Precision Predictions for Top-quark Width

王烨凡 (山东大学)

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in collaboration with 陈龙斌, 李海涛, 李钊, 王健, 吴泉锋

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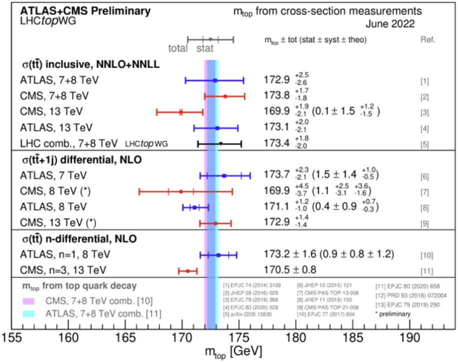
Motivation

Top-quark mass is one of the fundamental parameters in the Standard Model.

Summary of the top-mass analyses at the LHC.

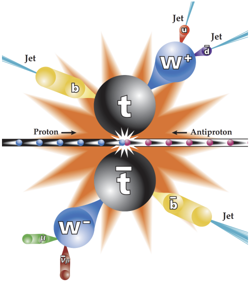


Motivation

Top decay width Γ_t is one of the fundamental properties of top-quark.

Due to its large mass, Γ_t is expected to be very large.

The measurement of Γ_t could hint new physics.



[Denisov, Vellidis 2015]

Motivation

The top-quark decays almost exclusively to Wb. $\Gamma_t = \Gamma_t(t \to Wb)$.

$$
\left|V_{\rm CKM}\right| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}
$$

[PDG 2022]

The most precise measurement for Γ_t by now is given by CMS

$\Gamma_t = 1.36 \pm 0.02\ (\text{stat.})^{+0.14}_{-0.11}\ (\text{syst.})$ GeV [CMS, 2014].

In the future e^+e^- collider, Γ_t can be measured with an uncertainty of 30 MeV [Martinez, Miquel 2019].

Theoretical Development

NLO QCD corrections [Jezabek, Kuhn 1989, Czarnecki 1990, Li, Oakes, Yuan 1991]

NLO EW corrections [Denner, Sack 1991, Eilam, Mendel, Migneron, Soni 1991]

Numerical result of full NNLO QCD corrections [Gao, Li, Zhu 2013, Brucherseifer, Caola, Melnikov 2013]

Numerical result of full N3LO QCD corrections [Chen, Chen, Guan, Ma 2023] – see Long Chen's talk

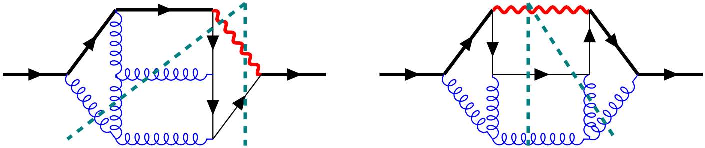
Analytic results of NNLO and N3LO QCD corrections [Chen, Li, Wang, Wang 2022, Chen, Li, Li, Wang, Wang, Wu 2023]

Optical Theorem

Consider the forward scattering amplitudes Σ for $t \to Wb \to t$

$$\Gamma_t = \frac{\mathrm{Im}(\Sigma)}{m_t} \tag{1}$$

For N3LO QCD corrections, the self-energy four-loop diagrams are considered. For example,



The imaginary part comes from cut diagrams.

The complicated phase space integration can be avoided.

Calculation Method

For $t \to Wb \to t$, b quark is considered as massless. We define $w = m_w^2/m_t^2$.

1. Generating diagrams and amplitudes.
2. Reducing amplitudes to master integrals by `FIRE` [Smirnov, Chuharev 2020].
3. Analytically calculating the master integrals.

Master Integrals Calculations

Method: canonical differential equations. For example,

$$\frac{\partial F_4(w,\epsilon)}{\partial w} = \frac{\epsilon\,(F_5 - 2F_4)}{w-1} - \frac{\epsilon\,(F_4+F_5)}{w}, \qquad w = \frac{m_W^2}{m_t^2}, \quad D = 4-2\epsilon \tag{2}$$

Two important ingredients:

1. Construct canonical form (ϵ form, $d\log$ form) – `Libra` [Lee 2021]

2. Boundary conditions – `AMFlow` [Liu, Ma 2022] and PSLQ method [Ferguson, Beiley, Arno 1992 1999]

Results: harmonic polylogarithms (HPLs).

Analytic calculations of three loop and four loop master integrals are non-trivial.

Analytic Results

QCD corrections of Γ_t up to N3LO.

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 + \left(\frac{\alpha_s}{\pi}\right)^3 X_3 \right], \tag{3}$$

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}. \tag{4}$$

The LO and NLO corrections are

$$\begin{aligned} X_0 &= (2w+1)(w-1)^2, \\ X_1 &= C_F \Bigg(X_0 \Big(-2H_{0,1}(w) + H_0(w)H_1(w) - \frac{\pi^2}{3} \Big) + \frac{1}{2}(4w+5)(w-1)^2 H_1(w) \\ &\quad + w(2w^2+w-1)H_0(w) + \frac{1}{4}(6w^3 - 15w^2 + 4w + 5) \Bigg) \end{aligned} \tag{5}$$

Analytic Results of NNLO

According to color structure,

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 \right], \tag{6}$$

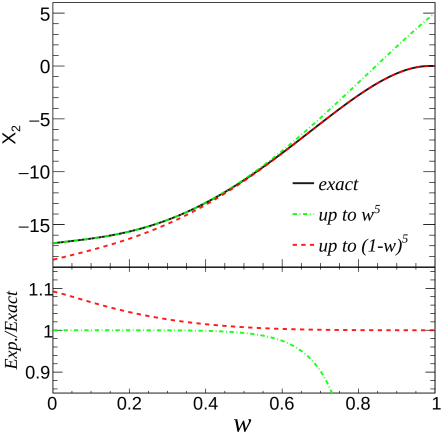
$$X_2 = C_F (T_R n_l X_l + T_R n_h X_h + C_F X_F + C_A X_A) \tag{7}$$

$$X_l = -\frac{X_0}{3} \left[H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3) \right] + g_l(w),$$

$$\begin{aligned}
X_F = {} & \frac{1}{12} X_0 \left[-6 \left(2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3) H_1(w) \right) - \pi^2 H_{1,0}(w) \right] \\
& + (X_0 + 4w) \left(-\frac{1}{6} \pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w) \right) \\
& + \frac{1}{12} \left(18w^3 - 3w^2 + 76w + 15 \right) \pi^2 H_{0,1}(w) - \frac{1}{2} \left(4w^3 - 2w^2 + 4w + 3 \right) H_{0,0,0,1}(w) \\
& + \frac{1}{2} \left(4w^3 - 2w^2 + 16w + 3 \right) H_{0,0,1,0}(w) + w \left(2w^2 - 7w - 16 \right) H_{0,0,1,1}(w) \\
& - \frac{1}{2} \left(2w^3 - 11w^2 - 28w - 1 \right) H_{0,1,1,0}(w) + \frac{1}{720} \pi^4 \left(42w^3 - 191w^2 - 328w - 11 \right) + g_F(w).
\end{aligned}$$

Analytic Results of NNLO

The result of NNLO expanded in $w = 0$ and $w = 1$ $(w = m_W^2/m_t^2)$



Off-Shell W Boson

Including the W boson width $\Gamma_W = 2.085$ GeV, the Γ_t becomes [Jezabek, Kuhn 1989]

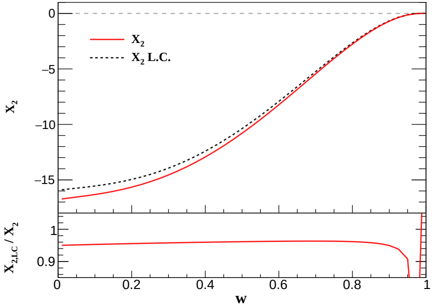
$$\tilde{\Gamma}_t \equiv \Gamma(t \to W^* b) = \frac{1}{\pi} \int_0^{m_t^2} dq^2 \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \Gamma_t(q^2/m_t^2), \tag{8}$$

Leading color contribution at NNLO

$$X_2 = C_F \left[C_F X_F + C_A X_A + T_R n_l X_l + T_R n_h X_h \right]$$
$$= C_F \left[N_c \left(X_A + \frac{X_F}{2} \right) + \frac{n_l}{2} X_l - \frac{1}{2N_c} X_F + \frac{n_h}{2} X_h \right] \quad (9)$$
$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad C_A = N_c, \quad T_R = \frac{1}{2},$$

$n_l(n_h) =$ number of light (heavy) quark flavors.

In the top quark decay, $N_c = 3, n_l = 5, n_h = 1$, $w \approx 0.22$, $X_{2,\mathrm{L.C}}/X_2 >$ 95%.



Analytic Calculations of N3LO

According to NNLO, the leading color contributions are dominant.

The analytic calculation of full N3LO is quite difficult.

$$X_3 = C_F\left[N_c^2 Y_A + \widetilde{Y}_A + \frac{\overline{Y}_A}{N_c^2} + n_l n_h Y_{lh} + n_l \left(N_c Y_l + \frac{\widetilde{Y}_l}{N_c}\right) + n_l^2 Y_{l2} \right. \\ \left. + n_h \left(N_c Y_h + \frac{\widetilde{Y}_h}{N_c}\right) + n_h^2 Y_{h2}\right]. \tag{10}$$

$$X_{3,\text{L.C}} = C_F\left[N_c^2 Y_A + n_l N_c Y_l + n_l^2 Y_{l2}\right]. \tag{11}$$

In leading color approximation, there are still 408 Feynman diagrams and 185 master integrals need to be calculated.

Numerical Results

Input parameters from [P.D.G 2022]

$$
\begin{aligned}
m_t &= 172.69\ \mathrm{GeV}, \quad m_b = 4.78\ \mathrm{GeV}, \\
m_W &= 80.377\ \mathrm{GeV}, \quad \Gamma_W = 2.085\ \mathrm{GeV}, \\
m_Z &= 91.1876\ \mathrm{GeV}, \quad G_F = 1.16638 \times 10^{-5}\ \mathrm{GeV}^{-2}, \\
|V_{tb}| &= 1, \quad \alpha_s(m_Z) = 0.1179.
\end{aligned} \tag{12}
$$

$\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

$$
\begin{aligned}
\Gamma_t = \Gamma_t^{(0)} [& (1 + \delta_b^{(0)} + \delta_W^{(0)}) \\
& + (\delta_b^{(1)} + \delta_W^{(1)} + \delta_{\mathrm{EW}}^{(1)} + \delta_{\mathrm{QCD}}^{(1)}) \\
& + (\delta_b^{(2)} + \delta_W^{(2)} + \delta_{\mathrm{EW}}^{(2)} + \delta_{\mathrm{QCD}}^{(2)} + \delta_{\mathrm{EW}\times\mathrm{QCD}}^{(2)}) \\
& + (\delta_b^{(3)} + \delta_W^{(3)} + \delta_{\mathrm{EW}}^{(3)} + \delta_{\mathrm{QCD}}^{(3)} + \delta_{\mathrm{EW}\times\mathrm{QCD}}^{(3)})],
\end{aligned} \tag{13}
$$

Numerical Results

Corrections in percentage ($\%$) normalized by the LO width $\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\rm EW}^{(i)}$	$\delta_{\rm QCD}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	—	—	1.459
NLO	0.126	0.132	1.683	-8.575	$1.361^{+0.0091}_{-0.0130}$
NNLO	$*$	0.030	$*$	-2.070	$1.331^{+0.0055}_{-0.0051}$
N^{3}LO	$*$	0.009	$*$	-0.667	$1.321^{+0.0025}_{-0.0021}$

QCD corrections are dominant.

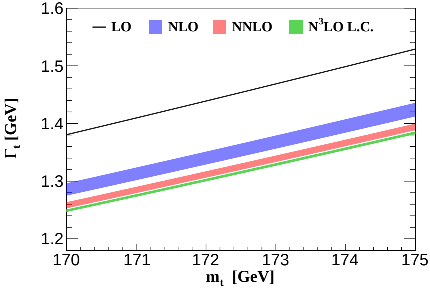
$X_{3,\rm L.C}/X_3 \approx 95\%$ compared with full numerical result. [Chen, Chen, Guan, Ma 2023]

NLO EW correction is 1.683%.

The off-shell W boson effect at NNLO and N3LO are further suppressed.

Numerical Results

m_t varies from 170 GeV to 175 GeV. μ varies from $\frac{m_t}{2}$ to $2m_t$.



The scale uncertainty at N3LO is reduced to $\pm 0.2\%$, only half of that at NNLO.

It is very convenient to use fitted function within this range

$$\Gamma_t(m_t) = 0.027037 \times m_t - 3.34801 \text{ GeV} \tag{14}$$

Mathematica program

TopWidth.m can be downloaded from https://github.com/haitaoli1/TopWidth. The package HPL is required [Maitre 2006].

```
<< TopWidth`

        (****** TopWidth-1.0 ******)
    Authors: Long-Bin Chen, Hai Tao Li, Jian Wang, YeFan Wang
    TopWidth[QCDorder, mbCorr, WwidthCorr, EWcorr, mu] is provided for top width calculations
    Please cite the paper for reference: arXiv:2212.06341

*-*-*-*-*-* HPL 2.0 *-*-*-*-*-*

Author: Daniel Maitre, University of Zurich

(* SetParameters[mt,mb,mw,Wwidth, GF] *)
```

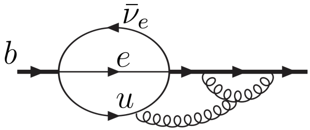
$$\texttt{SetParameters}\left[\frac{17269}{100}, \frac{478}{100}, 80377/1000, 2085/1000, 911876/10000, 11663788\times 10^{-12}\right];$$

$$\texttt{TopWidth}\left[3, 1, 1, 1, \frac{17269}{100}\right]$$

```
1.32073
```

Relations With Other Process

Integrating over w $(w = m_W^2/m_t^2)$ from 0 to 1, we obtain N3LO QCD corrections in semileptonic decay $\Gamma(b \to X_u e\bar{\nu}_e)$ [Ritbergen 1999].



$$\Gamma(b \to X_u e\bar{\nu}_e) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left[1 + \sum_{i=1} \left(\frac{\alpha_s}{\pi} \right)^i b_i \right]. \tag{15}$$

$$\begin{aligned} b_3 = {} & C_F \Bigg[N_c^2 \left(\frac{9651283}{82944} - \frac{1051339\pi^2}{62208} - \frac{67189\zeta(3)}{864} + \frac{4363\pi^4}{6480} + \frac{59\pi^2\zeta(3)}{32} + \frac{3655\zeta(5)}{96} \right. \\ & \left. - \frac{109\pi^6}{3780} \right) + n_l N_c \left(-\frac{729695}{27648} + \frac{48403\pi^2}{15552} + \frac{1373\zeta(3)}{108} + \frac{133\pi^4}{1728} - \frac{13\pi^2\zeta(3)}{72} - \frac{125\zeta(5)}{24} \right) \\ & + n_l^2 \left(\frac{24763}{20736} - \frac{1417\pi^2}{15552} - \frac{37\zeta(3)}{216} - \frac{121\pi^4}{6480} \right) + \text{subleading color} \Bigg] \\ = {} & (-195.3 \pm 9.8) C_F \,. \end{aligned} \tag{16}$$

Summary

We provide the first full analytic result QCD correction at NNLO and leading color QCD correction at N3LO, which can be used to perform fast numerical evaluations.

The NNLO and N3LO QCD corrections decrease the LO result by -2.07% and -0.667% with $m_t = 172.69$ GeV.

We derive the analytic N3LO QCD leading color predictions for the semileptonic $b \to u$ decay width.

Thanks !

$$\tilde{\Gamma}_t = \Gamma_0 \left[\tilde{X}_0 + \frac{\alpha_s}{\pi} \tilde{X}_1 + \left(\frac{\alpha_s}{\pi} \right)^2 \tilde{X}_2 \right], \quad r = \frac{\Gamma_W}{m_W}, \quad w = \frac{m_W^2}{m_t^2}$$

$$\begin{aligned}
\tilde{X}_0 &= \frac{1}{2\pi} \big(-(2(r-i)w - i((r-i)w+i)^2 G(w+irw,1)) \\
&\quad - ((r+i)w-i)^2 2(r+i)w + iG(w-irw,1) - 4r(1-2w)w \big),
\end{aligned} \tag{17}$$

$$\begin{aligned}
\tilde{X}_1 &= \frac{1}{18\pi} \big(((r+i)w-i)(2(4\pi^2-9)(r+i)^2 w^2 + (4\pi^2-27)(1-ir)w + 4\pi^2 - 15) G(w-iw,1) \\
&\quad + (r-i)w-i)(2(4\pi^2-9)(r-i)^2 w^2 + (4\pi^2-27)(1+ir)w + 4\pi^2 - 15) G(w+iw,1) \\
&\quad + \cdots \big)
\end{aligned} \tag{18}$$

HPLs

The analytical results of master integrals can be written as multiple polylogarithms (GPLs)

$$G_{a_1,a_2,\ldots,a_n}(x) \equiv \int_0^x \frac{\mathrm{d}t}{t-a_1} G_{a_2,\ldots,a_n}(t)\,, \tag{19}$$

$$G_{\vec{0}_n}(x) \equiv \frac{1}{n!} \ln^n x\,. \tag{20}$$

In our problem, we only need harmonic polylogarithms (HPLs).

$$H_{a_1,a_2,\ldots,a_n}(x) = G_{a_1,a_2,\ldots,a_n}(x)|_{a_i\in\{-1,0,1\}}. \tag{21}$$

For example,

$$H_0(x) = \ln x, \quad H_{1,0}(x) = \int_0^x \frac{\mathrm{d}t}{t-1} \ln t, \quad H_{-1,1,0}(x) = \int_0^x \frac{\mathrm{d}t}{t+1} H_{1,0}(t). \tag{22}$$

HPLs have good mathematical properties.