axion, magnetic monopole and detection of new axion couplings

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Outline

- Axion and magnetic monopole
- Quantum electromagnetodynamics (QEMD) and QEMD-axion modified Maxwell equations
- Solutions and numerical results
- Experimental setup and sensitivity to sub-μeV axion couplings
- Summary

1. Axion and magnetic monopole

- **The existence of magnetic monopole** $\oint \vec{B} \cdot d\vec{S} = 0$?

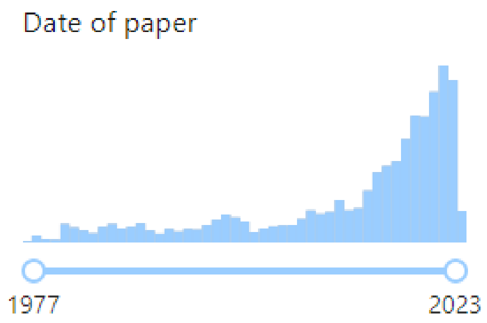
- In 1931, Dirac first proposed the notion of ‘monopole’ in quantum theory

  Charge quantization condition: $e \cdot q = 2\pi n \quad n \in \mathbb{Z}$

- Various proposed monopoles:

  Georgi-Glashow ('t Hooft, Polyakov 1974), QCD (Wu, Yang 1975), GUT (Dokos, Tomaras 1980), electroweak (Cho, Maison 1997)

- **Axion** (1970s)



- strong CP problem $\theta \frac{g_s^2}{32\pi^2} G^{a\,\mu\nu} \tilde{G}^a_{\mu\nu} \xrightarrow{\text{nEDM}} \bar{\theta} \lesssim 10^{-10}$

- Peccei-Quinn mechanism： $U(1)_{PQ}$ $\quad \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G\tilde{G} \longrightarrow \bar{\theta} \to \bar{\theta} + \frac{a}{f_a}$

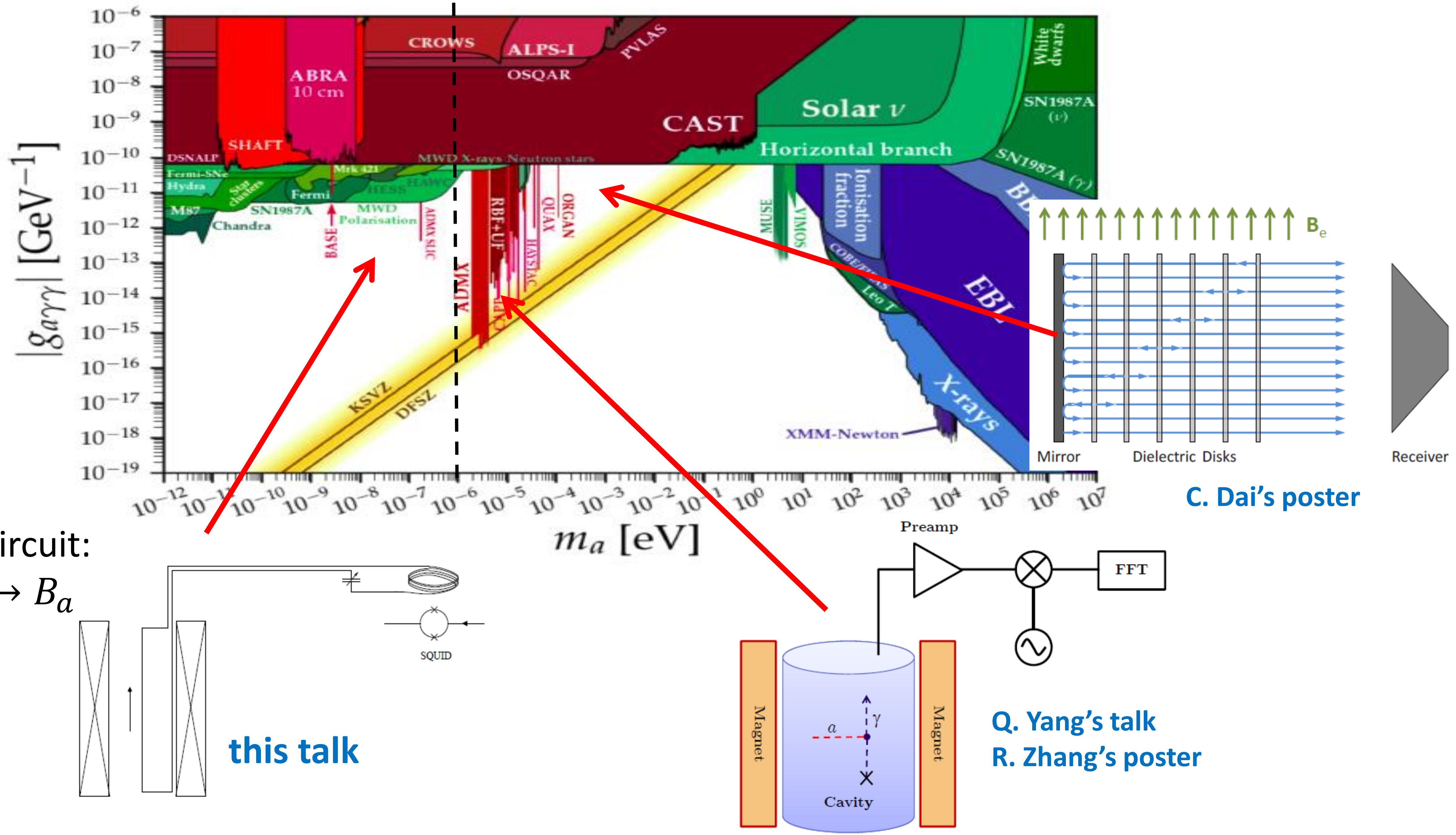
- candidate of dark matter $m_a \simeq 6 \cdot 10^{-6}\ \mathrm{eV}\ \frac{10^{12}\ \mathrm{GeV}}{f_a}$

- Axion-photon coupling

$$\boxed{\mathcal{L}_{a\gamma} = -\frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{a\gamma} a \vec{E}\cdot\vec{B}}$$

$$\Big\downarrow$$

$$\begin{aligned}
\boldsymbol{\nabla}\cdot\mathbf{E} &= \rho \underline{- g_{a\gamma\gamma}\mathbf{B}\cdot\boldsymbol{\nabla}a}\,,\\
\boldsymbol{\nabla}\cdot\mathbf{B} &= 0\,,\\
\boldsymbol{\nabla}\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}\,,\\
\boldsymbol{\nabla}\times\mathbf{B} &= \frac{\partial\mathbf{E}}{\partial t} + \mathbf{J} \underline{- g_{a\gamma\gamma}\left(\mathbf{E}\times\boldsymbol{\nabla}a - \frac{\partial a}{\partial t}\mathbf{B}\right)}
\end{aligned}$$



C. Dai's poster

LC circuit:

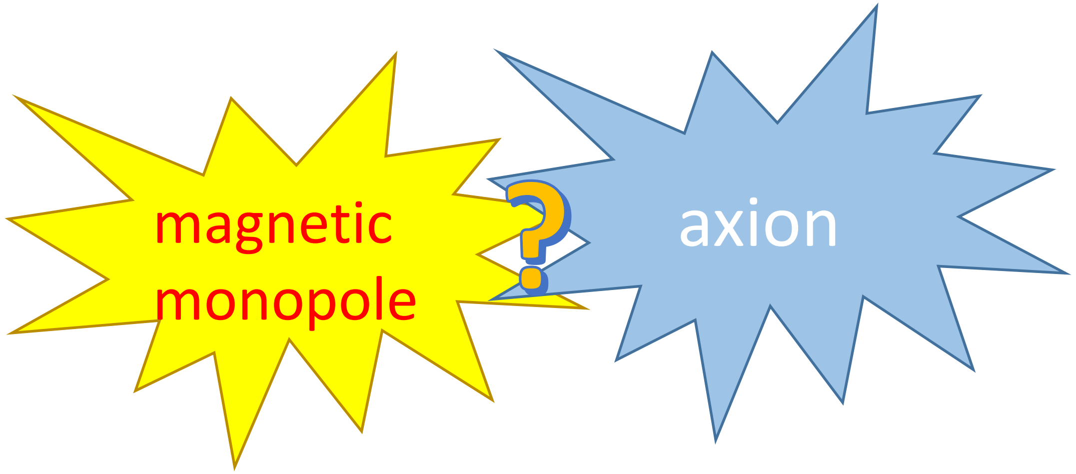
$B_0 \rightarrow B_a$

this talk

Q. Yang's talk

R. Zhang's poster

Axion-monopole interaction



magnetic monopole

axion

- A non-zero vacuum angle θ in the CP violating term $\theta F\tilde{F}$ introduces an electric charge proportional to θ for magnetic monopoles:

  monopole → dyon $$q = ne - \frac{\theta e}{2\pi}$$ (Witten, 1979)

  In axion theories, Witten effect implies a close relationship (interaction) between axions and magnetic monopoles

- The connection between axion and monopole was first derived under the classical theory of electromagnetism: $$\nabla \cdot \boldsymbol{E} = -(e^2/4\pi^2)\nabla \cdot (\theta \boldsymbol{B})$$ (Fischler et al, 1983)

- To properly quantize the axion-dyon dynamics, one needs to utilize the quantum electromagnetodynamics (QEMD) (Schwinger 1966, Zwanziger 1968)
- Based on QEMD, a generic axion-photon Lagrangian has been constructed in the framework of low-energy effective field theory (Sokolov, Ringwald, arXiv: 2205.02605)

- There exist more anomalous axion-photon couplings g_{aAA}, g_{aBB} and g_{aAB} in contrary to the ordinary axion EFT $g_{a\gamma\gamma}$ ⟶ new modified EoM ⟶ new axion-modified Maxwell equations

2. QEMD and QEMD-axion modified Maxwell equations

● QEMD and generic axion-photon Lagrangian

- QEMD takes such substitution for gauge group:

$U(1) \rightarrow U(1)_E \times U(1)_M$ to describe the coupling of electric charge, magnetic charge and photon.

A_μ B_μ ⟶ non-trivial commutation and opposite parity $(\partial \wedge X)^{\mu\nu} \equiv \partial^\mu X^\nu - \partial^\nu X^\mu$

$$\mathcal{L} = \frac{1}{2n^2}\left\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge\tilde{A})] - [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge\tilde{B})] - [n\cdot(\partial\wedge A)]^2 - [n\cdot(\partial\wedge B)]^2\right\} - j_e\cdot A - j_m\cdot B + \mathcal{L}_G$$

— Zwanziger's QEMD local-Lagrangian

The electromagnetic field strength tensors are introduced in this way:

$n\cdot F = n\cdot(\partial\wedge A)\,,\;\; n\cdot\tilde{F} = n\cdot(\partial\wedge B)$, where $n = (0, \vec{n}\,)$ is an arbitrary spatial vector

- The generic low-energy axion-photon EFT and anomalous coupling can be written by (arXiv:2205.02605)

$$\mathcal{L} \supset -\frac{1}{4}g_{aAA}\, a\, \mathrm{tr}[(\partial\wedge A)(\partial\wedge\tilde{A})] \boxed{-\frac{1}{4}g_{aBB}\, a\, \mathrm{tr}[(\partial\wedge B)(\partial\wedge\tilde{B})] - \frac{1}{2}g_{aAB}\, a\, \mathrm{tr}[(\partial\wedge A)(\partial\wedge\tilde{B})]}$$

$$\underbrace{U(1)_{\mathrm{PQ}}U(1)_{\mathrm{E}}^2 \qquad\qquad U(1)_{\mathrm{PQ}}U(1)_{\mathrm{M}}^2}_{\text{CP conserving}} \qquad\qquad U(1)_{\mathrm{PQ}}U(1)_{\mathrm{E}}U(1)_{\mathrm{M}}$$

CP violating

Witten effect induced

$$\mathcal{L} \supset -\left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\, j_m\right)\cdot A$$

QEMD-axion modified Maxwell equations

- The conventional EoM and axion-Maxwell equations are modified through the above anomalous axion-photon interactions. Given the interactions as well as free Lagrangian, one can derive QEMD axion-Maxwell equations by classical equations of motion:

$$\begin{cases} \vec{\nabla}\times\vec{B}_a-\dfrac{\partial\vec{E}_a}{\partial t}=\boxed{g_{aAA}(\vec{E}_0\times\vec{\nabla}a-\dfrac{\partial a}{\partial t}\vec{B}_0)}+g_{aAB}(\vec{B}_0\times\vec{\nabla}a+\dfrac{\partial a}{\partial t}\vec{E}_0)\ , \\ \vec{\nabla}\times\vec{E}_a+\dfrac{\partial\vec{B}_a}{\partial t}=-g_{aBB}(\vec{B}_0\times\vec{\nabla}a+\dfrac{\partial a}{\partial t}\vec{E}_0)-g_{aAB}(\vec{E}_0\times\vec{\nabla}a-\dfrac{\partial a}{\partial t}\vec{B}_0)\ , \\ \vec{\nabla}\cdot\vec{B}_a=-g_{aBB}\vec{E}_0\cdot\vec{\nabla}a+g_{aAB}\vec{B}_0\cdot\vec{\nabla}a\ , \\ \vec{\nabla}\cdot\vec{E}_a=\boxed{g_{aAA}\vec{B}_0\cdot\vec{\nabla}a}-g_{aAB}\vec{E}_0\cdot\vec{\nabla}a\ , \end{cases}$$

- One can calculate the coupling coefficients which are related to their corresponding anomaly coefficients E, M, D, respectively.

$$\boxed{g_{aAA}=\frac{Ee^2}{4\pi^2 v_{\mathrm{PQ}}}\ ,\quad g_{aBB}=\frac{Mg_0^2}{4\pi^2 v_{\mathrm{PQ}}}\ ,\quad g_{aAB}=\frac{Deg_0}{4\pi^2 v_{\mathrm{PQ}}}},$$

$$E=\sum_\psi q_\psi^2\cdot d(C_\psi),\ M=\sum_\psi g_\psi^2\cdot d(C_\psi),\ D=\sum_\psi q_\psi g_\psi\cdot d(C_\psi),$$

1. The DSZ quantization condition indicates:

$g_0 \gg e$

$g_{aAA} \ll |g_{aAB}| \ll g_{aBB}$

2. Assuming

$E \simeq |D| \simeq M$

$g_{aAA}/g_{aBB} \simeq (e/g_0)^2 \simeq 10^{-4}$

$|g_{aAB}|/g_{aBB} \simeq e/g_0 \simeq 10^{-2}$

3. The suppression from axion DM velocity

$|\vec{\nabla}a| \sim 10^{-3}\partial a/\partial t$

● Further simplified QEMD-axion modified Maxwell equations

$$\left\{\begin{aligned} &\vec{\nabla}\times\vec{B}_a-\frac{\partial\vec{E}_a}{\partial t}=0\ ,\\ &\vec{\nabla}\times\vec{E}_a+\frac{\partial\vec{B}_a}{\partial t}=-g_{aBB}(\vec{B}_0\times\vec{\nabla}a+\frac{\partial a}{\partial t}\vec{E}_0)+g_{aAB}\frac{\partial a}{\partial t}\vec{B}_0\ ,\\ &\vec{\nabla}\cdot\vec{B}_a=0\ ,\\ &\vec{\nabla}\cdot\vec{E}_a=0\ . \end{aligned}\right.$$

$$\boxed{\vec{j}^{m}_{\rm eff}=g_{aBB}\vec{B}_0\times\vec{\nabla}a-g_{aAB}\frac{\partial a}{\partial t}\vec{B}_0}$$

◆

Some conventions:

$$a(t,\vec{r})\ =\ a_0\cos(\omega_a t-\vec{k}_a\cdot\vec{r})$$

with $\omega_a\approx m_a$ and $\vec{k}_a\approx m_a\vec{v}_a$

- **Case I:** For ordinary haloscope experiments, they adopt a non-zero magnetic field $B_0\neq 0$ but vanishing electric field $E_0=0$

$$\begin{aligned} \nabla^2\vec{B}_a-\frac{\partial^2\vec{B}_a}{\partial t^2}&=g_{aBB}\vec{B}_0\times\vec{\nabla}\frac{\partial a}{\partial t}-g_{aAB}\frac{\partial^2 a}{\partial t^2}\vec{B}_0\ ,\\ \nabla^2\vec{E}_a-\frac{\partial^2\vec{E}_a}{\partial t^2}&=g_{aBB}(\vec{\nabla}a\cdot\vec{\nabla})\vec{B}_0-g_{aAB}\frac{\partial a}{\partial t}\vec{\nabla}\times\vec{B}_0\ . \end{aligned}$$

- **Case II:** Electric field $E_0\neq 0$ but vanishing magnetic field $B_0=0$

$$\begin{aligned} \nabla^2\vec{B}_a-\frac{\partial^2\vec{B}_a}{\partial t^2}&=g_{aBB}\frac{\partial^2 a}{\partial t^2}\vec{E}_0\ ,\\ \nabla^2\vec{E}_a-\frac{\partial^2\vec{E}_a}{\partial t^2}&=g_{aBB}\frac{\partial a}{\partial t}\vec{\nabla}\times\vec{E}_0\ . \end{aligned}$$

3. Solutions and numerical results

(arXiv: 1809.10709 Ouellet, Bogorad)

- Solutions in the static external magnetic/electric field

  - **Case I:** $B_0 \neq 0$, $E_0 = 0$ $\boxed{\vec{B}_0 = \theta(R-\rho)B_0\hat{z}}$

Assuming a static magnetic field B_0 along the $\hat{z}$ direction generated by a long solenoid with radius R

$$\nabla^2\vec{E}_a - \frac{\partial^2\vec{E}_a}{\partial t^2} = -\Big(g_{aBB}\frac{\partial a}{\partial\rho}\hat{z} + g_{aAB}\frac{\partial a}{\partial t}\hat{\phi}\Big)B_0\delta(\rho-R)\ .$$

$$\boxed{\vec{E}_{a,\phi}} \approx \begin{cases} i\Big[\frac{1}{2}g_{aAB}a_0B_0\omega_a\rho - \frac{1}{4}g_{aAB}a_0B_0\omega_a^3R^2\rho\Big(\gamma'(\omega_aR)-\frac{1}{2}\Big)\Big]e^{i\omega_at}\hat{\phi}, & \rho<R\ , \\ i\Big[\frac{1}{2}g_{aAB}a_0B_0\omega_a\frac{R^2}{\rho} - \frac{1}{4}g_{aAB}a_0B_0\omega_a^3R^2\rho\Big(\gamma'(\omega_aR)-\frac{1}{2}\Big)\Big]e^{i\omega_at}\hat{\phi}, & \rho>R\ , \end{cases}$$

$$\approx \begin{cases} i\frac{1}{2}g_{aAB}a_0B_0\omega_a\rho e^{i\omega_at}\hat{\phi}, & \rho<R\ , \\ i\frac{1}{2}g_{aAB}a_0B_0\omega_a\frac{R^2}{\rho}e^{i\omega_at}\hat{\phi}, & \rho>R\ , \end{cases}$$

Considering the limit of large Compton wavelengths $\lambda_a = \frac{2\pi}{\omega_a} \gg R$ and thus $\rho' = \omega_a R \ll 1$

- **Case II:** $E_0 \neq 0$, $B_0 = 0$ $\boxed{\vec{E}_0 = \theta(R-\rho)E_0\hat{z}}$

Similarly, a static electric field E_0 along $\hat{z}$ direction can be obtained by a pair of parallel plates with radius R

$$\nabla^2\vec{E}_a - \frac{\partial^2\vec{E}_a}{\partial t^2} = g_{aBB}E_0\frac{\partial a}{\partial t}\delta(\rho-R)\hat{\phi}\ .$$

The field solutions of **case II** are analogous to the results of $E_{a,\varphi}$ and $B_{a,z}$ in **case I**, only differing by the substitution of $g_{aAB}B_0 \rightarrow -g_{aBB}E_0$

- **Case I:** $B_0 \neq 0$, $E_0 = 0$ $\quad \vec{B}_0 = \theta(R-\rho)B_0\hat{z}$

$$\nabla^2\vec{B}_a - \frac{\partial^2\vec{B}_a}{\partial t^2} = g_{aBB}\vec{B}_0 \times \vec{\nabla}\frac{\partial a}{\partial t} - g_{aAB}\frac{\partial^2 a}{\partial t^2}\vec{B}_0$$

- **Case II:** $E_0 \neq 0$, $B_0 = 0$ $\quad \vec{E}_0 = \theta(R-\rho)E_0\hat{z}$

$$\nabla^2\vec{B}_a - \frac{\partial^2\vec{B}_a}{\partial t^2} = g_{aBB}\frac{\partial^2 a}{\partial t^2}\vec{E}_0$$

$g_{aAB}B_0 \rightarrow -g_{aBB}E_0$

$$\boxed{\vec{B}_{a,z}} \approx \begin{cases} g_{aAB}a_0B_0\left[\frac{(\omega_a R)^2}{2}\left(\gamma'(\omega_a R) - \frac{1}{2}\right)\left(1 - \frac{\omega_a^2\rho^2}{4}\right) + \frac{\omega_a^2\rho^2}{4}\right]e^{i\omega_a t}\hat{z}, & \rho < R\,, \\ g_{aAB}a_0B_0\frac{(\omega_a R)^2}{2}\left[\gamma'(\omega_a\rho) + \frac{1}{4}(1-\gamma'(\omega_a\rho))(\omega_a\rho)^2\right]e^{i\omega_a t}\hat{z}, & \rho > R\,, \end{cases}$$

$$\approx \begin{cases} g_{aAB}a_0B_0\left[\frac{(\omega_a R)^2}{2}\left(\gamma'(\omega_a R) - \frac{1}{2}\right) + \frac{\omega_a^2\rho^2}{4}\right]e^{i\omega_a t}\hat{z}, & \rho < R\,, \\ g_{aAB}a_0B_0\frac{(\omega_a R)^2}{2}\gamma'(\omega_a\rho)e^{i\omega_a t}\hat{z}, & \rho > R\,. \end{cases}$$

B_0 → to measure g_{aAB}

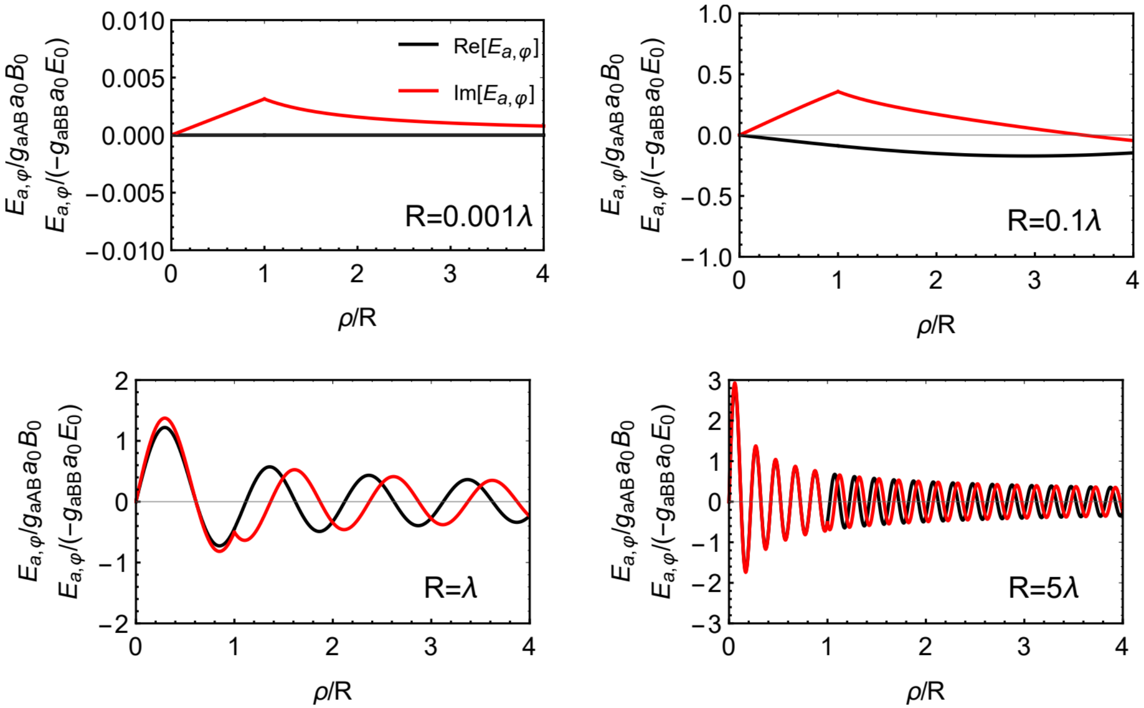
E_0 → to measure g_{aBB}

◆ The solutions of other components $E_{a,z}$, $B_{a,\varphi}$, $B_{a,\rho} \propto g_{aBB}$ are all suppressed by axion DM velocity $v{\sim}10^{-3}$

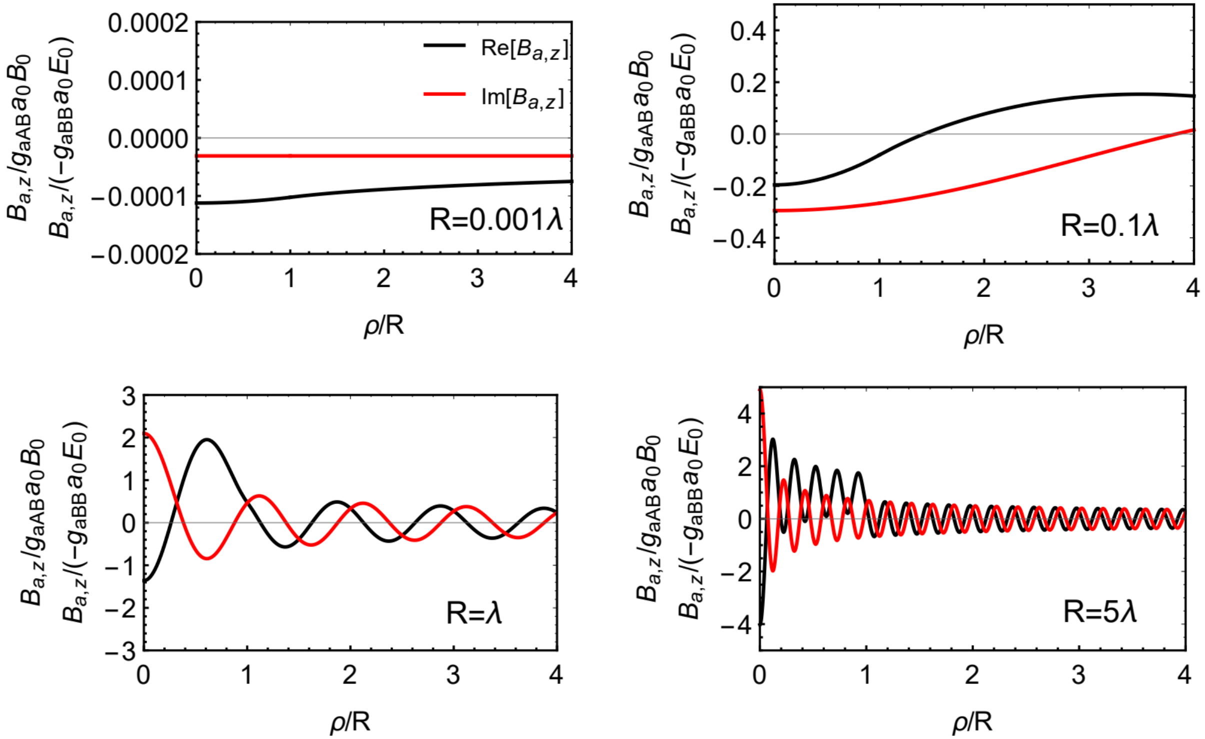
◆ The dominant solutions $E_{a,\varphi}$ and $B_{a,z}$ are not velocity-suppressed and equivalent to the solutions of arXiv: 1809.10709 only by replacing $E_{a,\varphi} \rightarrow B_{a,\varphi}$, $B_{a,z} \rightarrow -E_{a,z}$ in our results

Numerical results of axion-induced electromagnetic fields

$E_{a,\varphi}$



$B_{a,z}$



$$U_{E\phi}(\rho') = \begin{cases} a_{E\phi} J_1(\rho'), & \rho' < \omega_a R \;, \\ b_{E\phi} H_1^+(\rho'), & \rho' > \omega_a R \;, \end{cases}$$

$$a_{E\phi} = -\frac{\pi}{2} g_{aAB} a_0 B_0 \omega_a R H_1^+(\omega_a R) \;,$$

$$b_{E\phi} = -\frac{\pi}{2} g_{aAB} a_0 B_0 \omega_a R J_1(\omega_a R) \;.$$

$$U_{Bz}(\rho') = \begin{cases} a_{Bz} J_0(\rho') + g_{aAB} a_0 B_0, & \rho' < \omega_a R \;, \\ b_{Bz} H_0^+(\rho'), & \rho' > \omega_a R \;, \end{cases}$$

$$a_{Bz} = -\frac{i\pi}{2} g_{aAB} a_0 B_0 \omega_a R H_1^+(\omega_a R) \;,$$

$$b_{Bz} = -\frac{i\pi}{2} g_{aAB} a_0 B_0 \omega_a R J_1(\omega_a R) \;.$$

4. Experimental setup and sensitivity

Experimental setup

- The induction current in wire loop is: $I_a = \dfrac{2\pi R E_{a,\phi}(R)}{R_s}$
- The total noise can be divided into two parts: thermal noise T_c from circuit and the input white noise T_{Amp} from amplifiers.

$$\left\{\begin{aligned} P_{\rm signal} &= \langle I_a^2 R_s \rangle = \frac{Q_c \pi^4 g_{aAB}^2 \rho_{\rm DM} B_0^2 R^4 \left| H_1^+(\omega_a R) J_1(\omega_a R) \right|^2}{L\omega_a} \\ P_{\rm noise} &= \kappa_B T_N \sqrt{\frac{\Delta f}{\Delta t}} \end{aligned}\right.$$

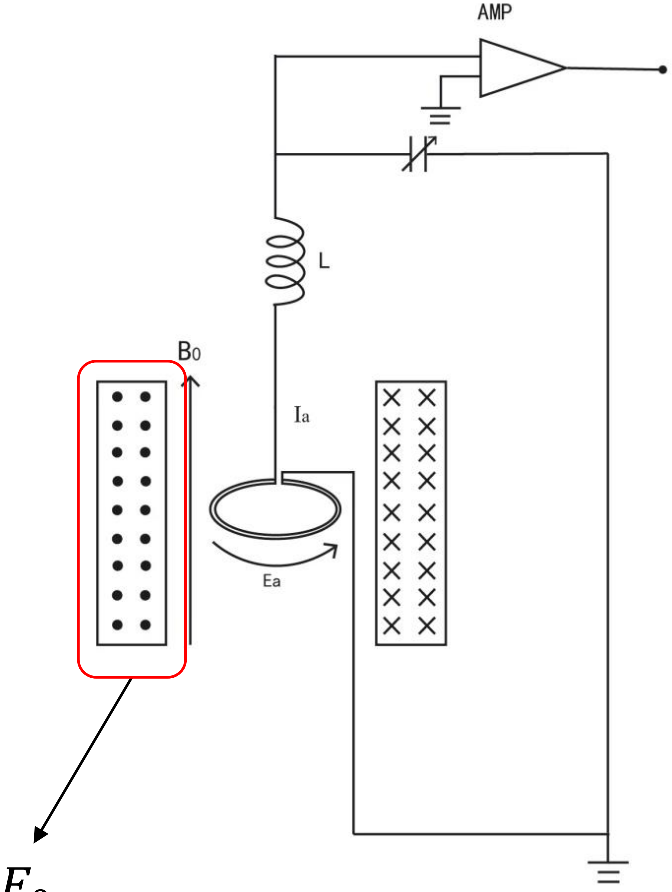
(arXiv: 2206.13543 Duan, Gao et al)

$$P_{\rm noise}/\Delta f \equiv k_B T_N \approx 4 k_B T_c + 4 k_B T_{\rm Amp}$$

$$\downarrow$$

$$\boxed{\mathrm{SNR} = \frac{P_{\rm signal}}{P_{\rm noise}} > 3}$$

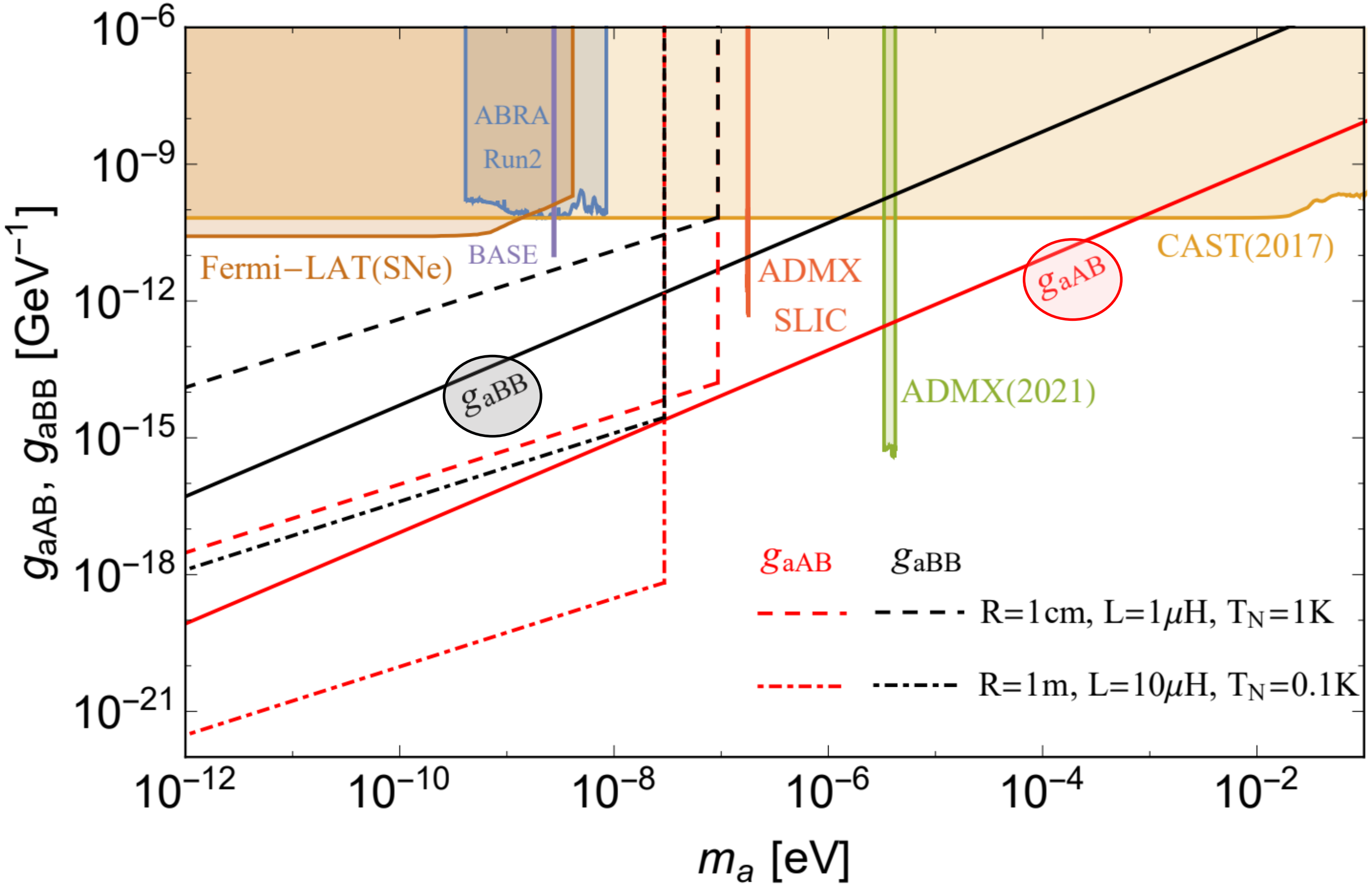
Sensitivity criterion



$g_{aAB} B_0 \rightarrow g_{aBB} E_0$

To measure g_{aBB}, the solenoid can be replaced by horizontally placed parallel plates.

- Expected sensitivity bounds



★ New strategies of sub-μeV axion detection

- The conventional LC axion detection :

$$B_0 \to B_a \text{ to search } g_{a\gamma}\ (g_{aAA})$$

- New strategies of QEMD axion detection:

$$B_0 \to E_{a,\varphi} \text{ to search } g_{aAB}$$

$$E_0 \to E_{a,\varphi} \text{ to search } g_{aBB}$$

5. Summary

- Quantum electromagnetodynamics (QEMD) gives generic anomalous axion-photon interactions in a low-energy EFT.

- Such Lagrangian provides new axion Maxwell equations. Their solutions imply the induced oscillating electric fields are always dominant compared with the axion magnetic field.

- New strategies are proposed to measure the axion-induced electric fields and new couplings for sub-μeV axion.

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- New strategies are proposed to measure the axion-induced electric fields and new couplings for sub-μeV axion.

Thank You

backups

- The equal-time commutators between the potentials:

$$[A^\mu(t,\vec{x}), B^\nu(t,\vec{y})] = i\epsilon^{\mu\nu}{}_{\rho 0}\, n^\rho \,(n\cdot\partial)^{-1}(\vec{x}-\vec{y}) \ ,$$

$$[A^\mu(t,\vec{x}), A^\nu(t,\vec{y})] = [B^\mu(t,\vec{x})\,, B^\nu(t,\vec{y})] = -i\,({g_0}^\mu n^\nu + {g_0}^\nu n^\mu)\,(n\cdot\partial)^{-1}(\vec{x}-\vec{y})$$

- QEMD axion Maxwell equations:

$$\partial_\mu F^{\mu\nu} - g_{a\mathrm{AA}}\,\partial_\mu a\, F^{d\,\mu\nu} + g_{a\mathrm{AB}}\,\partial_\mu a\, F^{\mu\nu} - \frac{e^2 a}{4\pi^2 v_a}\, j_m^{\phi\,\nu} \ = \ \bar{j}_e^{\,\nu} \ ,$$

$$\partial_\mu F^{d\,\mu\nu} + g_{a\mathrm{BB}}\,\partial_\mu a\, F^{\mu\nu} - g_{a\mathrm{AB}}\,\partial_\mu a\, F^{d\,\mu\nu} \ = \ j_m^{\nu} \,.$$