第九届中国LHC物理年会
The 9th China LHC Physics Workshop

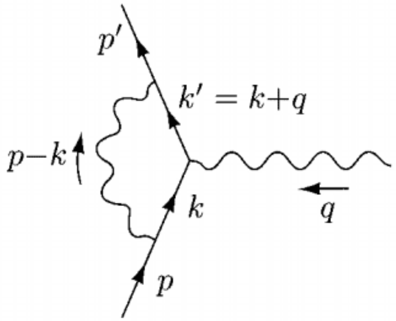
New perspectives on UV divergences of loops and the hierarchy problem

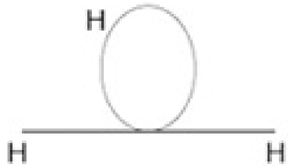
Lian-Bao Jia（贾连宝）

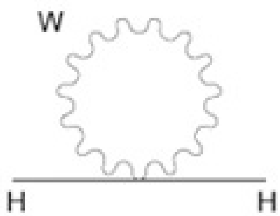
SWUST（西南科技大学）

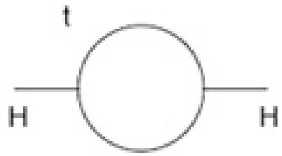
Shanghai 2023.11.17

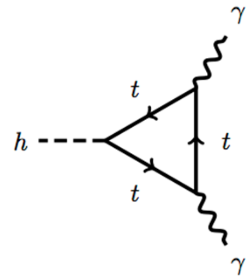
Based on arXiv:2305.18104

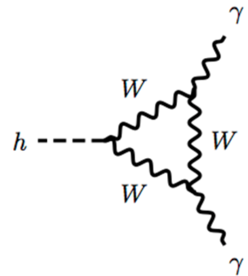






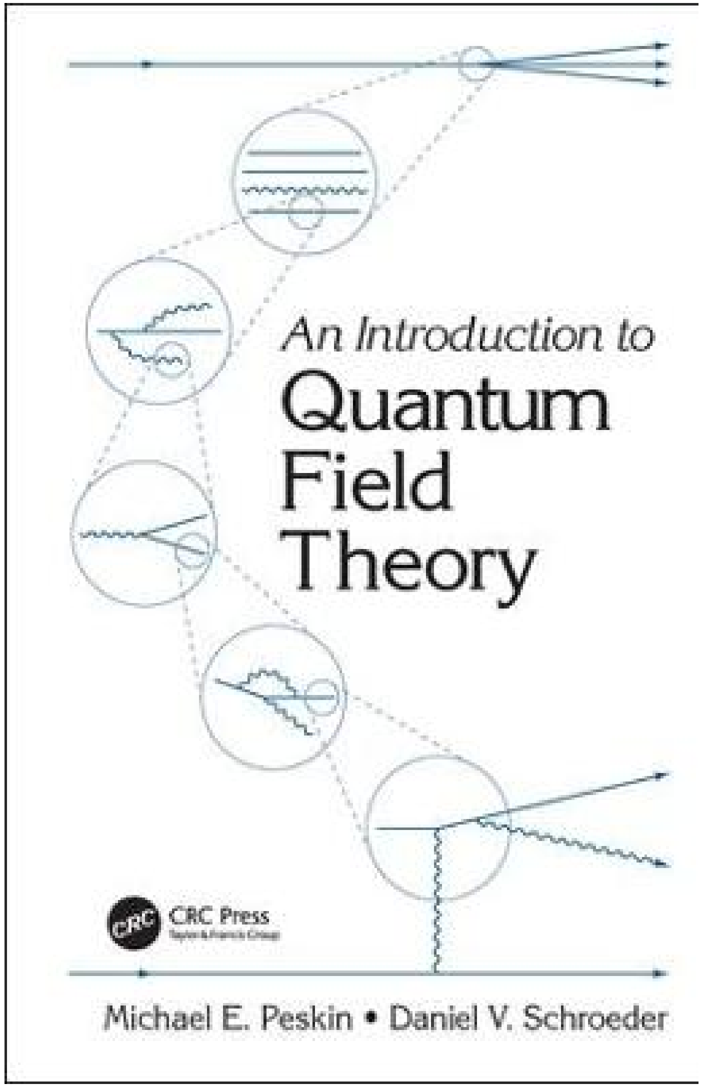






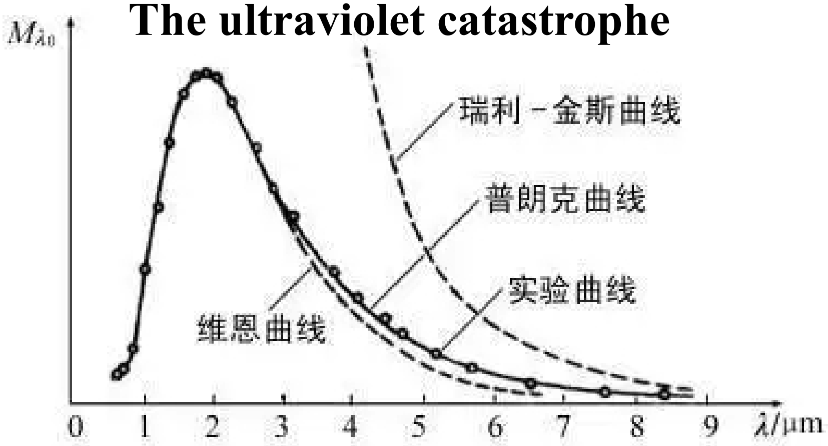
Outline:

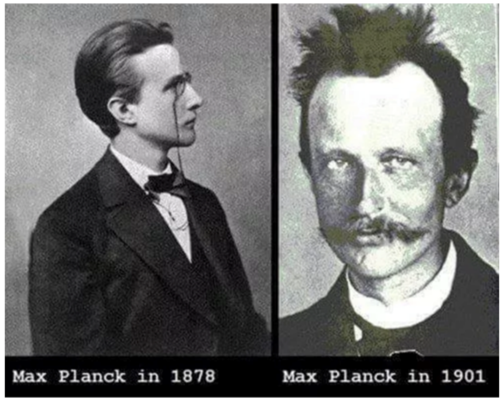
- I. Angel or Devil?--- Free flow of ideas on UV divergences
- II. New perspective --- UV-free scheme
- III. The hierarchy problem
- IV. Summary and outlook



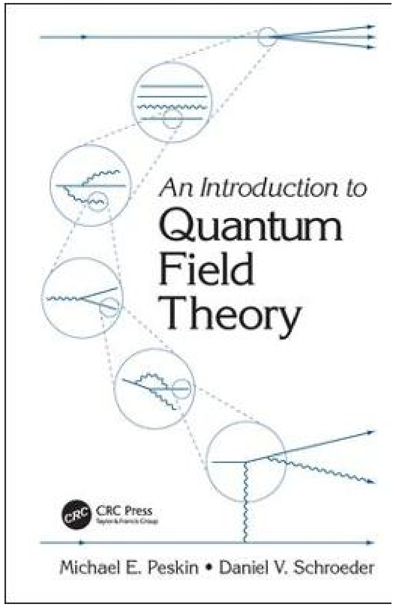
I. Angel or Devil?--- Free flow of ideas on UV divergences

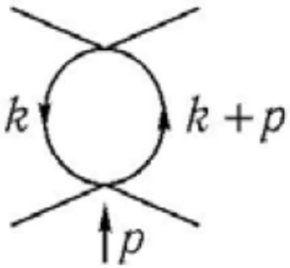
The ultraviolet catastrophe





UV divergence!

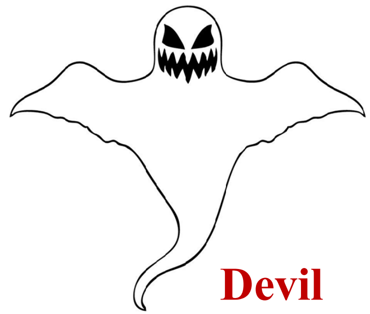




$$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \frac{i}{(k+p)^2 - m^2}$$

$$\equiv (-i\lambda)^2 \cdot iV(p^2).$$

$$V(p^2) \underset{d\to 4}{\longrightarrow} -\frac{1}{32\pi^2} \int_0^1 dx \Big(\frac{2}{\epsilon} - \gamma + \log(4\pi) - \log\big[m^2 - x(1-x)p^2\big] \Big)$$



Devil

∞

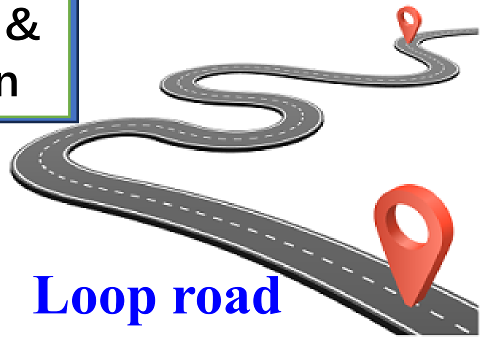
I. Angel or Devil?--- Free flow of ideas on UV divergences

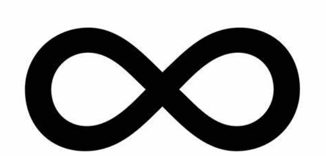
–

=

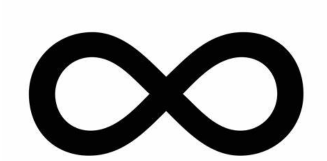
Regularization & renormalization

$\infty - \infty$





Regularization

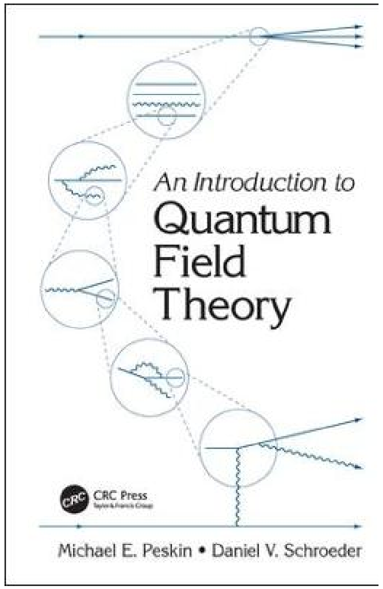


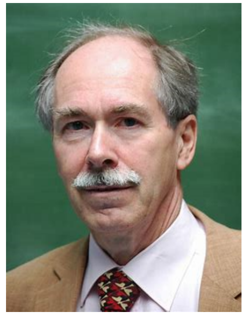
Renormalization

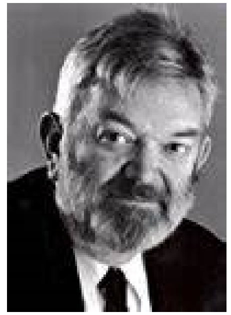
Paradigm procedure

Renormalization (counterterm)

Based on: BPHZ scheme







ELSEVIER

Nuclear Physics B

Volume 44, Issue 1, 1 July 1972, Pages 189-213

Regularization and renormalization of gauge fields

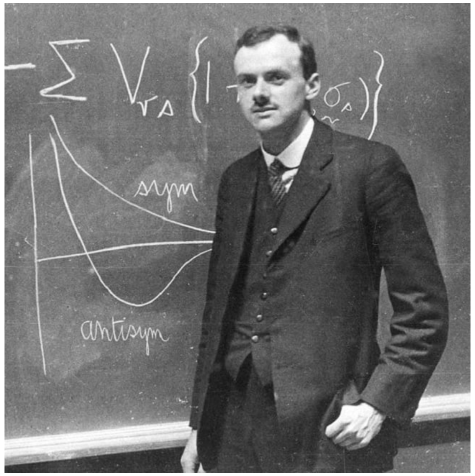
G. 't Hooft, M. Veltman

Issues/puzzles in Regularization and Renormalization (BPHZ):

a. Infinity (UV divergences)
b. Log divergence is OK, power-law divergence is not
c. γ^5 issue (DR)
d. Unitarity gauge vs Feynman-'t Hooft gauge

Free flow of ideas

Hence most physicists are very satisfied with the situation. They say: "Quantum electrodynamics is a good theory, and we do not have to worry about it any more." I must say that I am very dissatisfied with the situation, because this so-called "good theory" does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small—not neglecting it just because it is infinitely great and you do not want it!

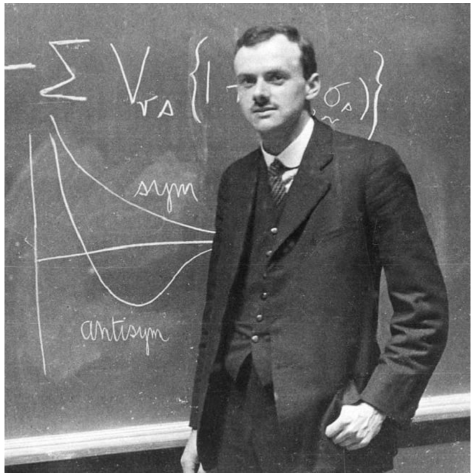


P.A.M. Dirac

I believe the successes of the renormalization theory will be on the same footing as the successes of the Bohr orbit theory applied to one-electron problems.

I disagree with most physicists at the present time just on this point. I cannot tolerate departing from the standard rules of mathematics. Of course, the proper inference from this work is that the basic equations are not right. There must be some drastic change introduced into them so that no infinities occur in the theory at all and so that we can carry out the solution of the equations sensibly, according to ordinary rules and without being bothered by difficulties. This requirement will necessitate some really drastic changes: simple changes will not do, just because the Heisenberg equations of motion in the present theory are all so satisfactory. I feel that the change required will be just about as drastic as the passage from the Bohr orbit theory to the quantum mechanics.

Go forward in the direction pointed by Dirac:

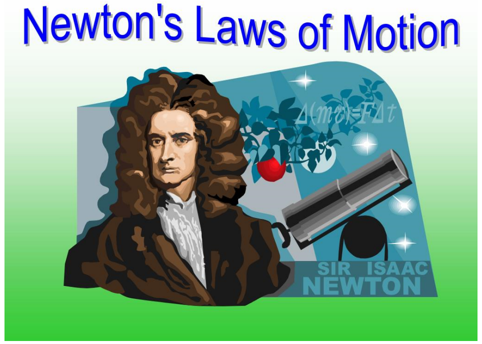


P.A.M. Dirac

a. **No UV divergence (mathematically well-defined).**
b. **A general method with both Log and power-law divergences being OK!**

Is it possible in a method?

Ⅱ. New perspective --- UV-free scheme



Negligible

Low-energy corrections

Loop

UV regions (Planck scale)

Loop

Negligible?!

A presumption:

The physical contributions of loops are finite with contributions from UV regions being insignificant.

UV-free scheme:

$$\mathcal{T}_{\rm P} = \Big[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\rm F}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\Big]_{\{\xi_1, \cdots, \xi_i\} \to 0} + C$$

Regularization & renormalization

Jia, arXiv:2305.18104

Tamed loops: A way to obtain finite loop results without UV divergences

Lian-Bao Jia[1,2,3,*]

UV-free scheme:

assume that the physical transition amplitude $\mathcal{T}_{\rm P}$ with propagators can be described by an equation of

$$\mathcal{T}_{\rm P}=\Big[\int d\xi_1\cdots d\xi_i\frac{\partial\mathcal{T}_{\rm F}(\xi_1,\cdots,\xi_i)}{\partial\xi_1\cdots\partial\xi_i}\Big]_{\{\xi_1,\cdots,\xi_i\}\to 0}+C\,,(1)$$

a. Tree-level:

the photon propagator $\frac{-ig_{\mu\nu}}{p^2+i\epsilon}$,

$$\mathcal{T}_{\rm F}(\xi)=\frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon},\ \frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}=\frac{-ig_{\mu\nu}(-1)}{(p^2+\xi+i\epsilon)^2},$$

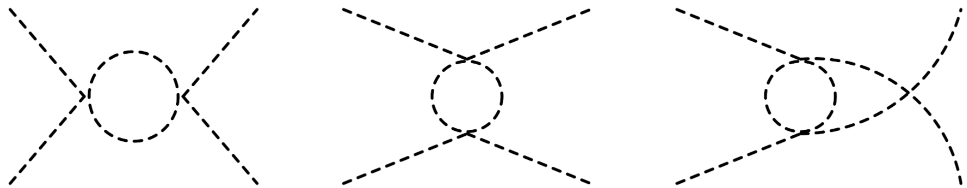
$$\Big[\int d\xi\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]=\frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon},\quad \text{with}\quad C=0$$

$$\mathcal{T}_{\rm P}=\Big[\int d\xi\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]_{\xi\to 0}=\frac{-ig_{\mu\nu}}{p^2+i\epsilon}$$

the gauge field propagator restored

where a Feynman-like amplitude $\mathcal{T}_{\rm F}(\xi_1,\cdots,\xi_i)$ is introduced, which is written by Feynman rules just with parameters $\xi_1,\cdots,\xi_i$ added into denominators of propagators. For the integral over ξ, here we introduce a definition of the primary antiderivative $[\int d\xi_1\cdots d\xi_i\frac{\partial\mathcal{T}_{\rm F}(\xi_1,\cdots,\xi_i)}{\partial\xi_1\cdots\partial\xi_i}]$ with the constant term being absorbed into C (for example, for the integral $\int xdx=\frac{x^2}{2}+C$, the primary antiderivative is $[\int xdx]=\frac{x^2}{2}$), and the supplement C is a boundary constant related to the transition process. After integration, $\mathcal{T}_{\rm P}$ will be obtained in the limit of parameters $\xi_1\to 0,\cdots,\xi_i\to 0$. If Eq. (1) is applied to tree-level and loop-level processes without UV divergences, $C=0$ is adopted. For loop processes with UV divergences, C can be set by renormalization conditions, symmetries and naturalness. The number of the parameter ξ_i introduced is as few as possible in the case of the loop integral becoming UV-converged. For a loop with UV divergences, one parameter ξ is introduced for logarithmic divergence, and two ξ parameters are introduced for quadratic divergence (three ξ parameters needed at most for a loop being converged). For multi-loops, a set of ξ parameters is introduced for each loop. The method above is UV-free scheme.

b. Loop-level Log:



ϕ^4 theory

Feynman-like scattering amplitude

$$\mathcal{T}_{\rm F}(\xi)=\frac{(-i\lambda)^2}{2}\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+\xi}\frac{i}{(k+q)^2-m^2}$$

The physical scattering amplitude

$$\mathcal{T}_{\rm P}(s)=\Big[\int d\xi\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]_{\xi\to 0}+C_1$$

$$=\Big[\frac{-\lambda^2}{2}\int d\xi\int\frac{d^4k}{(2\pi)^4}\frac{-i}{(k^2-m^2+\xi)^2}\frac{i}{(k+q)^2-m^2}\Big]_{\xi\to 0}+C_1\,,$$

$$\mathcal{T}_{\rm P}(s)=\frac{-i\lambda^2}{32\pi^2}\int_0^1 dx\log[m^2-x(1-x)s]+C_1\,.$$

An explanation why universal constant parts (γ_E, log(4π)) should be subtracted along with infinity in renormalization.

A freedom of ξ in propagators

No troublesome UV divergence in loop calculations!

Considering the renormalization conditions, the amplitudes are taken to be zero at $s=4m^2$, $t=u=0$.

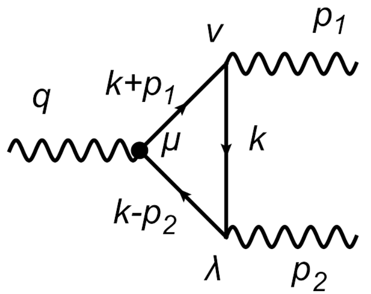
$$\Rightarrow\quad C_1=\frac{i\lambda^2}{32\pi^2}\int_0^1 dx\log[m^2-4m^2x(1-x)]\,.$$

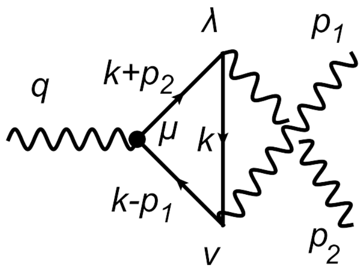
The total one-loop physical amplitude

$$\mathcal{T}_{\rm P}=\mathcal{T}_{\rm P}(s)+\mathcal{T}_{\rm P}(t)+\mathcal{T}_{\rm P}(u)$$

$$=\frac{-i\lambda^2}{32\pi^2}\int_0^1 dx\Big[\log\frac{m^2-x(1-x)s}{m^2-4m^2x(1-x)}+\log\frac{m^2-x(1-x)t}{m^2}+\log\frac{m^2-x(1-x)u}{m^2}\Big]$$

The axial anomaly





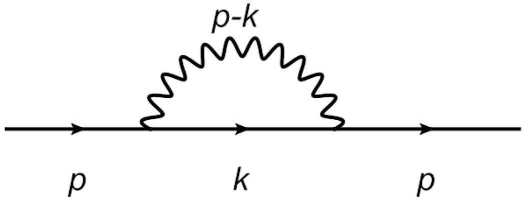
γ^5 **the original form**

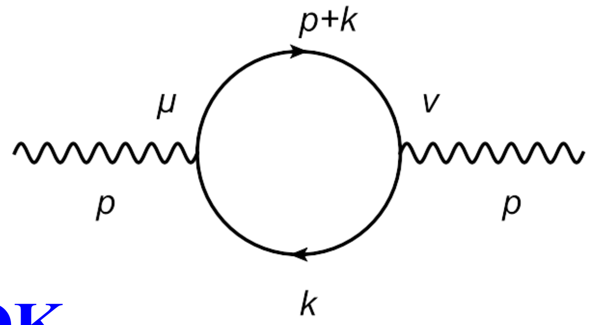
$$\partial_\mu j^{\mu 5} = i q_\mu \mathcal{T}_{\rm P}^{\mu\nu\lambda} \epsilon^*_\nu(p_1)\epsilon^*_\lambda(p_2)$$
$$= -\frac{e^2}{16\pi^2}(\frac{2}{3}+\frac{C_0}{2})\varepsilon^{\alpha\nu\beta\lambda}F_{\alpha\nu}F_{\beta\lambda}$$

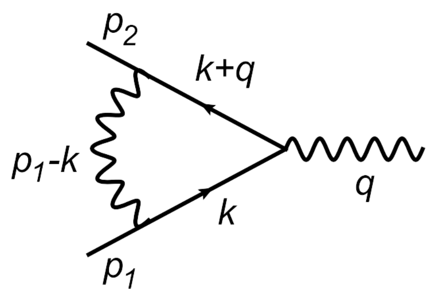
If $C_0 = \frac{2}{3}$ SM self-consistent

charge values of quarks

coincidence, or correlation?

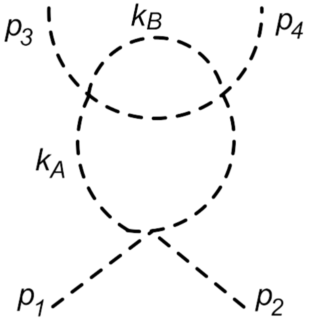






Log divergences are OK

two-loop transition



$$\mathcal{T}_{\rm P} = \Big[\int d\xi \frac{\partial \mathcal{T}_{\rm F}(\xi)}{\partial \xi}\Big]_{\xi\to 0} + C$$

$$= \Big[\frac{(-i\lambda)^3}{2}\int d\xi \int \frac{d^4k_A}{(2\pi)^4}\frac{d^4k_B}{(2\pi)^4}\frac{i}{k_A^2-m^2}\frac{i}{(k_A+q)^2-m^2}$$

$$\times \frac{-i}{(k_B^2-m^2+\xi)^2}\frac{i}{(k_B+k_A+p_3)^2-m^2}\Big]_{\xi\to 0} + C\,,$$

with $q=p_1+p_2$. After the k_B integral, one has

$$\mathcal{T}_{\rm P} = \Big[\frac{(-i\lambda)^3}{2}\int_0^1 dx \int d\xi \int \frac{d^4k_A}{(2\pi)^4}\frac{i}{k_A^2-m^2}\frac{i}{(k_A+q)^2-m^2}$$

$$\times \frac{x}{16\pi^2}\frac{i}{(k_A+p_3)^2 x(1-x)-m^2+x\xi}\Big]_{\xi\to 0} + C\,.$$

The expression can be rewritten as

$$\mathcal{T}_{\rm P} = \Big[\frac{(-i\lambda)^3}{2}\int_0^1 dx\int_0^1 dy\int d\xi\int \frac{d^4k_A}{(2\pi)^4}\frac{-1}{(k_A^2+2yk_A\cdot q+yq^2-m^2)^2}$$

$$\times \frac{x}{16\pi^2}\frac{i}{(k_A+p_3)^2x(1-x)-m^2+x\xi}\Big]_{\xi\to 0} + C$$

$$= \Big[\frac{(-i\lambda)^3}{2}\int_0^1 dx\int_0^1 dy\int_0^1 dz\int d\xi\int \frac{d^4k_A}{(2\pi)^4}\frac{-i}{16\pi^2(1-x)}$$

$$\times \frac{2(1-z)}{[zD_B+(1-z)D_A]^3}\Big]_{\xi\to 0} + C\,,$$

with $D_A = k_A^2+2yk_A\cdot q+yq^2-m^2$, $D_B = (k_A+p_3)^2-m^2/x(1-x)+\xi/(1-x)$. After evaluating the k_A integral, one has

$$\mathcal{T}_{\rm P} = \Big[\frac{(-i\lambda)^3}{2}\int_0^1 dx\int_0^1 dy\int_0^1 dz\int d\xi\frac{-ix}{16\pi^2}\frac{-i(1-z)}{16\pi^2(\Delta-xz\xi)}\Big]_{\xi\to 0} + C$$

$$= \frac{(-i\lambda)^3}{2(4\pi)^4}\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{(1-z)}{z}\log\Delta + C\,,$$

with $\Delta = [(y(1-z)q+zp_3)^2-(yq^2-m^2)(1-z)-p_3^2 z]x(1-x)+m^2 z$. Considering the renormalization conditions that the corrections should be zero at $q^2=4m^2$, the result can be written as

$$\mathcal{T}_{\rm P} = \frac{(-i\lambda)^3}{2(4\pi)^4}\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{1}{z}\Big[(1-z)\log\Delta$$

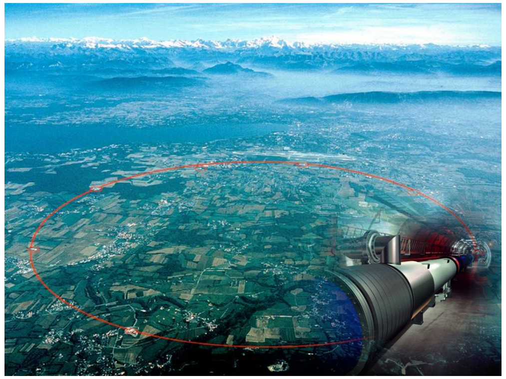
$$-\log[(y^2q^2-yq^2+m^2)x(1-x)]\Big] - C_0$$

$$= \frac{(-i\lambda)^3}{2(4\pi)^4}\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{1}{z}\Big[(1-z)\log\frac{\Delta}{\Delta_0}$$

$$-\log\frac{y^2q^2-yq^2+m^2}{(4y^2-4y+1)m^2}\Big]\,,$$

with $\Delta_0 = [y(1-z)(y+z-yz)4m^2+(z^2-z)p_3^2-(4y-1)m^2(1-z)]x(1-x)+m^2 z$.

III. The hierarchy problem (c. Loop-level Λ^2, Λ^4)

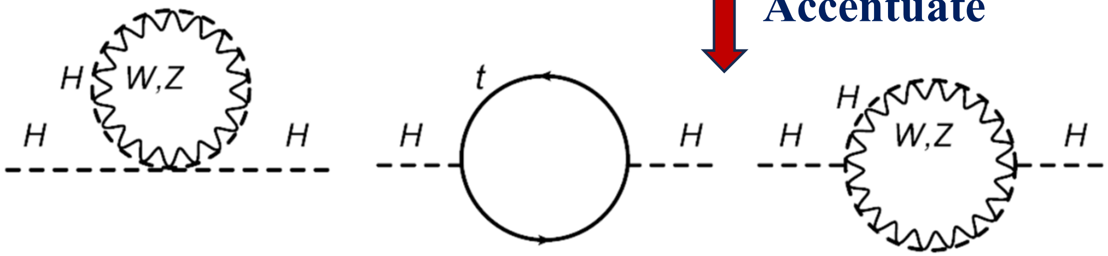


LHC

Higgs boson

125 GeV

Accentuate



The hierarchy problem

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

Fine-tuning!

power-law divergences (Λ^2, Λ^4)

For W, Z

In Feynman-'t Hooft gauge (Λ^2)

μ, k, ν: $$= \frac{-ig^{\mu\nu}}{k^2 - m_A^2};$$ k: $$= \frac{i}{k^2 - m_A^2}.$$

√

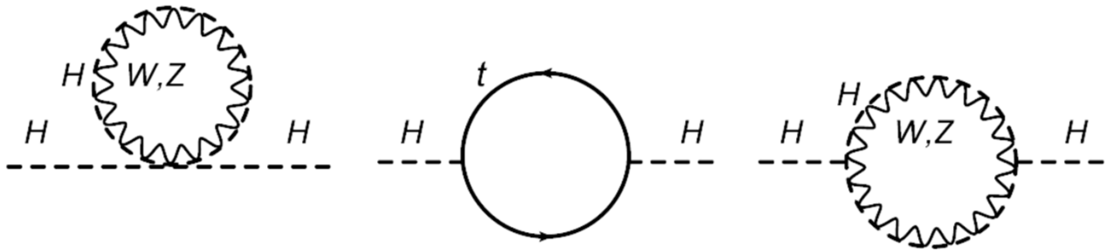
✕

Supersymmetry?

In unitarity gauge (Λ^4)

μ, k, ν: $$= \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$$

A real problem for renormalization!



power-law divergences (Λ^2, Λ^4)

In UV-free scheme

Higgs in the first diagram

$$\mathcal{T}_{\rm P}^{H1} = \Big[\int d\xi_1 d\xi_2 \frac{\partial \mathcal{T}_{\rm F}^{H1}(\xi_1,\xi_2)}{\partial\xi_1\partial\xi_2}\Big]_{\{\xi_1,\xi_2\}\to 0} + C$$
$$= \Big[(-3i)\frac{m_H^2}{2v^2}\int d\xi_1 d\xi_2 \int \frac{d^4k}{(2\pi)^4} \times \frac{2i}{(k^2-m_H^2+\xi_1+\xi_2)^3}\Big]_{\{\xi_1,\xi_2\}\to 0} + C\,.$$

After integral, one has

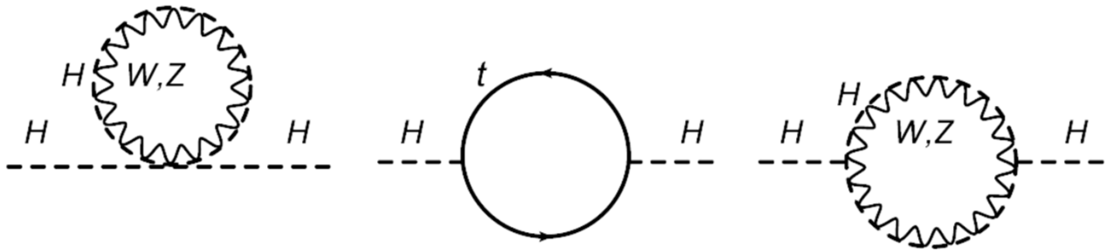
$$\mathcal{T}_{\rm P}^{H1} = i\frac{3m_H^4}{32\pi^2v^2}(\log\frac{1}{m_H^2}+1)+C$$
$$= i\frac{3m_H^4}{32\pi^2v^2}(\log\frac{\mu^2}{m_H^2}+1)\,.$$

V (V=W,Z) in **unitary gauge**

$$\mathcal{T}_{\rm P}^{V1} = \Big[\int d\xi_1 d\xi_2 d\xi_3 \frac{\partial \mathcal{T}_{\rm F}^{V1}(\xi_1,\xi_2,\xi_3)}{\partial\xi_1\partial\xi_2\partial\xi_3}\Big]_{\{\xi_1,\xi_2,\xi_3\}\to 0} + C$$
$$= \Big[i\frac{2m_V^2}{v^2 s_V}\int d\xi_1 d\xi_2 d\xi_3 \int \frac{d^4k}{(2\pi)^4} g_{\mu\nu} \times \frac{6i(g^{\mu\nu}-k^\mu k^\nu/m_V^2)}{(k^2-m_V^2+\xi_1+\xi_2+\xi_3)^4}\Big]_{\{\xi_1,\xi_2,\xi_3\}\to 0} + C\,,$$

where the symmetry factor s_V is $s_V = 1,\ 2$ for W, Z respectively. After integral, one has

$$\mathcal{T}_{\rm P}^{V1} = i\frac{2m_V^2}{v^2 s_V}\frac{m_V^2}{16\pi^2}(3\log\frac{1}{m_V^2}+\frac{5}{2})+C$$
$$= i\frac{2m_V^2}{v^2 s_V}\frac{3m_V^2}{16\pi^2}(\log\frac{\mu^2}{m_V^2}+\frac{5}{6})\,.$$



power-law divergences (Λ^2, Λ^4)

top quark loop

$$\mathcal{T}_{\rm P}^{t} = \Big[\int d\xi_1 d\xi_2 \frac{\partial \mathcal{T}_{\rm F}^{t}(\xi_1,\xi_2)}{\partial\xi_1\partial\xi_2}\Big]_{\{\xi_1,\xi_2\}\to 0} + C$$

$$= \Big[\frac{3m_t^2}{v^2}\int d\xi_1 d\xi_2 \int \frac{d^4k}{(2\pi)^4}$$

$$\times \mathrm{tr}\Big(\frac{2i(\not k + m_t)}{(k^2 - m_t^2 + \xi_1 + \xi_2)^3}\frac{i(\not p + \not k + m_t)}{(p+k)^2 - m_t^2}\Big)\Big]_{\{\xi_1,\xi_2\}\to 0} + C\,,$$

$$\mathcal{T}_{\rm P}^{t} = -\frac{3m_t^2}{v^2}\frac{i}{4\pi^2}\int_0^1 dx[m_t^2 - p^2 x(1-x)]$$

$$\times(3\log\frac{1}{m_t^2 - p^2 x(1-x)} + 2) + C$$

$$= -\frac{3m_t^4}{v^2}\frac{3i}{4\pi^2}\int_0^1 dx[1 - \frac{p^2}{m_t^2}x(1-x)]$$

$$\times(\log\frac{\mu^2}{m_t^2 - p^2 x(1-x)} + \frac{2}{3})\,.$$

Higgs in the third diagram

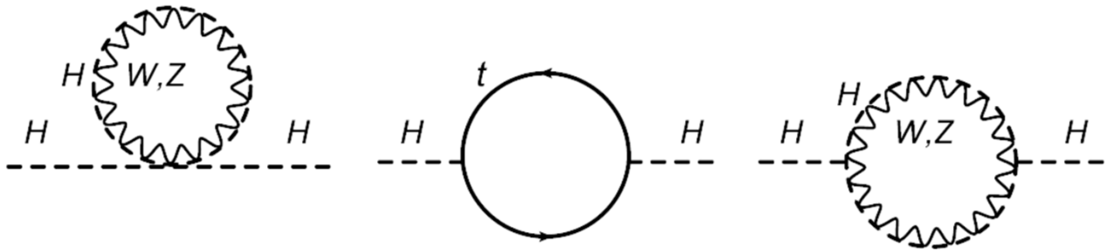
$$\mathcal{T}_{\rm P}^{H3} = \Big[\int d\xi_1 \frac{\partial \mathcal{T}_{\rm F}^{H3}(\xi_1)}{\partial\xi_1}\Big]_{\xi_1\to 0} + C$$

$$= \Big[(-3i)^2\frac{m_H^4}{2v^2}\int d\xi_1 \int \frac{d^4k}{(2\pi)^4}$$

$$\times \frac{-i}{(k^2 - m_H^2 + \xi_1)^2}\frac{i}{(k+p)^2 - m^2}\Big]_{\xi_1\to 0} + C$$

$$\mathcal{T}_{\rm P}^{H3} = \frac{9m_H^4}{2v^2}\frac{i}{16\pi^2}\int_0^1 dx\,\log\frac{1}{m_H^2 - x(1-x)p^2} + C$$

$$= i\frac{9m_H^4}{32\pi^2 v^2}\int_0^1 dx\,\log\frac{\mu^2}{m_H^2 - x(1-x)p^2}\,.$$



V (V=W,Z) **in the third diagram**

$$\mathcal{T}_{\rm P}^{V3}=\Big[\int d\xi_1 d\xi_2 d\xi_3 \frac{\partial \mathcal{T}_{\rm F}^{V3}(\xi_1,\xi_2,\xi_3)}{\partial\xi_1\partial\xi_2\partial\xi_3}\Big]_{\{\xi_1,\xi_2,\xi_3\}\to 0}+C$$

$$=\Big[-\frac{4m_V^4}{v^2 s_V}\int d\xi_1 d\xi_2 d\xi_3\int\frac{d^4k}{(2\pi)^4}\frac{6i(g^{\mu\nu}-k^\mu k^\nu/m_V^2)}{(k^2-m_V^2+\xi_1+\xi_2+\xi_3)^4}$$

$$\times\frac{-i(g_{\mu\nu}-(k+p)_\mu(k+p)_\nu/m_V^2)}{(k+p)^2-m_V^2}\Big]_{\{\xi_1,\xi_2,\xi_3\}\to 0}+C\,.$$

power-law divergences (Λ^2, Λ^4)

$$\mathcal{T}_{\rm P}^{V3}=\frac{4m_V^4}{v^2 s_V}\frac{6i}{16\pi^2}\int_0^1 dx\Big(\Big[\frac{1}{2}-\frac{p^2}{m_V^2}(x-x^2+\frac{1}{12})$$
$$+\frac{p^4}{m_V^4}\frac{x(1-x)}{12}(20x-20x^2-1)\Big]\log\frac{1}{m_V^2-x(1-x)p^2}$$
$$+\frac{1}{12}-\frac{p^2}{12m_V^2}(22x(1-x)-1)$$
$$-\frac{p^4x(1-x)}{12m_V^4}(-21x(1-x)+1)\Big)+C$$
$$=\frac{m_V^4}{v^2 s_V}\frac{3i}{2\pi^2}\int_0^1 dx\Big(\Big[\frac{1}{2}-\frac{p^2}{m_V^2}(x-x^2+\frac{1}{12})$$
$$+\frac{p^4}{m_V^4}\frac{x(1-x)(20x-20x^2-1)}{12}\Big]\log\frac{\mu^2}{m_V^2-x(1-x)p^2}$$
$$+\frac{1}{12}-\frac{p^2(22x(1-x)-1)}{12m_V^2}-\frac{p^4x(1-x)(-21x(1-x)+1)}{12m_V^4}\Big)$$

Considering μ in the electroweak scale,

125 GeV Higgs can be obtained without fine-tuning, i.e., an alternative interpretation within SM.

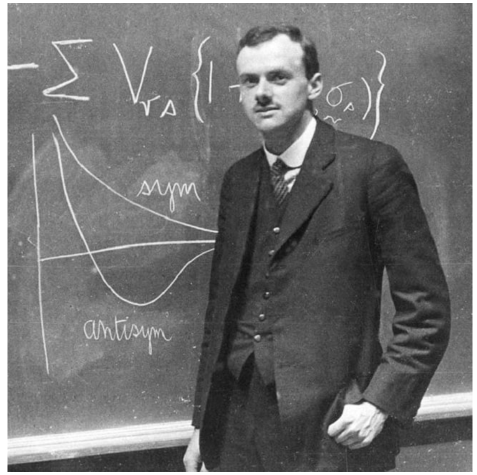
power-law divergences are OK in UV-free scheme!

Different schemes

As expected by Dirac!

Regularization & renormalization

$\infty - \infty$



P.A.M. Dirac

Schemes	Tree level	Loop Log	Loop Λ^2, Λ^4
Regularization & renormalization(BPHZ)		OK	
UV-free scheme	OK	OK	OK

UV-free scheme

$$\mathcal{T}_{\rm P} = \Big[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\rm F}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\Big]_{\{\xi_1, \cdots, \xi_i\} \to 0} + C$$

An alternative scheme for loops

IV. Summary and outlook

A. An alternative scheme --- UV-free scheme for loops was introduced, and finite loop results can be obtained **without UV divergences**. The γ^5 matrix remains the original form, and the unitary gauge can be adopted for gauge bosons with masses.

B. UV-free scheme can be adopted to describe tree-level transitions and Loop Log and **power−law (Λ^2, Λ^4) divergences**.

C. **To the hierarchy problem,** 125 GeV Higgs can be obtained in UV-free scheme **without fine-tuning**, i.e., an alternative interpretation **within SM**.

Outlook: It is the beginning of a new method.

Thank you!