Distribution, structure formation and the potential to distinguish thermal histories of dark matter

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arXiv: 2306.00065 [hep-ph] F. Huang, **Y-Z Li** and J-H Yu

Contents

Background and Motivation

Possible thermal histories of DM and its outcomes, Bottom-up approach to infer DM properties.

Dark Matter: Known and unknown

What do we know about DM at present?

1, Relic abundance $\Omega_{DM} \approx 0.25$.   2, Non-baryonic.   3, Non-relativisc today.

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We still want to know more about DM:

- Particle nature: Mass, spin, elementray or composite, ...   Direct/indirect detection.
- The thermal history: Production mechanism, temperature, ...   Cosmological observation.

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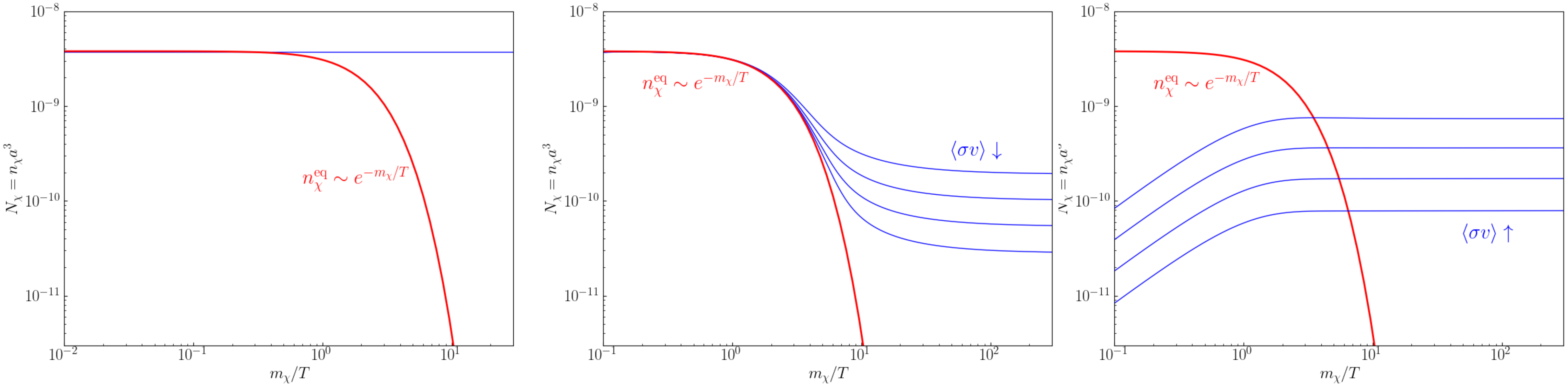
- Particle nature: Mass, spin, elementray or composite, ...   Direct/indirect detection.
- The **thermal history**: Production mechanism, temperature, ...   Cosmological observation.

1, Production or decoupling of DM.

2, Large scale structure formation.

Possible thermal histories of DM: freeze-out/-in and warm dark matter

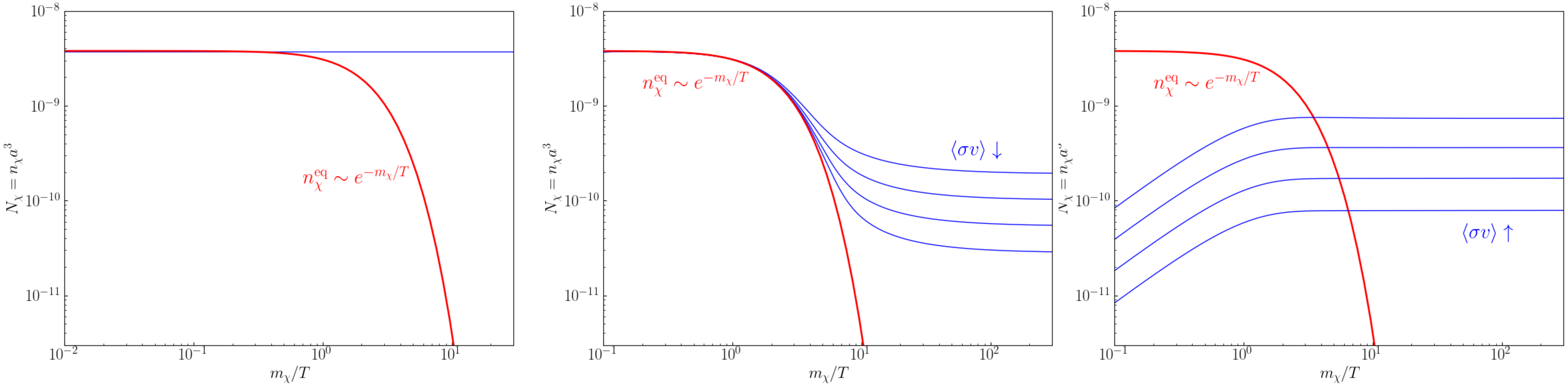


Relativistic freeze-out (WDM)

Non-relativistic freeze-out

Freeze-in

Possible thermal histories of DM: freeze-out/-in and warm dark matter



Relativistic freeze-out (WDM)

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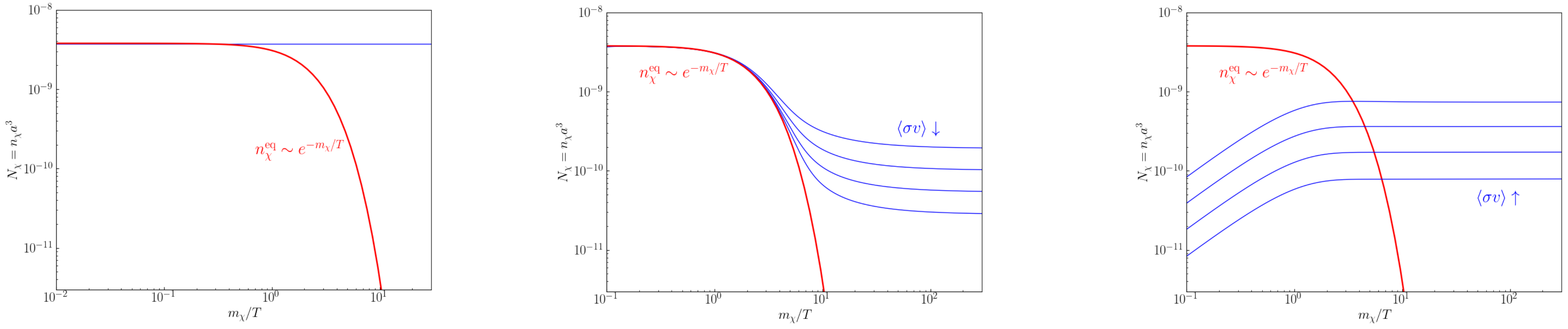
Is it possible to distinguish different thermal histories from the observation? Yes!

Effective phase space distributions for different thermal histories



Outcomes of different thermal histories:

1, Fixed **interaction couplings** to satisfy the relic abundance for given mass.

2, Different **phase space distributions** for DM.

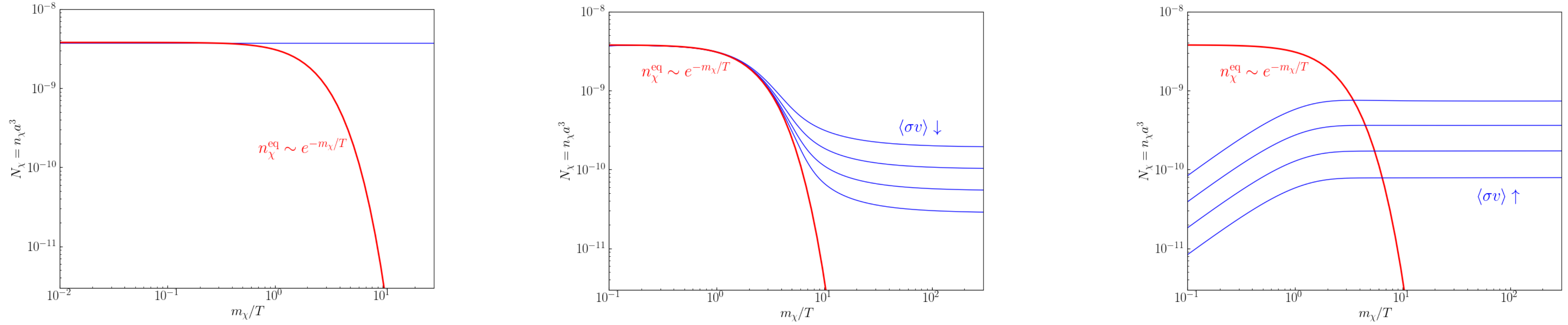
3, Significant imprints on the **large scale structures**.

Bottom-up approach: Structure observation → DM thermal history → DM interaction and coupling

Current Constraints on DM Thermal History: General Considerations

Connection between the mass and the DM thermal histories through the effective distributions.

Relation between the DM thermal history and DM mass

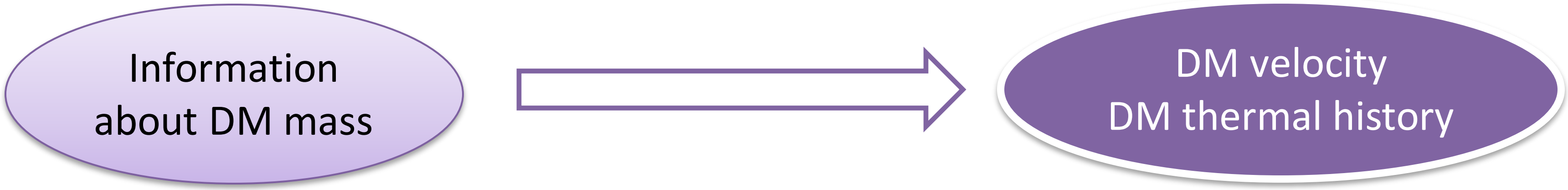


DM production need to satisfy the relic abundance condition:

$$g_\chi \, m_\chi \, n_{\chi,0}(m_\chi, T_{dec}, \eta_{dec}, \dots) = \rho_{crit} \Omega_{DM}$$

Result: The relation between the DM velocity and DM mass for fixed thermal history is specific.

The information about DM coupling hides behind this relation.



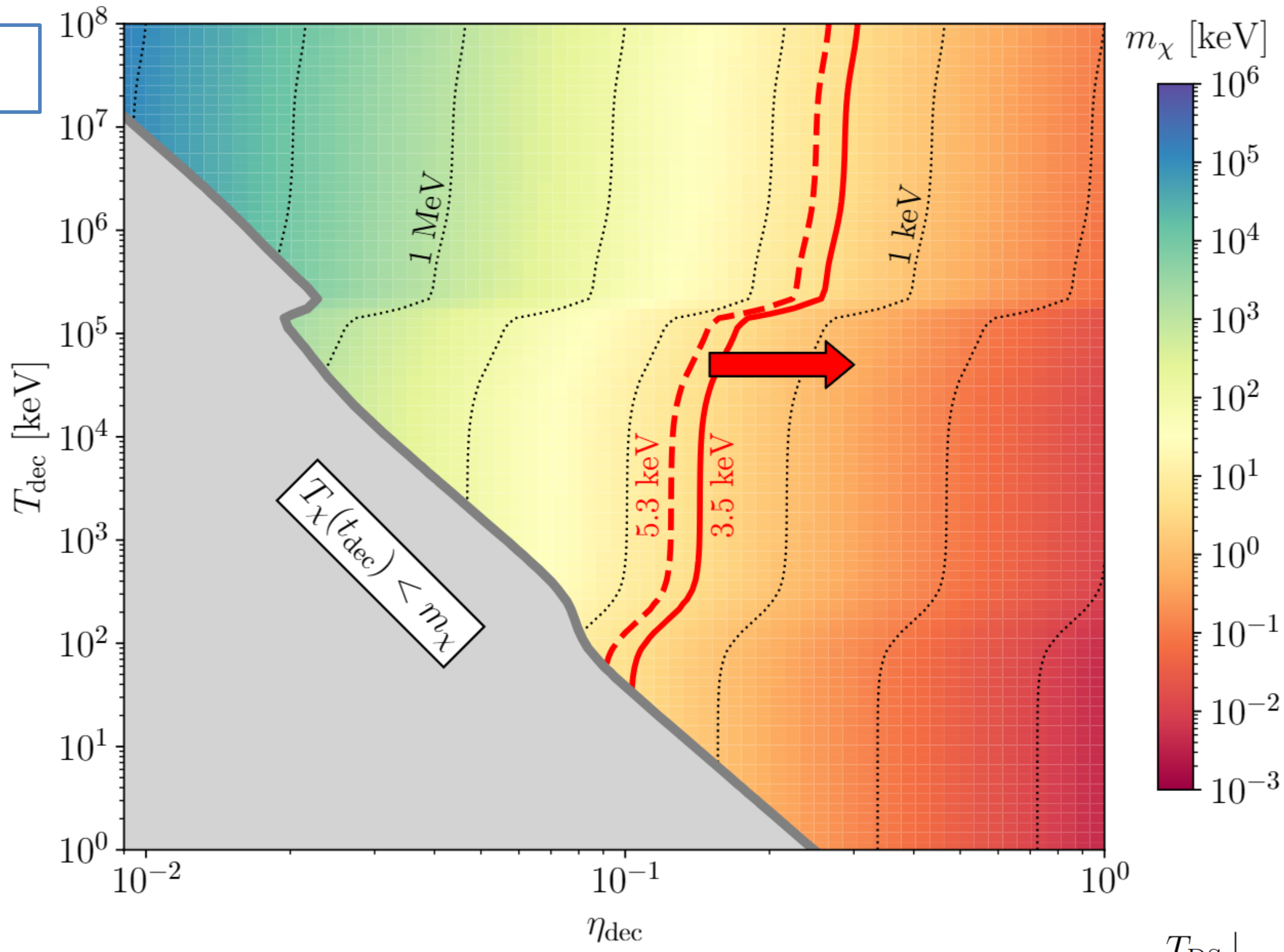
T_{dec}

$$\eta_{\rm dec} \equiv \left. \frac{T_{\rm DS}}{T_{\rm SM}} \right|_{T_{\rm SM}=T_{\rm dec}}$$

Lyman-α constraints on the DM thermal histories through average velocity

Warm dark matter

Currents Lyman-α constraints on WDM mass $\boldsymbol{m_{WDM} \gtrsim 3.5\,(5.3)\ keV}$ shows that WDM can only be generated from a colder dark sector.

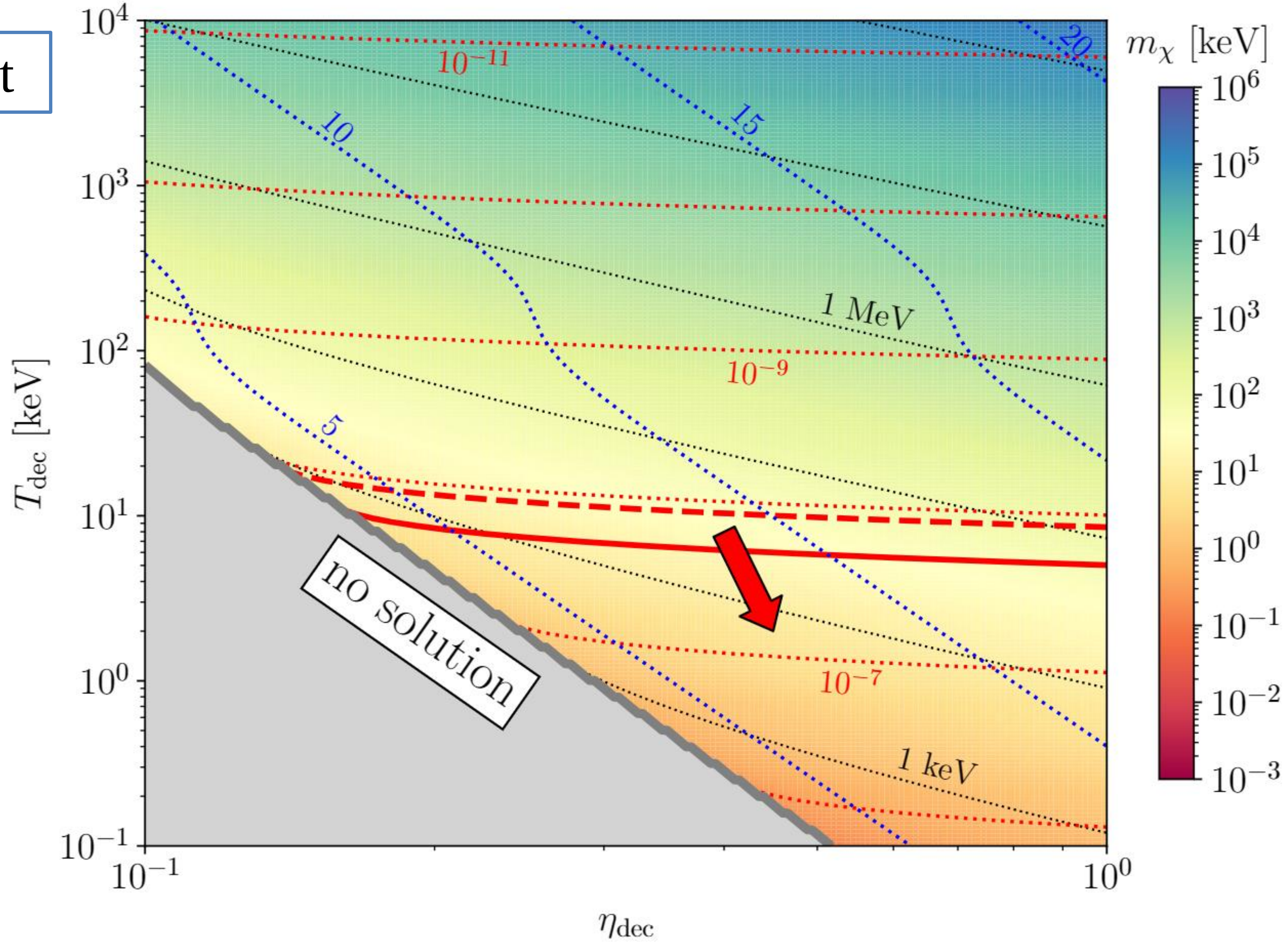


$$\eta_{\rm dec} \equiv \left.\frac{T_{\rm DS}}{T_{\rm SM}}\right|_{T_{\rm SM}=T_{\rm dec}}$$

Lyman-α constraints on the DM thermal histories through average velocity

Freeze-out

Freeze-out can happen in the SM thermal bath.

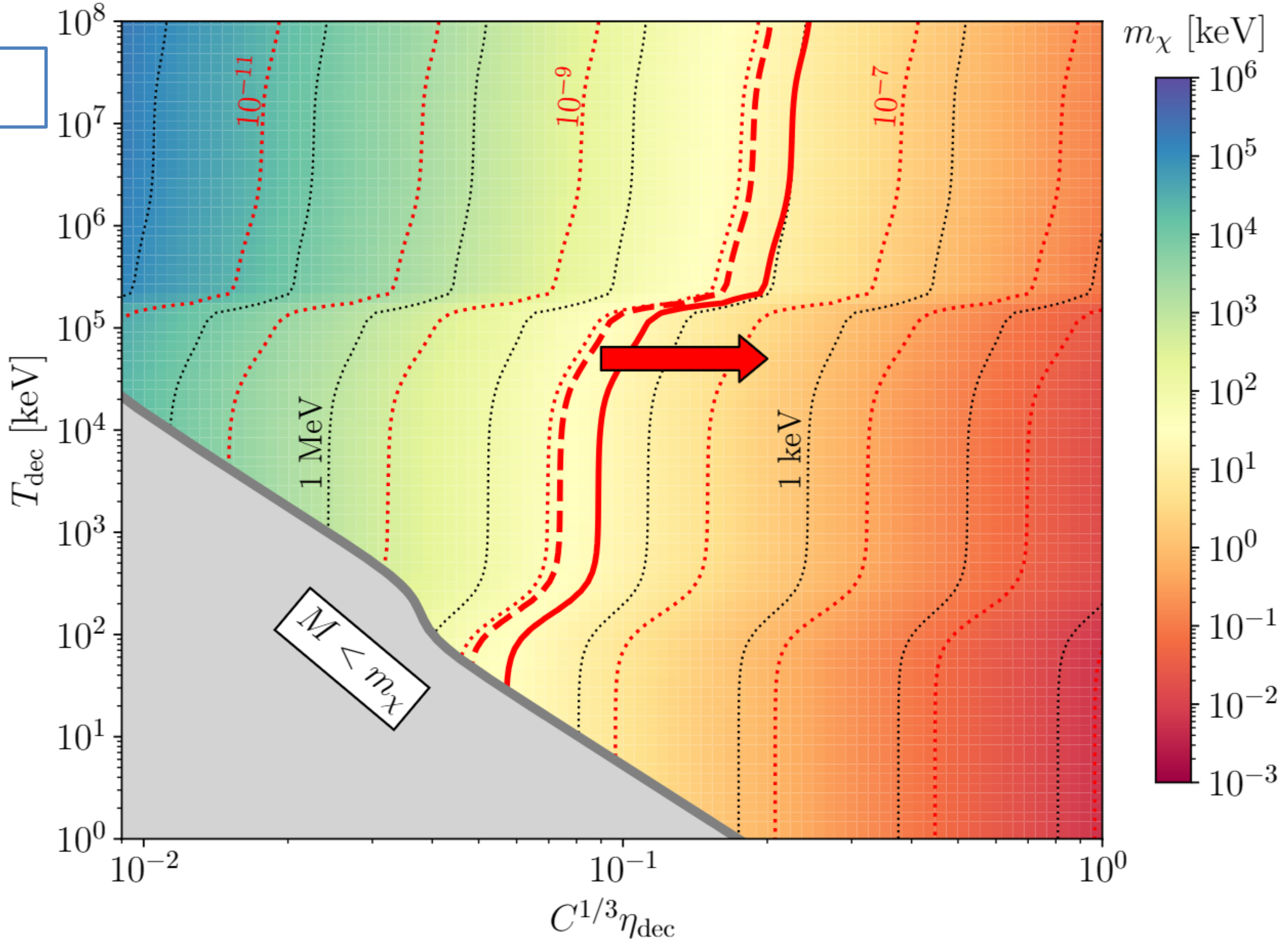


Lyman-α constraints on the DM thermal histories through average velocity

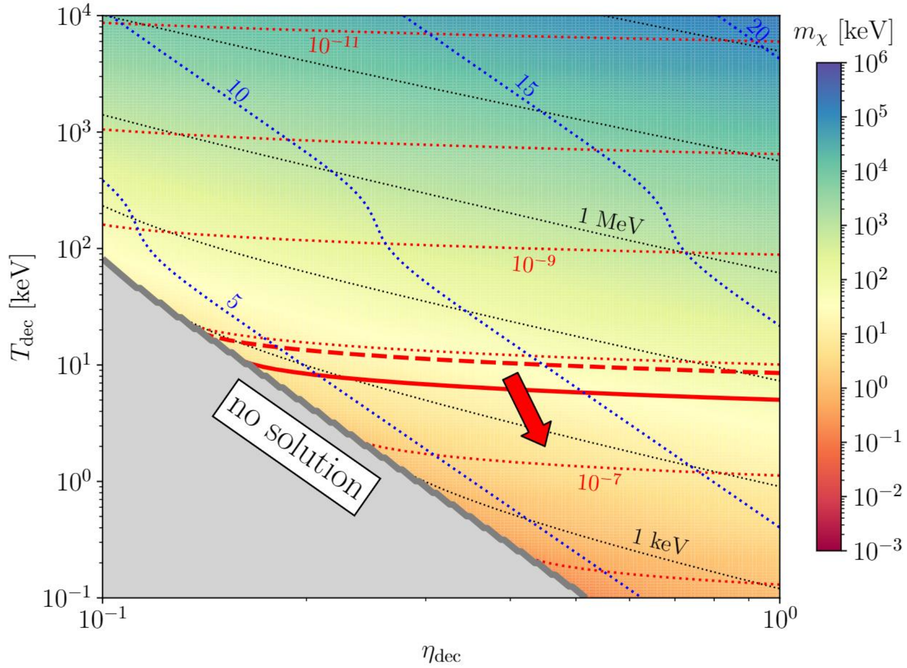
Freeze-in

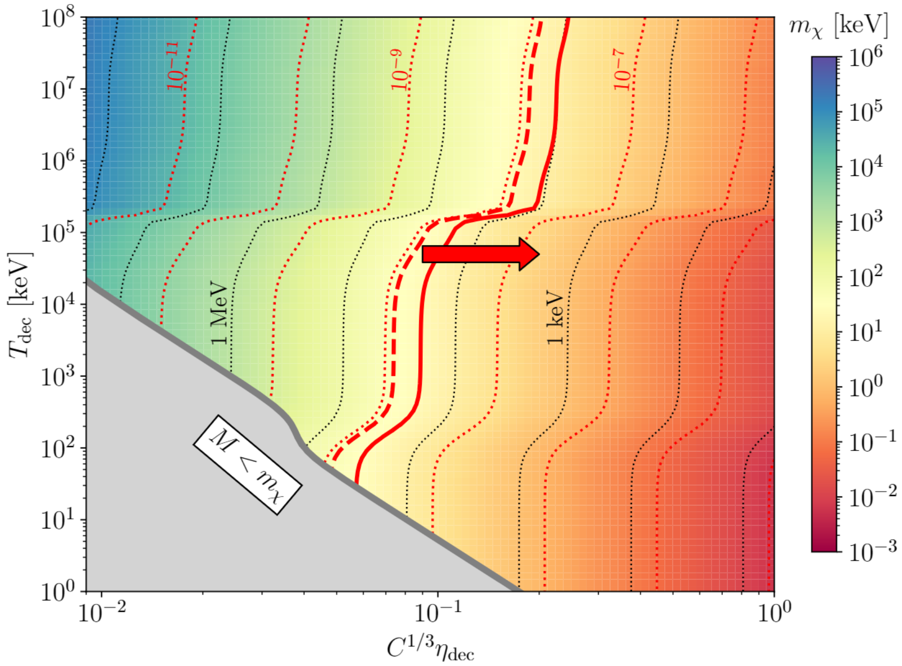
$$f(p,t) \simeq C \frac{1}{\sqrt{p/T_\chi^{\rm eff}}} e^{-p/T_\chi^{\rm eff}}$$

The presentation for the results of freeze-in depends on the choice of the additional freedom.



Shortcoming of current considerations





Shortcoming: Constraints for freeze-out/freeze-in are obtained through one single average velocity.

Maybe not precise enough!

Rigorous treatment: Calculate the full distribution and the corresponding large scale structure.

We must specify a model.

Current Constraints on DM Thermal History: An Explicit Model

Actual distributions, transfer functions and constraints on thermal history for freeze-in/freeze-out,

A simple model for Freeze-out/Freeze-in

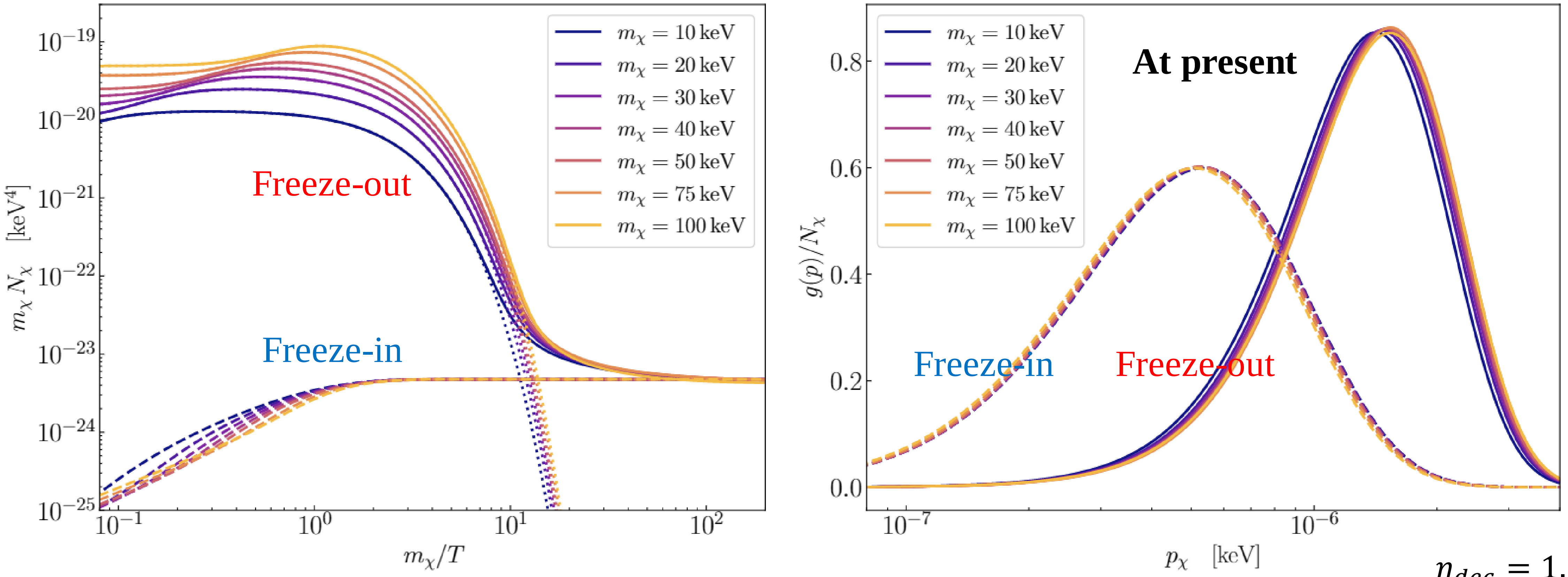
$$\psi + \psi \to \chi + \chi, \quad m_\psi \ll m_\chi, \quad \rho_{\rm DS} \ll \rho_{\rm SM}$$

The freeze-out and freeze-in scenarios are unified under one single framework.

A simple model for Freeze-out/Freeze-in

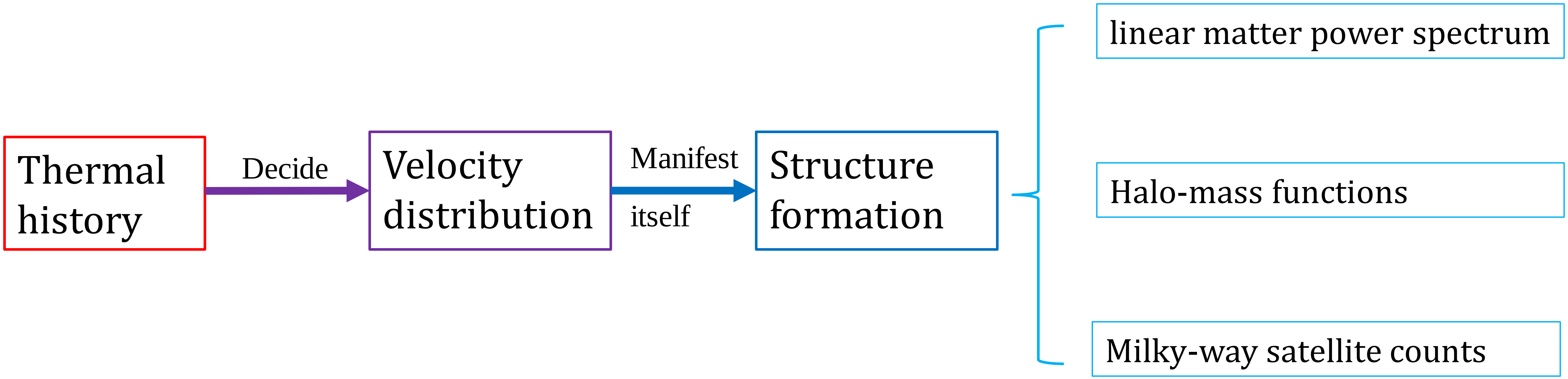
$$\psi + \psi \to \chi + \chi, \quad m_\psi \ll m_\chi, \quad \rho_{\rm DS} \ll \rho_{\rm SM}$$

The freeze-out and freeze-in scenarios are unified under one single framework.



$\eta_{dec} = 1.0$

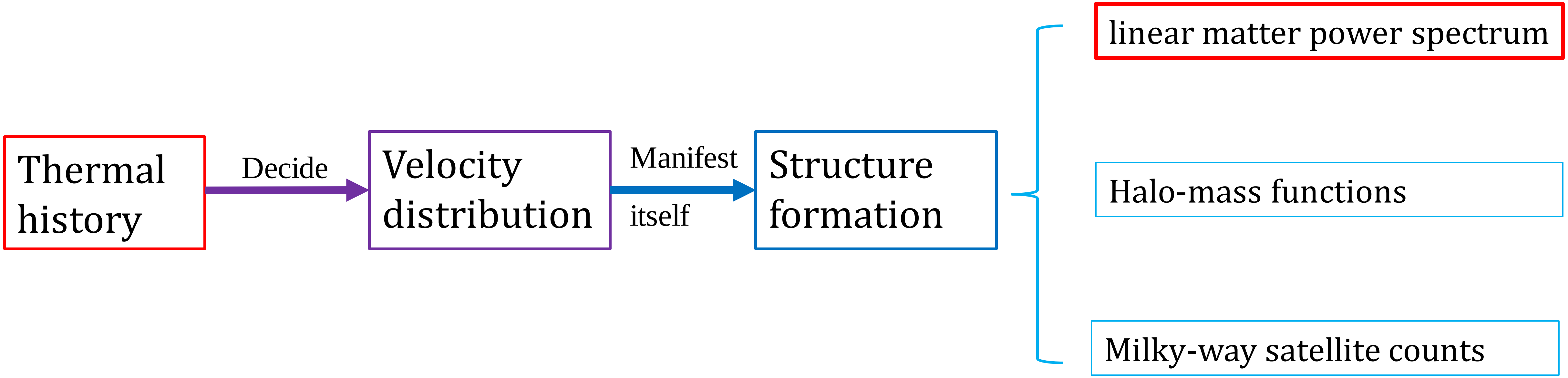
DM distribution and the large scale structure



Through the free-streaming of DM, the distribution is encoded in the large scale structure.

DM distribution and the large scale structure

$$P(k) \equiv \left|\delta_k^2\right| .$$

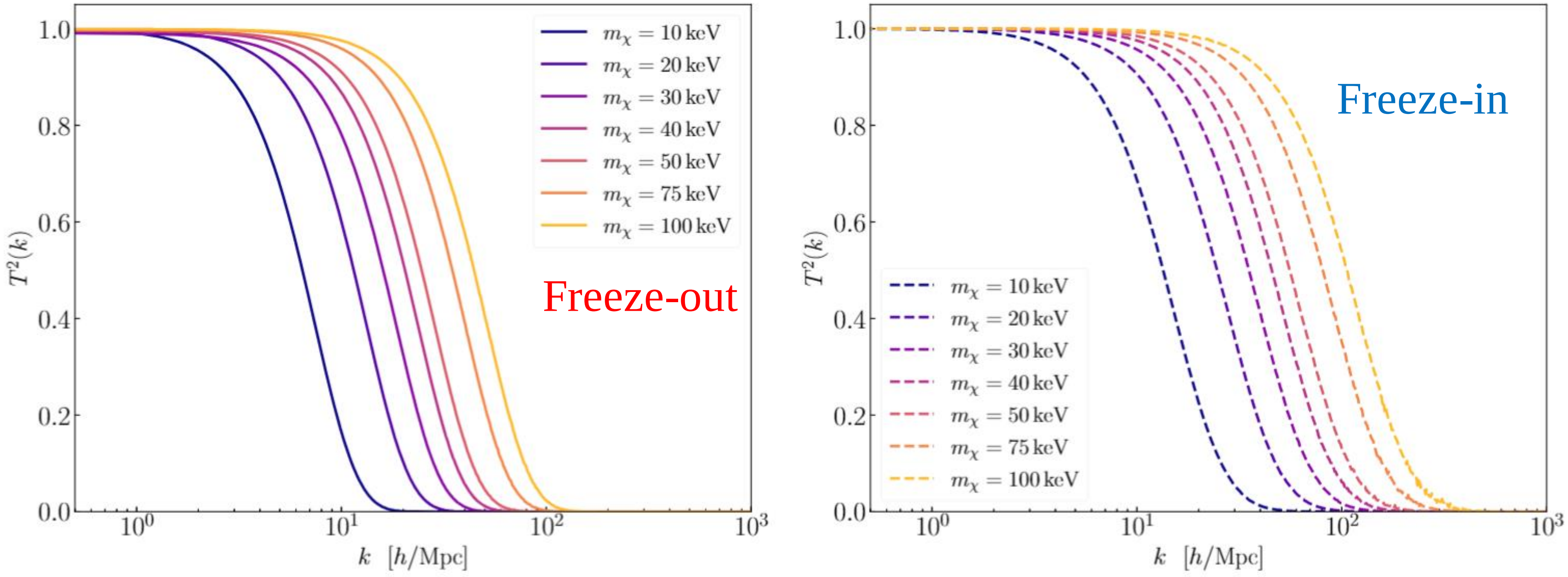


Through the free-streaming of DM, the distribution is encoded in the large scale structure.

Imprints of thermal history on linear matter power spectrum

$$P(k) \equiv \left|\delta_k^2\right| \, .$$

Focusing on the deviation from the CDM scenario: $T^2(k) \equiv \dfrac{P(k)}{P_{\mathrm{CDM}}(k)}$



The location of the transfer functions for different thermal histories and masses has significant difference.

We can recast the lyman-alpha constraints by **comparing the transfer functions**.

Lyman-α contraints: Two recasting mothods

Two methods to recast the Lyman-α constraints:
$\boldsymbol{m_{WDM} \gtrsim 3.5\ (5.3)\ keV}$

(1) Half-mode analysis,

$$T^2(k) \geq T^2_{\rm WDM}(k) \qquad \text{for all } 0 \leq k \leq k_{1/2}\,,$$

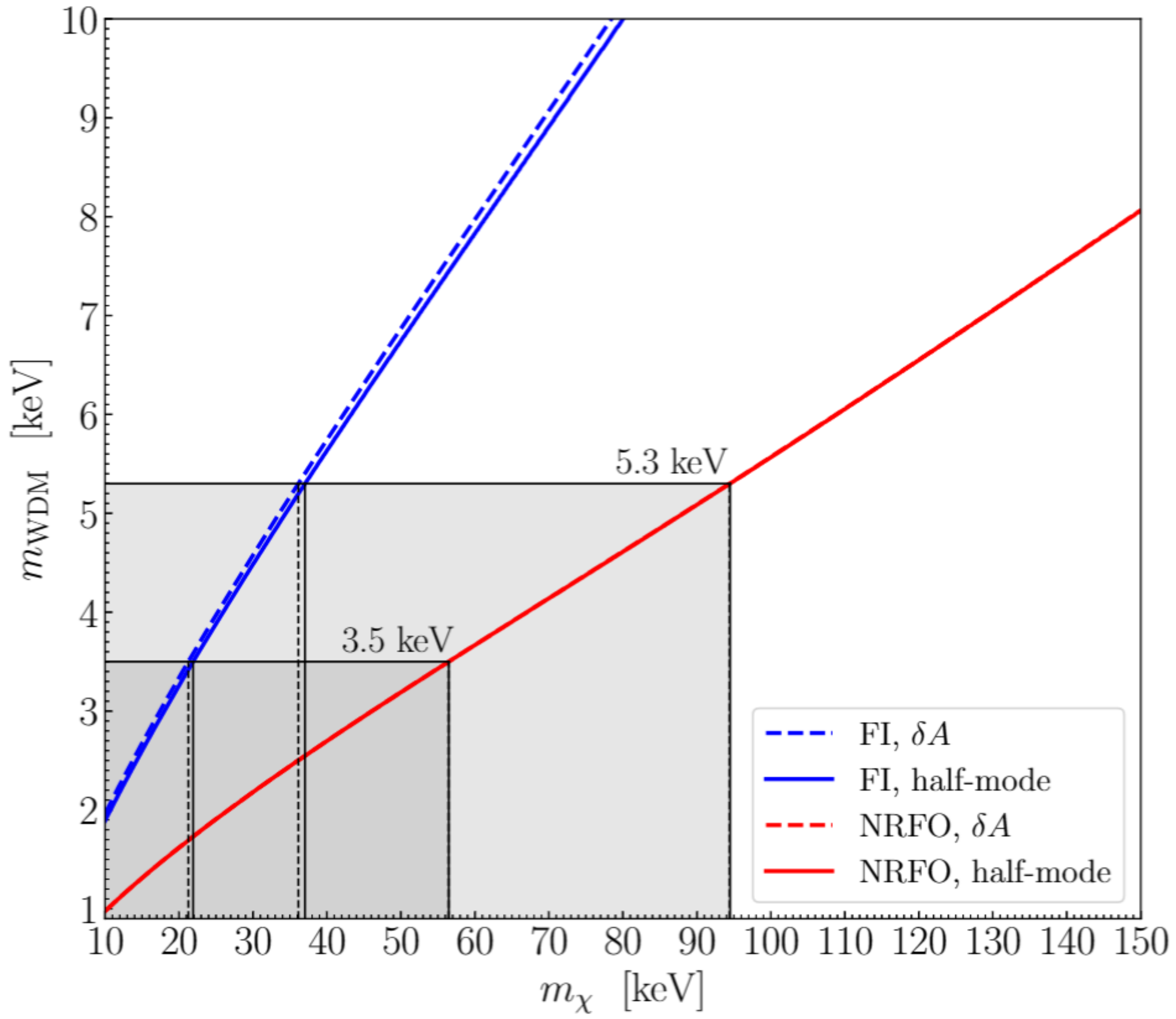
(2) δA analysis,

$$\delta A \leq \delta A_{\rm WDM},\ \ \delta A \equiv 1 - A/A_{\rm CDM}\,,$$

$$A \equiv \int_{k_{\rm min}}^{k_{\rm max}} dk \frac{P_{\rm 1D}(k)}{P_{\rm 1D}^{\rm CDM}(k)}\,, P_{\rm 1D}(k) \equiv \frac{1}{2\pi}\int_k^\infty dk'\ k' P(k')\,.$$

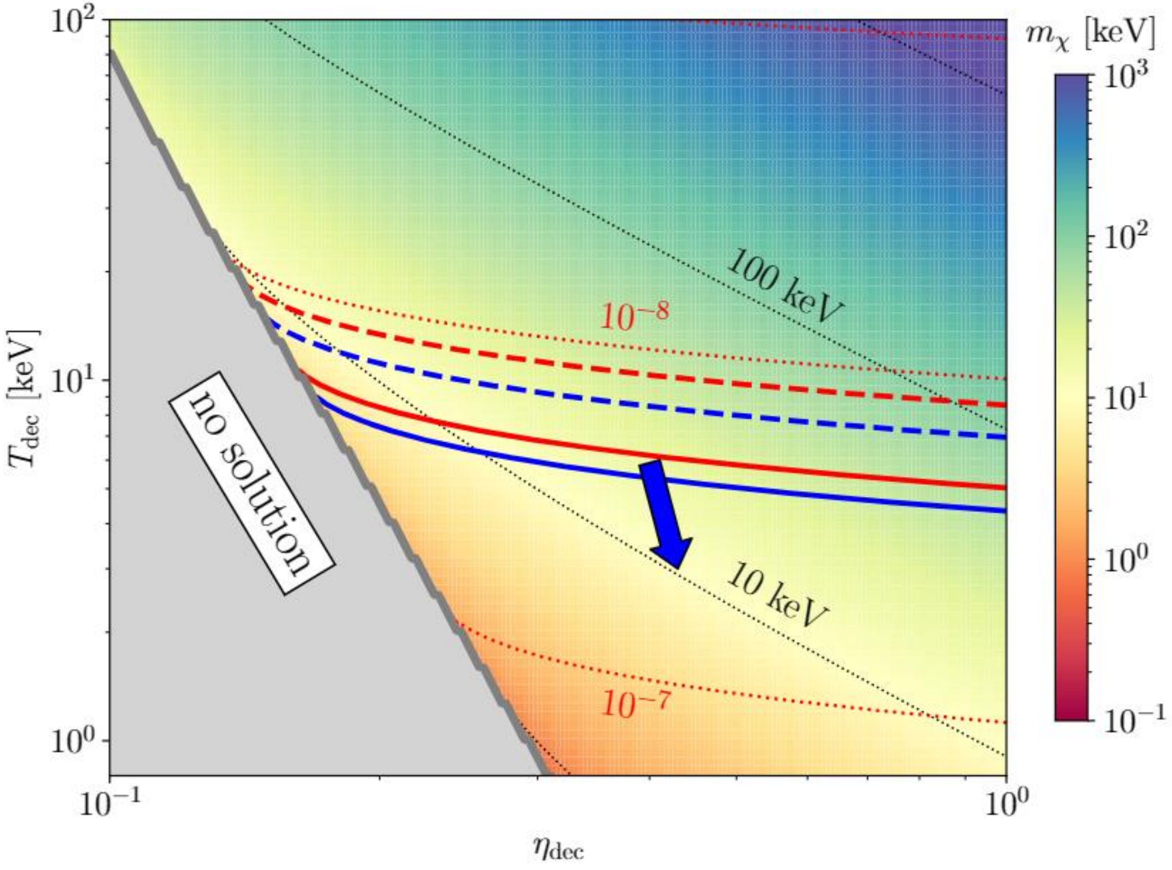
$\Rightarrow$ $\boldsymbol{m_{FI} \gtrsim 21.9(37.0)\ keV}$,

$\boldsymbol{m_{NRFO} \gtrsim 56.5\ (94.5)\ keV}$.

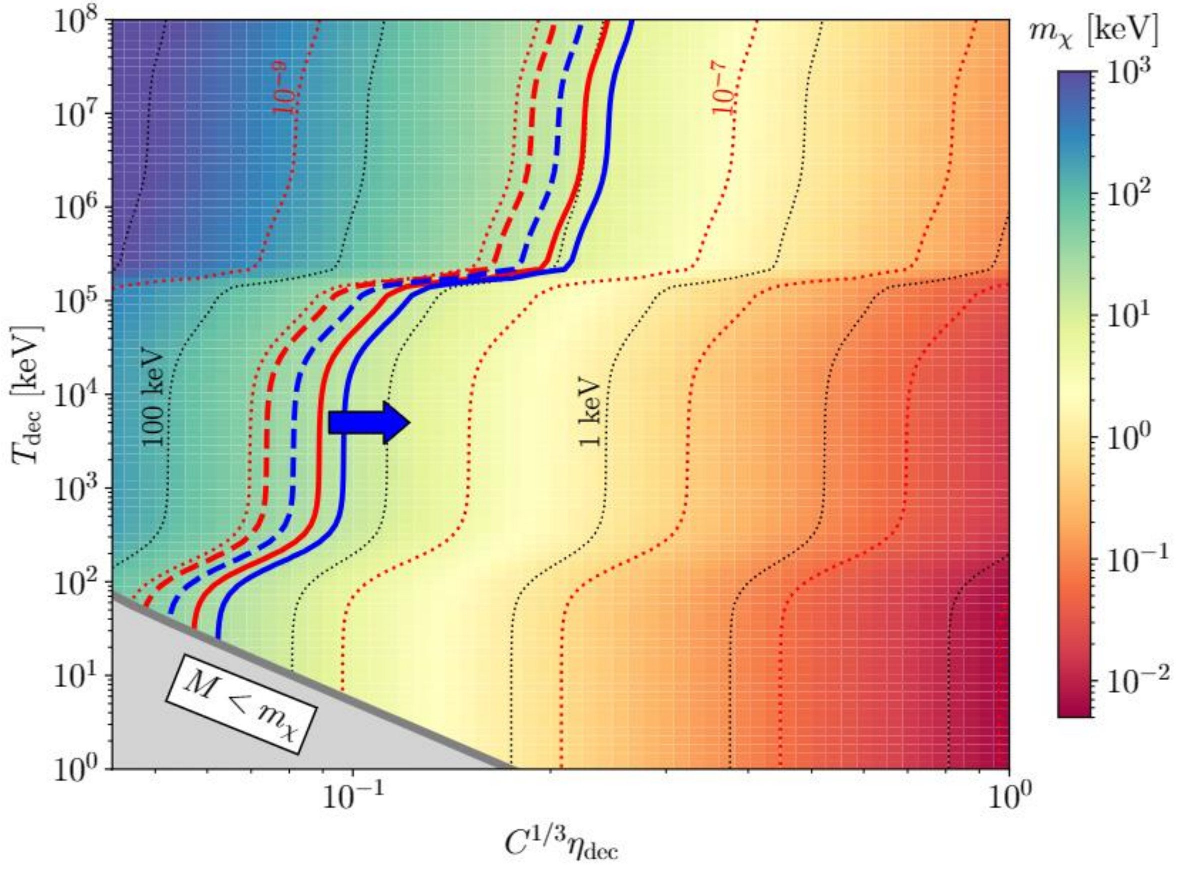


Precise lyman-α constraints on DM thermal histories

Freeze-out



Freeze-in

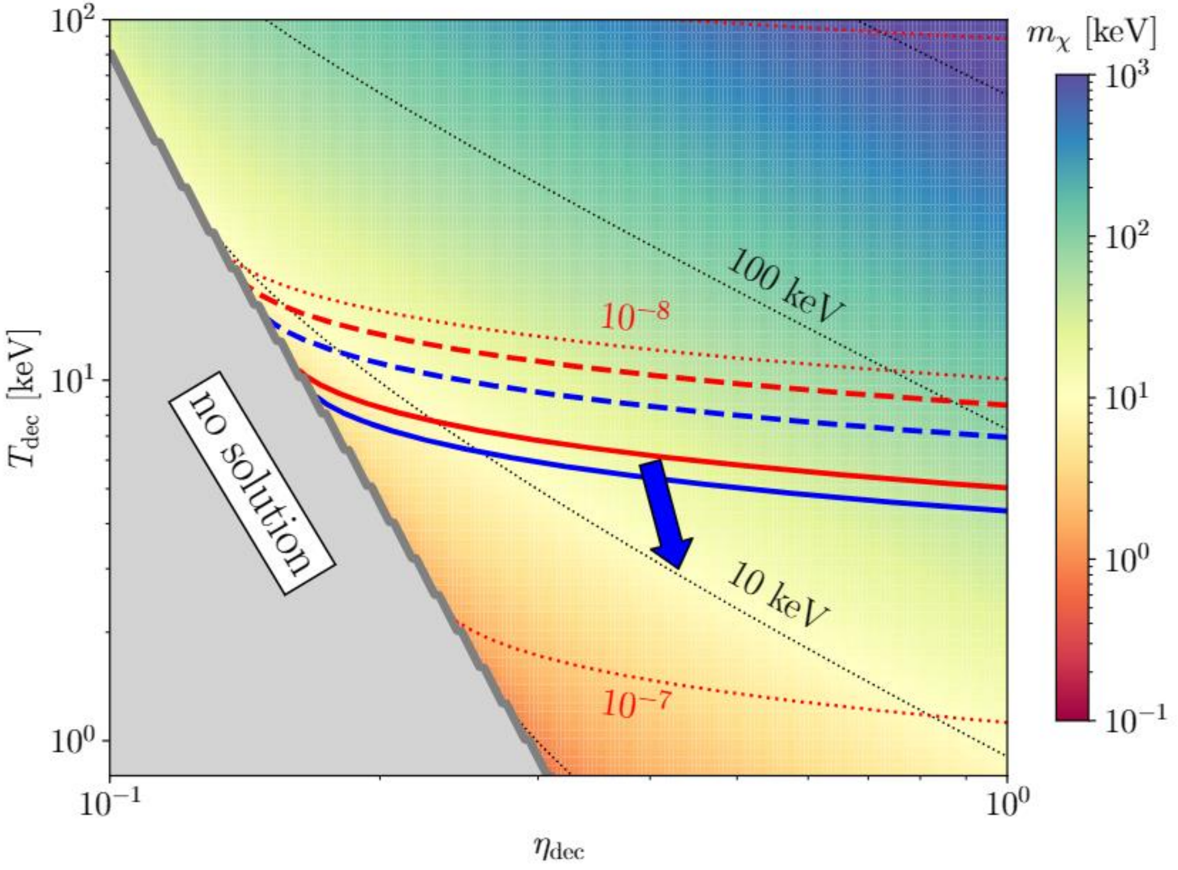


Red curves: Bounds from average velocities.

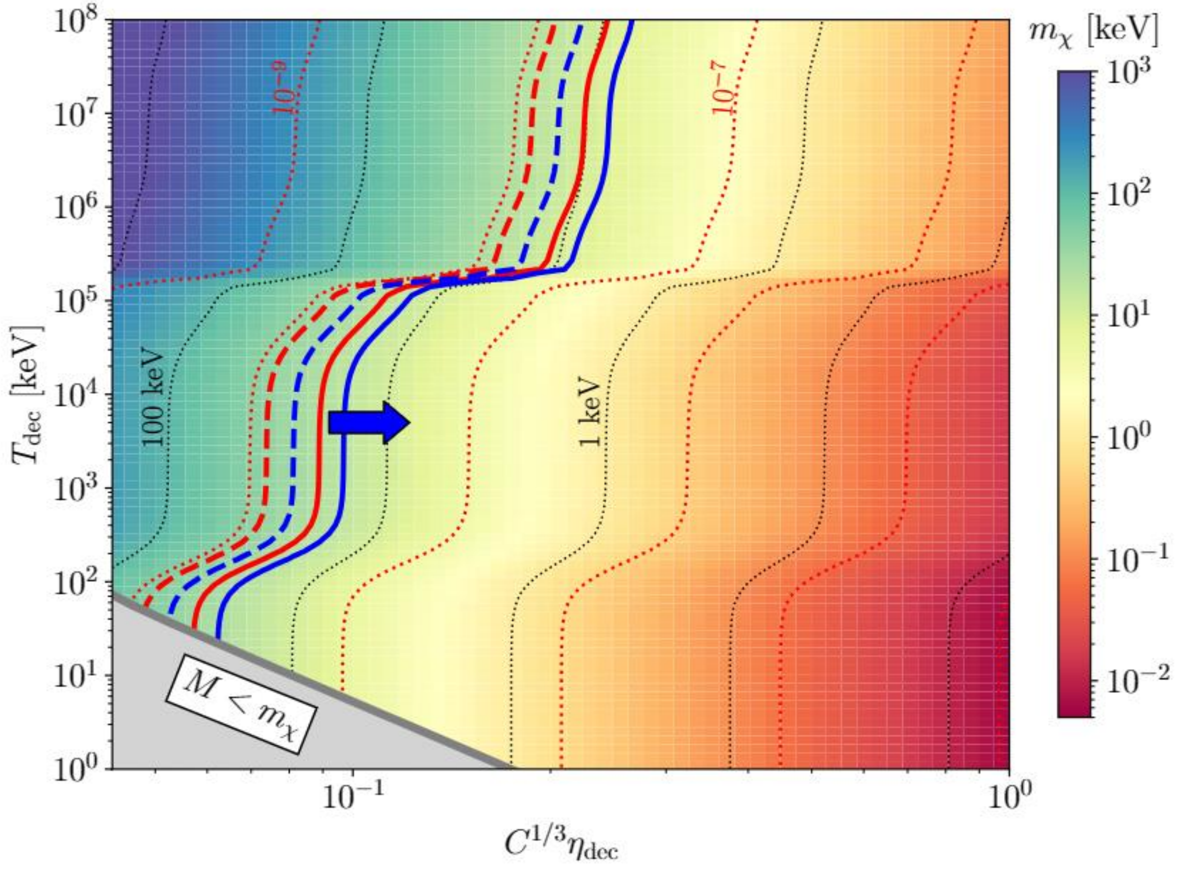
Blue cureves: Bounds from actual distributions.

Precise lyman-α constraints on DM thermal histories

Freeze-out



Freeze-in



Next question: What may be inferred from future observation?

Distinguishing Different Thermal History from Future Observation

Analysis with hypothetical future constraints on $P(k)$

Distinguishing freeze-in/freeze-out from future observations

Assumed future measurements on a physical quantity X(y):

$$X^+(y) > X(y) > X^-(y)\,, \quad X^{\pm}(y) = X_{\rm ref}[1 \pm \sigma_X^{\pm}(y)]\,,$$

Type I: analysis with a WDM mass range.

$$m_{\rm WDM}^{\rm ref}(1+\sigma_{m_{\rm WDM}}) > m_{\rm WDM} > m_{\rm WDM}^{\rm ref}(1-\sigma_{m_{\rm WDM}})\,.$$

Type II: analysis with future constraints on $P(k)$

$$P_{\rm WDM}(k)[1+\sigma_P^+(k)] > \ P(k) \ > P_{\rm WDM}(k)[1-\sigma_P^-(k)]\,,$$
$$T_{\rm WDM}^2(k)[1+\sigma_P^+(k)] > T^2(k) > T_{\rm WDM}^2(k)[1-\sigma_P^-(k)]\,.$$

Best-fit :
WDM model with mass m_{WDM}^{ref}.

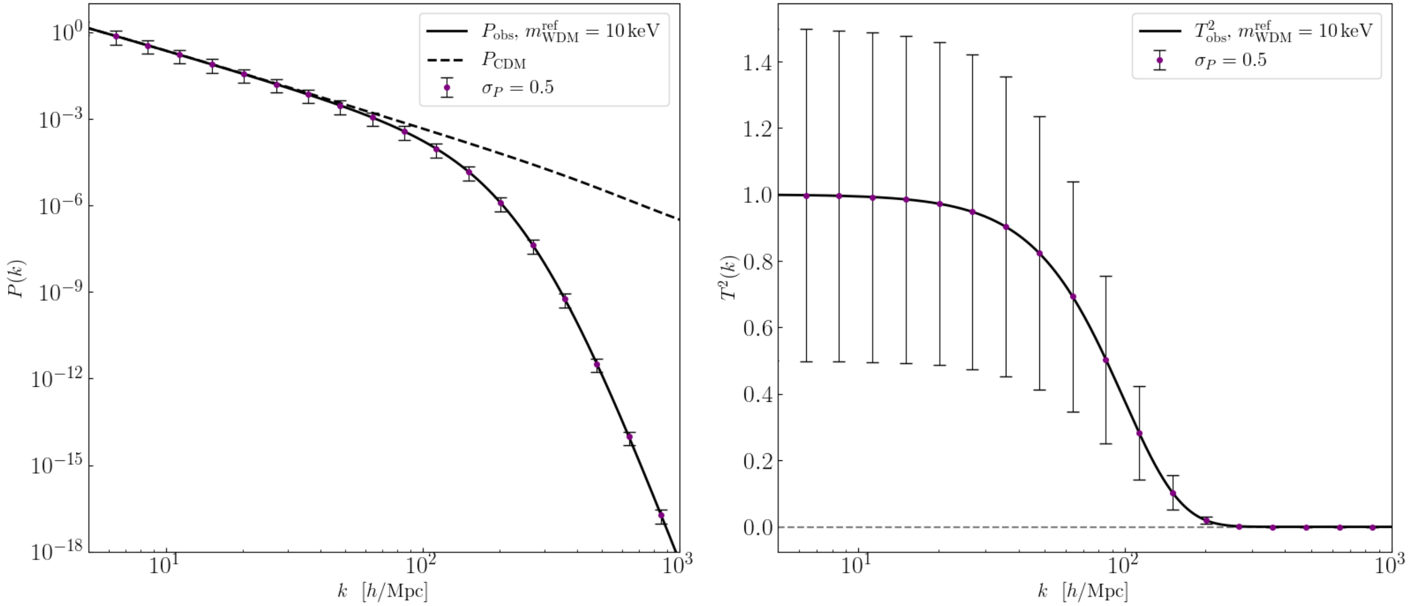
The location of the observational data: $k_{\rm min} \le k_i \le k_{\rm max},\ i = 1, \ldots, N$

Hypothetical future constraints on $P(k)$

Two possibilities of the observational uncertainty:

1, Constant symmetric relative errors on $P(k)$.

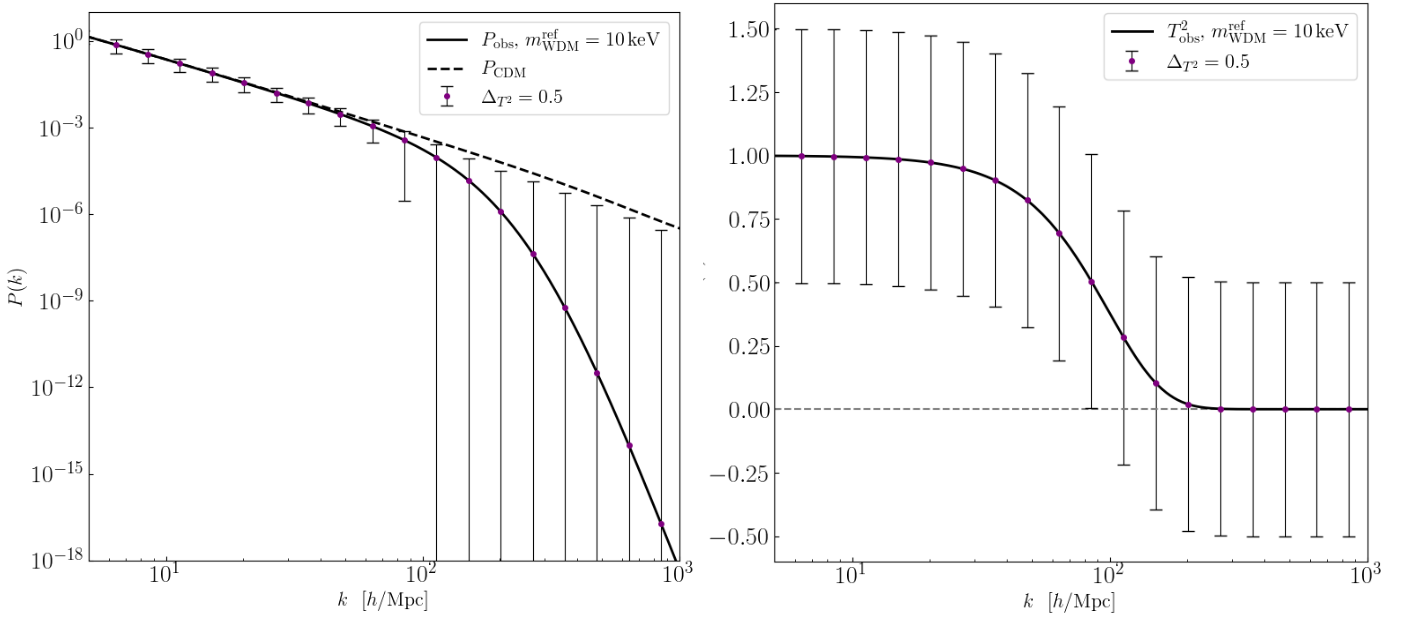
$$\sigma_P^{\pm}(k) = \sigma_P$$



2, Constant symmetric absolute errors on $T^2(k)$

$$\Delta_{T^2} \equiv \left| T^{2\pm}(k) - T^2_{\rm WDM}(k) \right|$$

$$\sigma_P(k) = \sigma_{T^2}(k) = \frac{\Delta_{T^2}}{T^2_{\rm WDM}(k)}\,.$$

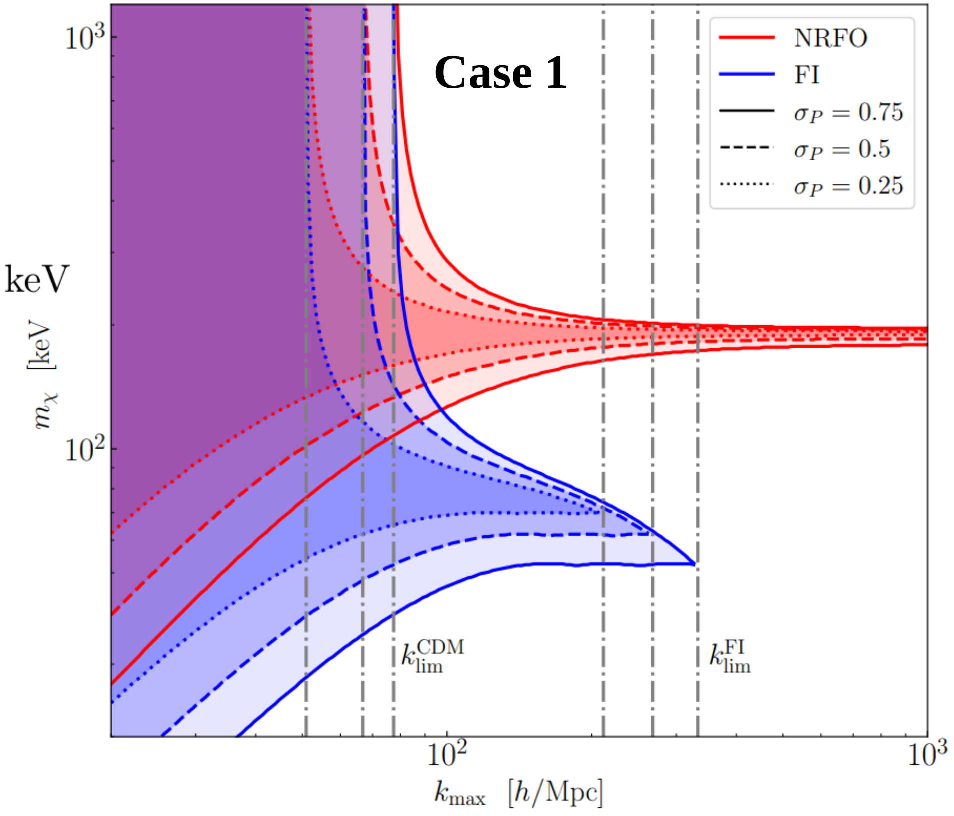


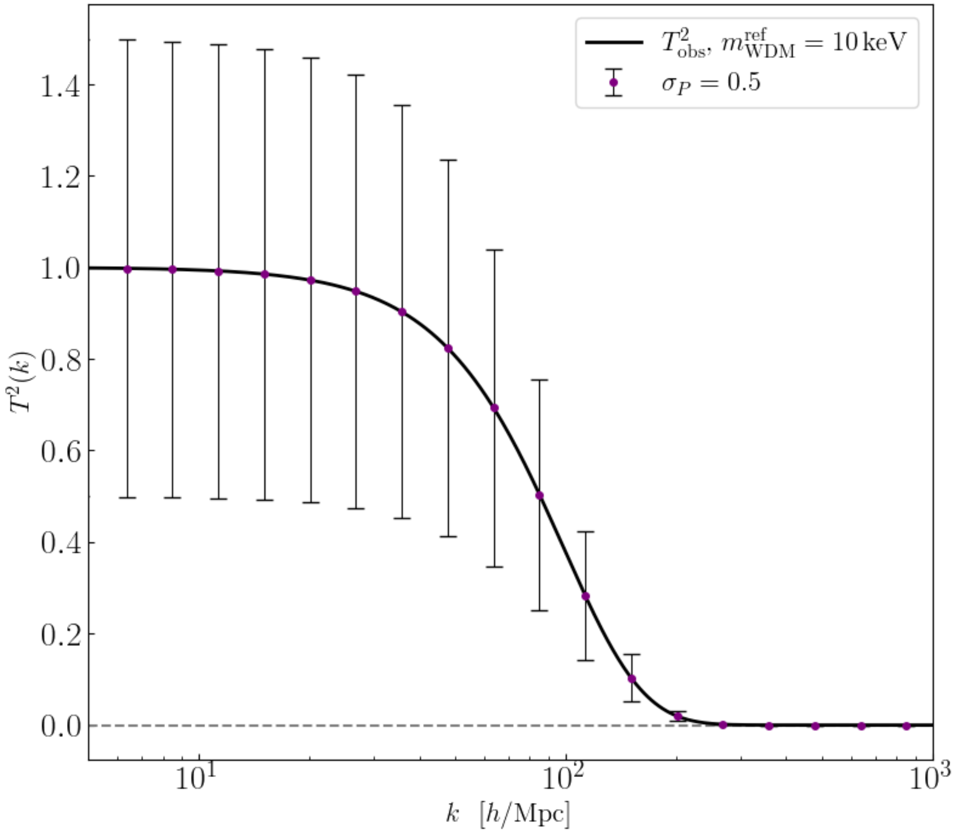
Allowed mass range from future constraints on $P(k)$, case 1

$k_{\rm min} = 1\ h/{\rm Mpc},\ \ {\rm N} = 20.$

The freeze-in scenario may be completely rulled out from accurate observation of $P(k)$, but the discrimination relies on the data uncertainties on small scales.

$m_{\rm WDM}^{\rm ref} = 10\,{\rm keV}$





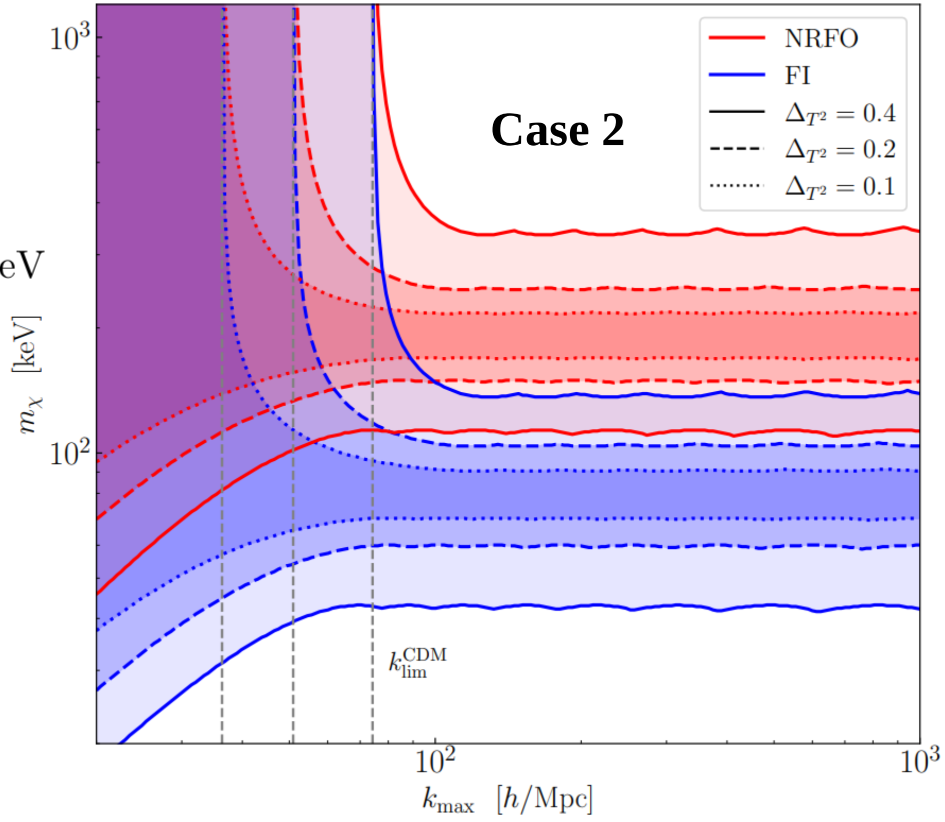
→ Varying the range of wavenumbers.

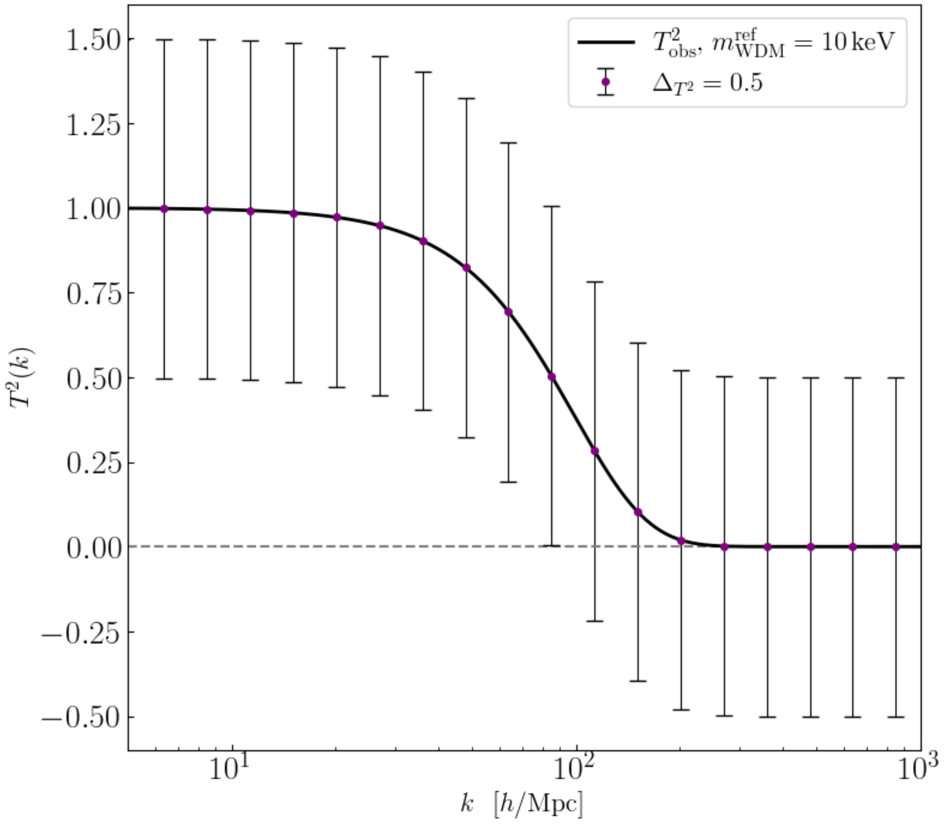
Allowed mass range from future constraints on $P(k)$, case 2

$k_{\min} = 1\ h/\mathrm{Mpc},\ \mathrm{N} = 20.$

The freeze-in scenario may be completely rulled out from accurate observation of $P(k)$, but the discrimination relies on the data uncertainties on small scales.

$m_{\mathrm{WDM}}^{\mathrm{ref}} = 10\,\mathrm{keV}$





→ Varying the range of wavenumbers.

Conclusion

Connection between the mass and the DM thermal histories, the potential to distinguish thermal histories of dark matter

Conclusion

1, We build the connection between the mass and the DM thermal histories and investigate the current Lyman-alpha constraints on DM mass and thermal histories .

2, We investigate possible future observations and find that future precise observation may uniquely identify the allowed parameter spaces for different scenarios, or even completely rule out one of the scenarios.

Thank you!

Backup: Velocity-mass and mass-thermal historiy relations for different thermal histories

We can build the velocity-mass relation for specific thermal history.

$$\langle v\rangle_0 \approx \begin{cases} 1.1\times 10^{-7}\times\left(\frac{2}{g_\chi}\right)^{\frac{1}{3}}\left(\frac{\Omega_\chi}{0.25}\right)^{\frac{1}{3}}\left(\frac{1\ \mathrm{keV}}{m_\chi}\right)^{\frac{4}{3}} & (\mathbf{RFO}) \\ 1.9\times 10^{-6}\times\left(\frac{2}{g_\chi}\right)^{\frac{1}{3}}\left(\frac{\Omega_\chi}{0.25}\right)^{\frac{1}{3}}\left(\frac{1\ \mathrm{keV}}{m_\chi}\right)^{\frac{4}{3}}\left(\frac{e^{x_\mathrm{dec}}}{e^{10}}\right)^{\frac{1}{3}} & (\mathbf{NRFO}) \\ 1.0\times 10^{-7}\times\left(\frac{2}{g_\chi}\right)^{\frac{1}{3}}\left(\frac{\Omega_\chi}{0.25}\right)^{\frac{1}{3}}\left(\frac{1\ \mathrm{keV}}{m_\chi}\right)^{\frac{4}{3}}\left(\frac{1}{C}\right)^{\frac{1}{3}} & (\mathbf{FI}) \end{cases}$$

We can also build the relation between mass and decoupling temperatures through the average velocity.

$$m_\chi \approx \begin{cases} 1.9\times 10^{-3}\ \mathrm{keV}\times\frac{g_{\star,s}(T_\mathrm{dec})}{g_\chi}\frac{\Omega_\chi}{0.25}\,\eta_\mathrm{dec}^{-3} & (\mathbf{RFO}) \\ 2.1\ \mathrm{keV}\times\frac{g_{\star,s}(T_\mathrm{dec})}{g_\chi}\frac{\Omega_\chi}{0.25}\,\eta_\mathrm{dec}^{-3}\left(\frac{10}{x_\mathrm{dec}}\right)^{\frac{3}{2}}\frac{e^{x_\mathrm{dec}}}{e^{10}} & (\mathbf{NRFO})\ . \\ 2.7\times 10^{-3}\ \mathrm{keV}\times\frac{g_{\star,s}(T_\mathrm{dec})}{g_\chi}\frac{\Omega_\chi}{0.25}\,C^{-1}\eta_\mathrm{dec}^{-3} & (\mathbf{FI}) \end{cases}$$

$$\eta_\mathrm{dec}\equiv\left.\frac{T_\mathrm{DS}}{T_\mathrm{SM}}\right|_{T_\mathrm{SM}=T_\mathrm{dec}}$$

Backup: Lyman-α contraints: Two recasting mothods

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$\boldsymbol{m_{WDM} \gtrsim 3.5\ (5.3)\ keV}$

(1) Half-mode analysis,

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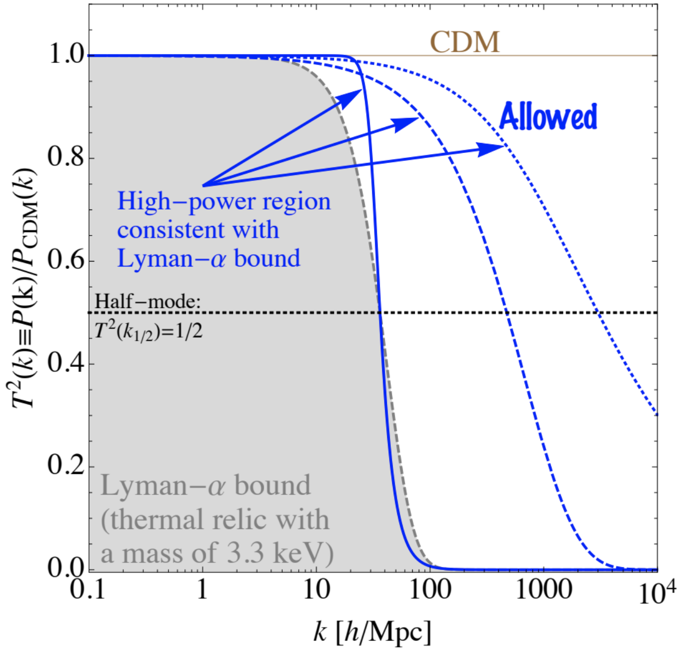
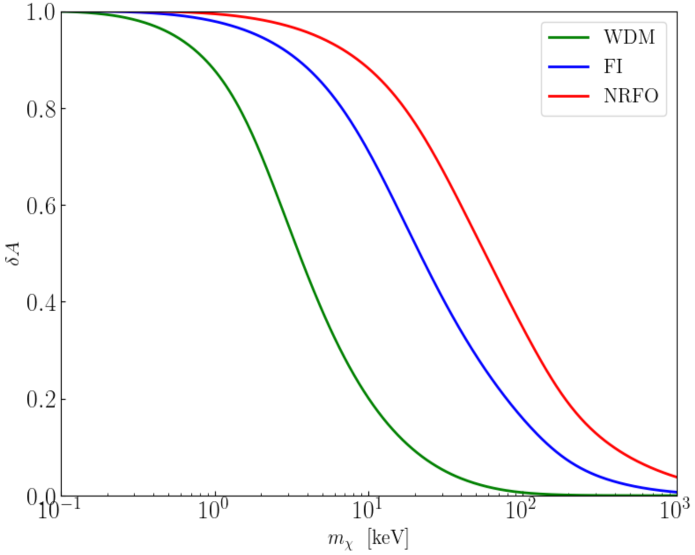


Konig, Merle, Totzauer, 1609.01289



Backup: Fitting function of the transfer functions

WDM: $T^2_{\rm WDM}(k) \simeq \left[1 + (\alpha_{\rm WDM} k)^{2\nu}\right]^{-10/\nu}$, $\alpha_{\rm WDM} = 0.049 \times \left(\frac{m_\chi^{\rm WDM}}{\rm keV}\right)^{-1.11} \left(\frac{\Omega_\chi}{0.25}\right)^{0.11} \left(\frac{h}{0.7}\right)^{1.22} \frac{\rm Mpc}{h}, \nu = 1.12.$

Another fitting formula: $T^2(k) = [1 + (\alpha\, k)^\beta]^{2\gamma}$. Decomposition of α: $\alpha = \alpha_1 \left(\frac{m_\chi}{\rm keV}\right)^{\alpha_2} \left(\frac{g_{\star,s}(T_0)}{g_{\star,s}(m_\chi)}\right)^{1/3} \frac{\rm Mpc}{h}$

Freeze-out: $\{\alpha_1, \alpha_2, \beta, \gamma\} = \{0.32, -0.80, 2.24, -4.46\}$. Same shape with WDM!

Freeze-in: $\{\alpha_1, \alpha_2, \beta, \gamma\} = \{0.30, -0.87, 2.28, -1.51\}$.

A translational symmetry: $m_\chi \to m'_\chi\,, \quad \alpha(m_\chi) \to \alpha' = \alpha(m'_\chi)\,,$

$$k \to k' = \frac{\alpha}{\alpha'} k\,, \quad T^2(k) \to T'^2(k') = T^2(k)\,.$$

Due to the (approximate) same shape of the normalized momentum distributions $g_p(p)/(N_\chi)$