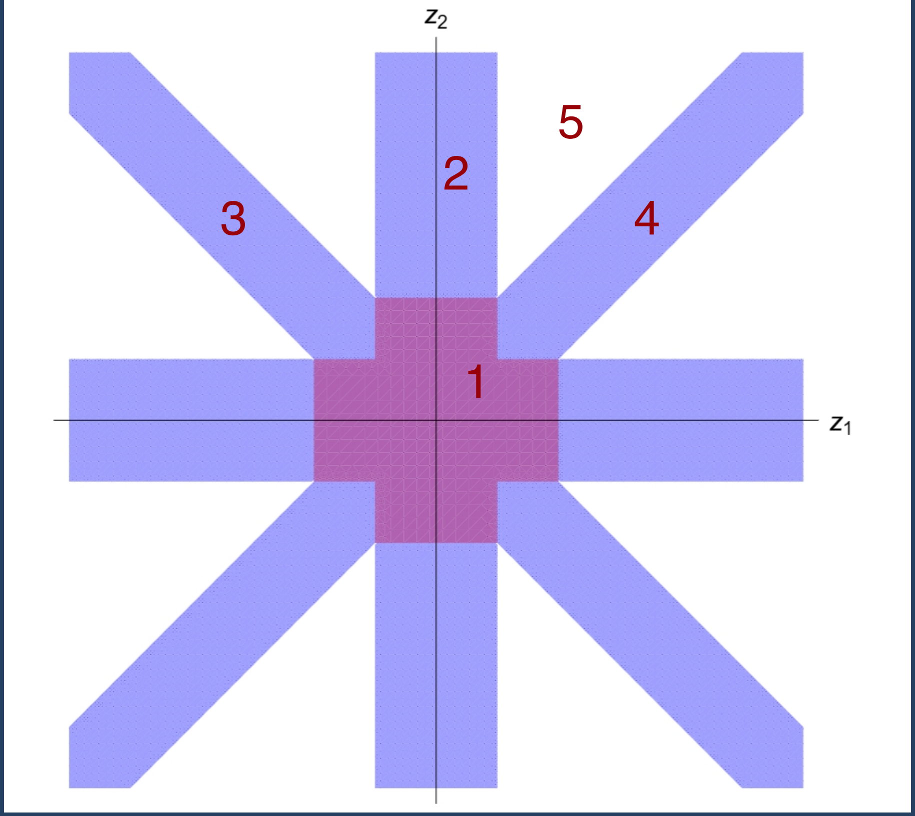
$$L_1 = \ln \frac{z_1^2 \mu^2 e^{2\gamma_E}}{4} \;\&\; L_2 = \ln \frac{z_2^2 \mu^2 e^{2\gamma_E}}{4} \;\&\; . L_{12} = \ln \frac{(z_1 - z_2)^2 \mu^2 e^{2\gamma_E}}{4}$$

$$\begin{aligned}
M_p(z_1, z_2, 0, P^z, \mu) &= \left\{1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}L_1^{\rm UV} + \frac{1}{2}L_2^{\rm UV} + \frac{1}{2}L_{12}^{\rm UV} + \frac{3}{2}\right)\right\} M_0\left(z_1, z_2, 0, P^z, \mu\right) \\
&- \frac{\alpha_s C_F}{8\pi}\int_0^1 d\eta_1 \int_0^{1-\eta_1} d\eta_2 \\
&\times \left\{\left(L_1^{\rm IR} - 1 + \frac{1}{\epsilon_{\rm IR}}\right) M_0\left(\left(1-\eta_1\right) z_1, z_2, \eta_2 z_1, P^z, \mu\right) + \left(L_2^{\rm IR} - 1 + \frac{1}{\epsilon_{\rm IR}}\right) M_0\left(z_1, \left(1-\eta_1\right) z_2, \eta_2 z_2, P^z, \mu\right)\right. \\
&\left. + 2\left(L_{12}^{\rm IR} - 3 + \frac{1}{\epsilon_{\rm IR}}\right) M_0\left(\left(1-\eta_1\right) z_1 + \eta_1 z_2, \left(1-\eta_2\right) z_2 + \eta_2 z_1, 0, P^z, \mu\right)\right\} \\
&- \frac{\alpha_s C_F}{4\pi}\int_0^1 d\eta \times \left\{ M_0\left((1-\eta) z_1 + \eta z_2, z_2, 0, P^z, \mu\right)\left\{\left(L_{12}^{\rm IR} + 1 + \frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ + 2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right. \\
&+ M_0\left(z_1, (1-\eta) z_2 + \eta z_1, 0, P^z, \mu\right)\left\{\left(L_{12}^{\rm IR} + 1 + \frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ + 2\left(\frac{\ln\eta}{\eta}\right)_+\right\} \\
&+ M_0\left((1-\eta) z_1, z_2, 0, P^z, \mu\right)\left\{\left(L_1^{\rm IR} + 1 + \frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ + 2\left(\frac{\ln\eta}{\eta}\right)_+\right\} \\
&+ M_0\left(z_1, (1-\eta) z_2, 0, P^z, \mu\right)\left\{\left(L_2^{\rm IR} + 1 + \frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ + 2\left(\frac{\ln\eta}{\eta}\right)_+\right\} \\
&- M_0\left(z_1, z_2, \eta z_1, P^z, \mu\right)\left\{\left(L_1^{\rm IR} + 1 + \frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ + 2\left(\frac{\ln\eta}{\eta}\right)_+\right\} \\
&\left. - M_0\left(z_1, z_2, \eta z_2, P^z, \mu\right)\left\{\left(L_2^{\rm IR} + 1 + \frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ + 2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right\},
\end{aligned} \tag{2.10}$$

Result and Outlook

One-loop calculation of ratio element:

$$\delta\hat{M}_H^{(1)}(z_1,z_2,\mu)=\hat{M}_p^{(1)}(z_1,z_2,0,0,\mu)\left(\theta(2z_s-|z_1|)\theta(z_s-|z_2|)+\theta(z_s-|z_1|)\theta(|z_2|-z_s)\theta(2z_s-|z_2|)\right)$$
$$+\hat{M}_p^{(1)}(z_1,\mathrm{sign}(z_2)2z_s,0,0,\mu)\,\theta(z_s-|z_1|)\theta(|z_2|-2z_s)+\hat{M}_p^{(1)}(\mathrm{sign}(z_1)2z_s,z_2,0,0,\mu)\,\theta(|z_1|-2z_s)\theta(z_s-|z_2|)$$
$$+\hat{M}_p^{(1)}(z_s+(z_1-z_2)\theta(z_1-z_2),z_s+(z_2-z_1)\theta(z_2-z_1),0,0,\mu)\,\theta(|z_1|-z_s)\theta(|z_2|-z_s)\theta(z_s-|z_1-z_2|)$$
$$+\hat{M}_p^{(1)}(z_s+(z_1+z_2)\theta(z_1+z_2),-z_s+(z_2+z_1)\theta(-z_2-z_1),0,0,\mu)\,\theta(|z_1|-z_s)\theta(|z_2|-z_s)\theta(z_s-|z_1+z_2|)$$
$$+\hat{M}_p^{(1)}(\mathrm{sign}(z_1)z_s,\mathrm{sign}(z_2)2z_s,0,0,\mu)\,\theta(|z_1|-z_s)\theta(|z_2|-z_s)\theta(|z_1-z_2|-z_s)\theta(|z_1+z_2|-z_s).$$



Result and Outlook

$$C^{hybrid}(x_1, x_2, y_1, y_2, \mu) = C^{\bar{MS}}(x_1, x_2, y_1, y_2, \mu) - \delta C(x_1, x_2, y_1, y_2, \mu)$$

$$\delta C_H^{(1)}(x_1,x_2,y_1,y_2,P^z,\mu) = (P^z)^2\frac{\alpha_s C_F}{2\pi}\Bigg[I_{\rm H}[(x_1-y_1)P^z,(x_2-y_2)P^z] + I_{\rm HSI}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+ I_{\rm HSII}[(x_1-y_1)P^z,(x_2-y_2)P^z] + I_{\rm HSIII}[(x_1-y_1)P^z,(x_2-y_2)P^z] + I_{\rm HSIV}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+ I_{\rm S}[(x_1-y_1)P^z,(x_2-y_2)P^z] + \delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{5}{2}\ln\left(\frac{\mu^2 e^{2\gamma_E}}{4}\right)+4\right)\Bigg], \quad (3.22)$$

$$I_{\rm H}[p_1,p_2] \equiv \int \frac{dz_1}{2\pi}\frac{dz_2}{2\pi}e^{ip_1z_1+ip_2z_2}\left[\frac{7}{8}\ln\left(z_1^2\right)+\frac{7}{8}\ln\left(z_2^2\right)+\frac{3}{4}\ln\left((z_1-z_2)^2\right)\right] \quad (B.7)$$
$$\times\left(\theta(2z_s-|z_1|)\theta(z_s-|z_2|)+\theta(z_s-|z_1|)\theta(|z_2|-z_s)\theta(2z_s-|z_2|)\right)$$
$$=\frac{7}{8}\left[\mathrm{I0}\left(\{2z_s,-2z_s\},p_2\right)\mathrm{I1}\left(\{z_s,-z_s\},p_1\right)+\left(\mathrm{I0}\left(\{2z_s,-2z_s\},p_1\right)-\mathrm{I0}\left(\{z_s,-z_s\},p_1\right)\right)\mathrm{I1}\left(\{z_s,-z_s\},p_2\right)\right.$$
$$\left.+\mathrm{I0}\left(\{z_s,-z_s\},p_2\right)\left(\mathrm{I1}\left(\{2z_s,-2z_s\},p_1\right)-\mathrm{I1}\left(\{z_s,-z_s\},p_1\right)\right)+\mathrm{I0}\left(\{z_s,-z_s\},p_1\right)\mathrm{I1}\left(\{2z_s,-2z_s\},p_2\right)\right]$$
$$+\frac{3}{8\pi\left(p_1+p_2\right)}\left[\sin\left(\left(p_1+p_2\right)z_s\right)\left[\mathrm{I1}\left(\{z_s,-z_s\},p_1\right)+\mathrm{I1}\left(\{z_s,-z_s\},-p_2\right)\right.\right.$$
$$\left.-\mathrm{I1}\left(\{2z_s,-2z_s\},p_1\right)-\mathrm{I1}\left(\{2z_s,-2z_s\},-p_2\right)+\mathrm{I1}\left(\{3z_s,-3z_s\},p_1\right)+\mathrm{I1}\left(\{3z_s,-3z_s\},-p_2\right)\right]$$
$$+\sin\left(2\left(p_1+p_2\right)z_s\right)\left[-\mathrm{I1}\left(\{z_s,-z_s\},p_1\right)-\mathrm{I1}\left(\{z_s,-z_s\},-p_2\right)\right.$$
$$\left.+\mathrm{I1}\left(\{3z_s,-3z_s\},p_1\right)+\mathrm{I1}\left(\{3z_s,-3z_s\},-p_2\right)\right]$$
$$+\cos\left(2\left(p_1+p_2\right)z_s\right)\left[-\mathrm{I1t}\left(\{z_s,-z_s\},p_1\right)+\mathrm{I1t}\left(\{z_s,-z_s\},-p_2\right)\right.$$
$$\left.+\mathrm{I1t}\left(\{3z_s,-3z_s\},p_1\right)-\mathrm{I1t}\left(\{3z_s,-3z_s\},-p_2\right)\right]$$
$$+\cos\left(\left(p_1+p_2\right)z_s\right)\left[-\mathrm{I1t}\left(\{z_s,-z_s\},p_1\right)+\mathrm{I1t}\left(\{z_s,-z_s\},-p_2\right)-\mathrm{I1t}\left(\{2z_s,-2z_s\},p_1\right)\right.$$
$$\left.\left.+\mathrm{I1t}\left(\{2z_s,-2z_s\},-p_2\right)+\mathrm{I1t}\left(\{3z_s,-3z_s\},p_1\right)-\mathrm{I1t}\left(\{3z_s,-3z_s\},-p_2\right)\right]\right],$$

Result and Outlook

Apply hybrid scheme to all leading twist LCDA:

$$f_N\{(\not{P}C)_{\alpha\beta}(\gamma_5 u_\Lambda)_\gamma V(z_i P\cdot n) + (\not{P}\gamma_5 C)_{\alpha\beta}(u_\Lambda)_\gamma A(z_i P\cdot n) + (i\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma_\mu\gamma_5 u_\Lambda)_\gamma T(z_i P\cdot n)$$

Does scales separate to all order? —A prerequisite to hybrid scheme

$$\hat{M}_p\left(z_1, z_2, 0, 0, \mu\right) = \left(1 + \sum_{n=1}\sum_{m=0}^{n}\alpha_s^n a_{n,m}L_1^m\right)\left(1 + \sum_{n=1}\sum_{m=0}^{n}\alpha_s^n b_{n,m}L_2^m\right)\left(1 + \sum_{n=1}\sum_{m=0}^{n}\alpha_s^n c_{n,m}L_{12}^m\right)$$

$$L_1 = \ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4} \;\&\; L_2 = \ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4} \;\&.\; L_{12} = \ln\frac{(z_1-z_2)^2\mu^2 e^{2\gamma_E}}{4}$$

Operator mixing problem.

Thank you for listening!