Can sub-GeV dark matter coherently scatter on the electrons in the atom?

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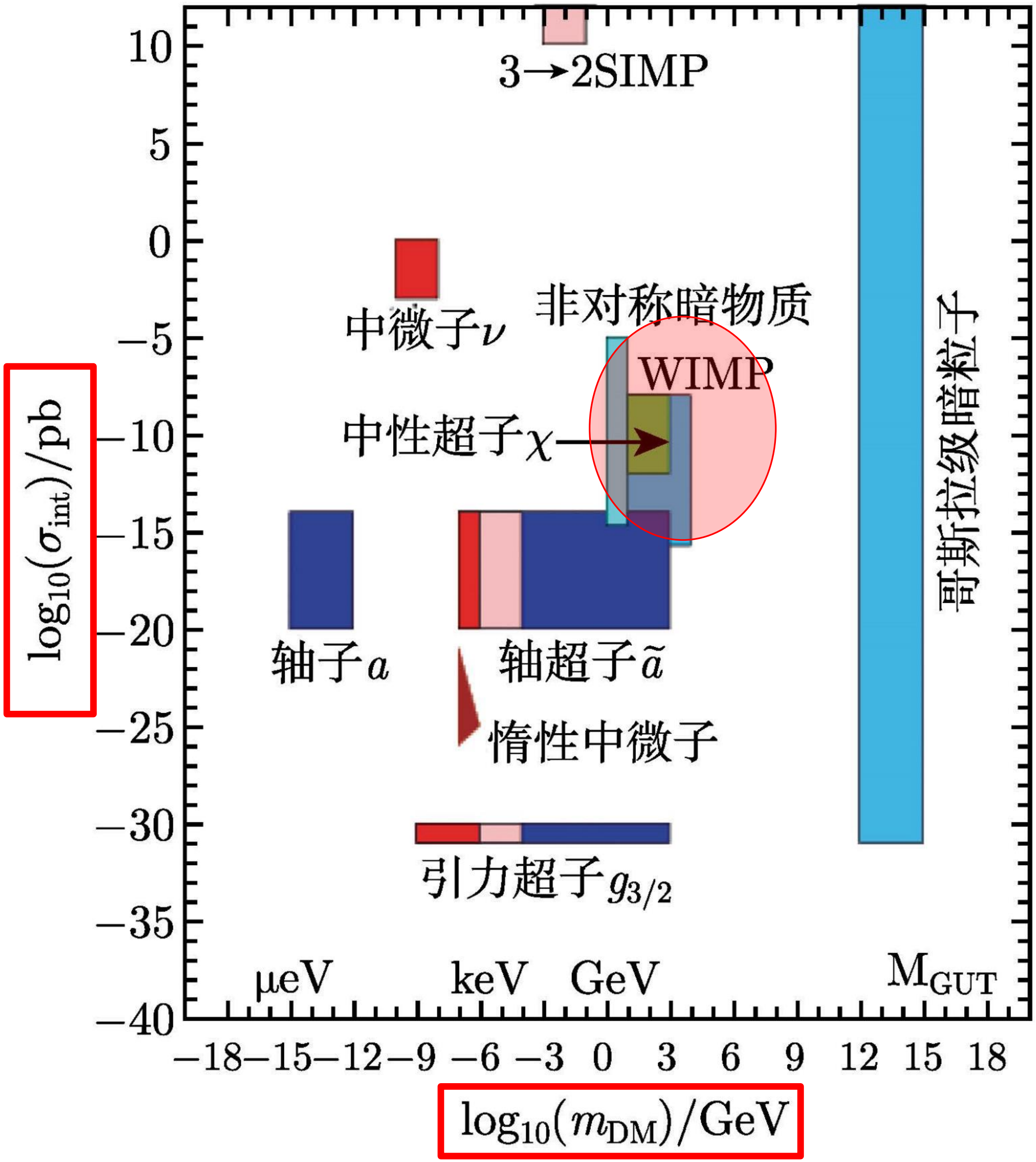
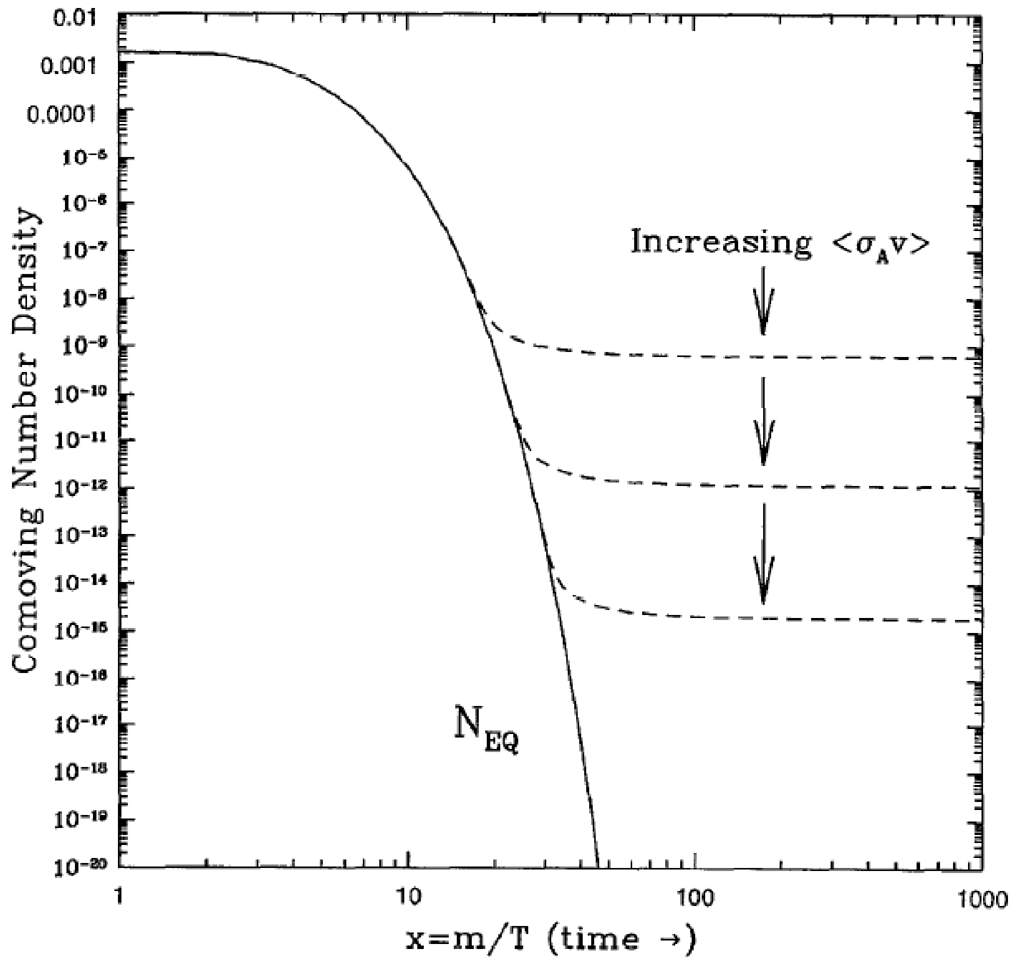
CONTENT

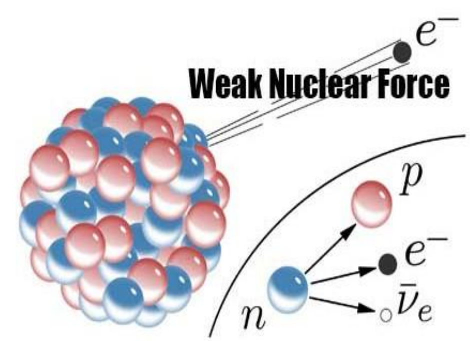
- Detection of WIMP
- Sub-GeV Dark matter
- Coherently scattering on the electrons in the atom
- Conclusion



3→2SIMP
中微子ν
非对称暗物质
WIMP
中性超子χ
轴子a
轴超子ã
惰性中微子
引力超子$g_{3/2}$
哥斯拉级暗粒子
μeV
keV
GeV
M_{GUT}
$\log_{10}(\sigma_{int})$/pb
$\log_{10}(m_{DM})$/GeV
10
5
0
−5
−10
−15
−20
−25
−30
−35
−40
−18 −15 −12 −9 −6 −3 0 3 6 9 12 15 18

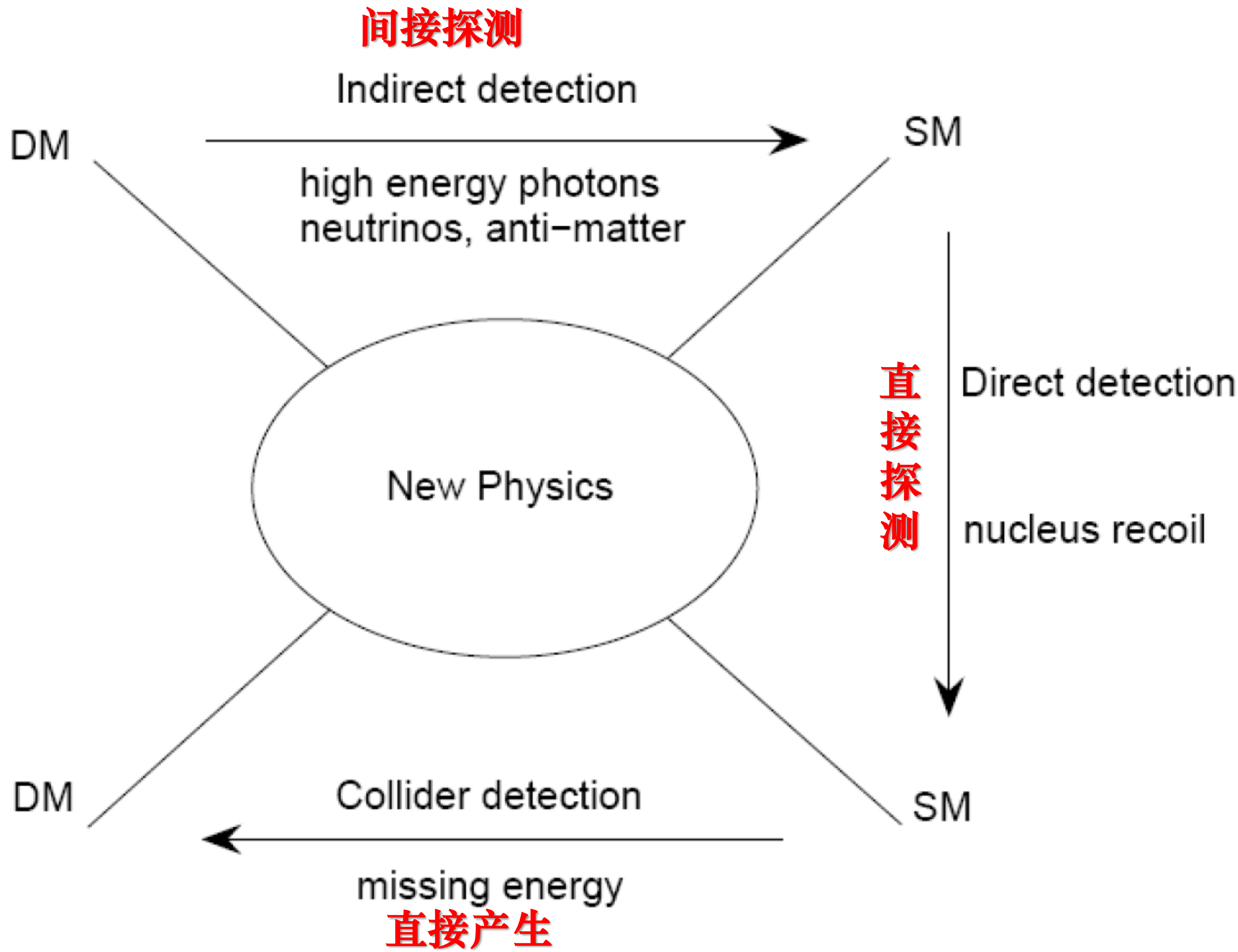


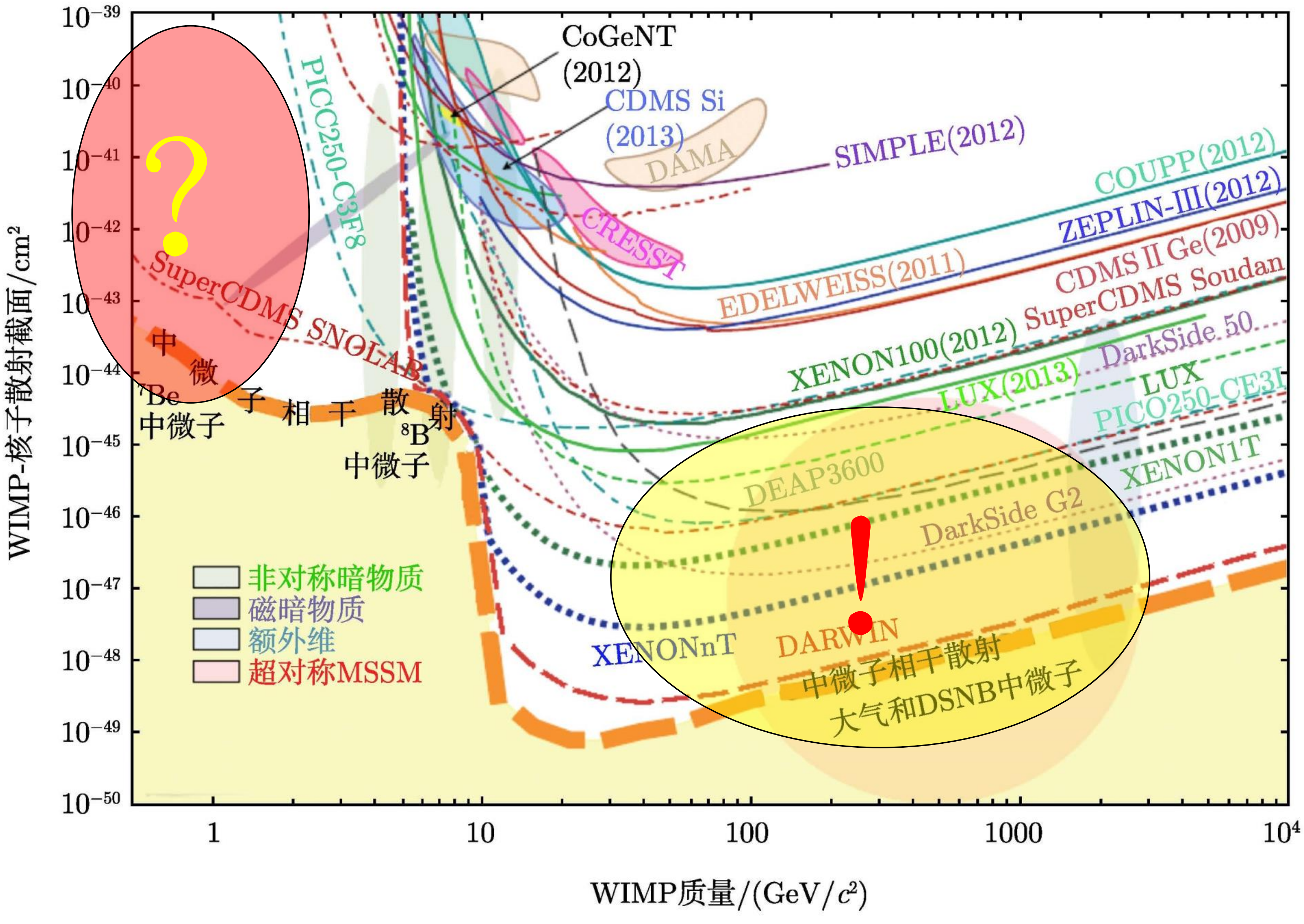
$$\Omega_\chi h^2 = 0.1 \times \left(\frac{x_f}{10}\right) \left(\frac{g_*}{100}\right)^{1/2} \frac{0.282 \text{ pb}}{a + \frac{3}{x_f} b},$$



暗物质探测手段

Detection of particle dark matter





CoGeNT (2012)
CDMS Si (2013)
DAMA
SIMPLE(2012)
COUPP(2012)
ZEPLIN-III(2012)
CDMS II Ge(2009)
SuperCDMS Soudan
CRESST
EDELWEISS(2011)
PICC250-C3F8
SuperCDMS SNOLAB
XENON100(2012)
LUX(2013)
DarkSide 50
LUX
PICO250-CE3I
XENON1T
DEAP3600
DarkSide G2
XENONnT
DARWIN
中微子相干散射
大气和DSNB中微子
^{7}Be 中微子
^{8}B 中微子
中微子相干散射
非对称暗物质
磁暗物质
额外维
超对称MSSM
?
!
WIMP-核子散射截面/cm^2
WIMP质量/(GeV/c^2)

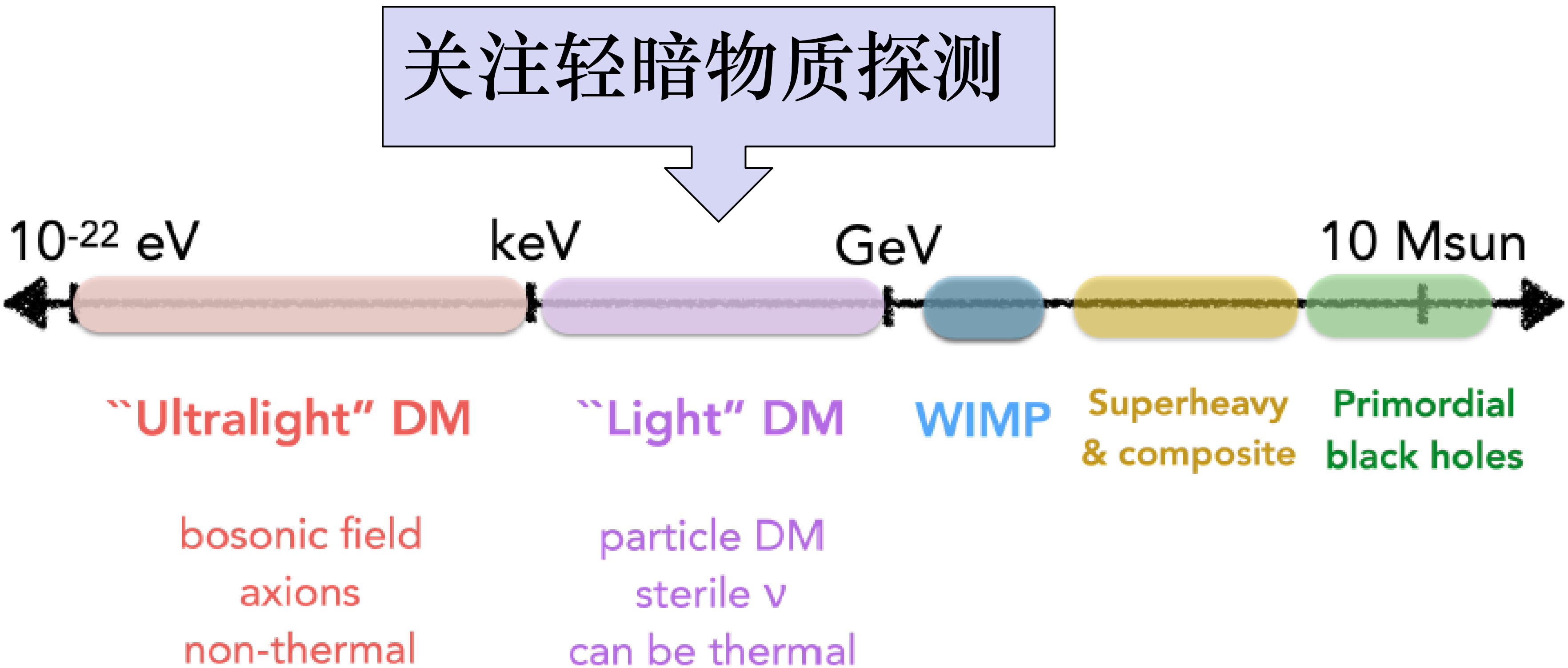
WIMP Search Status

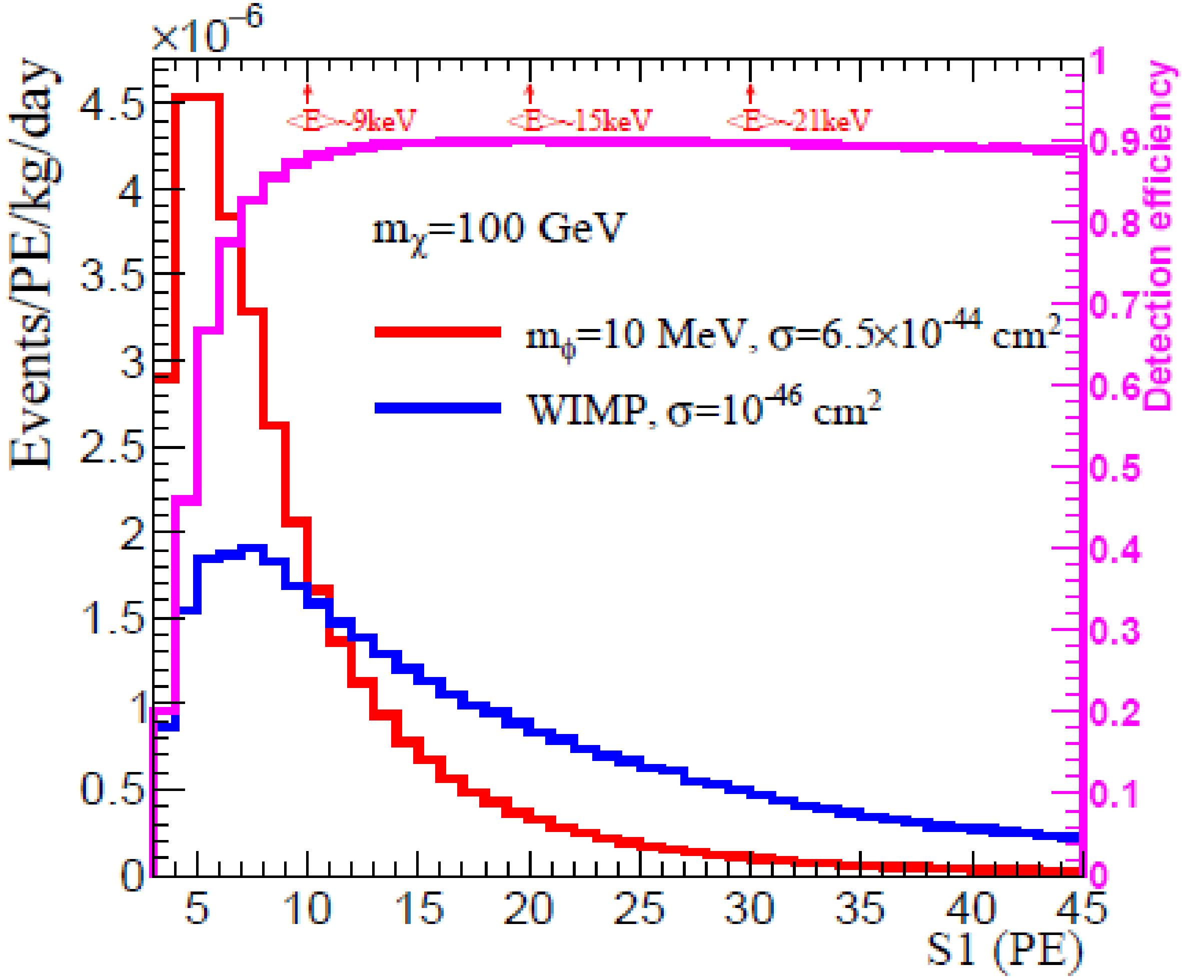
“上穷碧落下黄泉，两处茫茫皆不见。”白居易《长恨歌》

He exhausted all avenues in heaven and the nether world,
… he could not bring her existence to light.

A Song of Immortal Regret, Bai Juyi (772-846)

——某人的报告透明片


关注轻暗物质探测
10-22 eV
keV
GeV
10 Msun
``Ultralight" DM
``Light" DM
WIMP
Superheavy
& composite
Primordial
black holes
bosonic field
axions
non-thermal
particle DM
sterile ν
can be thermal



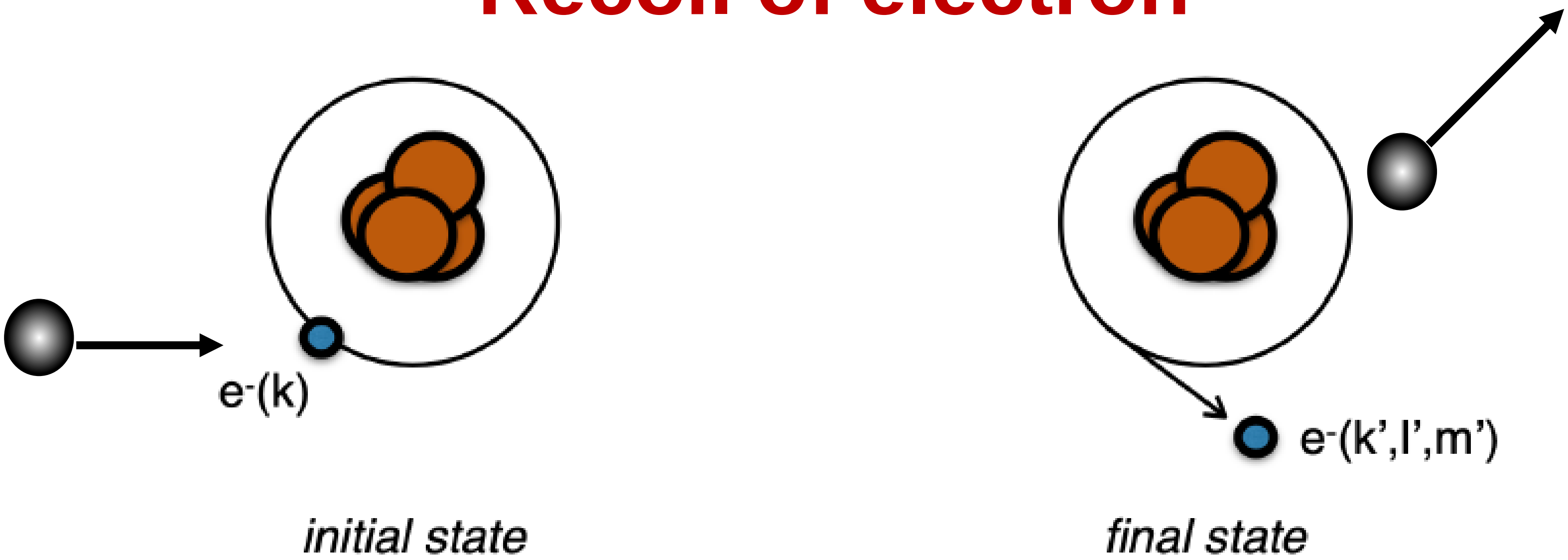
From PANDAX group Phys. Rev. Lett. 121. 021304

$$\frac{\mathrm{d}R}{\mathrm{d}E_r} = \frac{n_\chi \sigma_0}{4m_R^2 v_E} F^2(\sqrt{2m_A E_r}) \left(\mathrm{erf}\left(\frac{v_{\min}+v_E}{v_0}\right) - \mathrm{erf}\left(\frac{v_{\min}-v_E}{v_0}\right) \right)$$

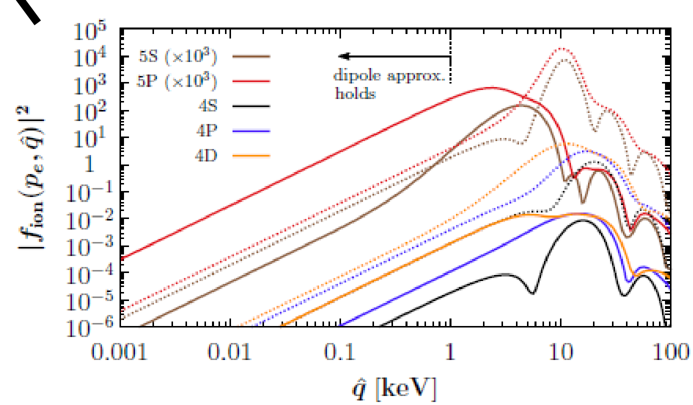
The signal of sub-GeV dark matter in WIMP detections

- **Other signal**
  1. scattering on electrons
  2. Migdal effect
  3. bremsstrahlung
- **Exotic source**
  1. Cosmic Rays boosted dark matter
  2. Atmospheric dark matter
  3. Annihilation， decay ……

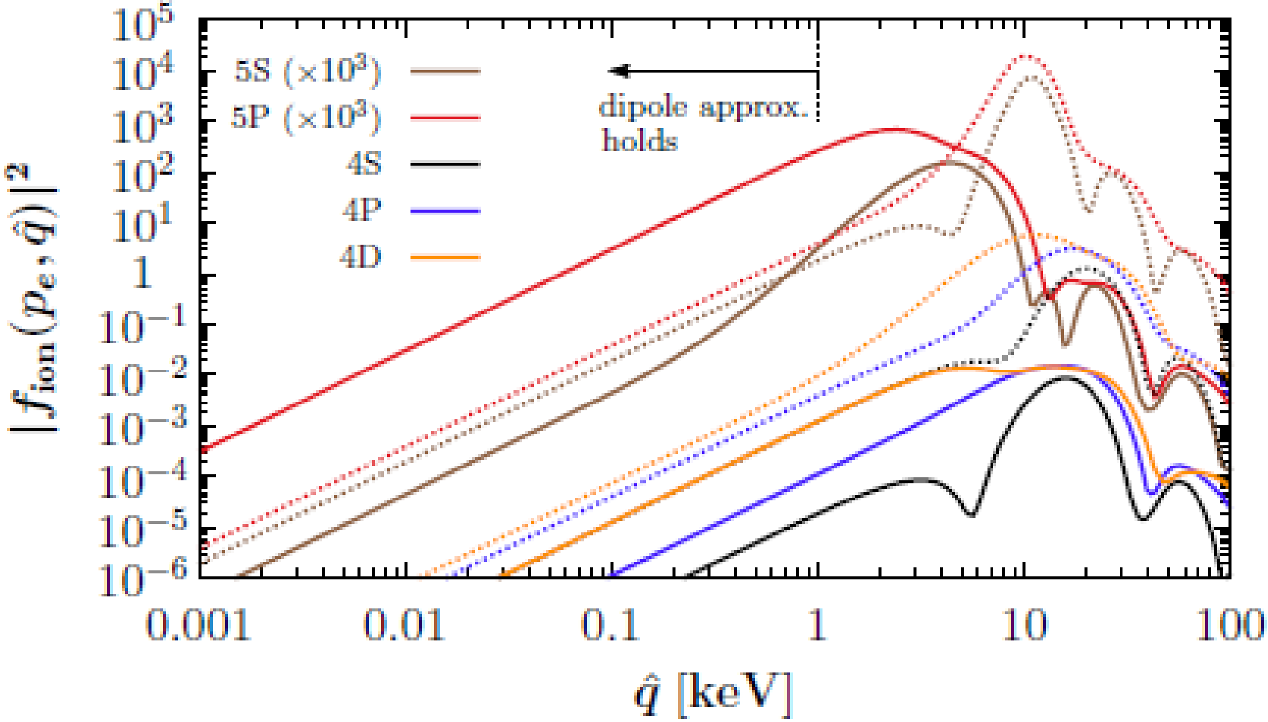
Recoil of electron



$$\frac{d\langle\sigma_{ion}^{nl}\rangle}{d\ln E_{er}} = \frac{\overline{\sigma}_e}{8\mu_{\chi e}^2}\int q\,dq\,|f_{ion}^{nl}(k',q)|^2|F_{\mathrm{DM}}(q)|^2\eta(v_{min}),$$



$$\frac{dR_{ion}}{d\ln E_{er}} = N_T\frac{\rho_\chi}{m_\chi}\sum_{nl}\frac{d\langle\sigma_{ion}^{nl}v\rangle}{d\ln E_{er}},$$



$$|f_{ion}^{nl}(k', q)|^2 = \frac{4k'^3}{(2\pi)^3} \sum_{l'L} (2l+1)(2l'+1)(2L+1)$$

$$\times \begin{bmatrix} l & l' & L \\ 0 & 0 & 0 \end{bmatrix}^2 \left| \int r^2 dr R_{k'l'}(r) R_{nl}(r) j_L(qr) \right|^2,$$

1. The quanta of recoil energy obey binomial distribution

$$N_q \sim \mathrm{Binom}\left(\varepsilon/W, L\right),$$

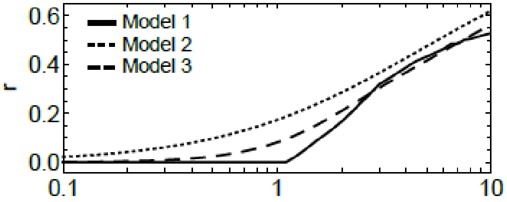
2. The quanta are divided in to ionization and stimulation

$$N_i \sim \mathrm{Binom}\left(N_q, \frac{1}{1+\langle N_{ex}/N_i\rangle}\right),$$
$$N_{ex} = N_q - N_i.$$

3. Ionization quanta are divided in to electron and photon

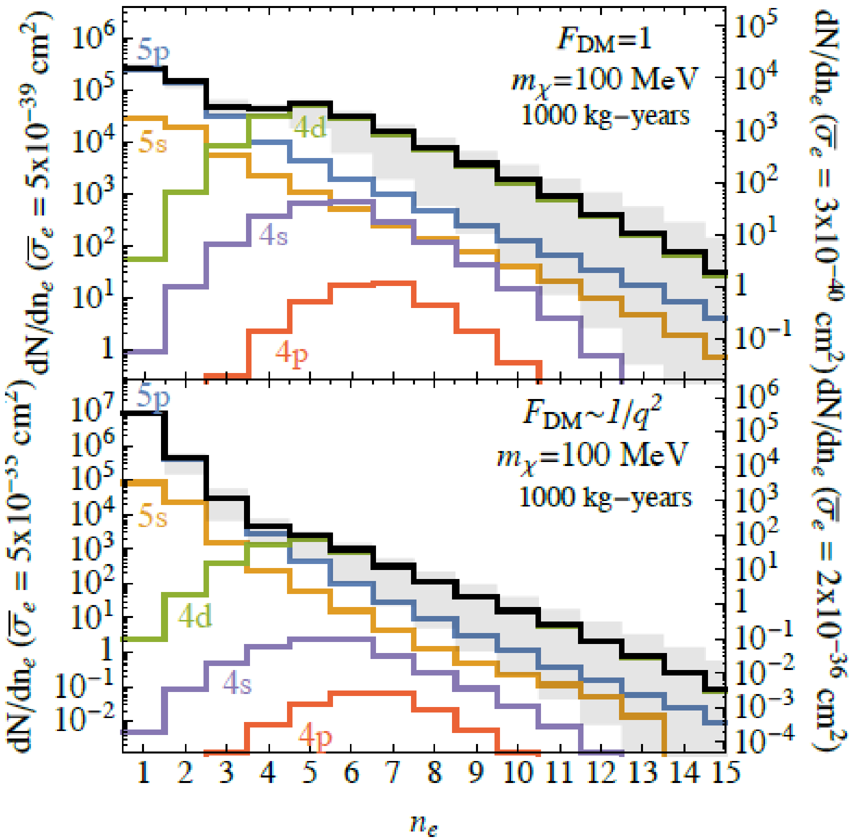
$$r \sim \mathrm{Gauss}\left(\langle r\rangle, \Delta r\right).$$

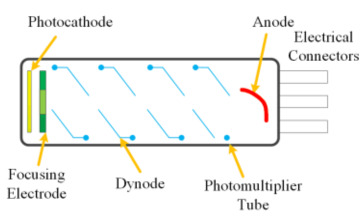
$$N_e \sim \mathrm{Binom}\left(N_i, 1-r\right),$$
$$N_\gamma = N_i - N_e + N_{ex},$$

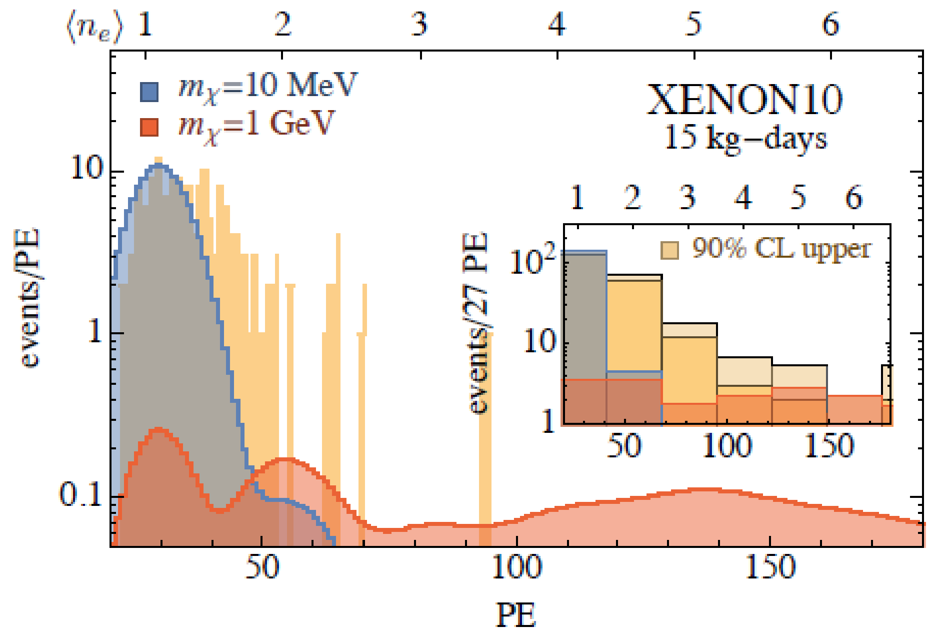


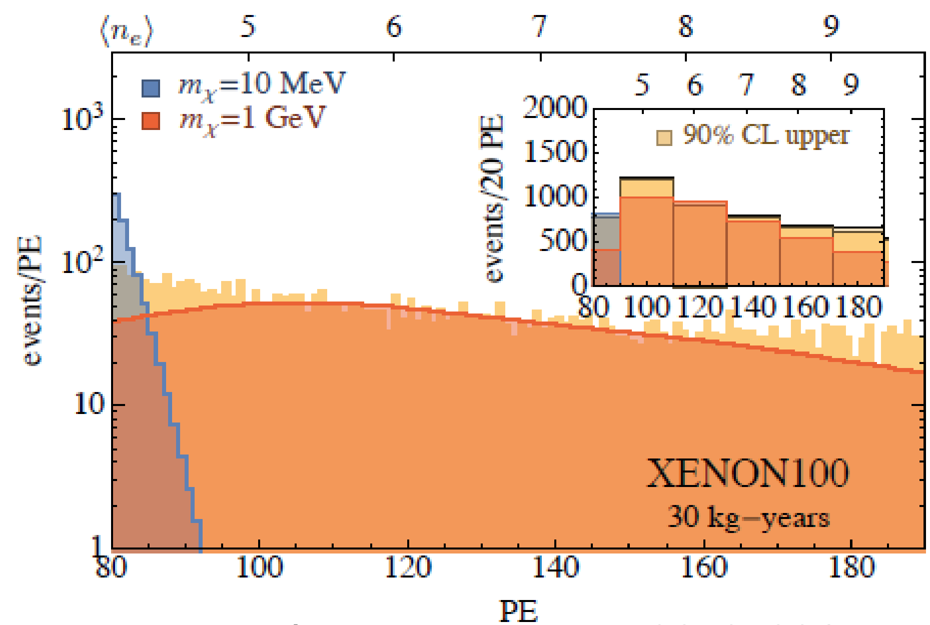
4. The ionized electron change into photon by gaussian distribution when they drift to S2

$$P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right) = \mathrm{gauss}\left(\mathrm{S2} \mid g_2 n_e^{\mathrm{s}}, \sigma_{\mathrm{S2}}\right)$$



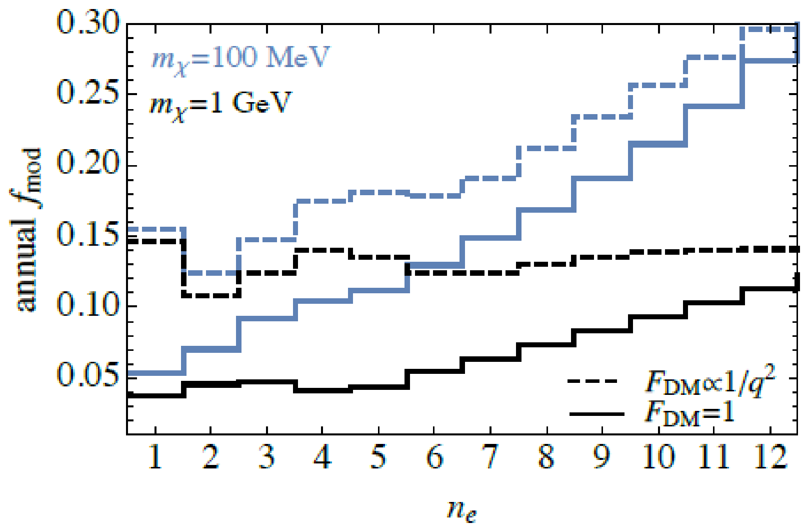


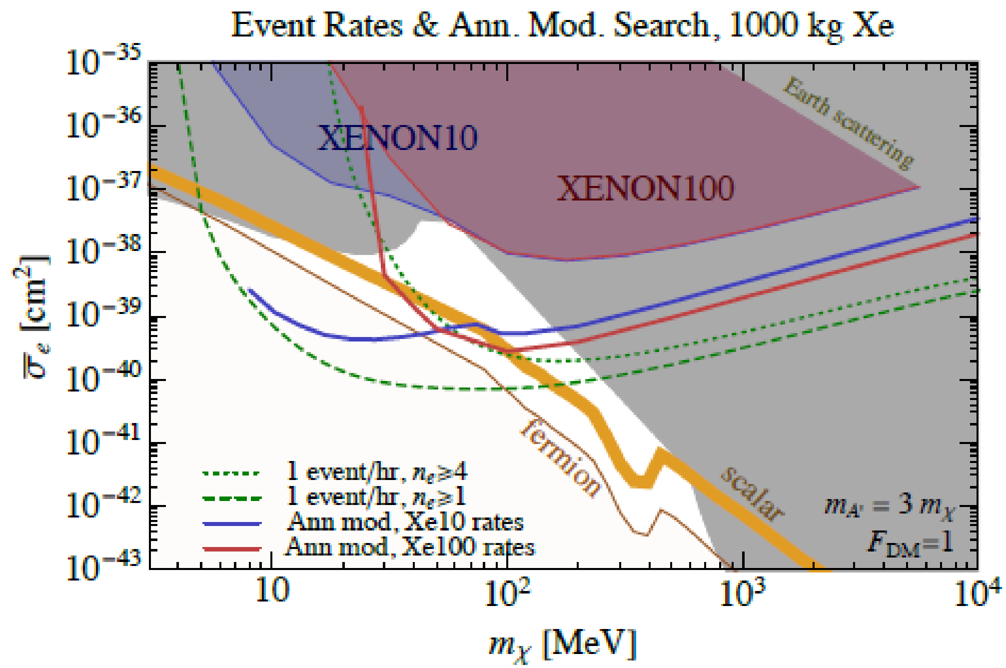


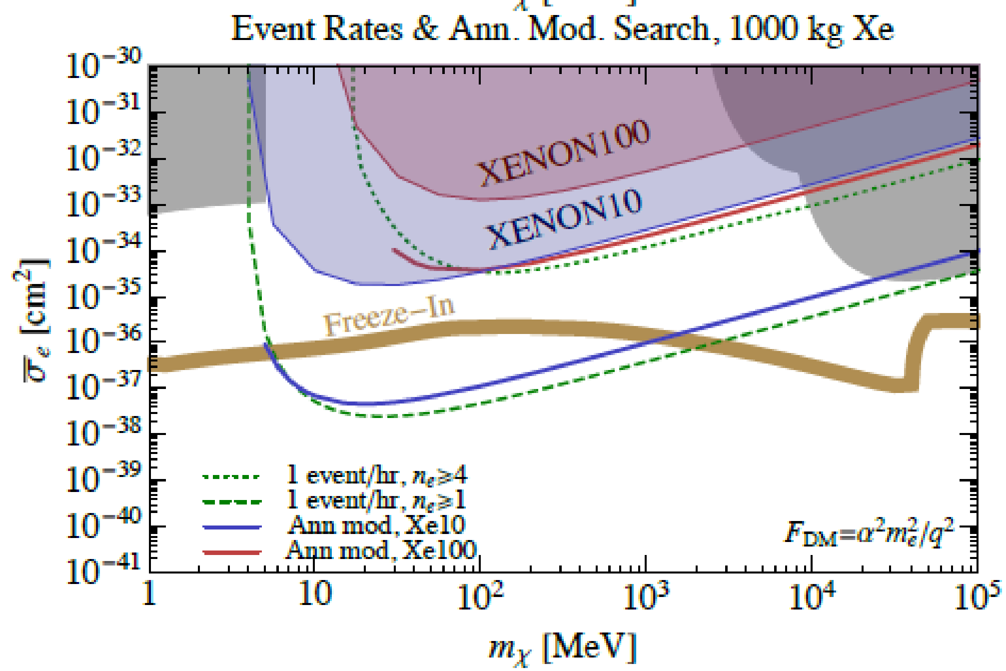


电离电子到光电子

From R. Essig … Phys. Rev. D 96. 043017



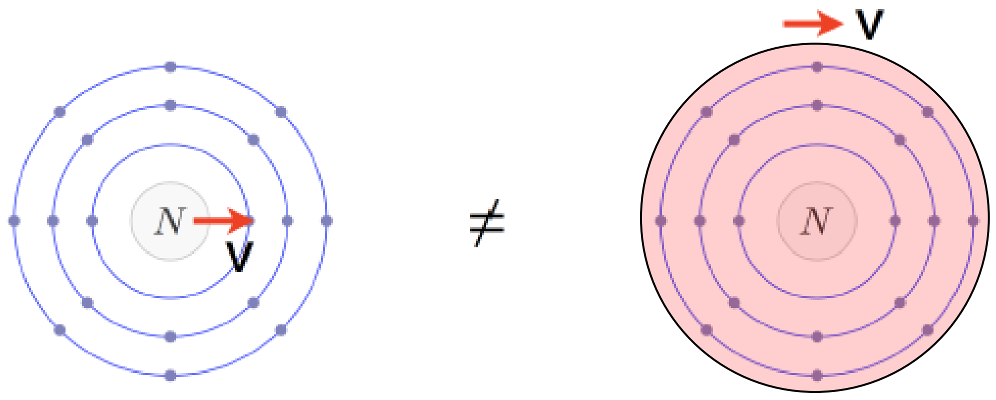




电子反冲对暗物质截面和质量的限制

From R. Essig … Phys. Rev. D 96. 043017

Migdal效应（核反冲导致的原子反冲）

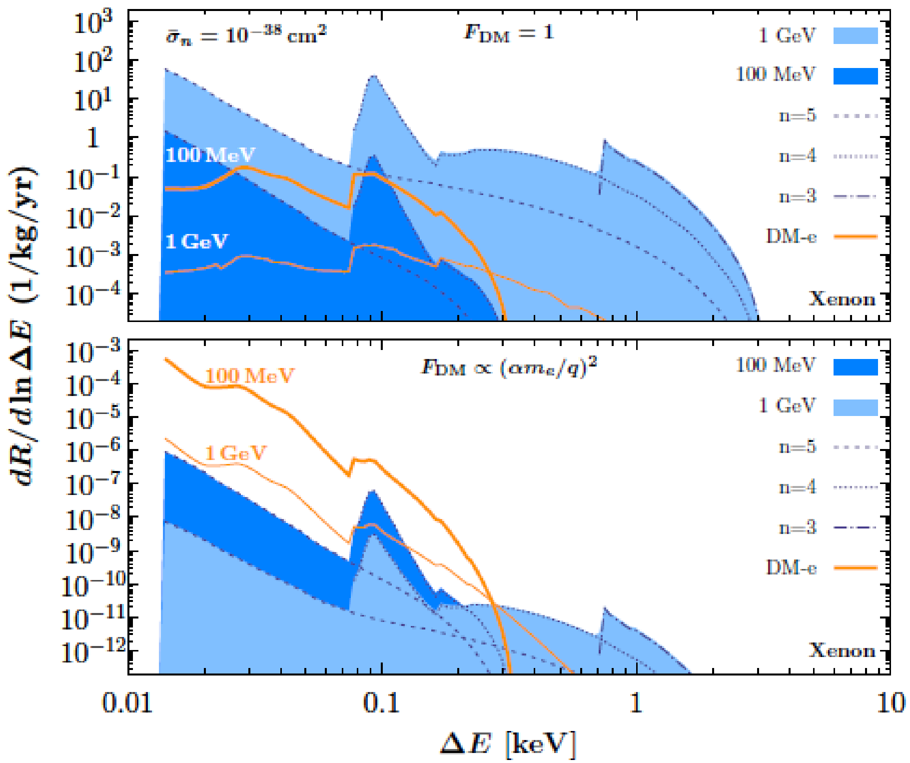


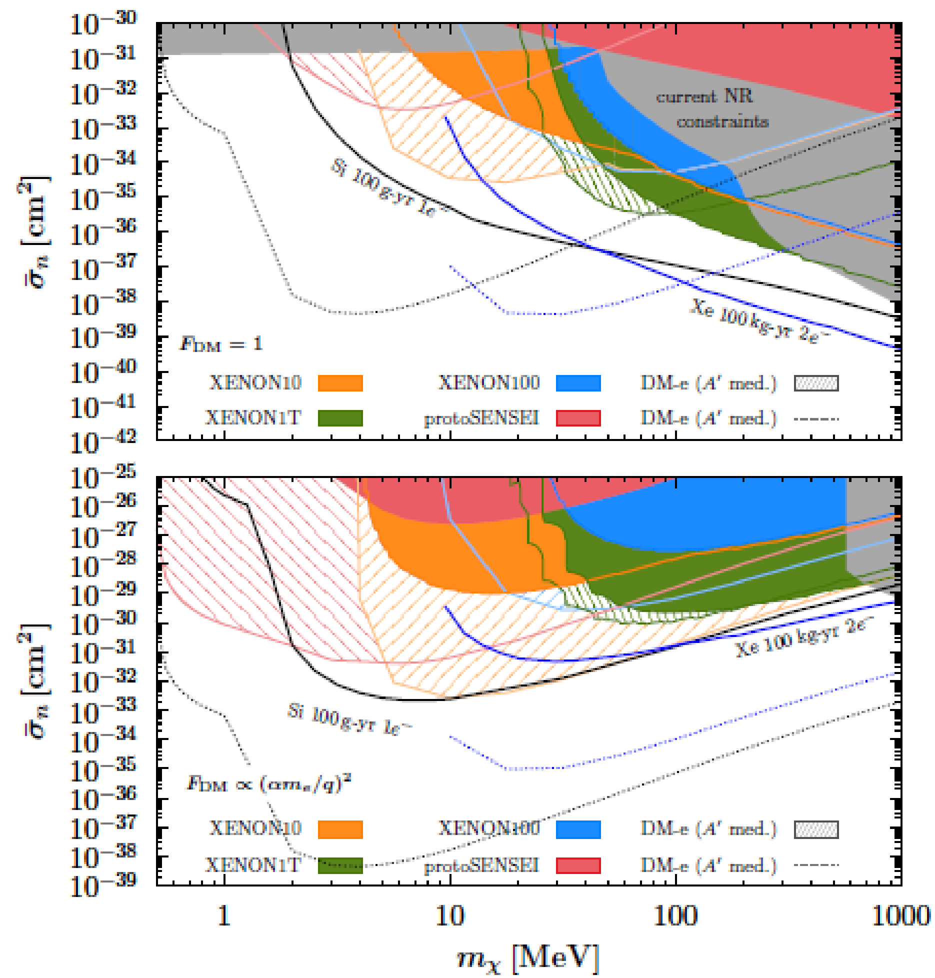
$$\left|\langle p_e, l'|e^{i\frac{m_e}{m_N}\boldsymbol{q}\cdot\sum_\alpha \boldsymbol{x}^{(\alpha)}}|n,l\rangle\right|^2 = \frac{1}{2\pi}\frac{dp_{nl\to E_e}(q)}{dE_e}\,.$$

$$\frac{dp_{nl\to E_e}}{d\ln E_e} = \frac{\pi}{2}|f_{nl}^{\rm ion}(p_e,q_e)|^2\,,$$

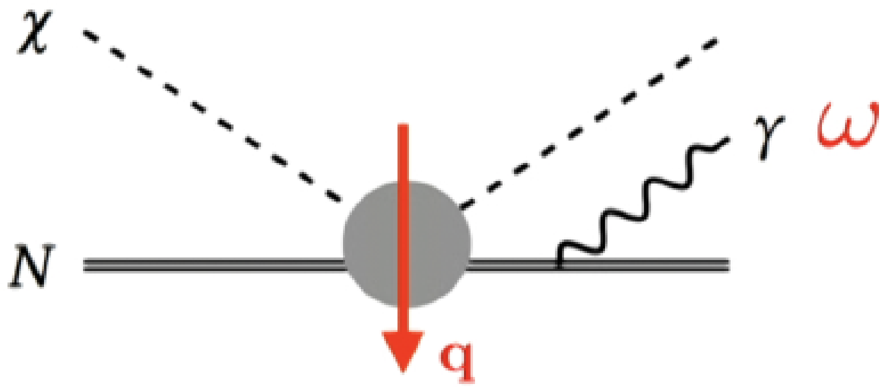
$$\frac{d\langle\sigma_{n,l}v\rangle}{d\ln E_e} = \frac{\bar{\sigma}_n}{8\mu_n^2}[f_pZ+f_n(A-Z)]^2\int dq\ \left[q|F_N(q)|^2 \times|F_{\rm DM}(q)|^2|f_{nl}^{\rm ion}(p_e,q_e)|^2\eta(v_{\rm min}(q,\Delta E_{n,l}))\right]$$

Migdal效应的限制结果

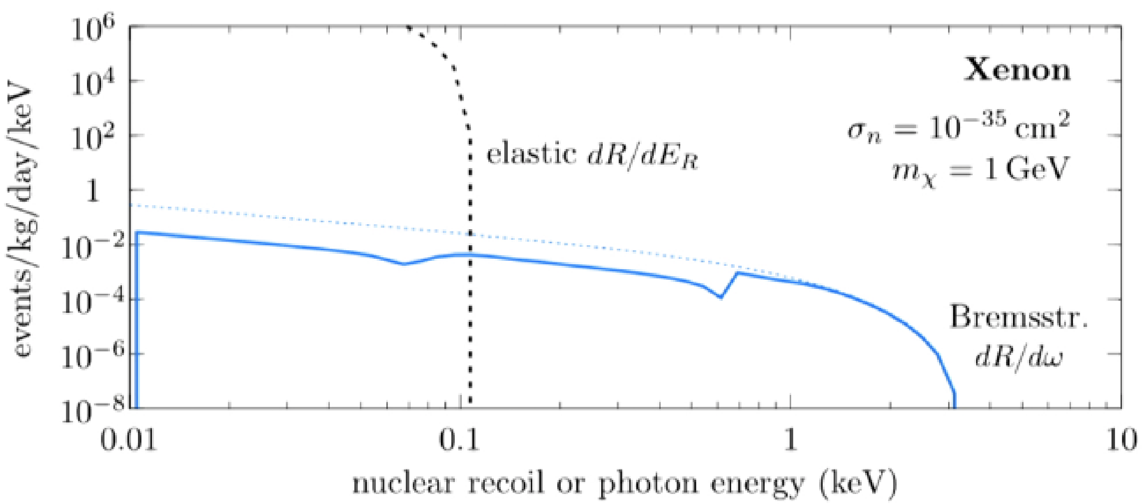


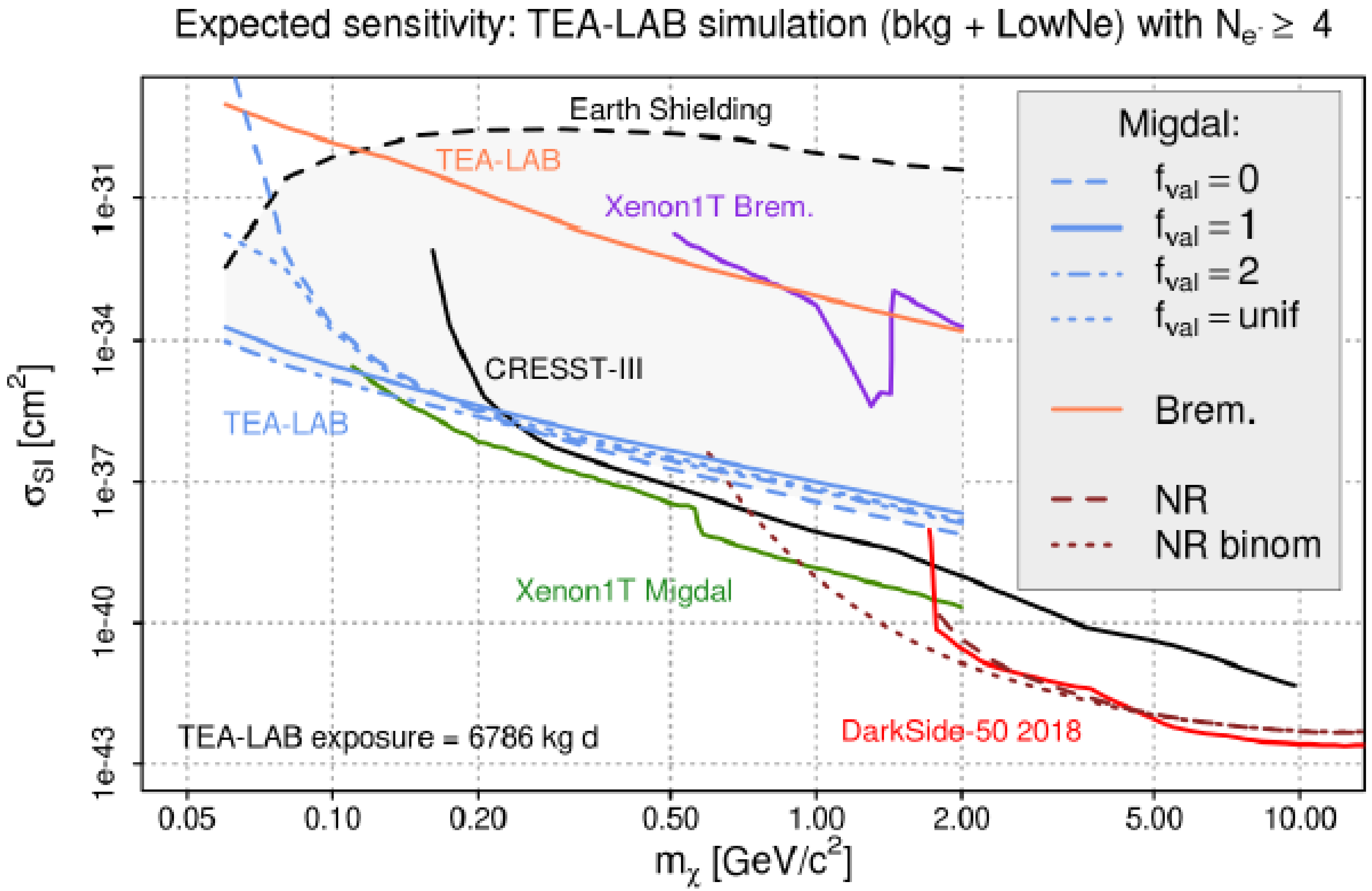


韧致辐射



$$\frac{d^2\sigma_{SI}}{dE_R d\omega} = \frac{4\alpha|f(\omega)|^2}{3\pi\omega}\frac{E_R}{m_N}\left(\frac{d\sigma_{SI}}{dE_R}\right)\Big|_{(2\to 2)},$$

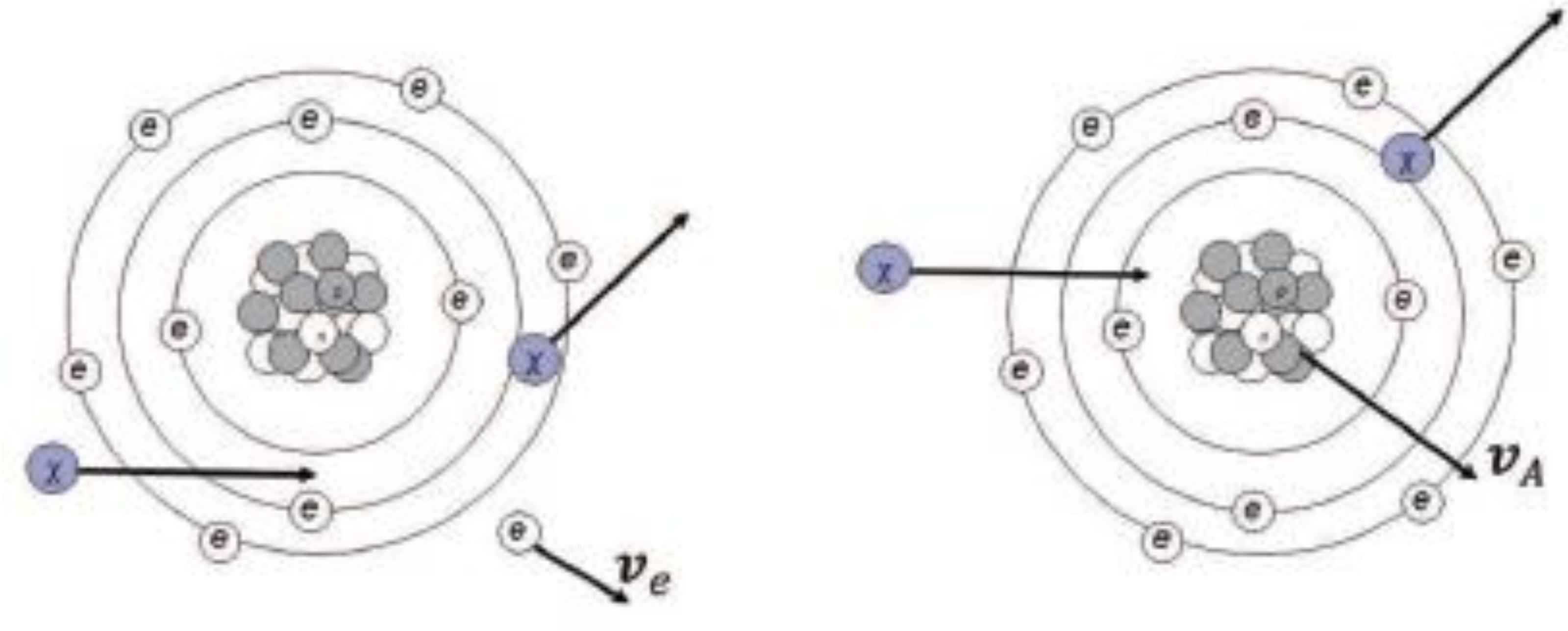




From G. Cortona, JHEP 11 (2020) 034

电子相干散射效应

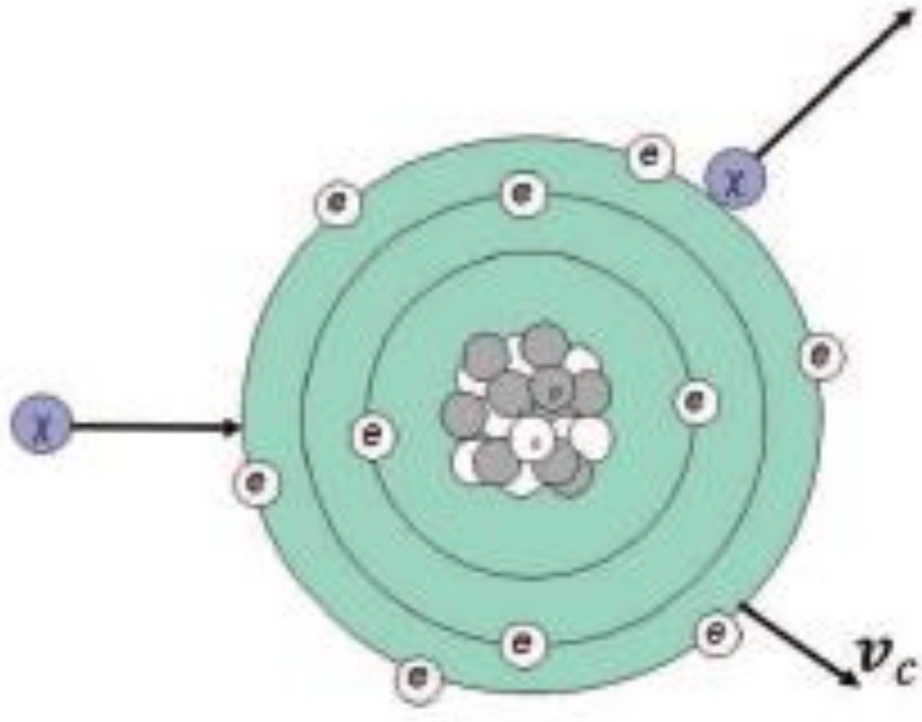
电子相干散射效应



(a) Recoil of Electron

(b) Recoil of Nucleus

电子相干散射效应

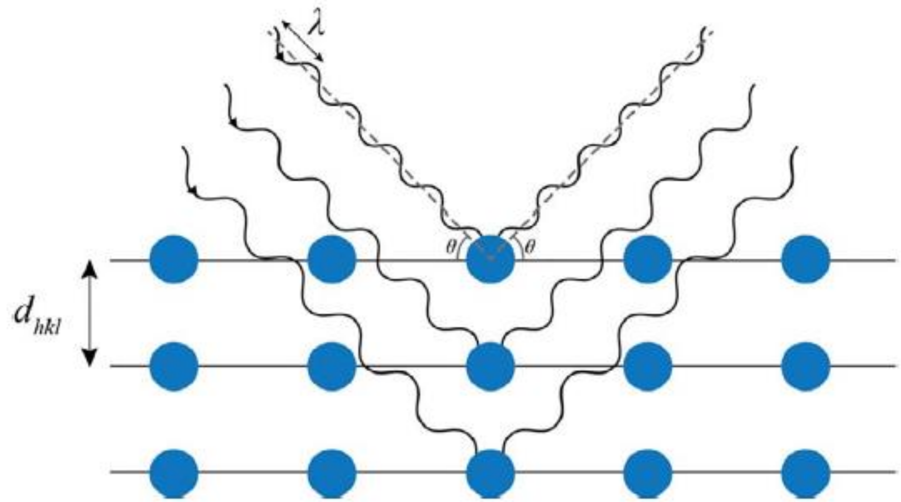


(c) Recoil of Electron Cloud

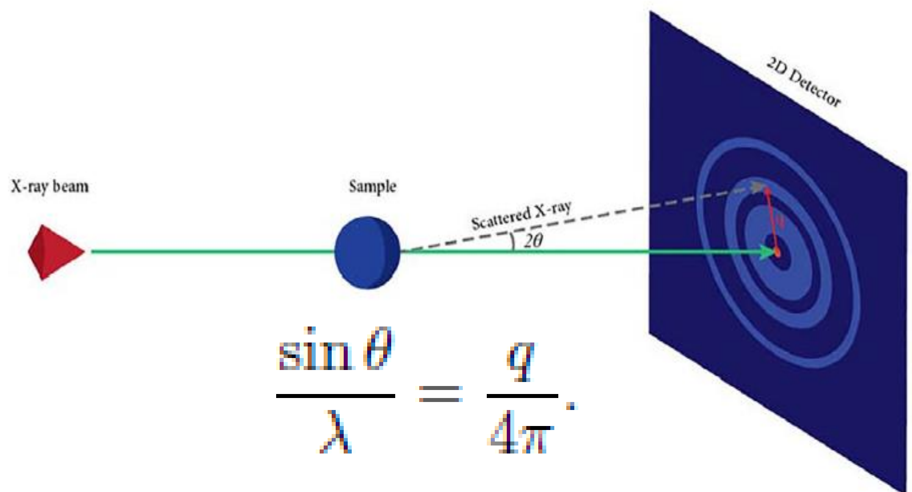
核外电子云被暗物质（相干散射）整体推动，被静止原子核拉回来的过程产生电离

XRD深入研究物质的微观结构

大角度XRD



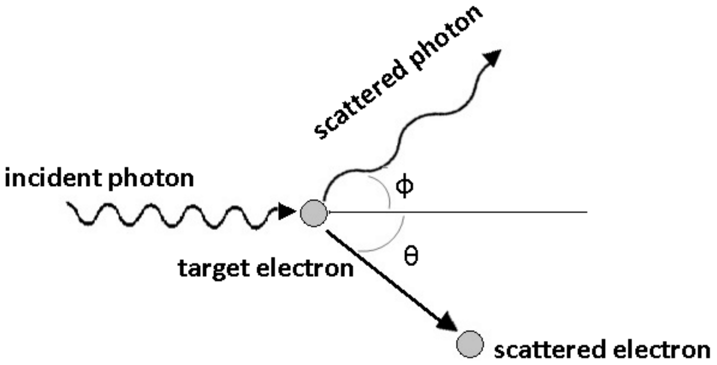
小角度XRD



$$\frac{\sin\theta}{\lambda} = \frac{q}{4\pi}.$$

小角度散射测量原子形状因子

- 非相干散射（康普顿散射）



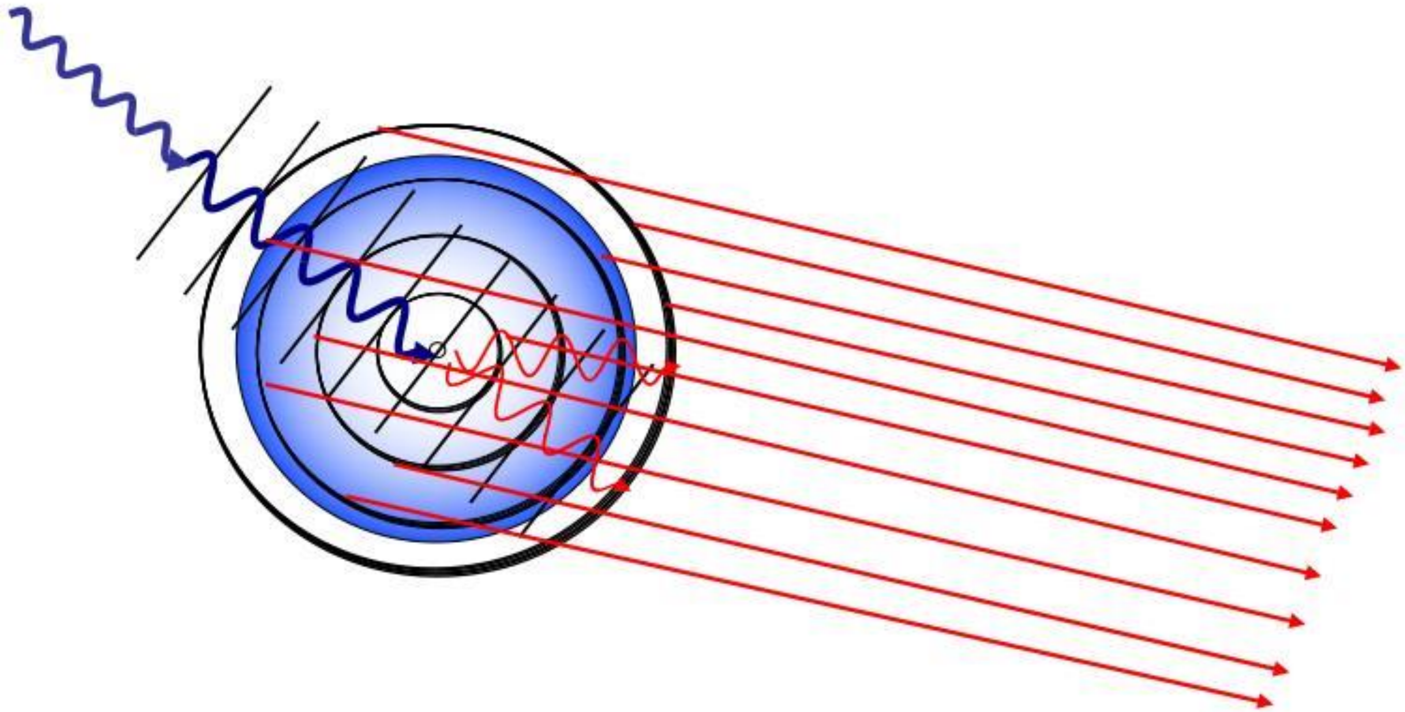
- 相干散射

$$F(q) = \sum_{j}^{Z} f_j = \sum_{j}^{Z} \int \rho(\boldsymbol{r}_j) \exp(\boldsymbol{q} \cdot \boldsymbol{r}_j) \mathrm{d}\boldsymbol{r}_j.$$

有效核外电荷分布

$$I_{\mathrm{coh}} = I_e \left(F(q)\right)^2 .$$

Rayleigh Scattering (Coherent Scattering)



1 . Photons are scattered by bound electrons

2. Atoms are neither excited or ionized

3. Scattering from different parts of electron cloud - coherent scattering

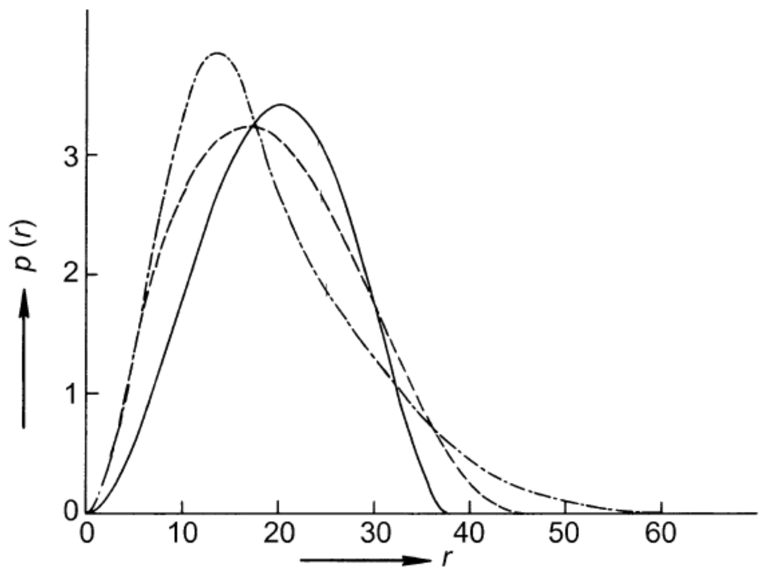


Fig. 2.6.1.4. Comparison of the $p(r)$ function of a sphere (———), ɛ prolate ellipsoid of revolution 1:1:3 (—·—·—), and an oblate ellipsoic of revolution 1:1:0.2 (- - - -) with the same radius of gyration.

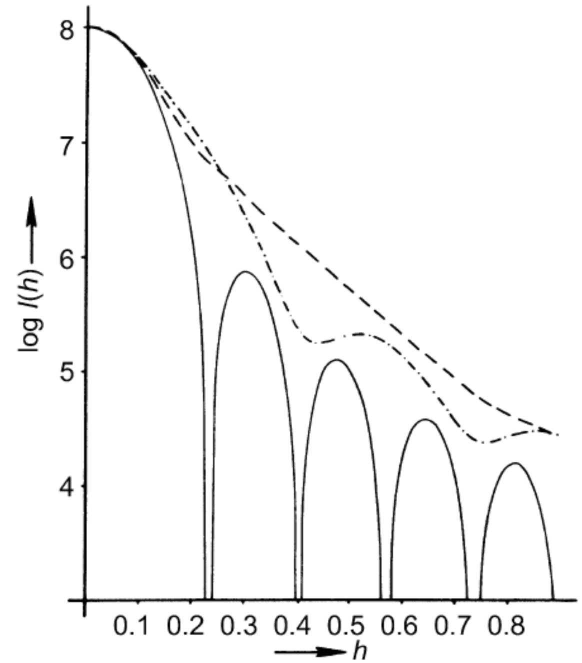


Fig. 2.6.1.5. Comparison of the $I(h)$ functions of a sphere, a prolate, and an oblate ellipsoid (see legend to Fig. 2.6.1.4).

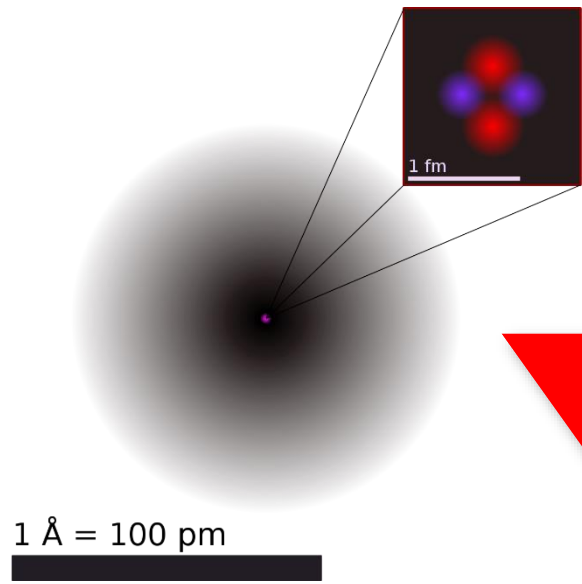
前向相干散射可以精确测量微观单粒子内部电荷分布

$$H_{\text{int, C}} = -\int \frac{\mathrm{d}^3\mathrm{q}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot(-\boldsymbol{x}_\chi+\boldsymbol{x}_e)} \frac{\mathcal{M}_{\chi C}(q)}{4m_\chi Z m_e},$$

$$|\mathcal{M}_{\chi C}|^2 = Z^2 |F_C(q)|^2 |\mathcal{M}_{\chi e}|^2,$$

如果存在相干散射意味着靶原子中所有的电子都贡献散射，散射截面有Z^2的抬高。通常认为这种抬高被电荷的空间分布压低，只要传递动量和原子尺寸可以比拟的时候才有效果。

通常的结果

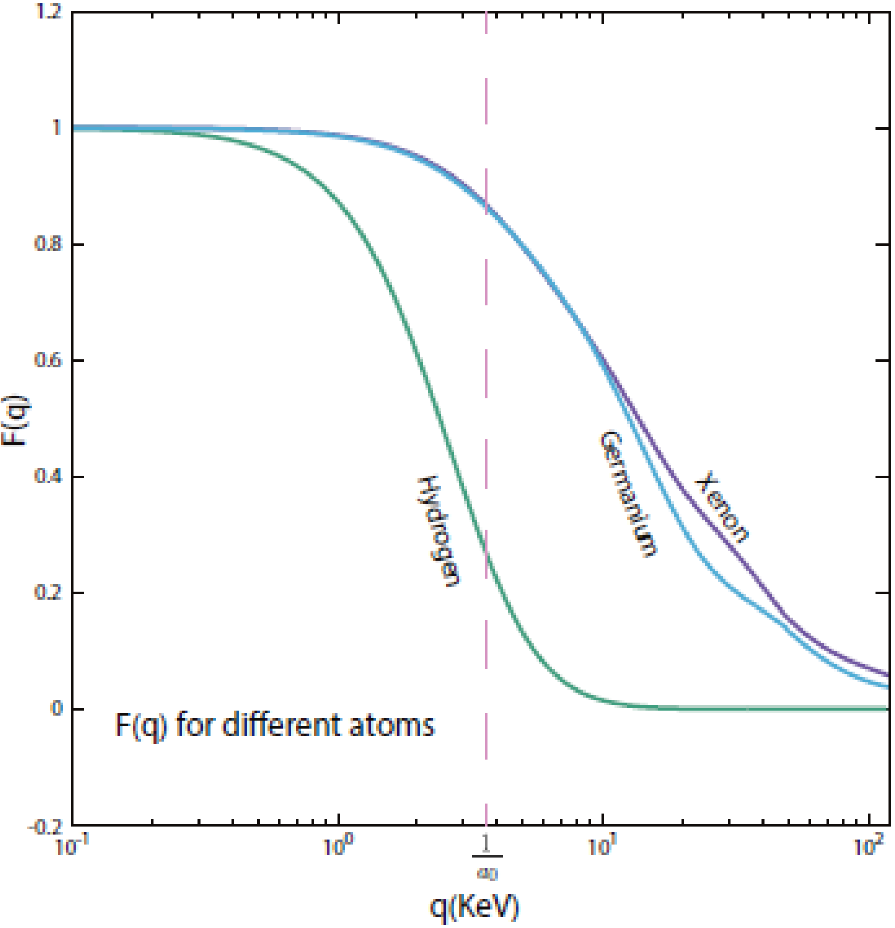


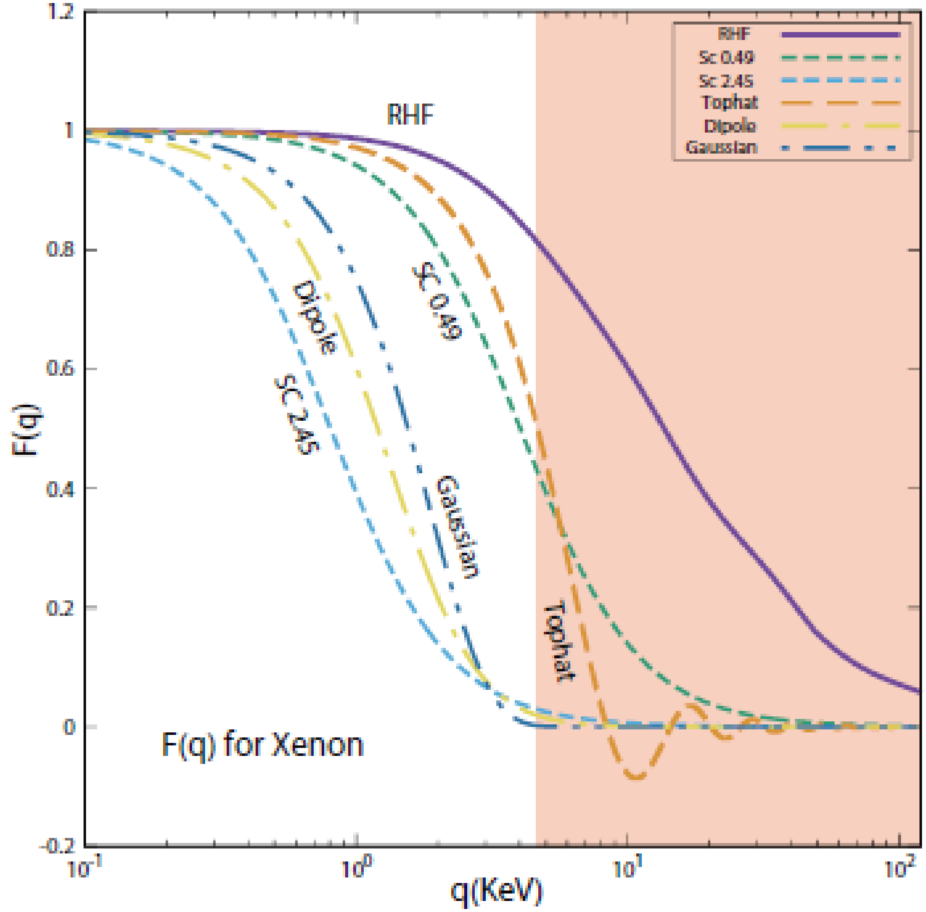
$$\phi(r) = \frac{Q}{4\pi r} e^{-\frac{r}{r_0}},$$

$$F[illegible] = \frac{1}{1 + q^2 r_0^2}.$$

Shape	$\rho(r)$	r_{Atom}	$F(q)$
Tophat	$\frac{3}{4\pi r_0^3}\theta(r_0$ [illegible]	$\sqrt{3/5}r_0$	[illegible] $\frac{\sin(r_0 q) - r_0 q \cos(r_0 q))}{r_0^3 q^3}$
Dipole	$\frac{e^{-r/}[illegible]}{8\pi r_0^3}$	$2\sqrt{3}r_0$	$\frac{1}{(1+r_0^2 q^2)^2}$
Gaussian	$\frac{1}{8 r_0^3 \pi^{3/2}} e^{-r^2/(4r_0^2)}$	$\sqrt{6}r_0$	$e^{-r_0^2 q^2}$

奇怪的相对论平均场论结果



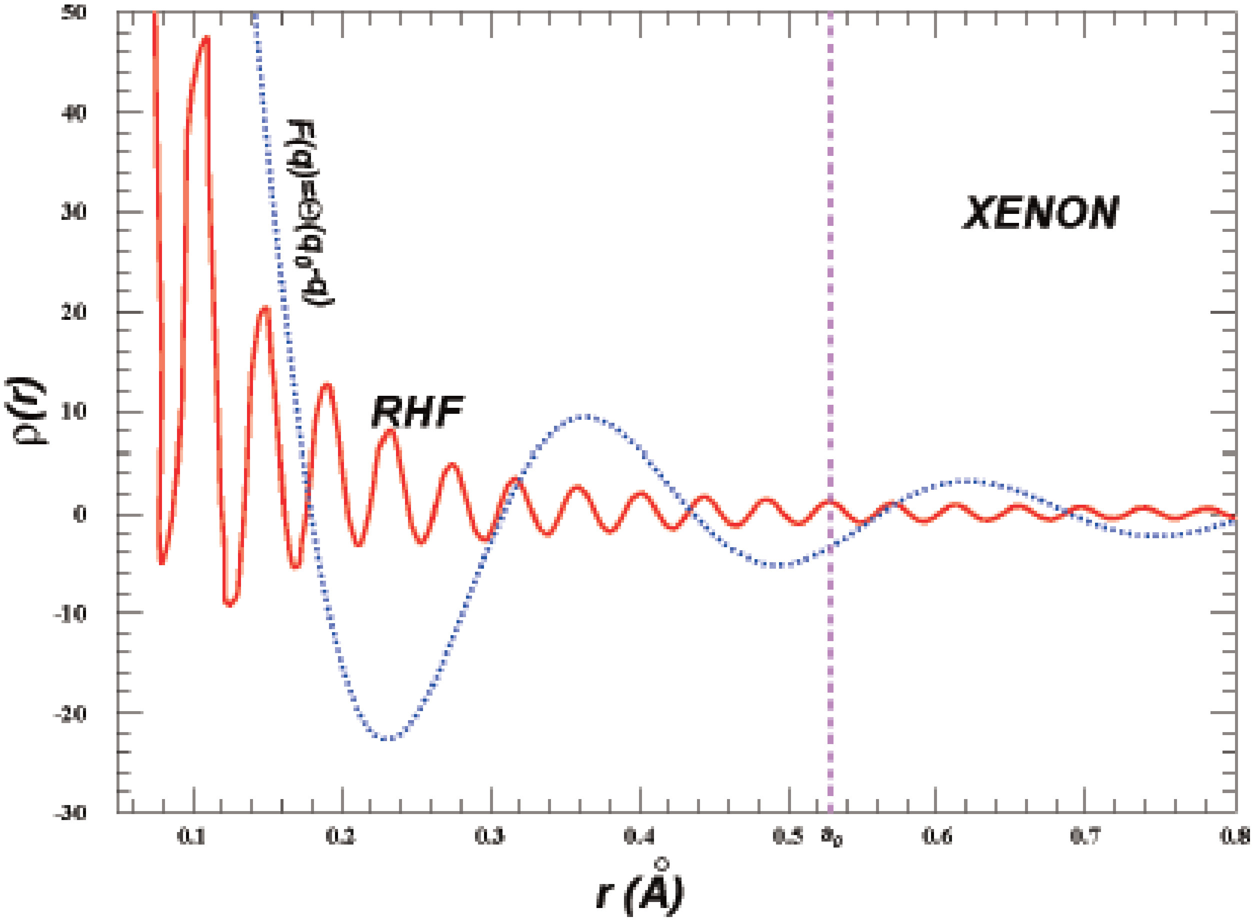


$$f(\sin\theta/\lambda) = \sum_{i=1}^{4} a_i \exp\left(-b_i \sin^2\theta/\lambda^2\right) + c.$$

Element	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	c
H	0.489918	20.6593	0.262003	7.74039	0.196767	49.5519	0.049879	2.20159	0.001305
H1-	0.897661	53.1368	0.565616	15.187	0.415815	186.576	0.116973	3.56709	0.002389
He	0.8734	9.1037	0.6309	3.3568	0.3112	22.9276	0.178	0.9821	0.0064
Li	1.1282	3.9546	0.7508	1.0524	0.6175	85.3905	0.4653	168.261	0.0377
Li1+	0.6968	4.6237	0.7888	1.9557	0.3414	0.6316	0.1563	10.0953	0.0167
Be	1.5919	43.6427	1.1278	1.8623	0.5391	103.483	0.7029	0.542	0.0385
Be2+	6.2603	0.0027	0.8849	0.8313	0.7993	2.2758	0.1647	5.1146	-6.1092
B	2.0545	23.2185	1.3326	1.021	1.0979	60.3498	0.7068	0.1403	-0.1932
C	2.31	20.8439	1.02	10.2075	1.5886	0.5687	0.865	51.6512	0.2156
Cval	2.26069	22.6907	1.56165	0.656665	1.05075	9.75618	0.839259	55.5949	0.286977
N	12.2126	0.0057	3.1322	9.8933	2.0125	28.9975	1.1663	0.5826	-11.529
O	3.0485	13.2771	2.2868	5.7011	1.5463	0.3239	0.867	32.9089	0.2508
O1-	4.1916	12.8573	1.63969	4.17236	1.52673	47.0179	-20.307	-0.01404	21.9412
F	3.5392	10.2825	2.6412	4.2944	1.517	0.2615	1.0243	26.1476	0.2776
F1-	3.6322	5.27756	3.51057	14.7353	1.26064	0.442258	0.940706	47.3437	0.653396
Ne	3.9553	8.4042	3.1125	3.4262	1.4546	0.2306	1.1251	21.7184	0.3515
Na	4.7626	3.285	3.1736	8.8422	1.2674	0.3136	1.1128	129.424	0.676
Na1+	3.2565	2.6671	3.9362	6.1153	1.3998	0.2001	1.0032	14.039	0.404
Mg	5.4204	2.8275	2.1735	79.2611	1.2269	0.3808	2.3073	7.1937	0.8584
Mg2+	3.4988	2.1676	3.8378	4.7542	1.3284	0.185	0.8497	10.1411	0.4853
Al	6.4202	3.0387	1.9002	0.7426	1.5936	31.5472	1.9646	85.0886	1.1151
Al3+	4.17448	1.93816	3.3876	4.14553	1.20296	0.228753	0.528137	8.28524	0.706786
Siv	6.2915	2.4386	3.0353	32.3337	1.9891	0.6785	1.541	81.6937	1.1407
Sival	5.66269	2.6652	3.07164	38.6634	2.62446	0.916946	1.3932	93.5458	1.24707
Si4+	4.43918	1.64167	3.20345	3.43757	1.19453	0.2149	0.41653	6.65365	0.746297
P	6.4345	1.9067	4.1791	27.157	1.78	0.526	1.4908	68.1645	1.1149
S	6.9053	1.4679	5.2034	22.2151	1.4379	0.2536	1.5863	56.172	0.8669
Cl	11.4604	0.0104	7.1964	1.1662	6.2556	18.5194	1.6455	47.7784	-9.5574
Cl1-	18.2915	0.0066	7.2084	1.1717	6.5337	19.5424	2.3386	60.4486	-16.378
Ar	7.4845	0.9072	6.7723	14.8407	0.6539	43.8983	1.6442	33.3929	1.4445
K	8.2186	12.7949	7.4398	0.7748	1.0519	213.187	0.8659	41.6841	1.4228
K1+	7.9578	12.6331	7.4917	0.7674	6.359	-0.002	1.1915	31.9128	-4.9978
Ca	8.6266	10.4421	7.3873	0.6599	1.5899	85.7484	1.0211	178.437	1.3751

RHF
结果

有效电荷分布



$$F(q) = \theta(q_0 - q), \quad \rho(r) = \frac{\sin(q_0 r) - q_0 r \cos(q_0 r)}{2\pi^2 r^3}.$$

RHF形状因子表明通常的原子的有效电荷主要集中在核心附近，与测量的原子半径不矛盾

相干散射

单粒子散射

$$S|\boldsymbol{p}\rangle = |\boldsymbol{p}\rangle + \int d^3\boldsymbol{p}'\,\delta\left(E_{\boldsymbol{p}} - E_{\boldsymbol{p}'}\right)\mathcal{M}\left(\boldsymbol{p},\boldsymbol{p}'\right)|\boldsymbol{p}'\rangle\,.$$

多个粒子

$$|\boldsymbol{p}\rangle \to USU^{\dagger}|\boldsymbol{p}\rangle = |\boldsymbol{p}\rangle + \int d^3\boldsymbol{p}'\,\delta\left(E_{\boldsymbol{p}} - E_{\boldsymbol{p}'}\right)e^{-i\boldsymbol{x}_i\cdot\boldsymbol{q}}\mathcal{M}\left(\boldsymbol{p},\boldsymbol{p}'\right)|\boldsymbol{p}'\rangle\,,$$

干涉效应

$$|\boldsymbol{p}\rangle \to |\boldsymbol{p}\rangle + \sum_{i=1}^{N}\int d^3\boldsymbol{p}'\,\delta\left(E_{\boldsymbol{p}} - E_{\boldsymbol{p}'}\right)e^{-i\boldsymbol{x}_i\cdot\boldsymbol{q}}\mathcal{M}\left(\boldsymbol{p},\boldsymbol{p}'\right)|\boldsymbol{p}'\rangle\,,$$

$$P(\boldsymbol{p}') \propto \left|\langle\boldsymbol{p}'|\psi\rangle\right|^2 \propto \sum_{ij}\left\langle e^{-i\Delta\boldsymbol{x}_{ij}\cdot\boldsymbol{q}}\right\rangle\left|\mathcal{M}\left(\boldsymbol{p},\boldsymbol{p}'\right)\right|^2,$$

目标粒子的聚集程度决定了干涉效应的大小

- **RHF形状因子表明通常的原子的有效电荷主要集中在核心附近，与测量的原子半径不矛盾**
- **原子核外电子云作为一个“compact ball”产生干涉效应**
- **散射过程时间尺度大概大于XRD的1000倍，但是只要mediator足够重，仍然可以使用impulse approximation**

散射截面

$$\bar{\sigma}_e = \frac{16\pi\epsilon^2\alpha\alpha_D\mu_{\chi e}^2}{(m_{A'}^2 + |q_0|^2)^2}.$$

运动学

$$\boldsymbol{q} \equiv m_\chi \boldsymbol{v} - \boldsymbol{p}'_\chi \simeq Z m_e \boldsymbol{v}_C.$$

$$E_{e,f} - E_{e,i} \equiv \Delta E_e = \frac{1}{2}m_\chi v^2 - \frac{|m_\chi \boldsymbol{v} - \boldsymbol{q}|^2}{2m_\chi} - \frac{\boldsymbol{q}^2}{2Zm_e} = \boldsymbol{q}\cdot\boldsymbol{v} - \frac{\boldsymbol{q}^2}{2\mu_{\chi C}},$$

动力学

$$H_{\text{int, C}} = -\int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot(-\boldsymbol{x}_\chi + \boldsymbol{x}_e)} \frac{\mathcal{M}_{\chi C}(q)}{4m_\chi Z m_e},$$

$$|\mathcal{M}_{\chi C}|^2 = Z^2 |F_C(q)|^2 |\mathcal{M}_{\chi e}|^2,$$

$$R_C \propto |\langle \Psi_f | \Psi_{\boldsymbol{v}_C}\rangle|^2 \sim \left|\langle \psi_f \left| e^{i\boldsymbol{q}_e\cdot\boldsymbol{x}} \right| \psi_i \rangle\right|^2$$

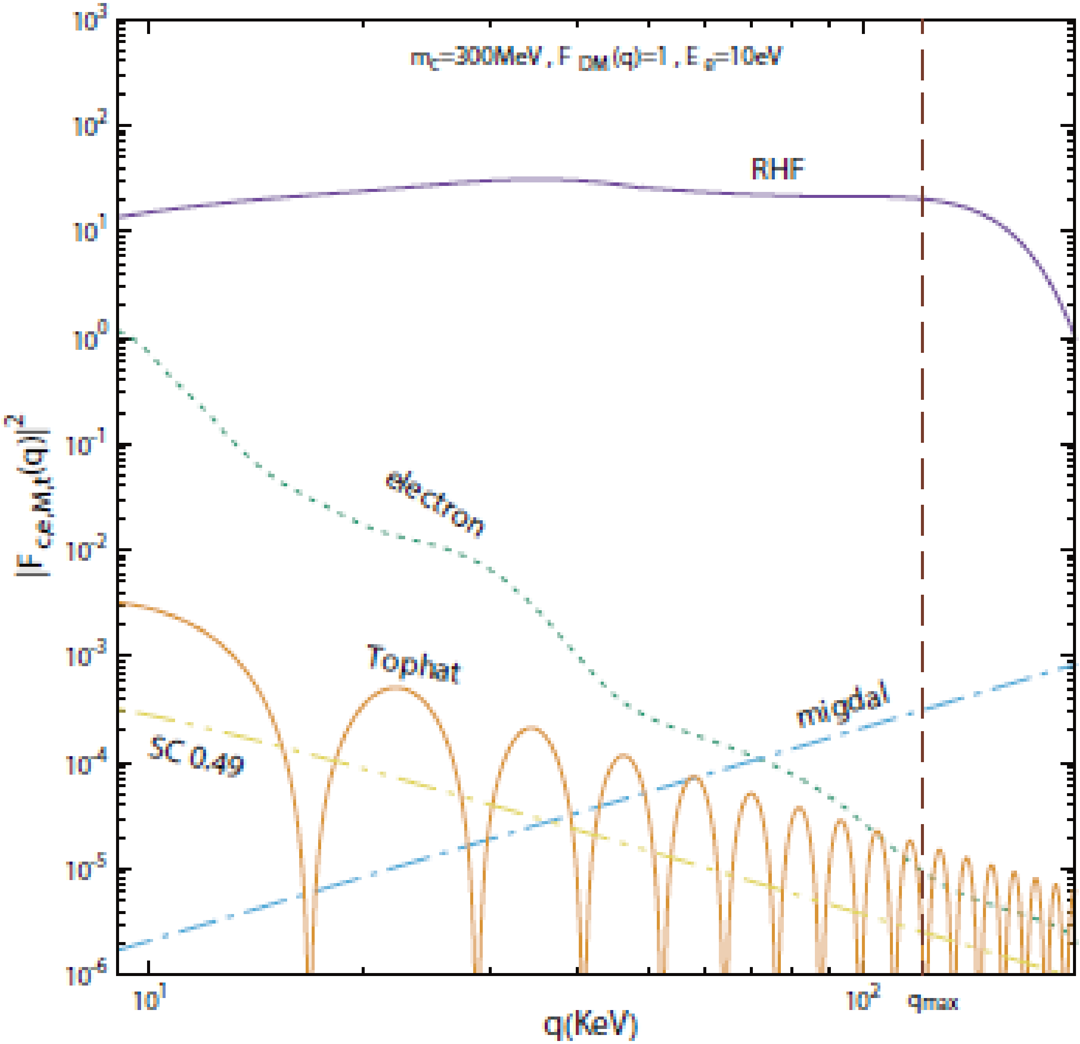
$$v_{min} = \frac{|E_{nl}| + E_e}{|\boldsymbol{q}|} + \frac{|\boldsymbol{q}|}{2\mu_{\chi C}}.$$

$$\boldsymbol{q}_e = m_e \boldsymbol{v}_C = \frac{\boldsymbol{q}}{Z}.$$

事例率

$$\frac{\mathrm{d}\langle \sigma_{\text{ion}}^{nl} v\rangle}{\mathrm{d}\ln E_e} = \frac{\bar{\sigma}_e}{8\mu_{\chi e}^2} \times \int q \mathrm{d}q \left| f_{\text{ion}}^{nl}(k_e, q)\right|^2 |F_{DM}(q)|^2 \eta(v_{min}),$$

$$\frac{\mathrm{d}R_{\text{ion}}}{\mathrm{d}\ln E_e} = N_T \frac{\rho_\chi}{m_\chi} \sum_{nl} \frac{\mathrm{d}\langle \sigma_{\text{ion}}^{nl} v\rangle}{\mathrm{d}\ln E_e},$$



$$|f_{\text{ion, C}}(E_e, q)|^2 = \frac{k_e^3}{4\pi^3} Z^2 |F_C(q)|^2 \times 2 \sum_{n,l,l',m'} \left| \left\langle \psi^f_{E_e} \left| e^{i\boldsymbol{q}_e \cdot \boldsymbol{x}} \right| \psi^i_{E_{nl}} \right\rangle \right|^2 .$$

- 电子电离因子随传递动量很快降为零
- **Migdal效应电离因子并不明显下降**
- 电子云电离因子比另外两种电离增大很多，正是因为干涉效应，但是积分区间有个上限。

$$q_{\rm max} = 2\mu_{\chi C} v_{\rm max} \simeq 127.43 {\rm KeV},$$

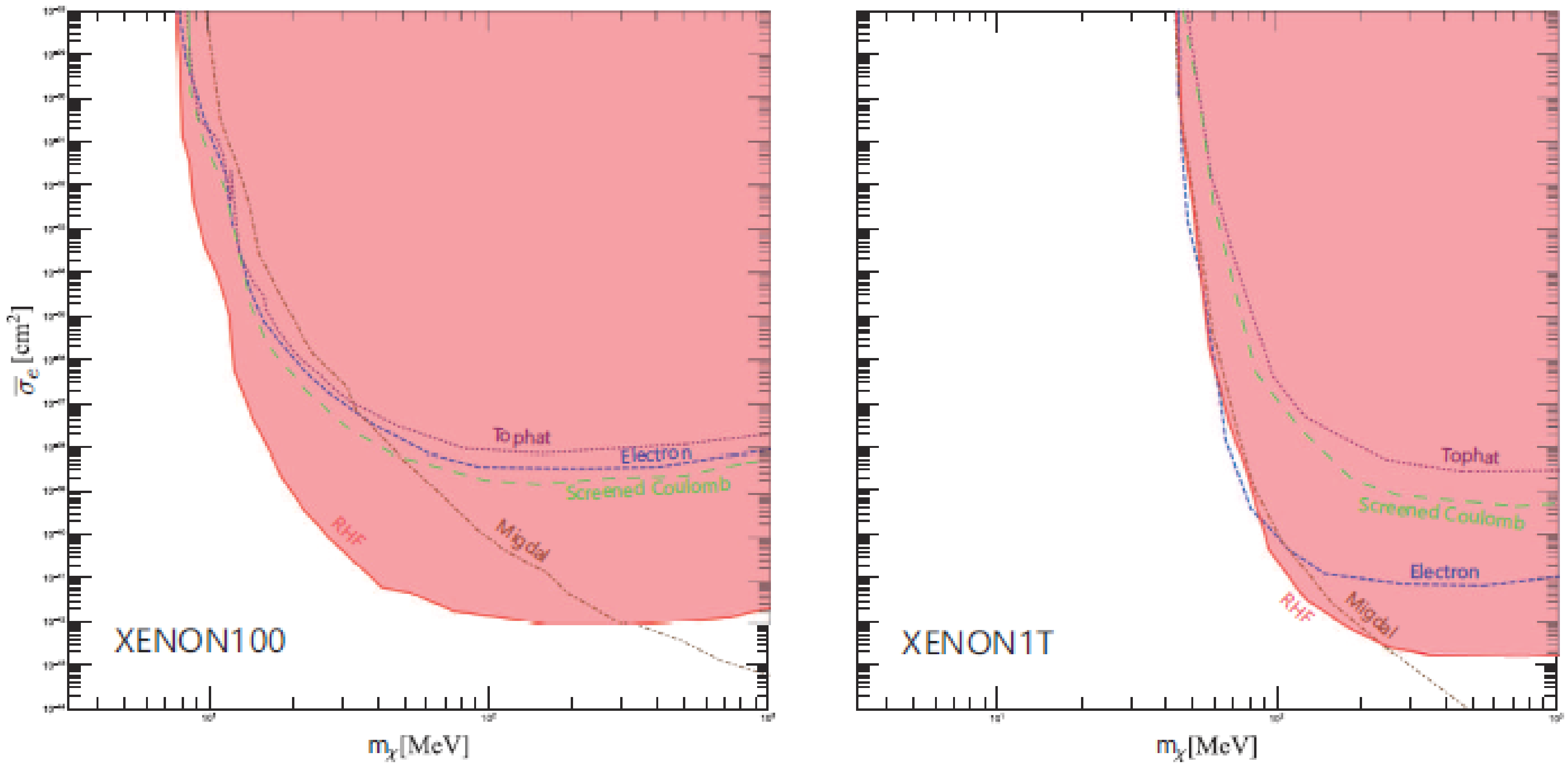


FIG. 7: The limits of the cross sections and dark matter mass from Xenon100 (left panel) and Xenon1T(right panel). The blue dash (dot dash) lines show limits on the recoil of single electron and the Migdal effect. The dot line shows the result of Tophat model of the electron cloud, while the long green dash line gives the exclusion area from the screened Coulomb potential model with the screening length $r_0 = 0.49$Å. The shaded regions show results of RHF form factor.

电子相干散射排除线远低于单电子散射区域，特别在轻暗物质区域

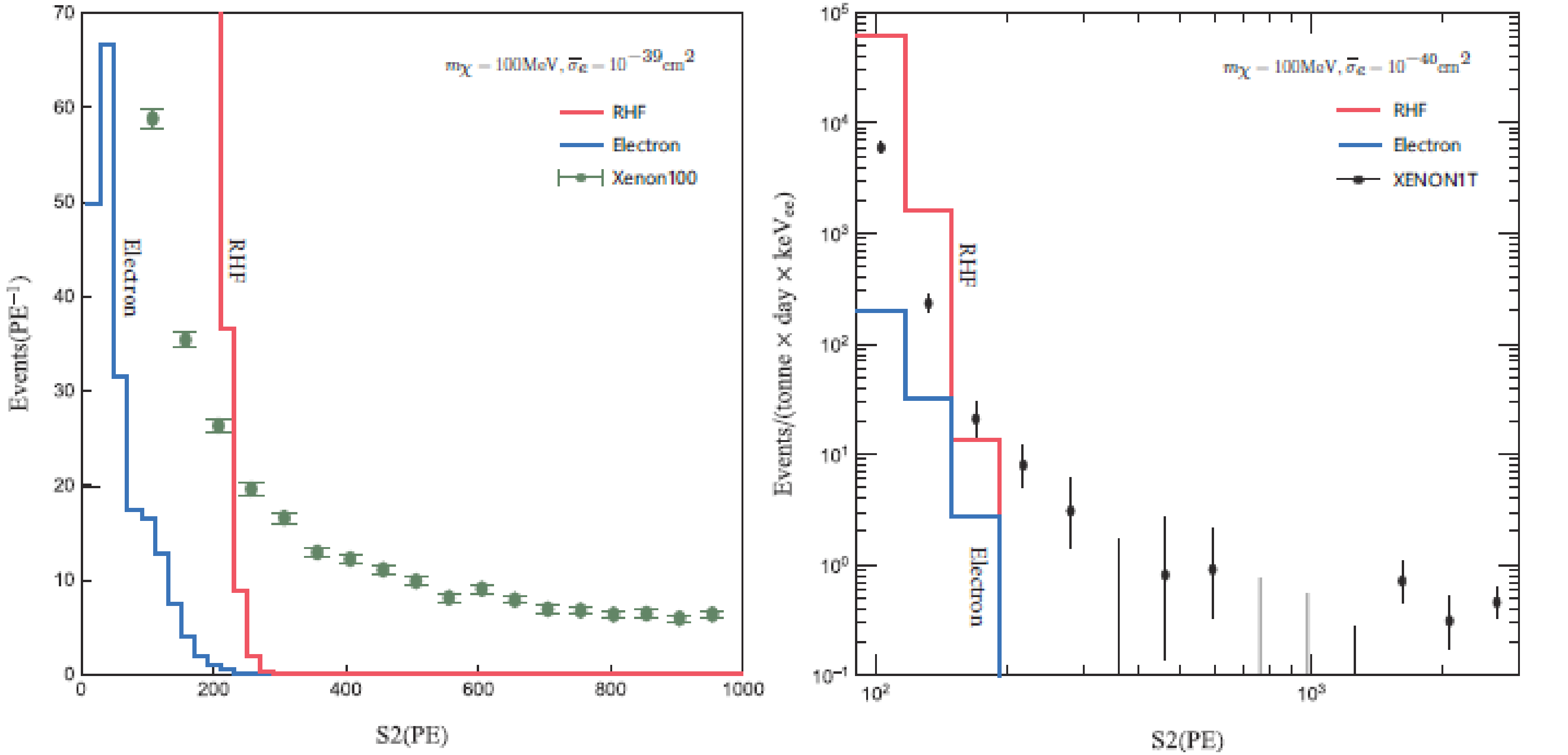


FIG. 8: Observed events versus photonelectrons (PE) from Xenon100 (left panel) and Xenon1T (right panel) data for m_χ = 100MeV and for Xenon100 $\bar{\sigma}_e = 1 \times 10^{-39}$cm^2 while for Xenon1T, $\bar{\sigma}_e = 1 \times 10^{-40}$cm^2.

总结

- WIMP很困难
- 电子反冲对轻暗物质很敏感
- 有很多电子反冲效应有待深入研究
- 通常测量的电子电离因子应该主要产生与电子云反冲然后被原子核拉回来过程的电离，而非电子直接电离过程

谢谢！