Z boson mixing and the mass of the W boson

Yu-Pan Zeng[1] 曾育盼

School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang 524088, China

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[1] zengyupan@gdou.edu.cn

Overview

Introduction

- The CDF Collaboration has measured the mass of the W boson to be 80.4335 ± 0.0094 GeV, which is deviated from the SM prediction of 80.357 ± 0.006 GeV.
- The W boson mass is connected to the Z boson mass, which can be affected by mixing the Z boson with an extra vector boson.
- In this work we consider two models which altering the W boson mass at tree level:
  - The DPDM model: the extra vector boson mixing with the Z boson directly.
  - The U(1) model: the extra vector boson mixing with the Z boson through kinetic mixing with the B boson.

Overview

GENERAL DISCUSSION

Masses in the SM

$$m_W^2 = \frac{1}{4}g^2v^2,\ m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2$$
$$G_F = \frac{1}{\sqrt{2}v^2},\ e = \sqrt{4\pi\alpha} = \frac{gg'}{\sqrt{g^2+g'^2}}$$

mass matrix of general mixing case: $\begin{pmatrix} m_Z^2 & b \\ b & a \end{pmatrix}$

$$m^2_{\hat{Z},\hat{Z}'} = \frac{1}{2}\left(m_Z^2 + a \pm \sqrt{(m_Z^2-a)^2+4b^2}\right)$$

GENERAL DISCUSSION

Masses in the SM

$$m_W^2 = \frac{1}{4}g^2v^2,\ m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2 = \frac{1}{4}\frac{g^2g'^2}{e^2}v^2$$

$$G_F = \frac{1}{\sqrt{2}v^2},\ e = \sqrt{4\pi\alpha} = \frac{gg'}{\sqrt{g^2+g'^2}} = \frac{1}{\sqrt{\frac{1}{g^2}+\frac{1}{g'^2}}}$$

mass matrix of general mixing case: $\begin{pmatrix} m_Z^2 & b \\ b & a \end{pmatrix}$

$$m^2_{\hat{Z},\hat{Z}'} = \frac{1}{2}\left(m_Z^2 + a \pm \sqrt{(m_Z^2 - a)^2 + 4b^2}\right)$$

$$m_W \uparrow \rightarrow g \uparrow \rightarrow m_Z \uparrow \rightarrow m_Z > m_{\hat{Z}} \rightarrow a > m_Z^2$$

General constraint

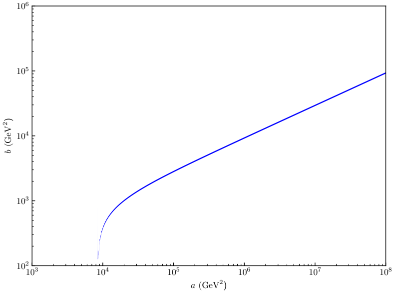


Figure: Band which gives the W mass between 80.4053 and 80.4617 GeV.

$$b^2 = \frac{4m_W^4}{4m_W^2 - e^2v^2}(a - m_{\hat{Z}}^2) + m_{\hat{Z}}^4 - m_{\hat{Z}}^2 a.$$

$m_W \to m_W - \delta m_W$, δm_W: loop corrections from SM.

Overview

BSM: Derivative Portal Dark Matter

$$
\begin{aligned}
\mathcal{L} = \quad & -\frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} - \frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} - \frac{\epsilon}{2}Z^{\mu\nu}Z'_{\mu\nu} \\
& + \sum_f Z_\mu \bar{f}\gamma^\mu(g_V - g_A\gamma^5)f + g_\chi Z'_\mu \bar{\chi}\gamma^\mu\chi \\
& + \frac{1}{2}m_Z^2 Z_\mu Z^\mu + \frac{1}{2}m_{Z'}^2 Z'_\mu Z'^\mu - m_\chi \bar{\chi}\chi.
\end{aligned}
$$

- DM connect to the SM through derivative portal (kinetic mixing Z and Z')
- naturally escape stringent DM direct detection constraint

DIAGONALIZATION

diagonalizing the kinetic mixing

$$K = \begin{pmatrix} -k_1 & k_2 \\ k_1 & k_2 \end{pmatrix}, \; k_1 = 1/\sqrt{2-2\epsilon}, \; k_2 = 1/\sqrt{2+2\epsilon}$$

mass matrix

$$\begin{pmatrix} k_1^2 M_1 & k_1 k_2 M_2 \\ k_1 k_2 M_2 & k_2^2 M_1 \end{pmatrix}, \; M_1 = m_Z^2 + m_{Z'}^2, \; M_2 = m_{Z'}^2 - m_Z^2$$

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \; \tan 2\theta = \frac{2k_1 k_2 M_2}{(k_2^2 - k_1^2) M_1}$$

$$m_Z^2 = \frac{1}{8k_1^2 k_2^2}(m_{\hat{Z}}^2 + m_{\hat{Z}'}^2) - \sqrt{\frac{1}{64 k_1^4 k_2^4}(m_{\hat{Z}}^2 + m_{\hat{Z}'}^2)^2 - \frac{1}{4k_1^2 k_2^2} m_{\hat{Z}}^2 m_{\hat{Z}'}^2}$$

STU from effective lagrangian

$$L_{NC,\hat{Z}ff} = \sum_f (-k_2 \sin\theta - k_1 \cos\theta)\hat{Z}_\mu \bar{f}\gamma^\mu (g_V - g_A\gamma^5) f$$

$$= \sum_f (-k_2 \sin\theta - k_1 \cos\theta)\hat{Z}_\mu \bar{f}\gamma^\mu \frac{e}{s_w c_w}(T_f^3 \frac{1-\gamma^5}{2} - Q_f s_w^2) f$$

effective lagrangian from burgess1994model:

$$\mathcal{L}_{CC,Wff} = -\frac{e}{\sqrt{2}\hat{s}_w}(1 - \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} + \frac{\hat{c}_w^2 \alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)} + \frac{\alpha U}{8\hat{s}_w^2}) \sum_{ij} V_{ij} \bar{f}_i \gamma^\mu \gamma_L f_j W_\mu^\dagger + \text{c.c.}$$

$$\mathcal{L}_{NC,\hat{Z}ff} = \frac{e}{\hat{s}_w \hat{c}_w}(1 + \frac{\alpha T}{2}) \sum_f \bar{f}\gamma^\mu [T_f^3 \frac{1-\gamma^5}{2} - Q_f(\hat{s}_w^2 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2 \hat{s}_w^2 \alpha T}{\hat{c}_w^2 - \hat{s}_w^2})] f \hat{Z}_\mu$$

$$\hat{s}_w \hat{c}_w m_{\hat{Z}} = s_w c_w \frac{1}{2}\sqrt{g^2 + g'^2} v = \frac{1}{2} e v = s_w c_w m_Z$$

STU and χ^2

$$\alpha T = 2\left(\frac{\hat{s}_w \hat{c}_w}{s_w c_w}(-k_2 \sin\theta - k_1 \cos\theta) - 1\right)$$

$$\alpha S = 4\hat{c}_w^2 \hat{s}_w^2 \alpha T + 4(\hat{c}_w^2 - \hat{s}_w^2)(s_w^2 - \hat{s}_w^2)$$

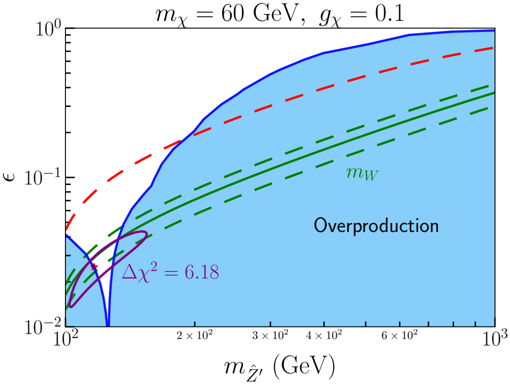
$$\alpha U = 8\hat{s}_w^2\left(\frac{\hat{s}_w}{s_w} - 1 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2 \alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)}\right)$$

$$\chi^2 = X Cov^{-1} X^T, \; X = \begin{pmatrix} S - 0.005 & T - 0.04 & U - 0.134 \end{pmatrix}$$

$$Cov = \begin{pmatrix} 0.097^2 & \rho_{ST} \times 0.097 \times 0.12 & \rho_{SU} \times 0.097 \times 0.087 \\ \rho_{ST} \times 0.097 \times 0.12 & 0.12^2 & \rho_{TU} \times 0.12 \times 0.087 \\ \rho_{SU} \times 0.097 \times 0.087 & \rho_{TU} \times 0.12 \times 0.087 & 0.087^2 \end{pmatrix}$$

global fit data are taken from deBlas:2022hdk

RESULTS



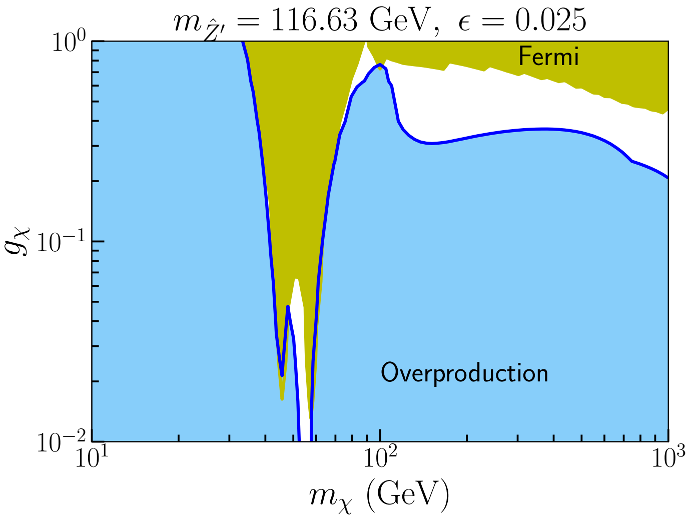
best fit (*):

$m_{\hat{Z}'} = 116.63$ GeV, $\epsilon = 0.025$, $\chi^2 = 3.21$

blue line: observed DM relic density

green lines: m_W within 3σ deviations

results: DM indirect detection



SUMMARY CONSTRAINTS ON ONE BMP

$$L_{NC,\hat{Z}'ff} = \sum_f \hat{Z}'_\mu \bar{f}\gamma^\mu (g'_V - g'_A\gamma^5) f$$

$$= \sum_f (-k_1 \sin\theta + k_2 \cos\theta)\hat{Z}'_\mu \bar{f}\gamma^\mu \frac{e}{s_w c_w}(T^3_f \frac{1-\gamma^5}{2} - Q_f s_w^2) f$$

TABLE I. Summary of phenomenological constraints for the benchmark point $m_{\hat{Z}'} = 116.63$ GeV, $\epsilon = 0.025$, $m_\chi = 1000$ GeV, $g_\chi = 0.21$.

	m_W	χ^2 of STU	$\Omega_{DM}h^2$
model value	80.4136	3.21	0.1235
constraint	80.4335 ± 0.0094 GeV [1]	d.o.f=3	0.1200±0.0012 [63]
	$< \sigma_{ann} v >$	Z'-electron couplings	DM-Xe scattering events
model value	4.5×10^{-26} cm^3/s	$g'_V = 0.0005$ and $g'_A = 0.0073$	1.2×10^{-9}
constraint	2.3×10^{-25} cm^3/s [66, 67]	$\sim O(10^{-2})$ [2]	7.9 [71]

BSM: The U(1) model

$$\mathcal{L}_{\mathrm{K}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}X^{\mu\nu}X_{\mu\nu} - \frac{\epsilon}{2}B^{\mu\nu}X_{\mu\nu}$$

$$\begin{aligned}
& \frac{1}{2}\begin{pmatrix} W^{3\mu} & B^{\mu} & X^{\mu} \end{pmatrix} \begin{pmatrix} g^2v^2/4 & -gg'v^2/4 & 0 \\ -gg'v^2/4 & g'^2v^2/4 & 0 \\ 0 & 0 & g_x^2v_s^2 \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \\ X_\mu \end{pmatrix} \\
= \quad & \frac{1}{2}\begin{pmatrix} W^{3\mu} & B^{\mu} & X^{\mu} \end{pmatrix} K^{-1T}OO^TK^T \begin{pmatrix} g^2v^2/4 & -gg'v^2/4 & 0 \\ -gg'v^2/4 & g'^2v^2/4 & 0 \\ 0 & 0 & g_x^2v_s^2 \end{pmatrix} KOO^TK^{-1} \begin{pmatrix} W^3_\mu \\ B_\mu \\ X_\mu \end{pmatrix} \\
= \quad & \frac{1}{2}\begin{pmatrix} A^{\mu} & \hat{Z}^{\mu} & \hat{Z}'^{\mu} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & m^2_{\hat{Z}} & 0 \\ 0 & 0 & m^2_{\hat{Z}'} \end{pmatrix} \begin{pmatrix} A_\mu \\ \hat{Z}_\mu \\ \hat{Z}'_\mu \end{pmatrix}
\end{aligned}$$

STU

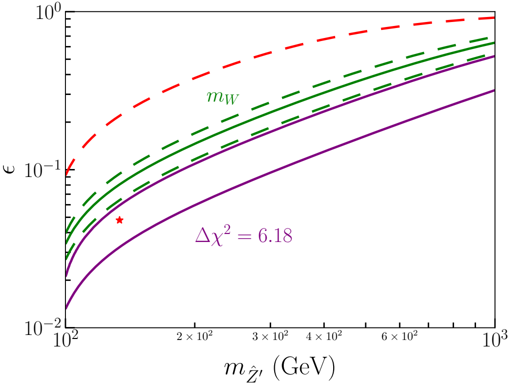
$$L_{NC,\hat{Z}ff} = \sum_f \hat{Z}_\mu \bar{f} \gamma^\mu (g_V - g_A \gamma^5) f, \; g_V = g_A + g'[KO]_{22} Q_f, \; g_A = \frac{T_f^3}{2}(-g'[KO]_{22} + g[KO]_{12})$$

$$\alpha T = \frac{2\hat{s}_w \hat{c}_w (-g'[KO]_{22} + g[KO]_{12})}{e} - 2$$

$$\alpha S = \frac{-4g'[KO]_{22}(\hat{c}_w^2 - \hat{s}_w^2)}{-g'[KO]_{22} + g[KO]_{12}} - 4\hat{s}_w^2(\hat{c}_w^2 - \hat{s}_w^2) + 4\hat{c}_w^2 \hat{s}_w^2 \alpha T$$

$$\alpha U = 8\hat{s}_w^2 \left(\frac{\hat{s}_w}{s_w} - 1 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2 \alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)}\right)$$

RESULTS



green lines: m_W within 3σ deviations
best fit (*):
$m_{\hat{Z}'} = 133.65$ GeV, $\epsilon = 0.048$, $\chi^2 = 24.94$,
only giving about 27 MeV extra mass

Overview

CONCLUSION

- We have explored the possibility of altering the W boson mass at tree level in two models: the DPDM model and the U(1) model.
- Apart from giving the W boson mass, we also discussed the electroweak oblique parameters constraints for both models, and explored DM relic density and DM indirect detection constraints for the DPDM model.
- We find that in both model the best fit value for the extra vector boson mass is around 120 GeV.
- While the best fit of the U(1) model can only contribute 27 MeV extra mass to the SM W boson mass, the best fit of the DPDM model can give the observed W boson mass as well as the observed DM relic density.
- The DPDM model can also escape stringent DM direct detection and the best fit of the DPDM can saturate the constraints from DM indirect detection and rough estimation of collider bounds.

Overview

Introduction

General discussion

BSM

Conclusion

Backups

Comparison of STU between the two models

STU of the U(1) model

$$\alpha T = -2c_\xi \frac{s_w c_w}{\hat{s}_w \hat{c}_w} - 2$$

$$\alpha S = 4(\hat{c}_w^2 - \hat{s}_w^2)(s_w^2 - \hat{s}_w^2 + \frac{s_w^2 c_w^2 - \hat{s}_w^2 \hat{c}_w^2}{s_w^2}) + 4\hat{c}_w^2 \hat{s}_w^2 \alpha T$$

$$\alpha U = 8\hat{s}_w^2 (\frac{\hat{s}_w}{s_w} - 1 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2 \alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)})$$

$$= 8\hat{s}_w^2 (\frac{\hat{s}_w}{s_w} - 1 + (s_w^2 - \hat{s}_w^2 + \frac{s_w^2 c_w^2 - \hat{s}_w^2 \hat{c}_w^2}{s_w^2})) - 4\hat{s}_w^2 \hat{c}_w^2 \alpha T$$

STU of the DPDM model

$$\alpha T = 2(\frac{\hat{s}_w \hat{c}_w}{s_w c_w}(-k_2 \sin\theta - k_1 \cos\theta) - 1)$$

$$\alpha S = 4\hat{c}_w^2 \hat{s}_w^2 \alpha T + 4(\hat{c}_w^2 - \hat{s}_w^2)(s_w^2 - \hat{s}_w^2)$$

$$\alpha U = 8\hat{s}_w^2 (\frac{\hat{s}_w}{s_w} - 1 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2 \alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)})$$

$$= 8\hat{s}_w^2 (\frac{\hat{s}_w}{s_w} - 1 + (s_w^2 - \hat{s}_w^2)) - 4\hat{s}_w^2 \hat{c}_w^2 \alpha T$$

EXPANSION AT $\hat{s}_w = s_w$

STU of the U(1) model

$$
\begin{aligned}
\alpha T &= -2c_\xi \frac{s_w c_w}{\hat{s}_w \hat{c}_w} - 2 \\
\alpha S &= -8\hat{c}_w^2(\hat{c}_w^2 - \hat{s}_w^2)\frac{\Delta s}{\hat{s}_w} + 4\hat{c}_w^2\hat{s}_w^2\alpha T \\
\alpha U &= -8\hat{s}_w^2(\hat{c}_w^2 - \hat{s}_w^2)\frac{\Delta s}{\hat{s}_w} - 4\hat{s}_w^2\hat{c}_w^2\alpha T
\end{aligned}
$$

$\rightarrow T = -\frac{2c_\xi s_w c_w}{\alpha \hat{s}_w \hat{c}_w} - \frac{2}{\alpha}$
S=-866.17$\Delta s + 0.716T$
U=-263.76$\Delta s - 0.716T$

$s_w = 0.48269 \rightarrow$desired $m_W \rightarrow \Delta s = 0.00046$

$\rightarrow S = 0.716T - 0.398,\ U = -0.716T - 0.121$

STU of the DPDM model

$$
\begin{aligned}
\alpha T &= 2(\frac{\hat{s}_w \hat{c}_w}{s_w c_w}(-k_2 \sin\theta - k_1 \cos\theta) - 1) \\
\alpha S &= 4\hat{c}_w^2\hat{s}_w^2\alpha T - 8\hat{s}_w(\hat{c}_w^2 - \hat{s}_w^2)\Delta s \\
\alpha U &= 8\hat{s}_w(\hat{c}_w^2 - \hat{s}_w^2)\Delta s - 4\hat{s}_w^2\hat{c}_w^2\alpha T
\end{aligned}
$$

$\rightarrow$
$T = (\frac{2\hat{s}_w \hat{c}_w(-k_2 \sin\theta - k_1 \cos\theta)}{s_w c_w \alpha} - \frac{2}{\alpha})$
S=0.716 T-263.76Δs
U=263.76$\Delta s - 0.716T$

$S = 0.716T - 0.121,\ U = -0.716T + 0.121$

UV complete DPDM model

$$
\begin{aligned}
\mathcal{L} \quad &= \mathcal{L}_{SM} + (D^{\mu}\Phi)^{\dagger} D_{\mu}\Phi + \mu_{\Phi}^2 |\Phi|^2 - \lambda_{\Phi}|\Phi|^4 + \lambda_{H\Phi}|H|^2|\Phi|^2 \\
&- \frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} + i\bar{\chi}\gamma^{\mu} D_{\mu}\chi - m_{\chi}\bar{\chi}\chi + i\psi_L \gamma^{\mu} D_{\mu}\psi_L \\
&+ iN_R \partial\!\!\!/ N_R - \frac{1}{2} M_N N_R^c N_R - Y_{\nu}\tilde{H} L_L N_R - Y_{\psi}\Phi\psi_L N_R + \text{h.c.}
\end{aligned}
$$

$$
\begin{aligned}
D_{\mu}\Phi &= (\partial_{\mu} - i g_{\chi} Z'_{\mu})\Phi \\
D_{\mu}\chi &= (\partial_{\mu} - i g_{\chi} n_{\chi} Z'_{\mu})\chi \\
D_{\mu}\psi_L &= (\partial_{\mu} - i g_{\chi} Z'_{\mu})\psi_L
\end{aligned}
$$

$$
\frac{1}{2}\begin{pmatrix} 0 & Y_{\nu} v_H & 0 \\ Y_{\nu} v_H & M_N & Y_{\psi} v_{\Phi} \\ 0 & Y_{\Phi} v_{\psi} & 0 \end{pmatrix}
$$