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# Buildd an Unified frameworsfor SU(N) Confinement Phase transition and thermodynamics with quasigluon

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Based on:
1. Zhao-Feng Kang, Shinya Matsuzaki, Jiang Zhu.JHEP09(2021)060
2. Jun Guo and Zhao-Feng Kang,J iang Zhu. PRD(2023) 107,076005.

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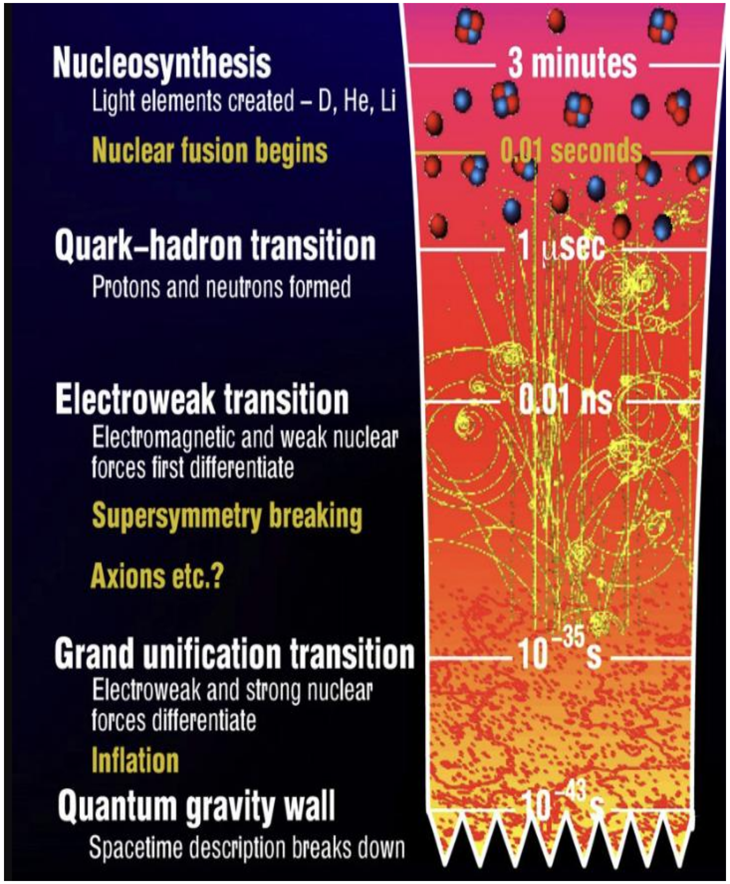
# Motivation

- **Pure Yang-Mills Theory**

History of the Universe




- ✓ EW-like Phase Transition based on Perturbation Calculation
- ✗ QCD-like Phase Transition which are highly Non-perturbation

Study the most Simplest FOPT in QCD–like PT

Pure Yang-Mills Theory is the **GROUND BASE** for other QCD-like phase transition

So, it is Very important to understand the Confinement Phase Transition in Pure Yang-Mills Theory

Also wieldly considered in New Physics

# Outline

1. Information of Confinement

2. Effective Model

3. The Quasi-Particle Model

4. Conclusion & Outlook

# Information of Confinement

- **Method to describe Confinement Phase Transition**

The free energy of static test quark placed in $\vec{r}$

**Order parameter: Fundamental Polyakov Loop**

K. Fukushima and V. Skokov, PPNP(2017)

$$\frac{1}{N}e^{-\beta F_q(\vec{x})} = \frac{1}{N}\mathrm{tr}\mathcal{P}\exp\left[ig\int_0^\beta dx_4 A_4(\vec{x}, x_4)\right] = l(\vec{x})$$

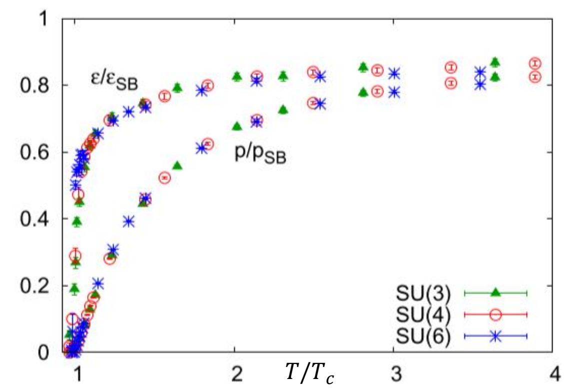
$l = 0$ is confinement
$l \neq 0$ deconfinement

$Z_N$ Center symmetry broken

$N = 2$ **Second OPT**
$N \geq 3$ **First OPT**

$$\frac{L_N}{(N^2-1)T_c^4} \simeq 0.388 - \frac{1.61}{N^2},$$

$$\frac{p_N}{N^2-1} \approx \frac{p_M}{M^2-1}, \qquad \frac{\epsilon_N}{N^2-1} \approx \frac{\epsilon_M}{M^2-1}$$




Polyakov Polynomial model: C. Ratti, M. A. Thaler, W. Weise. 0604025 Z. Kang, J. Zhu, S. Matsuzaki. JHEP(2021) W.-C. Huang, M. Reichert, F. Sannino, Z.-W. Wang.PRD(2021)

Haar Measure Polyakov Model: J. Kubo, M. Yamada. PRD(2004) J. Kubo, M. Yamada. PRD(2008) S. Roessner, C. Ratti, W. Weise. PRD(2007) J. Kubo, M. Yamada. JHEP(2018)

# Information of Confinement

- **Summary: Challenge/Requirement**

From the above information, we can sum up the conditions that a good and effective model should meet.

1. Lattice Thermodynamics for all $N$
2. Correct Latent Heat behavior for all $N$
3. First order Phase Transition for $N \geq 3$
4. Crossover Phase Transition for $N = 2$

(3 and 4:) As SSB for $Z_n$

(1–4:) **In a Unification Frameworks** ★★★★★

**So, to what extent has the existing model been achieved?**

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# Effective Model

- **Model for Confinement Phase Transition: Polynomial Model**

A Natural thinking: **Polynomial Potential**

Central Ideal: Landau-Ginzburg Phase Transition Theory

$$a(T) = a_0 + a_1\left(\frac{T_c}{T}\right) + a_2\left(\frac{T_c}{T}\right)^2$$

Potential should have $Z_n$ symmetry

**Well describe phase structure at any given temperature**

For $N = 3$[1]

$$V(l,T) = T^4\left[-\frac{a(T)}{2}ll^\dagger - \frac{b}{6}\left(l^3 + l^{\dagger 3}\right) + \frac{c}{4}\left(ll^\dagger\right)^2\right]$$

Only 3 term allowed by $Z_3$ symmetry

But it is hard to directly generate into $SU(N)$ case

To get model for any $N \geq 3$, one have to consider charge 2 PL $l_2$[2]

Modified PLM for $N \geq 3$[3]

$$V(l,T) = T^4 U(l,T) = T^4\left[-\frac{a(T)}{2}|l|^2 + b|l|^4 + c|l|^n\right]$$

But how about $N = 2$ ?


[1] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D73 (2006) 014019, [arXiv:0506234 [hep-ph]]
[2] R. D. Pisarski, Nucl. Phys. A 702, 151-158 (2002) [arXiv:0112037 [hep-ph]].
[3] Z. Kang, J. Zhu and S. Matsuzaki, JHEP 09, 060 (2021), [arXiv:2101.03795 [hep-ph]].

# Effective Model

## • Model for Confinement Phase Transition: Haar-Type

Different directions: **Haar-Type Potential**

Also a more theoretical based model

For $N = 3$[1] $V(l,T) = T^4\left\{-\frac{a(T)}{2}ll^\dagger + b(T)\log\left[1 - 6ll^\dagger - 3(ll^\dagger)^2 + 4(l^3 + l^{\dagger 3})\right]\right\}$

$$a(T) = a_0 + a_1\left(\frac{T_c}{T}\right) + a_2\left(\frac{T_c}{T}\right)^2 \qquad b(T) = b_3\left(\frac{T_c}{T}\right)^3.$$

Haar-measure of SU(3) come from the functional integrate

Can be generating to describe any $N$[2] $\mathcal{V}_{\text{Haar},1}(L,T) = -\frac{a(T)}{2}ll^\dagger + b(T)\log H_N[L],$ → Haar-measure of SU(N)

Naturally get crossover for $N = 2$ and FOPT for $N \geq 3$

**But the latent heat and thermal quantities can not be both agree with lattice data for any $N$ in this frameworks[3]**

Data Consistent in $N = 3$ ⇒ Data Inconsistent for $N = 4,5,6...$

[1] S. Roessner, C. Ratti, and W. Weise, Phys. Rev. D75 (2007) 034007, [arXiv:0609281 [hep-ph]].
[2] J. Kubo and M. Yamada, JHEP 10, 003 (2018), [arXiv:1808.02413 [hep-ph]].
[3] Z. Kang, J. Zhu and S. Matsuzaki, JHEP 09, 060 (2021), [arXiv:2101.03795 [hep-ph]].

# Effective Model

- **Summary: Effective Model**

Polynomial Model

| $N$ | Lattice Data | Latent Heat | PT | $Z_n$ |
|---|---|---|---|---|
| 2 | void | void | × | × |
| 3 | ✓ | ✓ | ✓ | ✓ |
| 4 | ✓ | ✓ | ✓ | ✓ |
| 6 | ✓ | ✓ | ✓ | ✓ |

Haar Type Model

| $N$ | Lattice Data | Latent Heat | PT | $Z_n$ |
|---|---|---|---|---|
| 2 | void | void | ✓ | ✓ |
| 3 | ✓ | ✓ | ✓ | ✓ |
| 4 | ✓ | × | ✓ | ✓ |
| 6 | ✓ | × | ✓ | ✓ |

So, the traditional PLM failed to achieve our goals

Fortunately, there is another model which can describe the $SU(N)$ lattice thermodynamics → Quasi-particle Model

# The Quasi-Particle Model

- **Quasi-Particle method**

From the hard-thermal-loop perturbation theory(HTLpt), the dispersion equation of $SU(N)$ theory is given by :

$$w^2 - k^2 - \Pi_t^*(w,k) = 0,$$

Transverse Self-Energy

**Just Like Photon in Plasma**

Build a phenomenon theory “absorbed” strong interactions into the Quasigluon mass

$$M_g^2(T) = \frac{N}{6} G^2(T) T^2, \quad G^2(T) = \frac{48\pi^2}{11N \log\left(\frac{T}{T_c/\lambda} + \frac{T_s}{T_c}\right)^2}$$

Explain lattice QCD thermodynamics successfully by QPM pressure [1,2] :

$$p(T) = \frac{g(T)}{6\pi^2} \int_0^\infty f_B(E_k) \frac{k^4}{E_k} dk - B(T)$$

Thermodynamics is ok **BUT** How about the Phase Transition?

[1] V. Goloviznin and H. Satz, Z. Phys. C 57, 671-676 (1993)
[2] A. Peshier, B. Kampfer, O. P. Pavlenko and G. Soff, Phys. Rev. D 54, 2399-2402 (1996).

# The Quasi-Particle Model

- **The Unified Frameworks from the First Principle with Quasi-Particle method**

We will add a thermal mass for gluon and this should contain all the non-perturbation effect

The Lagrangian: $$\mathcal{L} = -\frac{1}{2g^2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{D}_\mu \bar{c}^a D^\mu c^a + ih^a \bar{D}_\mu \hat{A}^{\mu,a} + \frac{1}{2}m_g^2(T)\hat{A}^a_\mu \hat{A}^{a,\mu},$$

Ghost field | Landau-DeWitt gauge Fix term | QPM Mass

Background Field Method: $$A_\mu = \bar{A}_\mu + \hat{A}_\mu$$

Background field | Fluctuation field

Temporal Background Field: $$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0}$$

$$\mathcal{L} = \frac{1}{2}\hat{A}^a_\mu (D^{-1})_{ab}\hat{A}^{\mu,b} + ih^a \bar{D}_\mu \hat{A}^{\mu,a} + \bar{D}_\mu \bar{c}^a \bar{D}^\mu c^a$$

Separate the action into $S = S_{A,h} + S_c$

$$S_{A,h} = \int d^4x \left[\frac{1}{2}\hat{A}^a_\mu (D^{-1})_{ab}\hat{A}^{\mu,b} + ih^a \bar{D}_\mu \hat{A}^{\mu,a}\right] \qquad S_c = \int d^4x [\bar{D}_\mu \bar{c}^a \bar{D}^\mu c^a],$$

# The Quasi-Particle Model

- **The Effective Potential**

One can compute the effective potential $V = T\log Z/V$ by finite temperature field theory

$$\mathcal{V}(s,T,N) = 3\mathcal{V}_{qp}(s,M_g,T,N) - \mathcal{V}_g(s,T,N) \qquad \mathcal{V}_g(s,T,N) = \frac{\pi^2 T^4}{90} + \frac{N^2\pi^2T^4}{90}\left[(s-1)^2(2s^2-2s-1) - \frac{5(s-1)^2s^2}{N^2} + \frac{s^3(3s-4)}{N^4}\right]$$

$$\mathcal{V}_{qp}(s,M_g,T,N) = \frac{T^4}{2\pi^2}\int_0^\infty dx x^2\left((N-1)\log[1-e^{-\hat{E}(x,M_g,T)}] + \sum_{i=1}^{N}(N-i)\log[1+e^{-2\hat{E}(x,M_g,T)} - 2e^{-\hat{E}(x,M_g,T)}\cos(2\pi\frac{is}{N})]\right)$$

$$l_N(s) = \frac{1}{N}\frac{\sin(\pi s)}{\sin(\pi s/N)}$$

Confinement Phase: $l = 0,\ s = 1$

Deconfinement Phase: $l \neq 0,\ s \neq 1$

$$\hat{E}(x,M_g,T) = \sqrt{x^2 + \left(\frac{M_g}{T}\right)^2}$$

- **PT around $T_c$**

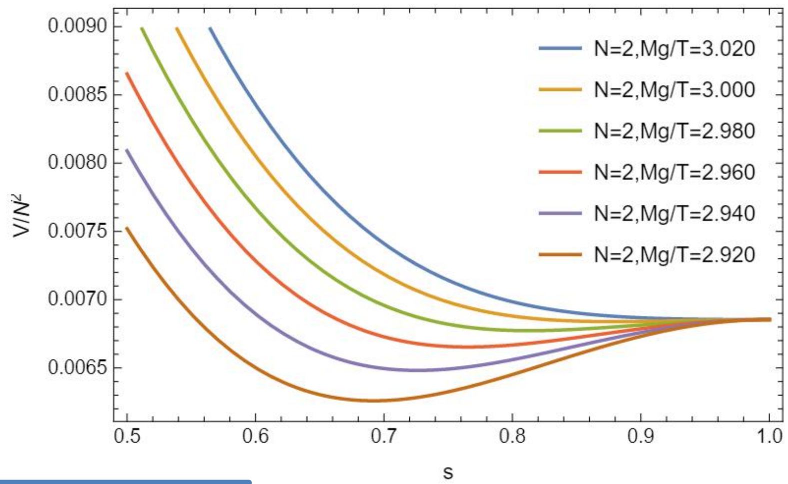
$$\mathcal{V}_{qp}(s,T) = -\frac{T^2M_g^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\mathrm{tr}(\hat{L}_A)^n K_2(nM_g/T). \qquad \hat{L}_A = \mathrm{diag}[1,1,...,1,e^{i2\pi q_{ij}},...,e^{-i2\pi q_{ij}}], \quad q_{ij} = \frac{i-j}{N}s$$

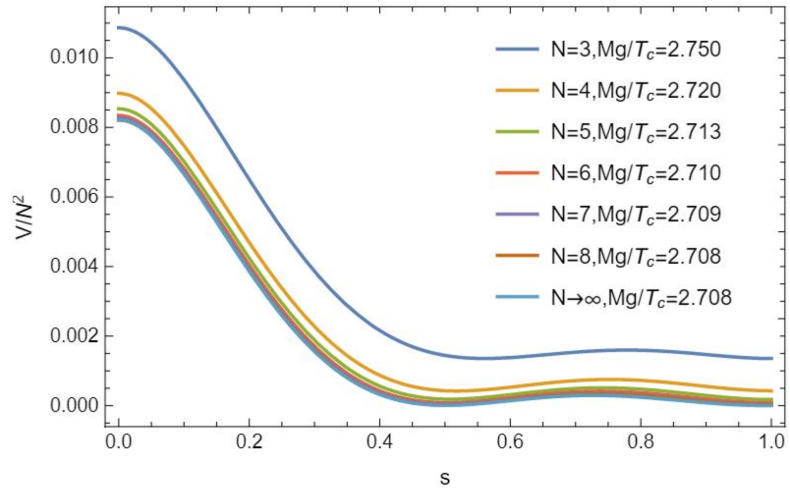
We guess $M_g/T$ is quite large around $T_c$ ★

$$\mathcal{V}_{eff} \simeq -\frac{N^2}{2}\left(\frac{3T^4}{\pi^2}(\frac{M_g}{T})^2K_2(M_g/T)l_N(s)^2 + \frac{\pi^2T^4}{45}\left[(-1+s)^2(-1-2s+2s^2) - \frac{5(-1+s)^2s^2}{N^2} + \frac{s^3(-4+3s)}{N^4}\right]\right)$$

# The Quasi-Particle Model

- **Phase Transition Behavior**




Vacuum Degenerate

The value of $M_g(T_c)/T_c \sim 2.7$ can be find in above diagram

$$\mathcal{V}_{qp}(s,T) = -\frac{T^2 M_g^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \mathrm{tr}(\hat{L}_A)^n K_2(nM_g/T).$$

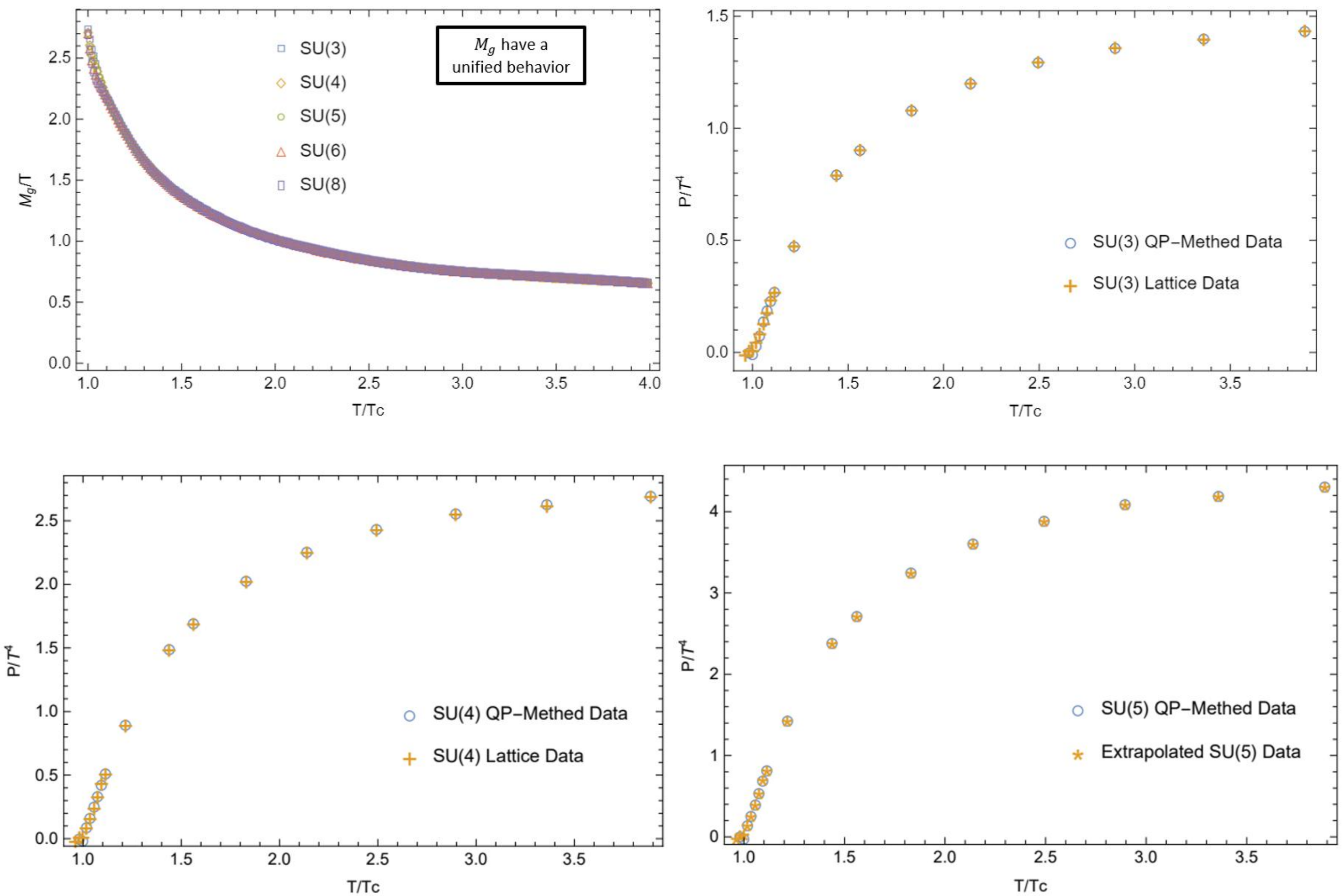
$\frac{K_2(2.7)}{K_2(5.6)} \sim 27$

Low temperature expansion to 1 term is valid

| Color number | $N=3$ | $N=4$ | $N=5$ | $N=6$ | $N=7$ | $N=8$ | $N\to\infty$ |
|---|---|---|---|---|---|---|---|
| $s_d$ | 0.5605 | 0.5186 | 0.5073 | 0.5033 | 0.5016 | 0.5009 | 0.5004 |
| $l_d$ | 0.5910 | 0.6300 | 0.6380 | 0.6398 | 0.6400 | 0.6396 | 0.6367 |
| $M_g(T_c)/T_c$ | 2.7499 | 2.7203 | 2.7126 | 2.7099 | 2.7088 | 2.7083 | 2.7077 |
| $dM_g(T_c)/dT_c$ | -5.7727 | -7.9951 | -9.2891 | -10.0965 | -10.6261 | -10.9954 | -12.3376 |
| $L_N/(N^2-1)T_c^4$ | 0.2091 | 0.2874 | 0.3236 | 0.3433 | 0.3551 | 0.3628 | 0.3880 |

# The Quasi-Particle Model

- **The Unified Frameworks from the First Principle with Quasi-Particle method**



# Conclusion

- **The Summary**

1. We Build a Quasi-gluon model from Lagrangian and this model can describe the $SU(N)$ thermodynamics and confinement phase transition in a unified frameworks.

2. We found this model we only need one parameter $M_g(T)$ to give the confinement phase transition and this function have a unified behavior for different color number.

This Model

| $N$ | Lattice Data | Latent Heat | FOPT | $Z_n$ |
|---|---|---|---|---|
| 2 | void | void | ✓ | ✓ |
| 3 | ✓ | ✓ | ✓ | ✓ |
| 4 | ✓ | ✓ | ✓ | ✓ |
| 6 | ✓ | ✓ | ✓ | ✓ |