QUANTUM CALCULATION OF AXION-PHOTON TRANSITION IN ELECTROMAGNETODYNAMICS FOR CAVITY HALOSCOPE

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Introduction

The Witten effect [1] implies the presence of electric charge of magnetic monopole and possible relationship between axion and dyon, which can be reliably built based on the quantum electromagnetodynamics (QEMD) [2-3]. In QEMD theory, the photon is described by two four-potentials A_μ and B_μ, and the gauge group of QEMD also becomes $U(1)_E \times U(1)_M$.

Recently, based on the QEMD framework, Ref. [4] constructed a more generic axion-photon Lagrangian in the low-energy axion effective field theory (EFT), which gives anomalous axion-photon interactions and couplings assuming the existence of heavy PQ-charged fermions with electric and magnetic charges:

CP-conserving

$$\mathcal{L} \supset -\frac{1}{4} g_{aAA}\, a\, \mathrm{tr}[(\partial\wedge A)(\partial\wedge A)^d] - \frac{1}{4} g_{aBB}\, a\, \mathrm{tr}[(\partial\wedge B)(\partial\wedge B)^d]$$
$$-\frac{1}{2} g_{aAB}\, a\, \mathrm{tr}[(\partial\wedge A)(\partial\wedge B)^d]\ .$$

CP-violating

$$g_{aBB} \gg |g_{aAB}| \gg g_{aAA}$$

Model: axion-QEMD

In terms of classical electromagnetic fields, the above axion-photon Lagrangian becomes:

$$\mathcal{L} \supset -\frac{1}{4}(g_{aAA} - g_{aBB})\, a\, F_{\mu\nu}F^{d\,\mu\nu} + \frac{1}{2} g_{aAB}\, a\, F_{\mu\nu}F^{\mu\nu}$$
$$= (g_{aAA} - g_{aBB})\, a\, \vec{H}\cdot\vec{E} + g_{aAB}\, a\, (\vec{H}^2 - \vec{E}^2)\ .$$

Taking care of the above anomalies, one can calculate the coupling coefficients as

$$g_{aAA} = \frac{Ee^2}{4\pi^2 v_{\rm PQ}}\ ,\quad g_{aBB} = \frac{Mg_0^2}{4\pi^2 v_{\rm PQ}}\ ,\quad g_{aAB} = \frac{Deg_0}{4\pi^2 v_{\rm PQ}}$$

■ Generalized symmetry realization

In fact, the above generic axion couplings can naturally arise when an axion-Maxwell theory couples to a $\mathbb{Z}_n$ topological QFT (TQFT) [5] and one can introduce two two-form background gauge fields of higher-form symmetries in a more general way, i.e. A_μ and B_μ as those in Zwanziger theory.

Methods

QUANTUM CALCULATION OF AXION-PHOTON TRANSITION IN QEMD

A. Magnetic background

Suppose an external magnetic field H_0 along the z-direction, the axion-photon interaction can be written as

$$\mathcal{L}_{a\gamma\gamma} = (g_{aAA} - g_{aBB})a\vec{E}\cdot\vec{H}_0 + g_{aAB}a\vec{H}\cdot\vec{H}_0$$

The Hamiltonian for the above interaction is

$$H_I = -\int d^3x \mathcal{L}_{a\gamma\gamma} = \frac{\sqrt{2\rho_a}}{m_a} H_0 \cos(\omega_a t)$$
$$\times\left[(g_{aBB} - g_{aAA})\int d^3x \hat{z}\cdot\vec{E} - g_{aAB}\int d^3x \hat{z}\cdot\vec{H}\right]$$

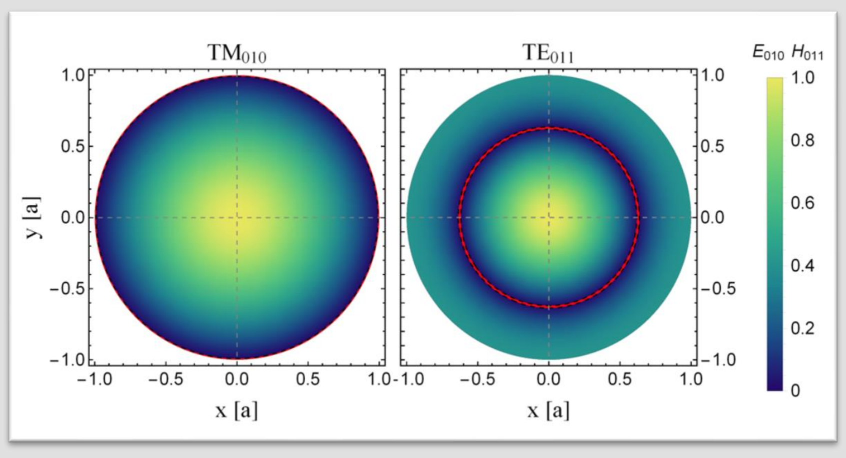
One can then calculate the $|0\rangle \to |1\rangle$ photon transition matrix element and the transition probability $P \approx \left|\left\langle 1\right| \int_0^t dt H_I \left|0\right\rangle\right|^2$ inside the cavity with an external magnetic field $\vec{H}_0$ [6]. Up to the first order, we define three form factors that characterize the coupling stre-ngth of cavity mode k to axions:

$$C_k^E = \frac{|\int d^3x \hat{z}\cdot\mathbf{u}_k^E|^2}{V\int d^3x |\mathbf{u}_k^E|^2},\ C_k^H = \frac{|\int d^3x \hat{z}\cdot\mathbf{u}_k^H|^2}{V\int d^3x |\mathbf{u}_k^H|^2},\ C_k^{EH} = \frac{\mathbf{Re}\left[\int d^3x\hat{z}\cdot\mathbf{u}_k^E \int d^3x \hat{z}\cdot\mathbf{u}_k^{H*}\right]}{V\sqrt{\int d^3x|\mathbf{u}_k^E|^2 \int d^3x |\mathbf{u}_k^H|^2}}$$

Assuming the transition emission process of a single photon is expected to take a long time t, the transition rate in the cavity can be obtained as

$$R_{\rm TE} = \frac{\pi}{2}\frac{\rho_a}{m_a^2} H_0^2 VQ g_{aAB}^2 C_{\omega_a}^H,\ \text{with } C^E = C^{EH} = 0\ ,$$
$$R_{\rm TM} = \frac{\pi}{2}\frac{\rho_a}{m_a^2} H_0^2 VQ (g_{aBB} - g_{aAA})^2 C_{\omega_a}^E,\ \text{with } C^H = C^{EH} = 0$$

Based on different types of cavity mode, the transition rate R can then be simplified. For the TM modes, only when m = p = 0, the integrals are non-zero, thus we only consider TM_{010} mode. For the TE modes, one can verify that in a cylindrical cavity TE mode has no coupling with axion, i.e. $C^H = 0$, and R_{TE} is zero.



This figure shows the distribution of TM_{010} and TE_{011}, the field strength H_{011} decreases to zero at the red circle and instead increases outside the circle, resulting in the cancellation of the cavity response to the axion.

Quantization

QUANTIZATION OF ELECTROMAGNETIC FIELD IN QEMD

In QEMD, the magnetic field and electric field can be given by using the curl of two vector potentials $\vec{A}$ and $\vec{B}$, respectively.

$$\vec{H} = \nabla\times\vec{A},\ \vec{E} = -\nabla\times\vec{B}$$

The vector potentials $\vec{A}$ and $\vec{B}$ can be expanded in terms of creation and annihilation operators as well as the mode functions $\boldsymbol{u}_k(\boldsymbol{x})$ satisfying the boundary condition inside the cavity.

$$\vec{A}(\mathbf{x},t) = \sum_k \frac{1}{\sqrt{2\omega_k V}}\left(a_k \mathbf{u}_k^{(A)}(\mathbf{x})e^{-i\omega_k t} + a_k^\dagger \mathbf{u}_k^{(A)*}(\mathbf{x})e^{i\omega_k t}\right)\ ,$$
$$\vec{B}(\mathbf{x},t) = \sum_k \frac{1}{\sqrt{2\omega_k V}}\left(a_k \mathbf{u}_k^{(B)}(\mathbf{x})e^{-i\omega_k t} + a_k^\dagger \mathbf{u}_k^{(B)*}(\mathbf{x})e^{i\omega_k t}\right)\ ,$$

Although the exact forms of mode functions are unknown, their curl in a cavity can be given by

$$\nabla\times\mathbf{u}_k^{(A)} = \omega_k \mathbf{u}_k^H,\quad \nabla\times\mathbf{u}_k^{(B)} = \omega_k\mathbf{u}_k^E\ ,$$

where $\boldsymbol{u}_k{}^H$ and $\boldsymbol{u}_k{}^E$ are the actual electromagnetic field modes inside the cavity with the normalization $\frac{1}{V}\int d^3x \left|\boldsymbol{u}_k{}^{E,H}\right|^2 = 1$.

Cavity Modes

TRANSVERSE WAVE MODES IN CAVITY

The dominant modes existing in a cylindrical cavity include TE modes (transverse electric modes with $E_z = 0, H_z \neq 0$) and TM modes (transverse magnetic modes with $H_z = 0, E_z \neq 0$). The distribution of the electromagnetic field must satisfy Helmholtz equation as well as the corresponding boundary conditions.

$$\nabla^2 u(r,\phi,z) + k^2 u(r,\phi,z) = 0,\ \vec{E}(r,\phi,z) \text{ or } \vec{H}(r,\phi,z) = \hat{z}u(r,\phi,z)$$

The solutions of the above differential equations yield a series of possible electromagnetic resonant modes inside the cavity:

$$\mathrm{TE}_{mnp}:\ \hat{z}\cdot\mathbf{u}_k^H(r,\phi,z) = H_z(r,\phi,z)_{mnp} = H_{mnp}J_m(k_\rho r)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}\sin\left(\frac{p\pi z}{L}\right)$$
$$\mathrm{TM}_{mnp}:\ \hat{z}\cdot\mathbf{u}_k^E(r,\phi,z) = E_z(r,\phi,z)_{mnp} = E_{mnp}J_m(k_\rho r)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}\cos\left(\frac{p\pi z}{L}\right)$$

where H_{mnp} and E_{mnp} are dimensionless coefficients to ensure the mode normalization.

It means under an external magnetic field, only the coupling $(g_{aBB} - g_{aAA}) \approx g_{aBB}$ can be measured through TM mode. This approach is exactly the same as that measuring the conventional axion coupling $g_{a\gamma\gamma}$ in axion electrodynamics. For illustration, we show the transition rate R_{TM} in terms of practical units as

$$R_{\rm TM} \approx 3.63\ \mathrm{Hz}\left(\frac{\rho_a}{7.1\times10^{-25}\ \mathrm{g/cm^3}}\right)\left(\frac{10^{-5}\ \mathrm{eV}}{m_a}\right)^2\left(\frac{H_0}{10\ \mathrm{mT}}\right)^2$$
$$\cdot\left(\frac{V}{0.001\ \mathrm{m^3}}\right)\left(\frac{Q}{10^5}\right)\left(\frac{g_{aBB}}{10^{-12}\ \mathrm{GeV^{-1}}}\right)^2\left(\frac{C_{\omega_a}^E}{1}\right)\ .$$

B. Electric background

We find that replacing H_0 with E_0 is associated with exchanging the couplings constrained by TE and TM modes. Now, the sensitivity to g_{aAB} is still given by the transverse magnetic wave which couples to axion.

$$R_{\rm TM} = 0.41\ \mathrm{Hz}\left(\frac{\rho_a}{7.1\times10^{-25}\ \mathrm{g/cm^3}}\right)\left(\frac{10^{-5}\ \mathrm{eV}}{m_a}\right)^2\left(\frac{E_0}{10^3\ \mathrm{kV/m}}\right)^2$$
$$\cdot\left(\frac{V}{0.001\ \mathrm{m^3}}\right)\left(\frac{Q}{10^5}\right)\left(\frac{g_{aAB}}{10^{-12}\ \mathrm{GeV^{-1}}}\right)^2\left(\frac{C_{\omega_a}^E}{1}\right)\ .$$

Result

SENSITIVITY OF RESONANT CAVITY TO AXION-PHOTON COUPLINGS IN QEMD

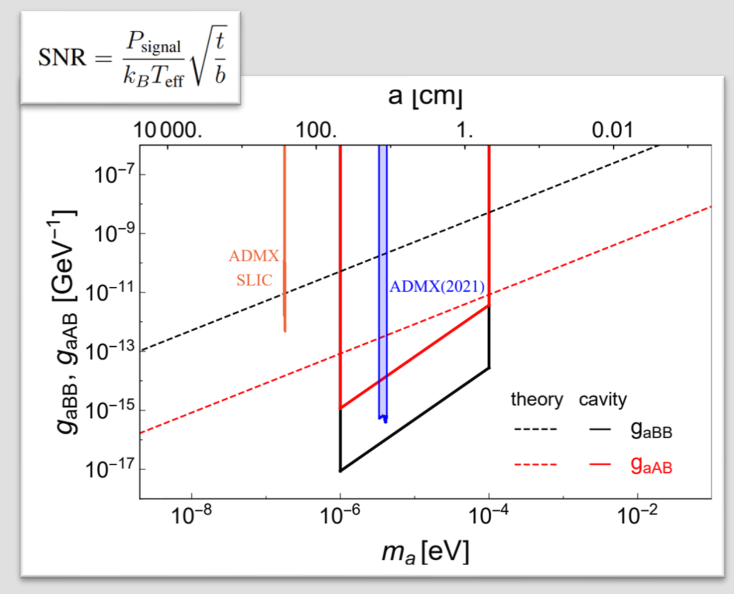
In an external magnetic field, the signal power is given by

$$P_{\rm signal} = m_a R_{\rm TM} = \frac{\pi}{2}\frac{\rho_a}{m_a} H_0^2 VQ g_{aBB}^2 C_{010}^E$$

where the cavity volume V is regarded as a function of m_a, i.e. $V = \pi a^2 L = \pi L(x_{0,1}/m_a)^2$, to ensure that m_a corresponding to each sensitivity is always at the resonant point. Similarly, for external electric field, it can be obtained by making a replacement $g_{aBB}H_0 \to g_{aAB}E_0$.

To estimate the sensitivity of cavity experiment to g_{aBB} or g_{aAB}, we take $Q = 10^5$ and limit the SNR to 5.

$${\rm SNR} = \frac{P_{\rm signal}}{k_B T_{\rm eff}}\sqrt{\frac{t}{b}}$$



■ **SETUP**

- observation time: 7 days
- cavity length: 1 m
- magnetic field: $H_0 = 10$ T with **Teff = 0.5 K**
- electric field: $E_0 = 10^4$ kV/m with **Teff = 0.1 K**

Conclusion

- In this work, we provide a complete quantum calculation of axion-single photon transition rate inside a homogeneous electromagnetic field in terms of the new axion interaction Hamiltonian in QEMD. It clearly implies the enhancement of conversion rate through resonant cavity in axion haloscope experiments.
- We provide the basic method for the generic cavity search of new axion-photon couplings in QEMD: an external magnetic field H_0 can be set to measure $(g_{aBB} - g_{aAA}) \approx g_{aBB}$ coupling or an external electric field E_0 to measure g_{aAB} coupling.
- The corresponding sensitivity bounds are given by the cavity experimental configuration and the form factor C_{010}^E in TM mode for the QEMD axion couplings.

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